Threshold resummation for high-transverse-momentum Higgs production at the LHC

This content has been downloaded from IOPscience. Please scroll down to see the full text.
JHEP02(2006)047
(http://iopscience.iop.org/1126-6708/2006/02/047)
View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 54.191.40.80
This content was downloaded on 31/08/2017 at 23:02
Please note that terms and conditions apply.

You may also be interested in:

Single production of charged gauge bosons from little Higgs models in association with top quark at the LHC
Chong-Xing Yue, Shuo Yang and Li-Hong Wang

Implications of the Higgs discovery in the MSSM golden region
Ian Low and Shashank Shalgar

Uncertainties of the inclusive Higgs production cross section at the tevatron and the LHC
Alexander Belyaev, Jon Pumplin, Wu-Ki Tung et al.

Difficult scenarios for NMSSM Higgs discovery at the LHC
Ulrich Ellwanger, John F. Gunion and Cyril Hugonie

The Higgs Machine Learning Challenge
C Adam-Bourdarios, G Cowan, C Germain-Renaud et al.

Soft gluons in Higgs plus two jet production
Jeff Forshaw and Malin Sjödahl
Threshold resummation for high-transverse-momentum Higgs production at the LHC

Daniel de Florian
Departamento de Física, FCEYN, Universidad de Buenos Aires
(1428) Pabellón 1 Ciudad Universitaria, Capital Federal, Argentina
E-mail: deflo@df.uba.ar

Anna Kulesza
Institut für Theoretische Teilchenphysik, Universität Karlsruhe
D-76128, Karlsruhe, Germany
E-mail: ania@particle.uni-karlsruhe.de

Werner Vogelsang
Physics Department and RIKEN BNL Research Center
Brookhaven National Laboratory, Upton, NY 11973, U.S.A.
E-mail: vogelsan@quark.phy.bnl.gov

ABSTRACT: We study the resummation of large logarithmic QCD corrections for the process $pp \to H + X$ when the Higgs boson $H$ is produced at high transverse momentum. The corrections arise near the threshold for partonic reaction and originate from soft gluon emission. We perform the all-order resummation at next-to-leading logarithmic accuracy and match the resummed result with the next-to-leading order perturbative predictions. The effect of resummation on the Higgs transverse momentum distribution at the LHC is discussed.

KEYWORDS: NLO Computations, Hadronic Colliders, QCD.
1. Introduction

To date, the Higgs boson, responsible for the mechanism of the electroweak symmetry breaking, remains the only undetected ingredient of the Standard Model. The search for the Higgs boson is one of the highest priorities for both the CERN Large Hadron Collider (LHC) and Fermilab Tevatron physics programs [1]. Direct searches at LEP and fits to electroweak precision data indicate that it might be light, with a lower bound of 114.4 GeV [2] and an upper limit of $m_H < 260$ GeV at 95% CL [3].

The detection of the Higgs boson in the low mass range, $m_H < 140$ GeV, though feasible, will not be a simple task even at the LHC [4, 5]. The dominant production channel at hadron colliders is the gluon fusion, mediated at lowest order in the SM by a heavy (mainly top) quark loop. In the considered mass range, experimental searches at the LHC will concentrate on the rare two-photon decay mode $H \rightarrow \gamma + \gamma$. In the absence of any constraints imposed on the events, the bulk of the cross section will be at relatively low transverse momenta of the photon pair, where the background is large. Thus, despite the high production cross section, the detection of the signal is considered a challenging task.

A possible way to improve the signal significance for Higgs discovery in the considered mass range is to study the less inclusive $\gamma + \gamma + \text{jet(s)}$ final states,\(^1\) which offer several advantages [7]. In this case the photons are more energetic than for the inclusive channel,\(^1\)

---

\(^1\)The study of Higgs production in association with a jet was first suggested in the context of improving the $\tau$ reconstruction in the $\tau^+\tau^-$ decay channel [6].
and the reconstruction of the jet in the calorimeter allows a more precise determination of the interaction vertex, improving the efficiency and mass resolution. Furthermore, the existence of a jet in the final state allows for a new type of event selection and a more efficient background suppression. Also theoretical considerations make the process appear more favorable regarding its background: while for the fully inclusive channel the $gg \to \gamma\gamma$ background contribution that first enters at NNLO is as sizable as the Born cross section for the Higgs production \[8\] and thus complicates the organization of the perturbative calculation, it is significantly suppressed and less relevant at large transverse momentum of the photon pair \[9\]. One therefore expects that the background is under better theoretical control than that for the inclusive cross section.

Quite generally, on the theoretical side, signal and background cross sections need to be calculated with the highest possible accuracy, minimizing the theoretical uncertainties. In the case of Higgs boson production cross-sections, the main theoretical uncertainties come from two sources: the parton distributions, and the use of QCD perturbation theory for the partonic hard-scattering. Typical uncertainties in the relevant parton distributions are only of the order of a few percent, and further improvements are expected from new data that will become available from the usual Standard Model processes at the LHC. Regarding the status of perturbation theory for the partonic cross-sections for Higgs production, a very slow convergence of the perturbative expansion was observed for the fully inclusive Higgs production cross section, for which the next-to-leading-order (NLO) contributions were found to be as large as the leading-order (LO) term \[10, 11\]. Consequently, an enormous effort was devoted to obtaining the next order (NNLO) in perturbative QCD, which turned out to be under better control, albeit still sizeable \[12\]. Due to the high complexity of the calculations (the lowest order is already a one-loop process because a top quark loop is required to couple the gluons to the Higgs), the results for the NNLO corrections were obtained in the large-top-mass $m_t$ limit, i.e. $m_t \to \infty$. In this limit the top quark loops may be replaced by point-like vertices, and the Feynman rules are given by an effective Lagrangian. At NLO, the method is known to provide a very good approximation of the exact result for $m_H < 2m_t$ \[13\].

The origin of the large size of the higher order contributions to the perturbative partonic cross sections can be traced to the presence of large logarithmic terms, referred to as “threshold logarithms”. These result from the emission of soft and collinear gluons near the edges of phase space. It turns out that the threshold logarithms, along with terms from purely virtual corrections, account for more than 90% of the total cross section for inclusive Higgs production at the LHC \[14, 15\]. The most important (leading) logarithms at the $n$th order in perturbation theory are of the form $\alpha_s^k \left( \ln^{2k-1}(1 - z)/(1 - z) \right)_+$, where $1 - z = m_H^2/\hat{s}$, \(\hat{s}\) being the partonic center-of-mass energy. Sufficiently close to the partonic threshold at $\hat{s} = m_H^2$ or $z = 1$, where the initial state partons have just enough energy to produce the Higgs boson, fixed-order approximations of the partonic cross sections are bound to fail, no matter how small the coupling constant. These effects of multiple soft gluon emission can, however, be taken into account to all orders in perturbation theory by performing a resummation of the threshold logarithms. For inclusive Higgs boson production, the resummation is completely known to the next-to-next-to-leading logarithmic
(NNLL) accuracy \cite{16}.\footnote{Many of the ingredients needed to perform the resummation to complete N^{3}LL accuracy became available recently \cite{17,18}.} Thanks to the interplay of the partonic cross sections with the parton distributions, threshold resummation considerably improves the predictive power of the theoretical calculations even in situations where one is not very close to the hadronic threshold $s = m_{H}^{2}$. As a result, the theoretical uncertainties from perturbation theory for the Higgs cross section at the LHC are reduced to a level of about 10%, sufficient for Higgs discovery.

Motivated by all of the above, we will study in the present paper the cross section for Higgs production at large transverse momentum $p_{T}$, typically $m_{H} < p_{T} < \text{few} \times m_{H}$, and perform the resummation of threshold logarithms at NLL accuracy. Ideally, as explained earlier, we would have in mind here the process $pp \rightarrow H + \text{jet} + X$, with an observed jet at high transverse momentum that roughly balances that of the Higgs boson. For simplicity, we will for now only discuss the more inclusive reaction $pp \rightarrow H + X$ at large $p_{T}$, without explicit reference to a jet. Since in most cases a high-$p_{T}$ Higgs will indeed be accompanied by a recoiling jet, we expect that this process will share many features with $H + \text{jet}$ production, in particular regarding the relevance of perturbative higher-order QCD corrections and resummation that we wish to study here.

The LO predictions for single-inclusive Higgs cross section at large $p_{T}$, including the full dependence on the mass of the top quark, have been known for some time by now \cite{6,20}. Several different NLO calculations \cite{21,22} exist within the large-top-mass approximation. Two of these \cite{21,22} used numerical integration techniques, while the other two derived and provided analytical results \cite{23,24}. As in the fully inclusive case, threshold logarithms also dominate the cross section when the transverse momentum of the Higgs boson is large, even though they are of a somewhat different form. In the $p_{T}$ distribution, when the cross section is integrated over all rapidities of the Higgs, they occur in the partonic cross sections as $\alpha_{S}^{2} \ln^{m} (1 - \hat{y}_{T}^{2})$, $m \leq 2k$, where $\hat{y}_{T} = (p_{T} + m_{T})/\sqrt{s}$ with $m_{T} = \sqrt{p_{T}^{2} + m_{T}^{2}}$. Also, unlike the fully-inclusive case, for a Higgs produced at large $p_{T}$ there needs to be a recoiling parton already in the Born process, whose color charge plays a role for the structure of the resummed expression. As is customarily done, we will treat the gluon-Higgs interaction in the large $m_{t}$ limit, i.e. by replacing the top loop with an effective $ggH$ coupling. Even though this approximation is not as accurate at large transverse momentum as in case of the fully inclusive cross section \cite{22}, it is certainly expected to be good for the ratio between higher order calculations and the Born term, because the dominant large logarithms are completely independent of the structure of the $ggH$ coupling.

We note in passing that kinematically, and conceptually, the resummation of the Higgs cross section at large $p_{T}$ is close to that for high-$p_{T}$ $W$ or $Z$ production in hadronic collisions, considered in \cite{26}. Besides the obvious differences related to the different final state considered, we also differ from ref. \cite{26} in our technical treatment of the resummed formulas. In ref. \cite{26} a NNLO expansion of the resummed expression is obtained and used, while in the present work we keep the full NLL-resummed expression. This, as we shall see, involves a treatment of the whole cross section in Mellin-moment space. We
also emphasize that the logarithms we are resumming are different from those occurring at small $p_T$ ($p_T \ll m_H$), which have received much attention in the literature since the bulk of the inclusive events is in this regime. Here, the resummation has been carried out through NNLL, and also a formalism was applied that allows a NLL resummation of the logarithms at low $p_T$ jointly with the threshold logarithms present in the inclusive ($p_T$-integrated) Higgs cross section. Finally, we mention that for a very light Higgs and/or at high $p_T$, yet another class of logarithms could become important, arising through “fragmentation” production of the Higgs by a final-state gluon. The logarithms in this kinematic regime have been studied in for the Drell-Yan process. They are not relevant in the threshold situation we are considering in this work, for which typically $m_H < p_T < \text{few} \times m_H$ and $\hat{y}_T \sim 1$.

The paper is organized as follows: in section 2 we discuss the structure of the expressions for the Higgs $p_T$ distribution in fixed-order perturbation theory and discuss the role of the threshold region. Section 3 is concerned with the analytical results for the threshold-resummed distribution in Mellin-moment space. We also describe there the matching of the resummed to the fixed-order result, and the prescription for the inverse Mellin transform. Finally, in section 4 the phenomenological effects of threshold resummation on the Higgs $p_T$ distribution at the LHC are studied.

2. Perturbative cross section

We consider Higgs production in hadronic collisions,

$$h_1 + h_2 \rightarrow H + X,$$

at large transverse momentum $p_T$ of the Higgs boson $H$. The factorized cross section, differential in $p_T$ and the Higgs rapidity $y_H$, can be written as

$$\frac{d\sigma}{dp_T^2 dy_H} = \sum_{a,b} \int_0^1 dx_1 f_{a/h_1}(x_1, \mu_F^2) \int_0^1 dx_2 f_{b/h_2}(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}}{dp_T^2 dy_H},$$

where the perturbative partonic cross section is expanded as

$$\frac{d\hat{\sigma}_{ab}}{dp_T^2 dy_H} = \sigma_0 \left[ \frac{\alpha_S}{2\pi} G_{ab}^{(1)} + \left( \frac{\alpha_S}{2\pi} \right)^2 G_{ab}^{(2)} + \cdots \right],$$

with the partonic center-of-mass energy $\hat{s} = s x_1 x_2$. The Born cross section, computed within the large-top-mass approximation, is given by

$$\sigma_0 = \frac{\pi}{64} \left( \frac{\alpha_S}{3\pi v} \right)^2$$

with $v$ representing the Higgs vacuum expectation value, $v^2 = 1/(\sqrt{2}G_F)$. In the expressions above, $\mu_F$ is the factorization scale and the coupling constant $\alpha_S \equiv \alpha_S(\mu_R^2)$ is computed at the renormalization scale $\mu_R$. Explicit expressions for the (factorization and renormalization scale independent) LO coefficients $G_{ab}^{(1)}$ and the (scale-dependent) NLO contributions $G_{ab}^{(2)}$ can be found in [24].
The following three partonic channels contribute to this process at the lowest order: \( gg \to gH, gq \to qH, q\bar{q} \to gH \), the first one being dominant – as it is to be expected due to the large gluon-gluon luminosity at hadron colliders.

In this paper we will for simplicity focus just on the transverse momentum distribution of the Higgs boson and integrate over the full range of allowed rapidities

\[
\frac{d\sigma}{dp_T^2} = \int_{y_H^+}^{y_H^-} \frac{d\sigma}{dp_T^2 dy_H},
\]

where

\[
y_H^+ = -y_H^- = \frac{1}{2} \ln \left( \frac{1 + \sqrt{1 - 4 s m_T^2 / (s + m_H^2)^2}}{1 - \sqrt{1 - 4 s m_T^2 / (s + m_H^2)^2}} \right),
\]

\[
m_T = \sqrt{m_H^2 + p_T^2}
\]

with \( m_T \) denoting the transverse mass. In our calculation we will express the \( p_T \) distribution as a function of the hadronic threshold variable \( y_T \) defined as

\[
y_T = \frac{p_T + m_T}{\sqrt{s}},
\]

i.e. \( d\sigma/dp_T^2 = d\sigma/dp_T^2(y_T) \). The limit \( y_T \to 1 \) represents the hadronic threshold, i.e., when the hadronic center-of-mass energy is just enough to produce the Higgs boson with a given transverse momentum.

Using the expressions for \( G_{ab}^{(1)} \) in \([2]\) we obtain the (rapidity integrated) partonic cross sections at the lowest order:

\[
\frac{d\hat{\sigma}_{ab}^{(1)}}{dp_T^2} = \frac{\alpha_S}{2\pi} \frac{N_{ab}(\hat{y}_T, r)}{p_T^2 \sqrt{1 - \hat{y}_T^2}},
\]

where the partonic threshold variable \( \hat{y}_T \) is defined as \( \hat{y}_T = y_T/\sqrt{x_1 x_2} \). The square-root factor in the denominator is a Jacobean from the rapidity integration. The coefficients \( N_{ij}(\hat{y}_T, r) \) are regular at \( \hat{y}_T = 1 \). They also depend on the “fixed” quantity \( r \equiv p_T/m_T \) and are given in appendix \([5]\).

At the next-to-leading order, the integration over rapidity of the term \( G_{ab}^{(2)} \) leads to an expression for the partonic cross section that can be written as

\[
\frac{d\hat{\sigma}_{ab}^{(2)}}{dp_T^2} = \frac{\alpha_S}{2\pi} \frac{d\hat{\sigma}_{ab}^{(1)}}{dp_T^2} \left[ g_{2,ab}(p_T) \ln^2(1 - \hat{y}_T^2) + g_{1,ab}(p_T) \ln(1 - \hat{y}_T^2) + g_{0,ab}(p_T) \right] + f_{ab}(p_T, \hat{y}_T).
\]

The function \( f_{ab}(p_T, \hat{y}_T) \) represents terms that vanish in the limit \( \hat{y}_T \to 1 \).

As we discussed in the Introduction, at the \( k \)th order of perturbation theory for the \( \hat{\sigma}_{ij} \), there are logarithmically-enhanced contributions of the form \( \alpha_S^k \ln^m(1 - \hat{y}_T^2) \), with \( m \leq 2k \). In analogy with the inclusive total Higgs cross section, these logarithmic terms are due to soft-gluon radiation and dominate the perturbative expansion when the process is kinematically close to the partonic threshold. We emphasize that \( \hat{y}_T \) assumes particularly large values when the partonic momentum fractions approach the lower ends of their
ranges. Since the parton distributions rise steeply towards small argument, this generally increases the relevance of the threshold regime, and the soft-gluon effects are relevant even for situations where the hadronic center-of-mass energy is much larger than the produced transverse mass of the final state. For this particular process at the LHC, it has been explicitly checked in [25] that an approximation based on setting \( f_{a,b}(p_T, \hat{y}_T) = 0 \) in eq. (2.9) gives the bulk of the NLO contribution. In the following, we discuss the resummation of the large logarithmic corrections to all orders in \( \alpha_s \).

3. Resummation

The resummation of the soft-gluon contributions is carried out in Mellin-\( N \) moment space, where the convolutions in eq. (2.2) between parton distributions and subprocess cross sections factorize into ordinary products. We take Mellin moments in the scaling variable \( y_T^2 \) as

\[
\int_0^1 dy_T^2 (y_T^2)^{N-1} \frac{d\sigma}{dp_T^2} = \sum_{a,b} f_a(N+1, \mu_F^2) f_b(N+1, \mu_F^2) \tilde{\sigma}_{ab}(N),
\]

where the corresponding moments of the partonic cross sections are

\[
\tilde{\sigma}_{ab}(N) = \int_0^1 dy_T^2 (y_T^2)^{N-1} \frac{d\tilde{\sigma}_{ab}}{dp_T^2},
\]

and the \( f_{a,b}(N+1, \mu_F^2) \) are the usual moments of the parton distributions in their momentum fractions. The threshold limit \( \hat{y}_T^2 \to 1 \) corresponds to \( N \to \infty \), and the leading soft-gluon corrections arise as terms \( \propto \alpha_k^2 \ln 2k N \). The NLL resummation procedure discussed in this work deals with the “towers” \( \alpha_k^2 \ln^m N \) for \( m = 2k, 2k - 1, 2k - 2 \).

3.1 Resummation to NLL

In Mellin-moment space, threshold resummation results in exponentiation of the soft-gluon corrections. In case of the Higgs cross section at high \( p_T \), the resummed cross section reads [32, 33]:

\[
\sigma^{(\text{res})}_{ab\to c H}(N-1) = C_{ab\to c H} \Delta^a_N \Delta^b_N J_N \Delta^{(\text{int})ab\to c H}_N \sigma^{(1)}_{ab\to c H}(N-1).
\]

Each of the “radiative factors” \( \Delta^a_N, \Delta^b_N, \Delta^{(\text{int})ab\to c H}_N \) is an exponential. The factors \( \Delta^a_N \) represent the effects of soft-gluon radiation collinear to initial partons \( a \) and \( b \). The function \( J_N \) embodies collinear, soft or hard, emission by the non-observed parton \( c \) that recoils against the Higgs. Large-angle soft-gluon emission is accounted for by the factors \( \Delta_N^{(\text{int})ab\to c H} \), which, at variance with the universal \( \Delta^a_N \) and \( J_N \) functions, depend on the partonic process under consideration. Finally, the coefficients \( C_{ab\to c H} \) contain \( N \)-independent hard contributions arising from one-loop virtual corrections and non-logarithmic soft corrections. As we mentioned earlier, the structure of the resummed expression is similar to that for the large-\( p_T \) \( W \) production cross section [26] or, in the massless limit, to that for prompt-photon production in hadronic collisions [34].
The expressions for the radiative factors are

\[
\ln \Delta_N^a = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \int \frac{1}{\mu^2} \frac{d^2 q}{q^2} A_a(\alpha_S(q^2)) , \\
\ln J_N^a = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \left[ \int \frac{(1-z)Q^2}{q^2} A_a(\alpha_S(q^2)) + \frac{1}{2} B_a(\alpha_S((1-z)Q^2)) \right] , \\
\ln \Delta_N^{(int)}_{ab \rightarrow cH} = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} D_{ab \rightarrow cH}(\alpha_S((1-z)^2 Q^2)) .
\]

The relevant scale \( Q \) for this process is given by \( Q^2 = p_T^2 (1 + r)/r \). The coefficients \( C = A_a, B_a, D_{ab \rightarrow cH} \) each are a power series in the coupling constant \( \alpha_S \), \( C = \sum_{i=1}^{\infty} (\alpha_S/\pi)^i C^{(i)} \). The universal LL and NLL coefficients \( A_a^{(1)}, A_a^{(2)} \) and \( B_a^{(1)} \) are well known \[35, 36\]:

\[
A_a^{(1)} = C_a , \quad A_a^{(2)} = \frac{1}{2} C_a K , \quad B_a^{(1)} = \gamma_a
\]

with

\[
K = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f
\]

where \( C_q = C_A = N_c = 3, C_q = C_F = (N_c^2 - 1)/2N_c = 4/3, \gamma_q = -3/2C_F = -2 \) and \( \gamma_g = -2\pi b_0 \). Here, \( b_0 \) is the first coefficient of the QCD \( \beta \)-function:

\[
b_0 = \frac{1}{12\pi} (11C_A - 2N_f) .
\]

The process-dependent coefficient \( D_{ab \rightarrow cH}^{(1)} \) can be obtained either by expanding the resummed formula in eq. (3.3) to first order in \( \alpha_S \) and comparing to the fixed-order NLO result in \[24\], or by explicit computation as outlined in \[37\]. We have checked that both approaches result in

\[
D_{ab \rightarrow cH}^{(1)} = (C_a + C_b - C_c) \ln \frac{r + 1}{r} .
\]

The coefficient is evidently just proportional to a combination of the color factors for each hard parton participating in the process. This simplicity is due to the fact that there is only one color structure for a process with only three external partons. In the “massless” limit \( r \to 1 \) we recover the known expression for the case of prompt-photon production \[34\].

The final ingredients for the resummed cross section (3.3) are the lowest order partonic cross sections in Mellin-moment space, \( \sigma_{ab \rightarrow cH}^{(res)}(N - 1) \), and the coefficients \( C_{ab \rightarrow cH} \). The expressions for the former are presented in appendix A. Regarding the latter, at NLL accuracy, we only need to know the first-order term in the expansion \( C_{ab \rightarrow cH} = 1 + \sum_{i=1}^{\infty} (\alpha_S/\pi)^i C_{ab \rightarrow cH}^{(i)} \). We derive it by comparing the expansion of the resummed expression in eq. (3.3) with the fixed-order NLO calculation in \[24\], after going to moment space. Our results for the one-loop coefficients \( C_{ab \rightarrow cH}^{(1)} \) are listed in appendix B.
In order to organize the resummation according to the logarithmic accuracy of the Sudakov exponents it is customary to expand the latter as

\[ \ln N = \ln \left( \alpha_S \ln N \right) + O \left( \alpha_S \ln N \right)^2 + O \left( \alpha_S \ln N \right)^3 \]

\[ \ln J_N = \ln \left( \alpha_S \ln N \right) + O \left( \alpha_S \ln N \right)^2 + O \left( \alpha_S \ln N \right)^3 \]

\[ \ln \Delta_N^{\text{int}} = \ln \left( \alpha_S \ln N \right) + O \left( \alpha_S \ln N \right)^2 + O \left( \alpha_S \ln N \right)^3 \]

with \( \lambda = b_0 \alpha_S \ln N \). The LL and NLL auxiliary functions \( h^{(1,2)} \) and \( f^{(1,2)} \) are

\[ h^{(1)}_a(\lambda) = + \frac{A_a^{(1)}}{2 \pi^2 b_0 \lambda} \left[ 2 \lambda - (1 - 2 \lambda) \ln(1 - 2 \lambda) \right] \]

\[ h^{(2)}_a(\lambda, Q^2/\mu_R^2; Q^2/\mu_F^2) = \frac{A_a^{(2)}}{2 \pi^2 b_0^2} \left[ 2 \lambda + \ln(1 - 2 \lambda) \right] - \frac{A_a^{(1)} \gamma_E}{\pi b_0} \ln(1 - 2 \lambda) \]

\[ + \frac{A_a^{(2)}}{2 \pi^2 b_0^2} \left[ 2 \lambda + \ln(1 - 2 \lambda) \right] \ln \left( \frac{Q^2}{\mu_R^2} \right) - \frac{A_a^{(1)}}{2 \pi^2 b_0} \lambda \ln \left( \frac{Q^2}{\mu_F^2} \right) \]

\[ f^{(1)}_a(\lambda) = - \frac{A_a^{(1)}}{2 \pi b_0 \lambda} \left[ (1 - 2 \lambda) \ln(1 - 2 \lambda) - 2(1 - \lambda) \ln(1 - \lambda) \right] \]

\[ f^{(2)}_a(\lambda, Q^2/\mu_R^2; Q^2/\mu_F^2) = - \frac{A_a^{(2)}}{2 \pi^2 b_0^2} \left[ 2 \ln(1 - \lambda) - \ln(1 - 2 \lambda) - \ln^2(1 - \lambda) \right] \]

\[ + \frac{B_a^{(1)}}{2 \pi b_0} \ln(1 - \lambda) - \frac{A_a^{(1)} \gamma_E}{\pi b_0} \left[ \ln(1 - \lambda) - \ln(1 - 2 \lambda) \right] \]

\[ - \frac{A_a^{(2)}}{2 \pi^2 b_0^2} \left[ 2 \ln(1 - \lambda) - \ln(1 - 2 \lambda) \right] + \frac{A_a^{(1)}}{2 \pi^2 b_0} \left[ 2 \ln(1 - \lambda) - \ln(1 - 2 \lambda) \right] \ln \left( \frac{Q^2}{\mu_R^2} \right) \]

\[ b_1 = \frac{1}{24 \pi^2} \left[ (17 C_A^2 - 5 C_A N_f - 3 C_F N_f) \right] \]

3.2 Matching and inverse Mellin transform

When performing the resummation, one of course wants to make full use of the available fixed-order cross section, which in our case is NLO. Therefore, it is appropriate to match the resummed result with the fixed-order expression. This is achieved by expanding the resummed cross section to \( O(\alpha_S^3) \), subtracting the expanded result from the resummed one, and adding the full NLO cross section:

\[ \frac{d\sigma^{(\text{match})}(y_T)}{dp_T^2} = \sum_{a,b} \int_{C_{\text{MP}} - i \infty}^{C_{\text{MP}} + i \infty} \frac{dN}{2 \pi i} \left( y_T^2 \right)^{-N} f_{a/h_1} (N + 1, \mu_R^2) f_{b/h_2} (N + 1, \mu_F^2) \]

\[ \times \left[ \delta^{(\text{res})}_{ab \to cH}(N) - \delta^{(\text{res})}_{ab \to cH}(N) \right] + \frac{d\sigma^{(\text{NLO})}(y_T)}{dp_T^2} \]

\[ \text{(3.15)} \]
where $\hat{\sigma}_{ab\to cH}^{\text{(res)}}$ is the resummed cross section for the partonic channel $ab \to cH$ as given in eq. (3.3). In this way, NLO is taken into account in full, and the soft-gluon contributions beyond NLO are resummed to NLL. Any double-counting of perturbative orders is avoided.

Since the resummation is achieved in Mellin-moment space, one needs an inverse Mellin transform, in order to obtain a resummed cross section in $y_T$ space. This requires a prescription for dealing with the singularities at $\lambda = 1/2$ and $\lambda = 1$ in eqs. (3.9)–(3.13), which are a manifestation of the singularity in the perturbative strong coupling constant at scale $\Lambda_{\text{QCD}}$. We will use the “Minimal Prescription” developed in ref. [33], which relies on use of the NLL expanded forms eqs. (3.9)–(3.13), and on choosing a Mellin contour in complex-$N$ space that lies to the left of the poles at $\lambda = 1/2$ and $\lambda = 1$ in the Mellin integrand:

$$
\frac{d\sigma^{\text{(res)}}(y_T)}{dy_T^2} = \int_{C_{MP} - i\infty}^{C_{MP} + i\infty} \frac{dN}{2\pi i} \left( \frac{y_T^2}{N} \right)^{-N} \hat{\sigma}^{\text{(res)}}(N) ,
$$

(3.16)

where $b_0 \alpha_S(\mu_R^2) \ln C_{MP} < 1/2$, but all other poles in the integrand are as usual to the left of the contour. The result defined by the minimal prescription has the property that its perturbative expansion is an asymptotic series that has no factorial divergence and therefore no “built-in” power-like ambiguities.

4. Higgs transverse momentum distribution at the LHC

Having discussed the resummation formulas, we are now ready to present results for the high-$p_T$ production of Higgs bosons in the process $pp \to H + X$ at the LHC at $\sqrt{s} = 14$ TeV, choosing $m_H = 125$ GeV as an example. In our analysis we use the latest MRST2004 set [38] of parton distribution functions. Unless otherwise stated, we fix the the factorization and renormalization scales to $\mu_F^2 = \mu_R^2 = p_T^2 + m_H^2$. The considered $p_T$ spectrum starts above $p_T = 80$ GeV where the effects of small transverse momentum logarithms treated in [27–30] are less important.

First, we confirm that the soft (and virtual) contributions, corresponding to the terms entering through the $g$ functions in eq. (2.9), indeed dominate the cross section. For this we compare the fixed-order NLO calculation [24] to the $O(\alpha_s^2)$ expansion of the resummed expression (the second term in eq. (3.15)). Only in the kinematical region where both contributions are similar can one argue that threshold resummation is useful. Figure 1 shows the comparison. As can be seen, the soft and virtual terms faithfully reproduce the full NLO cross section to better than 10% over the whole $p_T$ range considered. Towards “lower” $p_T$, the agreement deteriorates slightly, which is expected since pieces in the cross section that are not logarithmic in $y_T^2$ will become more and more important there. At very large values of transverse momentum ($p_T > 200$ GeV), the process moves kinematically closer to threshold, and the soft approximation becomes nearly perfect.

One of the virtues of threshold resummation is the reduction of the scale dependence of the computed cross sections. For instance, the scale dependent term $\propto \lambda \ln(Q^2/\mu_F^2)$ in eq. (3.11) cancels the diagonal part of the DGLAP-evolution of the gluon distribution at large $N$. To verify this feature for the case of Higgs production we show in figure 2 the
Figure 1: Comparison between the full NLO result \[24\] and the NLO expansion of the resummed Higgs cross section (corresponding to the soft-virtual approximation at NLO), at $\sqrt{s} = 14$ TeV and $m_H = 125$ GeV. The insert plot shows the corresponding ratio.

NLO and the NLL resummed (matched) results computed for two different values of the scales, $\mu_F^2 = \mu_R^2 = \xi^2(p_T^2 + m_H^2)$ with $\xi = 1/2, 2$. A reduction of the scale dependence by about a factor of two is seen when NLL resummation is taken into account. The net effect of the NLL resummation relative to the NLO cross section, the “$K_{\text{NLL}/\text{NLO}}$-factor”

$$K_{\text{NLL}/\text{NLO}} = \frac{\sigma_{\text{NLL}}(p_T)}{\sigma_{\text{NLO}}(p_T)}, \quad (4.1)$$

is therefore scale dependent. While fixed-order and resummed expressions are very similar for $\xi = 1/2$, one finds $K_{\text{NLL}/\text{NLO}} > 1$ at larger factorization and renormalization scales. Overall, we find that threshold resummation does not introduce very large corrections beyond NLO to the high-$p_T$ Higgs cross section, which is somewhat at variance with what was found for the case of fully-inclusive Higgs production \[16\].

We have mentioned before that for the present calculation we are using the large-top-mass approximation for the coupling of two gluons to the Higgs. At large $p_T$, $p_T \gtrsim m_t$, this approximation is known to deteriorate \[6, 20\], and a full calculation that includes all effects from the top quark loop will be required.\(^3\) Fortunately, the large logarithms we are resumming are insensitive to the structure of the Higgs-gluon coupling since they are associated only with emission of soft and collinear gluons from the external lines. Therefore, even though our cross sections shown in figure 4 will not be good predictions anymore at

\(^3\)In order to extend the validity of the results in the soft-virtual approximation to larger values of $p_T$, one could replace \[25\] the LO order cross section calculated in the large $m_t$ limit by the known LO cross section for arbitrary $m_t$. 

– 10 –
large $p_T$, we can be confident that K-factors generally will be. In other words, the product between the full Born cross section (including all effects from the heavy quark loop) as derived in [6, 20] and our calculated K-factors

$$K_{\text{NLO/LO}} = \frac{d\sigma^{\text{NLO}}}{dp_T} / \frac{d\sigma^{\text{LO}}}{dp_T}$$

(4.2)

and

$$K_{\text{NLL/LO}} = \frac{d\sigma^{\text{NLL}}}{dp_T} / \frac{d\sigma^{\text{LO}}}{dp_T}$$

(4.3)

should provide a reliable description of the full NLO and NLL cross sections. In figure 3 we present these K-factors along with $K_{\text{NLL/NLO}}$ for our default scale choice $\mu = \sqrt{p_T^2 + m_H^2}$. Here the LO result is obtained using the corresponding MRST LO set of parton distributions [39] and the one-loop expression for the strong coupling constant. As can be seen from the dotted line for $K_{\text{NLL/NLO}}$, resummation predicts an increase of about 10% of the cross section beyond NLO. The results presented in figure 3 should be taken into account in the analysis of future LHC data.

We finally recall that for our predictions we have integrated over all rapidities of the Higgs. The dependence on rapidity could be taken into account in the resummation using the techniques developed in [40] for the case of prompt-photon production. In that study it was found that the higher-order corrections show very little dependence on rapidity unless one considers situations with very forward or backward production. We expect the same to be true for the present case. The K-factors shown in figure 3 will therefore also apply to the case where the cross section is integrated over a finite bin at central rapidities, for example.
5. Conclusions and summary

The process $pp \rightarrow H(\rightarrow \gamma\gamma) + X$ offers an enticing possibility of improving the signal-to-background ratio for Higgs detection at the LHC. In this work we have studied the NLL resummation of the logarithmic threshold corrections to the partonic cross sections relevant for this process. We have presented analytical expressions for the resummed cross section in Mellin-moment space. In particular, we have derived the process-dependent perturbative coefficients necessary for the NLL resummation. We report a correction of $\mathcal{O}(10\%)$ to the NLO $p_T$ distribution in the range $80\text{ GeV} < p_T < 300\text{ GeV}$ for $M_H = 125\text{ GeV}$. The resummed result exhibits less dependence on the factorization and renormalization scales than the NLO cross section, implying a reduction of the theoretical uncertainties for this process.

Acknowledgments

We are grateful to G. Sterman for valuable discussions and for reading the manuscript, and to M. Grazzini for comments on the manuscript. The work of A.K. was supported by the Deutsche Forschungsgemeinschaft in the Sonderforschungsbereich/Transregio SFB/TR-9 “Computational Particle Physics” and BMBF Grant No. 05HT4VKA/3. The work of D.dF. was supported in part by Fundación Antorchas, CONICET and UBACyT. W.V. is grateful to RIKEN, Brookhaven National Laboratory and the U.S. Department of Energy (contract number DE-AC02-98CH10886) for providing the facilities essential for the completion of his work.

A. LO cross sections

Using the variable $r \equiv p_T/m_T$, the coefficients of the LO cross sections after rapidity
integration in eq. (2.8) are given in terms of

\[ N'_{ab}(\hat{y}_T, r) \equiv N_{ab}(\hat{y}_T, r) \left(1 + r\right)^3 \sqrt{(1 + r)^2 - (1 - r)^2 \hat{y}_T^2} \]

as:

\[ N'_{gg}(\hat{y}_T, r) = 4N_c \left((1 + r)^4 - 2(1 + r)^2 \hat{y}_T^2 + 3(1 - r)^2 \hat{y}_T^4 - (3 - 2r^2) \hat{y}_T^6 - 2(1 - r)^4 \hat{y}_T^8\right), \]
\[ N'_{gg}(\hat{y}_T, r) = C_F \left(r + 1\right) \left(2(1 + r)^3 - (1 + r) (2 + r) (2 - r) \hat{y}_T^2 + 3 (1 - r) \hat{y}_T^4 - (1 - r)^3 \hat{y}_T^6\right), \]
\[ N'_{gg}(\hat{y}_T, r) = N'_{gg}(\hat{y}_T, r), \]
\[ N'_{qg}(\hat{y}_T, r) = 4C_F r^2 \hat{y}_T^2 \left((1 + r)^2 - 2\hat{y}_T^2 + (1 - r)^2 \hat{y}_T^4\right). \] (A.1)

The explicit expressions for the Mellin moments of the LO partonic cross sections are:

\[ \hat{\sigma}^{(1)}_{gg-H}(N) = \frac{2\alpha_s N_c \sigma_0}{\sqrt{\pi} p_T^2 (1 + r)^4} \left[ \frac{(1 + r)^4 F_N(0, z) \Gamma(N)}{\Gamma(\frac{N}{2} + N)} - \frac{2(1 + r)^2 F_N(1, z) \Gamma(1 + N)}{\Gamma(\frac{N}{2} + N)} \right. \]
\[ \left. + \frac{(3 - 2r^2) F_N(2, z) \Gamma(2 + N)}{\Gamma(\frac{N}{2} + N)} - \frac{2(1 - r)^2 F_N(3, z) \Gamma(3 + N)}{\Gamma(\frac{N}{2} + N)} \right] \]
\[ + \frac{(1 - r)^4 F_N(4, z) \Gamma(4 + N)}{\Gamma(\frac{N}{2} + N)} \right), \]
\[ \hat{\sigma}^{(1)}_{gg-H}(N) = \frac{\alpha_s C_F \sigma_0}{2\sqrt{\pi} p_T^2 (1 + r)^4} \left[ \frac{2(1 + r)^3 F_N(0, z) \Gamma(N)}{\Gamma(\frac{N}{2} + N)} - \frac{2(1 + r)^2 F_N(1, z) \Gamma(1 + N)}{\Gamma(\frac{N}{2} + N)} \right. \]
\[ \left. - \frac{(1 + r) (2 + r) (2 - r) F_N(1, z) \Gamma(1 + N)}{\Gamma(\frac{N}{2} + N)} \right] \]
\[ + \frac{3 (1 - r) F_N(2, z) \Gamma(2 + N)}{\Gamma(\frac{N}{2} + N)} - \frac{(1 - r)^3 F_N(3, z) \Gamma(3 + N)}{\Gamma(\frac{N}{2} + N)} \right], \]
\[ \hat{\sigma}^{(1)}_{qg-H}(N) = \frac{2\alpha_s C_F r^2 \sigma_0}{\sqrt{\pi} p_T^2 (1 + r)^4} \left[ \frac{(1 + r)^2 F_N(1, z) \Gamma(1 + N)}{\Gamma(\frac{N}{2} + N)} - \frac{2 F_N(2, z) \Gamma(2 + N)}{\Gamma(\frac{N}{2} + N)} \right. \]
\[ \left. + \frac{(1 - r)^2 F_N(3, z) \Gamma(3 + N)}{\Gamma(\frac{N}{2} + N)} \right], \]
\[ \hat{\sigma}^{(1)}_{qg-H}(N) = \hat{\sigma}^{(1)}_{gg-H}(N), \] (A.2)

where \( F_N(n, z) \equiv 2F_1(1/2, N + n, N + (n + 1)/2; z) \) and \( z \equiv (r - 1)^2/(r + 1)^2 \). For large \( p_T \), the variable \( r \) is close to 1, and for numerical purposes it is therefore sufficient to expand the Hypergeometric function \( 2F_1 \) to second order in \( z \):

\[ 2F_1(a, b, c; z) = 1 + \frac{ab}{c} z + \frac{a(a + 1)b(b + 1)}{2c(c + 1)} z^2 + O(z^3). \] (A.3)
B. One loop coefficients

The one-loop coefficients $C_{ob\rightarrow cH}^{(1)}$ for the three different subprocesses read

\[
C_{gg\rightarrow gH}^{(1)} = \frac{11}{2} - \frac{9C_F}{4} + \frac{22C_A}{9} - \frac{5N_f}{9} - \frac{C_F\pi^2}{4} + \frac{2C_A\pi^2}{3} + \frac{5C_A\pi^2}{12} + \frac{7N_f}{36} + \frac{5C_A\pi^2}{12} + \pi b_0\gamma_E + \frac{3C_A\gamma_E^2}{2} + (C_A - N_f) \frac{1 - 2r + 10r^2}{12(1 + 6r^2 + 2r^4)} + 2C_A\text{Li}_2(1 - r) + C_A\text{Li}_2\left(\frac{2r}{1 + r}\right) + 2C_A\ln(1 - r) \ln r - \frac{C_A\ln^2 r - C_A\ln r \ln(1 + r) + \frac{C_A}{2}\ln^2(1 + r)}{r} - C_A\gamma_E \ln \frac{1 + r}{r} + 2(\pi b_0 - C_A\gamma_E) \ln \frac{Q^2}{\mu_F^2} - 3\pi b_0 \ln \frac{Q^2}{\mu_R^2}, \quad (B.1)
\]

\[
C_{qg\rightarrow gH}^{(1)} = \frac{11}{2} - \frac{9C_F}{4} + \frac{22C_A}{9} - \frac{5N_f}{9} - \frac{C_F\pi^2}{4} + \frac{2C_A\pi^2}{3} + \frac{5C_A\pi^2}{12} + \frac{7N_f}{36} + \frac{5C_A\pi^2}{12} + \pi b_0\gamma_E + \frac{3C_A\gamma_E^2}{2} + \frac{5C_A\gamma_E^2}{2}\ln(1 + r) + (C_F + C_A)\ln(1 - r) \ln r - \frac{C_A\ln^2 r - C_F\ln r \ln(1 + r) + \frac{C_F}{2}\ln^2(1 + r) + (C_F + C_A)\text{Li}_2(1 - r)}{r} + C_F\text{Li}_2\left(\frac{2r}{1 + r}\right) + \left(\pi b_0 - \frac{3}{4}C_F - C_F\gamma_E - C_A\gamma_E\right) \ln \frac{Q^2}{\mu_F^2} - 3\pi b_0 \ln \frac{Q^2}{\mu_R^2}, \quad (B.2)
\]

\[
C_{qg\rightarrow gH}^{(1)} = \frac{11}{2} - \frac{9C_F}{4} + \frac{79C_A}{12} - \frac{5N_f}{6} + \frac{4C_F\pi^2}{3} - \frac{11C_A\pi^2}{12} + \frac{C_A - C_F}{2r} + \gamma_E\pi b_0 + 2C_F\ln(1 - r) \ln r + \frac{C_A}{2}\ln^2 \frac{1 + r}{r} - C_F\ln^2 r + \left(\frac{3}{2}C_F - 2\pi b_0\right) \ln \frac{1 + r}{r} + 2C_F\text{Li}_2(1 - r) + C_A\text{Li}_2\left(\frac{2r}{1 + r}\right) - 3\pi b_0 \ln \frac{Q^2}{\mu_R^2} + (C_A - 2C_F)\gamma_E \ln \frac{1 + r}{r} + \gamma_E^2 \left(2C_F - \frac{C_A}{2}\right) + C_F\left(\frac{3}{2} - 2\gamma_E\right) \ln \frac{Q^2}{\mu_F^2}, \quad (B.3)
\]

where $b_0$ is given in eq. (B.4).

References


Soft gluon effects in perturbative quantum chromodynamics, report SLAC-PUB-2934, 1982;


