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Compact gauge fields for supersymmetric lattices

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Compact gauge fields for supersymmetric lattices

Mithat Ünsal

Department of Physics, Boston University
Boston, MA 02215, U.S.A.
Email: unsal@buphy.bu.edu

Abstract: We show that a large class of euclidean extended supersymmetric lattice gauge theories constructed in [1 − 3] can be regarded as compact formulations by using the polar decomposition of the complex link fields. In particular, the gauge part of the supersymmetric lattice action is the standard Wilson action. This formulation facilitates the construction of gauge invariant operators.

Keywords: Lattice Gauge Field Theories, Extended Supersymmetry.

Contents

1. Introduction and results

In [4, 1−3], the spatial and euclidean spacetime lattices whose continuum limits are extended supersymmetric gauge theories in various dimensions are constructed. These lattices are obtained by the orbifold projection of supersymmetric matrix models. The projection generates a lattice, while preserving a subset of supersymmetry. But, a priori, a dimensionful parameter which can be identified as a lattice spacing is absent. However, these theories possess a moduli space of vacua, along which the potential vanishes, and the distance from the origin of the moduli space is interpreted as an inverse lattice spacing. Moving to infinity in the moduli space corresponds to taking the lattice spacing to zero. The construction of the supersymmetric lattices is inspired by orbifold projection [5] and the deconstruction of the supersymmetric theories [6−8].

In [4, 1−3], the fluctuations of the complex bosonic link fields around a particular configuration in moduli space are split into an hermitian and antihermitian matrix, as in the cartesian decomposition of a complex number. In the continuum limit, these matrices give rise to scalars and the gauge bosons of the target theory respectively. This construction provides a formulation in which the gauge boson is noncompact. However, this expansion is not the most natural one for two reasons. In terms of the shifted variables, certain global symmetries and the local gauge symmetry become hidden. Let us explain both in turn. The orbifold matrix theory, i.e, the lattice action before the shift, possesses at least a $\mathrm{U}(1)^d$ symmetry[1] which corresponds to rotations of the complex link fields as $z_{a,\mathbf{n}} \to e^{i\alpha_b \delta_{ab}} z_{a,\mathbf{n}}$. In terms of shifted variables, this symmetry is not manifest in the action. The latter issue is gauge covariance. The complex link boson $z_{a,\mathbf{n}}$ is in bifundamental representation and transforms covariantly under gauge transformation. Consequently, the orbifold matrix theory action is manifestly gauge invariant. However, in terms of shifted fields, the gauge

[1] There are other continuous symmetries of the orbifold matrix theory, for example, the R-symmetry.

	$\mathcal{Q}=4$	$\mathcal{Q}=8$	$\mathcal{Q}=16$
$d=1$	2	4	8
$d=2$	1	2	4
$d=3$	$\times$	1	2
$d=4$	$\times$	$\times$	1

Table 1: The euclidean lattices for extended SYM theories. $\mathcal{Q}$ is the amount of supersymmetry in both the mother theory and the target theory. The numbers in the boxes corresponds to the number of exactly realized supersymmetries on the euclidean lattice. The boxes with $\times$ are the theories that could not be reached due to insufficiency of R-symmetry within our approach.

invariance is a nonlinearly realized symmetry. It is, therefore, desirable, to make both the global symmetry and gauge symmetry manifest.

In this note, we use a parametrization of the complex link fields which preserves the exact gauge invariance of the orbifold matrix theory and keeps global symmetries manifest. It relies on the polar decomposition of a complex matrix into radial and angular variables. We expand our action around the same point in the moduli space. This expansion generates a lattice action which is manifestly gauge invariant, and the gauge fields are compact, unitary matrices, hence there is no need for gauge-fixing. However, having a compact gauge integration is not one of the advantages, since the moduli spaces of our target theories are noncompact. On the other hand, the polar decomposition preserves the exact global symmetries of the orbifold matrix theory and the observables of the theory are charged under these symmetries. Such observables are the order parameters for the corresponding symmetries.[2]

To facilitate the identification of the gauge part of our supersymmetric lattice action with the standard Wilson action [9], we choose to work with hypercubic lattices on the moduli space. As explained in detail in [4, 3], different points in moduli space correspond to different structures of the unit cell on the lattice. For example, in the two dimensional lattice for the $\mathcal{Q}=16$ supercharge target theory, different points in moduli space correspond to a hexagonal lattice, square lattice with a diagonal or asymmetric lattices such as a rectangular lattice with diagonal. Each of these lattices has a different point group symmetry. There are trajectories in moduli space which respect a particular type of point group symmetry. To take the continuum limit, one moves along a particular trajectory out to infinity with an appropriate scaling of the number of sites on the lattice. We chose previously [4] the most symmetric lattices to minimize the number of relevant and marginal operators. Our purpose here is to show a correspondence with the Wilson action, and the hypercubic trajectories are better suited for that.

The approach that has been presented here is applicable to all of the euclidean spacetime lattices listed in table 1, in particular, including all the $\mathcal{Q}=16$ target theories in $d \leq 4$ dimensions.[3] In table 1, each box is associated with a pair $(d, \mathcal{Q})$, a d dimensional

[2]We want to point out that our construction is not a new lattice formulation of the supersymmetric lattice gauge theories; it is merely a reparametrization of the complex link field.

[3]These theories are sometimes named with respect to the multiplicity of their minimal spinor dimensions in the corresponding dimension. More conventional names for the $\mathcal{Q}=4$, $\mathcal{Q}=8$, $\mathcal{Q}=16$ theories are: in $d=$

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target SYM theory with $\mathcal{Q}$ supersymmetries. The number within the box (associated with $(d, \mathcal{Q})$ theory) corresponds to the number of exactly realized supersymmetries by the d dimensional lattice action. Among these, there is also the $\mathcal{N} = 4$ SYM theory in $d = 4$ dimensions [3]. The theories with $\times$ sign are the ones that we are unable to reach, within our approach, due to insufficiency of the R-symmetry of the mother theory. The details of each supersymmetric lattice gauge theory can be quite different. For example, the number of exactly realized supersymmetries, the supersymmetry algebra, the multiplet structures, the point group symmetries of the spacetime lattices can be different in each case. The application in section 2 is aimed to show the realization of the ideas in one simple example, the $\mathcal{Q} = 4$ target theory in $d = 2$ dimensions. Generalization to other supersymmetric lattice theories listed in table 1 is also possible.

Before showing the realizations of these ideas explicitly in a simple example, we first want to point out the other recent approach to the supersymmetic lattice, which is sometimes known under the rubric twisted supersymmetry. (For references to earlier work, see [4, 10, 11]). Catterall and collaborators implemented the supersymmetry on the lattice by keeping one or more nilpotent supersymmetries exactly realized on the lattice [12, 13]. This led to the construction of the two dimensional Wess-Zumino and supersymmetric sigma models in various dimensions. [14, 15] In [16], the general criteria under which a nilpotent symmetry can be carried consistently with a latticization had been analyzed(also see [17, 18], which advocates more, and [19] for the recent results on $N = 1$ Wess-Zumino in $d = 4$). Recently, Sugino [20−22] generalized this approach to the gauge theories with compact gauge fields in the formulation. He showed that a continuum supersymmetric gauge theory written in a Q-exact form, can be carried to the lattice and gave the lattice counterpart of Q-transformations. However, this procedure is not unique. And as a result, he encounters a vacuum degeneracy problem on the gauge field sector, as well as spurious zero modes in the theory. Sugino argues that both problems can be solved. (see [21, 22] for details.) More recently, Catterall [23] constructed the two dimensional four supercharge SYM theory by using a geometric approach that is free of the problems encountered in [20]. (Also see [24]). However, the lattice theory he constructs requires a complexification (hence doubling) of all the degrees of freedom. This turns, for example, the unitary compact variables, into noncompact variables. Then, he conjectures that, one can restrict the path integral to a real line in the field space by preserving the Ward identities associated with twisted supersymmetry, and also recovers the desired target theory. The approach that we will follow does not encounter this problem.

2. Application: Compact lattice action for $\mathcal{N} = (2, 2)$ SYM theory

To illustrate these ideas explicitly, we work through a simple example in detail. The target theory is the $\mathcal{N} = (2, 2)$ SYM theory in $d = 2$ dimensions. The lattice construction is examined in detail in [1], and we refer the reader there for more details. The arguments in this section can be easily applied to any of the other target theories listed in table 1.

4 dimensions $\mathcal{N} = 1, 2, 4$, in $d = 3$ dimensions $\mathcal{N} = 2, 4, 8$, and in $d = 2$ dimensions $\mathcal{N} = (2, 2), (4, 4), (8, 8)$ respectively.

The lattice action for the $\mathcal{N} = (2,2)$ SYM target theory with gauge group $\mathrm{U}(k)$, in $\mathcal{Q} = 1$ superfield formulation is given by [1]

$$S = \frac{1}{g^2}\sum_{\mathbf{n}} \mathrm{tr} \int d\theta \Big[-\frac{1}{2}\mathbf{\Lambda}_{\mathbf{n}}\partial_\theta \mathbf{\Lambda}_{\mathbf{n}} - \mathbf{\Lambda}_{\mathbf{n}} \left(\bar{z}_{a,\mathbf{n}-\hat{\mathbf{e}}_a}\mathbf{Z}_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - \mathbf{Z}_{a,\mathbf{n}}\bar{z}_{a,\mathbf{n}}\right) - \\ - \mathbf{\Xi}_{12,\mathbf{n}}(\mathbf{Z}_{1,\mathbf{n}}\mathbf{Z}_{2,\mathbf{n}+\hat{\mathbf{e}}_1} - \mathbf{Z}_{2,\mathbf{n}}\mathbf{Z}_{1,\mathbf{n}+\hat{\mathbf{e}}_2})\Big], \tag{2.1}$$

where $\mathbf{n}$ is a two component integer vector labeling sites on the lattice and $\hat{\mathbf{e}}_{\mathbf{a}}$ is the unit vector in a'th direction where $a = 1, 2$.[4] The $\mathcal{Q} = 1$ supersymmetric multiplets in terms of their component fields can be expressed as

$$\begin{aligned}
\mathbf{\Lambda}_{\mathbf{n}} &= \lambda_{\mathbf{n}} - i\theta d_{\mathbf{n}}\,, \\
\mathbf{Z}_{a,\mathbf{n}} &= z_{a,\mathbf{n}} + \sqrt{2}\,\theta\,\psi_{a,\mathbf{n}}\,, \\
\bar{z}_{a,\mathbf{n}} &= \bar{z}_{a,\mathbf{n}}\,, \\
\mathbf{\Xi}_{12,\mathbf{n}} &= \xi_{12,\mathbf{n}} - 2\theta\,\left(\bar{z}_{2,\mathbf{n}+\hat{\mathbf{e}}_1}\bar{z}_{1,\mathbf{n}} - \bar{z}_{1,\mathbf{n}+\hat{\mathbf{e}}_2}\bar{z}_{2,\mathbf{n}}\right).
\end{aligned} \tag{2.3}$$

The fermi multiplet $\mathbf{\Lambda}_{\mathbf{n}}$ lives on the site $\mathbf{n}$. The bosonic multiplet $\mathbf{Z}_{a,\mathbf{n}}$ lives on the oriented link, starting at $\mathbf{n}$ and ending at $\mathbf{n} + \hat{\mathbf{e}}_{\mathbf{a}}$. The supersymmetry singlet $\bar{z}_{a,\mathbf{n}}$ lives on the oppositely oriented link, starting at $\mathbf{n} + \hat{\mathbf{e}}_{\mathbf{a}}$ and ending at $\mathbf{n}$, and the diagonal fermi multiplet $\mathbf{\Xi}_{\mathbf{n}}$ resides on the diagonal link, starting at $\mathbf{n} + \hat{\mathbf{e}}_{\mathbf{1}} + \hat{\mathbf{e}}_{\mathbf{2}}$ and ending at $\mathbf{n}$. The lowest component fermi multiplet $\mathbf{\Xi}_{12,\mathbf{n}}$ is antisymmetric under the exchange of its subscripts; $\xi_{12} = -\xi_{21}$, as its supersymmetric partner. By substituting the multiplets into the action eq. (2.1), we obtain the action in component fields. For convenience, we split the action into the bosonic and fermionic parts. The bosonic part is

$$S_b = \frac{1}{g^2}\sum_{\mathbf{n}} \mathrm{tr}\left[\frac{1}{2}d_{\mathbf{n}}^2 + id_{\mathbf{n}}\left(\bar{z}_{a,\mathbf{n}-\hat{\mathbf{e}}_a}z_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - z_{a,\mathbf{n}}\bar{z}_{a,\mathbf{n}}\right) + 2|(z_{1,\mathbf{n}}z_{2,\mathbf{n}+\hat{\mathbf{e}}_1} - z_{2,\mathbf{n}}z_{1,\mathbf{n}+\hat{\mathbf{e}}_2})|^2\right], \tag{2.4}$$

and the fermionic part is

$$S_f = \frac{\sqrt{2}}{g^2}\sum_{\mathbf{n}} \mathrm{tr}\Big[\lambda_{\mathbf{n}}\left(\bar{z}_{a,\mathbf{n}-\hat{\mathbf{e}}_a}\psi_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - \psi_{a,\mathbf{n}}\bar{z}_{a,\mathbf{n}}\right) + \xi_{ab,\mathbf{n}}(z_{a,\mathbf{n}}\psi_{b,\mathbf{n}+\hat{\mathbf{e}}_\mathbf{a}} - \psi_{b,\mathbf{n}}z_{a,\mathbf{n}+\hat{\mathbf{e}}_\mathbf{b}})\Big]. \tag{2.5}$$

The construction, symmetries and the continuum limit of this lattice action had been examined in detail in [1]. The symmetries are the $\mathrm{U}(k)$ gauge symmetry, the discrete translations $Z_N \times Z_N$ of the lattice, a Z_2 point group symmetry, the $\mathcal{Q} = 1$ supersymmetry

[4] The lattice action eq. (2.1) can also be expressed in a Q-exact form (also noted in [25]), which is given by

$$S = \frac{1}{g^2}\sum_{\mathbf{n}} \mathrm{tr}Q\left[+\frac{1}{2}\lambda_{\mathbf{n}}(id_{\mathbf{n}}) - \lambda_{\mathbf{n}}\left(\bar{z}_{a,\mathbf{n}-\hat{\mathbf{e}}_a}z_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - z_{a,\mathbf{n}}\bar{z}_{a,\mathbf{n}}\right) - \xi_{12,\mathbf{n}}(z_{1,\mathbf{n}}z_{2,\mathbf{n}+\hat{\mathbf{e}}_1} - z_{2,\mathbf{n}}z_{1,\mathbf{n}+\hat{\mathbf{e}}_2})\right], \tag{2.2}$$

The action of Q on the components can be read of from the supermultiplets eq. (2.3). For example, $Qz_{a,\mathbf{n}} = \sqrt{2}\psi_{a,\mathbf{n}}$, $Q\psi_{a,\mathbf{n}} = 0$ etc. The supersymmetry algebra is $Q^2\cdot = 0$. This is, in the sense of supersymmetry algebra, the difference with the Catterall's construction [23], in which the square of the twisted supersymmetry generator is an infinitesimal field dependent gauge rotation, $Q^2\cdot = \delta_{\mathrm{gauge}}\cdot$.

	$\mathbf{Z}_{1,\mathbf{n}}$	$\overline{z}_{1,\mathbf{n}}$	$\mathbf{Z}_{2,\mathbf{n}}$	$\overline{z}_{2,\mathbf{n}}$	$\mathbf{\Lambda}_{\mathbf{n}}$	$\mathbf{\Xi}_{\mathbf{n}}$	θ
r_1	$+1$	-1	0	0	0	-1	0
r_2	0	0	$+1$	-1	0	-1	0
Y	0	0	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$

Table 2: The $\mathrm{U}(1)^3$ global symmetry charges of the lattice theory. The $r_{1,2}$ are the ones that are used in orbifold projection. The last one, denoted by Y, is the exact R-symmetry on the lattice.

and a $\mathrm{U}(1)^3$ global symmetry given in table 2. The last U(1) denoted as Y in table 2 is an R-symmetry on the lattice, i.e., it does not commute with supersymmetry since the the superspace coordinate θ is charged under it. The global symmetries will be an essential part of the discussion of observables.

In [1], we expanded the lattice action eq. (2.1) around a point in moduli space, where the vacuum expectation value of the link field is interpreted as the inverse lattice spacing. Thus, the complex link fields had been written as

$$z_{a,\mathbf{n}} = \frac{1}{\sqrt{2}a}\mathbf{1}_k + \frac{h_{a,\mathbf{n}} + iv_{a,\mathbf{n}}}{\sqrt{2}} \tag{2.6}$$

where $h_{a,\mathbf{n}}$ and $v_{a,\mathbf{n}}$ are the hermitean matrices, which become the scalar and vector boson of the continuum theory. However, in terms of shifted fields, the $\mathrm{U}(1)^2$ subgroup of the $\mathrm{U}(1)^3$, associated with (r_1, r_2) charges in table 2, is hidden. Even though these symmetries are there, they become obscured, and we can not benefit from them easily. (To better appreciate these symmetries, one should address the observables, see section 3) It is preferable to make these symmetries manifest. The other point is gauge covariance. The lattice action eq. (2.1) is gauge invariant and its constituents transforms covariantly under gauge rotations.. However, the shifted fields hide manifest gauge covariance and it is hard to construct manifestly gauge invariant objects. In this sense, both a $\mathrm{U}(1)^2$ subset of the global symmetries and gauge symmetry are nonlinearly realized when the lattice action is expressed in terms of shifted fields. It is, therefore, preferable to make the global symmetries and gauge invariance manifest.

Here, we show that there is a more natural decomposition of the complex bosonic link fields, which makes all the global symmetries manifest and generates manifestly gauge invariant hopping terms. It is the polar decomposition of the complex link matrices. This also provides a formulation of the supersymmetric lattice gauge theory in which the gauge fields are compact, group valued matrices. However, it is not a new formulation, it is reexpressing the eq. (2.1) in a new parametrization.

Given a complex nonsingular matrix $z_{a,\mathbf{n}}$, we can always write $z_{a,\mathbf{n}} = H_{a,\mathbf{n}}U_{a,\mathbf{n}}$ where $H_{a,\mathbf{n}}$ is a hermitian nonnegative matrix and $U_{a,\mathbf{n}}$ is a unitary matrix. This decomposition is unique (modulo left-right decomposition, we chose left.) up to a set of measure zero. It is not unique for matrices with zero eigenvalues. However, this is not a problem for us, because we do an expansion about a point which is far from the origin of the moduli space. The vacuum that we are expanding around is $\langle H_{a,\mathbf{n}}\rangle = \frac{1}{\sqrt{2}a}1_k$ and $U_{a,\mathbf{n}} = 1_k$. Thus, we express the complex link matrices as

$$z_{a,\mathbf{n}} = \frac{1}{\sqrt{2}} H_{a,\mathbf{n}} U_{a,\mathbf{n}} = \frac{1}{\sqrt{2}} \left(\frac{\mathbf{1}_k}{a} + h_{a,\mathbf{n}} \right) U_{a,\mathbf{n}} \tag{2.7}$$

where $H_{a,\mathbf{n}}$ is a hermitean matrix with nonnegative eigenvalues, $U_{a,\mathbf{n}}$ is a unitary matrix and $h_{a,\mathbf{n}}$ is a hermitean matrix. The $\sqrt{2}$ in the denominator is for normalization. Notice that for the small gauge field configurations, one can expand the unitary matrix $U_{a,\mathbf{n}} = 1 + iav_{a,\mathbf{n}} + O(a^2)$ and the expression for the polar decomposition reduce to cartesian decomposition eq. (2.6). Thus, for small gauge field configurations, the corresponding lattice action reduces to the action we examined in [1]. For this reason, we will not reexamine the classical and the quantum continuum limit and the matching of the target theory fields to the lattice fields.

Now, let us analyze briefly the gauge transformation properties. Notice that the first term in eq. (2.7) dictates the gauge transformation property of the unitary link matrix $U_{a,\mathbf{n}}$. Let $g_{\mathbf{n}}$ denote a unitary gauge rotation matrix. Since under a gauge transformation, the complex link matrix transforms as bifundamental $z_{a,\mathbf{n}} \rightarrow g_{\mathbf{n}} z_{a,\mathbf{n}} g^{\dagger}_{\mathbf{n}+\hat{\mathbf{e}}_a}$, the unitary link field transforms the same way as well, $U_{a,\mathbf{n}} \rightarrow g_{\mathbf{n}} U_{a,\mathbf{n}} g^{\dagger}_{\mathbf{n}+\hat{\mathbf{e}}_a}$ and the hermitian matrix as an adjoint of site $\mathbf{n}$, $h_{a,\mathbf{n}} \rightarrow g_{\mathbf{n}} h_{a,\mathbf{n}} g^{\dagger}_{\mathbf{n}}$. Basically, the polar decomposition places the scalar $h_{a,\mathbf{n}}$ on the site $\mathbf{n}$ and $U_{a,\mathbf{n}}$ on the oriented link $(\mathbf{n}, \mathbf{n} + \hat{\mathbf{e}}_a)$, where the first entry is the starting point and the latter is the termination point.

The global $\mathrm{U}(1)^2$ symmetry, which becomes hidden in the case of cartesian decomposition, is manifest now. For example, under the first two $\mathrm{U}(1)^2$, the variable $z_{1,\mathbf{n}}$ has a global charge $(1,0)$. This imposes the charge of the unitary link field $U_{1,\mathbf{n}}$ as $(1,0)$, the charge of the hermitian scalar $h_{1,\mathbf{n}}$ as $(0,0)$. The charge of the fermionic superpartner of $z_{1,\mathbf{n}}$, the $\psi_{1,\mathbf{n}}$, is unchanged and equal to $(1,0)$.

Next, we will show that expanding the action by using polar decomposition, eq. (2.7) will reproduce the Wilson action for the gauge fields and gauge invariant hopping terms for fermions and scalars. Before we do that, let us rewrite the superfields eq. (2.3) by using the new parametrization. The superfield $\mathbf{\Lambda}_{\mathbf{n}}$ is same as above and the others are

$$\begin{aligned}
\mathbf{Z}_{a,\mathbf{n}} &= \frac{1}{\sqrt{2}} \left(\frac{1}{a} U_{a,\mathbf{n}} + h_{a,\mathbf{n}} U_{a,\mathbf{n}} \right) + \sqrt{2}\, \theta\, \psi_{a,\mathbf{n}} , \\
\bar{z}_{a,\mathbf{n}} &= \frac{1}{\sqrt{2}} \left(\frac{1}{a} U^{\dagger}_{a,\mathbf{n}} + U^{\dagger}_{a,\mathbf{n}} h_{a,\mathbf{n}} \right) , \\
\mathbf{\Xi}_{12,\mathbf{n}} &= \xi_{12,\mathbf{n}} - \theta \Bigg[\frac{1}{a^2} (U^{\dagger}_{2,\mathbf{n}+\hat{\mathbf{e}}_1} U^{\dagger}_{1,\mathbf{n}} - U^{\dagger}_{1,\mathbf{n}+\hat{\mathbf{e}}_2} U^{\dagger}_{2,\mathbf{n}}) + \frac{1}{a} (U^{\dagger}_{2,\mathbf{n}+\hat{\mathbf{e}}_1} U^{\dagger}_{1,\mathbf{n}} h_{1,\mathbf{n}} - \\
&\qquad\qquad\qquad\qquad - U^{\dagger}_{1,\mathbf{n}+\hat{\mathbf{e}}_2} h_{1,\mathbf{n}+\hat{\mathbf{e}}_2} U^{\dagger}_{2,\mathbf{n}}) + \\
&\qquad + \frac{1}{a} (U^{\dagger}_{2,\mathbf{n}+\hat{\mathbf{e}}_1} h_{2,\mathbf{n}+\hat{\mathbf{e}}_1} U^{\dagger}_{1,\mathbf{n}} - U^{\dagger}_{1,\mathbf{n}+\hat{\mathbf{e}}_2} U^{\dagger}_{2,\mathbf{n}} h_{2,\mathbf{n}}) + \\
&\qquad + (U^{\dagger}_{2,\mathbf{n}+\hat{\mathbf{e}}_1} h_{2,\mathbf{n}+\hat{\mathbf{e}}_1} U^{\dagger}_{1,\mathbf{n}} h_{1,\mathbf{n}} - U^{\dagger}_{1,\mathbf{n}+\hat{\mathbf{e}}_2} h_{1,\mathbf{n}+\hat{\mathbf{e}}_2} U^{\dagger}_{2,\mathbf{n}} h_{2,\mathbf{n}}) \Bigg] .
\end{aligned} \tag{2.8}$$

The superfields are covariant under gauge transformations and the components transform homogeneously under all global symmetries. Also notice that the θ component of the fermi multiplet $\mathbf{\Xi}_{\mathbf{n}}$ involves the square root of the standard Wilson action found in lattice QCD and its decoration with scalars insertions. The θ component is depicted in figure 1.

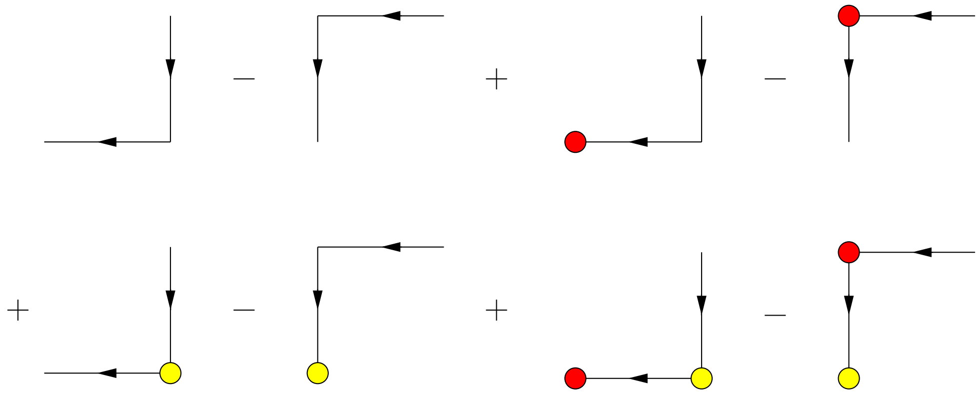

Figure 1: The $-\theta$ component of the fermi multiplet, $\boldsymbol{\Xi}_{12,\mathbf{n}}$. The vertical and horizontal arrows are unitary link fields, the red (yellow) circle is the h_1 (h_2) scalar. The arrow starts at site $\mathbf{n}+\hat{\mathbf{e}}_1+\hat{\mathbf{e}}_2$ and terminates at site $\mathbf{n}$. The modulus square of the sum generates both Wilson action and part of the scalar action.

2.1 The boson lattice action

Gauge fields. Our first goal is to show that our action involves the Wilson action for lattice gauge theory. The Wilson action is hidden in the third term of the bosonic action eq. (2.4). The $d_{\mathbf{n}}$ term does not contribute to the gauge field action. One easy geometric way to see that is, the $d_{\mathbf{n}}$ term is coupled to composite fields which do not surround an area, and the Wilson action is a sum over trace of the elementary square plaquettes, (which can be associated with the exponential of the Aharonov-Bohm flux in U(1) target theories.)

For our purpose, it is more useful to rewrite the third term in the bosonic action eq. (2.4) in a more suggestive form which makes the geometric visualization easier:

$$\begin{aligned} S_{b2} &= \frac{2}{g^2}\sum_{\mathbf{n}} \mathrm{tr}\Big[|(z_{1,\mathbf{n}}z_{2,\mathbf{n}+\hat{\mathbf{e}}_1} - z_{2,\mathbf{n}}z_{1,\mathbf{n}+\hat{\mathbf{e}}_2})|^2\Big] \\ &= \frac{2}{g^2}\sum_{\mathbf{n}} \mathrm{tr}\Big[z_{1,\mathbf{n}}z_{2,\mathbf{n}+\hat{\mathbf{e}}_1}\bar{z}_{1,\mathbf{n}+\hat{\mathbf{e}}_2}\bar{z}_{2,\mathbf{n}} - z_{1,\mathbf{n}}z_{2,\mathbf{n}+\hat{\mathbf{e}}_1}\bar{z}_{2,\mathbf{n}+\hat{\mathbf{e}}_1}\bar{z}_{1,\mathbf{n}} + \mathrm{h.c.}\Big] \end{aligned} \tag{2.9}$$

where h.c. stands for hermitian conjugate. By plugging the expansion eq. (2.7) into above expression, we obtain (among other things which will be explained momentarily)

$$\begin{aligned} S_{\mathrm{gauge}} &= \frac{1}{2g^2a^4}\sum_{\mathbf{n}} \mathrm{tr}\Big[|(U_{1,\mathbf{n}}U_{2,\mathbf{n}+\hat{\mathbf{e}}_1} - U_{2,\mathbf{n}}U_{1,\mathbf{n}+\hat{\mathbf{e}}_2})|^2\Big] \\ &= \frac{1}{2g^2a^4}\sum_{\mathbf{n}} \mathrm{tr}\Big[U_{1,\mathbf{n}}U_{2,\mathbf{n}+\hat{\mathbf{e}}_1}U^\dagger_{1,\mathbf{n}+\hat{\mathbf{e}}_2}U^\dagger_{2,\mathbf{n}} - 1 + \mathrm{h.c.}\Big]\,, \end{aligned} \tag{2.10}$$

This is exactly the Wilson action for the pure lattice gauge theory. The constant term in the action sets the action to zero for the vacuum state. The first term is a square plaquette variable. We will refer to the constant term as "flipped L" ($\lrcorner$) since it is a product of unitary link fields $U_{1,\mathbf{n}}U_{2,\mathbf{n}+\hat{\mathbf{e}}_1}U^\dagger_{2,\mathbf{n}+\hat{\mathbf{e}}_1}U^\dagger_{1,\mathbf{n}} = 1$. Even though it seems useless to introduce such a name for identity, it will be a convenient tool when we incorporate scalars. The scalars enter as decorations on plaquette and flipped L terms. Since the action eq. (2.10) is expressed in terms of the angular variables, the gauge fixing becomes unnecessary.

Scalar-gauge and scalar-scalar interactions. The scalar hopping terms and the the scalar potential term originate from two sources. One is the third term in scalar action eq. (2.4), which also gives rise the Wilson action, and the other is the $d_{\mathbf{n}}$ term. We examine both in turn, starting with the second part eq. (2.9).

The second part of the action eq. (2.9) is the trace of the modulus square of the θ component of the fermi multiplet $\Xi_{12,\mathbf{n}}$ (See figure 1). The first pair in figure 1 generates the field strength for the gauge field. The second pair generates gauge covariant hopping terms for scalar h_1 in the $\hat{\mathbf{e}}_{\mathbf{2}}$ direction. Similarly, the third pair generates the hopping terms for the scalar h_2 in the $\hat{\mathbf{e}}_{\mathbf{1}}$ direction. The last term give rise to the scalar-scalar interactions. In the continuum limit, these terms add up to $iv_{12} + D_2 h_1 - D_1 h_2 + [h_1, h_2] + O(a)$ where v_{12} is the gauge field strength and $D_a = \partial_a + i[v_a, \cdot\,]$ is the gauge covariant derivative.

Notice that this is not the most economical way to express the gauge covariant derivatives or scalar-scalar interactions on the lattice. For example, consider the h_2 field. A more economical covariant derivative in direction one would be $h_{2,\mathbf{n}+\hat{\mathbf{e}}_{\mathbf{1}}} - U^{\dagger}_{1,\mathbf{n}} h_{2,\mathbf{n}} U_{1,\mathbf{n}}$. However, the exact supersymmetry dictates something different then that. Nevertheless, both turn out to yield the same gauge covariant derivative in the continuum, where the difference between the two is suppressed by lattice spacing. Also, the scalar-scalar interaction term $[h_1, h_2]^2$ in the action does not follow from a local potential on the lattice. The way it arises seems a bit extravagant. For example, it emerges from a plaquette and flipped L decorated with four scalars at various sites. This is slightly nonlocal, unlike our usual way of writing the scalar interactions. But similar to the covariant derivative, the difference of the slightly nonlocal potential and local potential is suppressed with the lattice spacing toward the continuum limit.

The full expression for the eq. (2.9), expressed in terms of unitary link fields and hermitian scalars, is the trace of the modulus square of the sum of figures presented in figure 1. The square involves both the plaquettes (denoted as $\square$) and the flipped L's (denoted as $\lrcorner$) with scalar decorations. Hence, it is more convenient to assemble eq. (2.9) into a simple (symbolic) expression given as:

$$S_{b2} = \sum_{k=0}^{4} \frac{1}{2a^{4-k}g^2} \sum_{(\,\square,\lrcorner\,)} \mathrm{tr}\Big[(U[\,\square, k] - U[\lrcorner, k]) + \mathrm{h.c.} \Big]\,. \tag{2.11}$$

where k is the number of the scalar insertions onto the plaquette or the flipped L. The rule for the scalar insertions was indeed fixed when we chose the left decomposition in polar decomposition eq. (2.7). The scalar $h_{a,\mathbf{n}}$ can only be inserted at the starting point of $U_{a,\mathbf{n}}$ and at the end point of $U^{\dagger}_{a,\mathbf{n}}$.

The $k = 0$ term is the Wilson action as discussed. There are harmless $k = 1$ and $k = 3$ terms, the decorations with one and three scalars respectively. Examination of these terms shows that due to exact cancellations among the various components in the sum, the lowest dimensional operators arising from these terms are dimension five (in the normalization used in [1]), suppressed by lattice spacing, and hence irrelevant. The $k = 2$ terms in the sum provide gauge invariant hopping terms for scalars, namely h_1 (h_2) hopping in the $\hat{\mathbf{e}}_{\mathbf{2}}$ ($\hat{\mathbf{e}}_{\mathbf{1}}$) direction. There are two types of Lorentz symmetry violating cross term, which cancels by the contribution coming from the $d_{\mathbf{n}}$ term. The $k = 4$ sector generates the quartic scalar interaction of the continuum theory.

The auxiliary $d_{\mathbf{n}}$ field enters into the action quadratically and can be integrated out. The equation of motion gives

$$\begin{aligned} d_{\mathbf{n}} &= -i\sum_a (\bar{z}_{a,\mathbf{n}-\hat{\mathbf{e}}_a} z_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - z_{a,\mathbf{n}}\bar{z}_{a,\mathbf{n}}) \\ &= -i\sum_a \left(\frac{1}{a}(U^\dagger_{a,\mathbf{n}-\hat{\mathbf{e}}_a} h_{a,\mathbf{n}-\hat{\mathbf{e}}_a} U_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - h_{a,\mathbf{n}}) + \frac{1}{2}(U^\dagger_{a,\mathbf{n}-\hat{\mathbf{e}}_a} h^2_{a,\mathbf{n}-\hat{\mathbf{e}}_a} U_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - h^2_{a,\mathbf{n}})\right) \end{aligned} \tag{2.12}$$

which generates the gauge covariant hopping term for scalars. In the continuum, this becomes $D_1h_1 + D_2h_2 + O(a)$. Substituting the eq. (2.12) into the first part of the bosonic action, we obtain

$$S_{b1} = \frac{1}{2g^2}\sum_{\mathbf{n}} \mathrm{tr}\left[\left(\sum_a \frac{1}{a}(U^\dagger_{a,\mathbf{n}-\hat{\mathbf{e}}_a} h_{a,\mathbf{n}-\hat{\mathbf{e}}_a} U_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - h_{a,\mathbf{n}})\right)^2 + \cdots\right] \tag{2.13}$$

where the ellipsis indicates the terms which becomes irrelevant in the infrared . The continuum limit of the action eq. (2.13), generates hopping terms for h_a in $\hat{\mathbf{e}}_{\mathbf{a}}$ directions and a mixed Lorentz symmetry violating term. The latter cancels exactly with the cross term arising from the eq. (2.11). Upon adding these two parts, $S_{b1} + S_{b2}$, we obtain the bosonic part of the target theory action, the $\mathcal{N} = (2, 2)$ SYM theory in $d = 2$ dimensions. Namely,

$$\begin{aligned} S_b &= \frac{1}{2g_2^2}\int d^2x \mathrm{tr}\Big[(D_1h_1 + D_2h_2)^2 + |(D_1h_2 - D_2h_1) + i(v_{12} - i[h_1, h_2])|^2\Big] \\ &= \frac{1}{g_2^2}\int d^2x \mathrm{tr}\Big[\frac{1}{4}v^2_{\mu\nu} + \frac{1}{2}(D_\mu h_a)^2 - \frac{1}{4}[h_a, h_b]^2\Big] \end{aligned} \tag{2.14}$$

where $\mu, \nu = 1, 2$ and $a, b = 1, 2$. The three cross terms add up to a Lorentz invariant surface term proportional to

$$\epsilon^{\mu\nu}\left(D_\mu h_1 D_\nu h_2 - \frac{i}{2}v_{\mu\nu}[h_1, h_2]\right),$$

which can be shown to be the dimensional reduction of the topological term, $\mathrm{tr}\vec{E}.\vec{B}$ from $d = 4$ to $d = 2$ dimensions.

2.2 The fermion lattice action

The fermion gauge field interactions and the hopping terms are particularly simple. Substituting eq. (2.7) into fermion action eq. (2.5), we find the fermion-gauge field and the fermion-scalar interaction terms. The former is

$$S_{f-g} = \frac{1}{ag^2}\sum_{\mathbf{n}} \mathrm{tr}\Big[\lambda_{\mathbf{n}}(U^\dagger_{a,\mathbf{n}-\hat{\mathbf{e}}_a}\psi_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - \psi_{a,\mathbf{n}}U^\dagger_{a,\mathbf{n}}) + \xi_{ab,\mathbf{n}}(U_{a,\mathbf{n}}\psi_{b,\mathbf{n}+\hat{\mathbf{e}}_\mathbf{a}} - \psi_{b,\mathbf{n}}U_{a,\mathbf{n}+\hat{\mathbf{e}}_\mathbf{b}})\Big], \tag{2.15}$$

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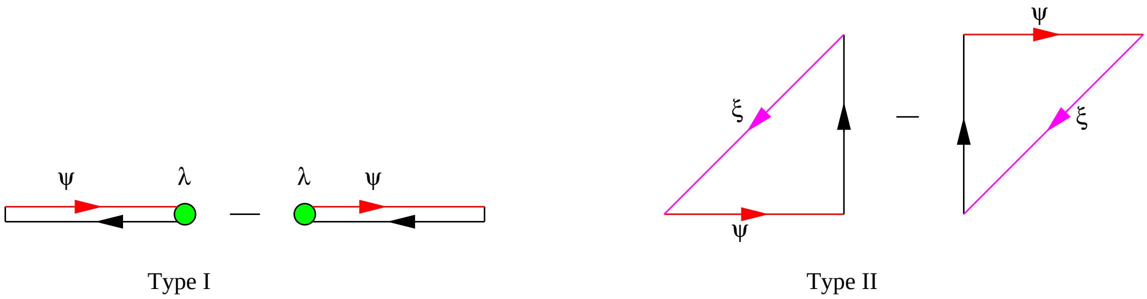

Figure 2: Representatives of type-I and type-II fermionic hopping terms. The black lines are unitary link fields. The green circle represents a site fermion $\lambda_{\mathbf{n}}$. Red lines are link fermions, $\psi_{a,\mathbf{n}}$. The diagonal of the triangular plaquette is the diagonal link fermion $\xi_{\mathbf{n}}$ starting at $\mathbf{n}+\hat{\mathbf{e}}_1+\hat{\mathbf{e}}_2$ and terminating at site $\mathbf{n}$.

These hopping terms can be classified in two types. The first type is the usual hopping term, which does not surround an area, and the second type makes a triangular plaquette. Both are manifestly gauge invariant. In the type-I case in figure 2, the unitary link fields properly parallel transport the link fermions $\psi_{a,\mathbf{n}}$ and $\psi_{a,\mathbf{n}-\hat{\mathbf{e}}_a}$ to the site $\mathbf{n}$, by a backward and forward parallel transport, respectively. The signed sum of these two terms couples to the site fermion $\lambda_{\mathbf{n}}$. In the type-II case, $\xi_{ab,\mathbf{n}}$ resides on the diagonal of the unit cell. The gauge invariant hopping term is a signed sum of two triangular plaquettes, whose two sides are fermionic and one side is the unitary link field.

Unlike the traditional hopping terms for adjoint fermions, which typically involves two parallel transports to make a gauge invariant hopping term, such as $\mathrm{Tr}(\chi_{\mathbf{n}} U_{\mathbf{n},a} \chi_{\mathbf{n}+\hat{\mathbf{e}}_a} U^{\dagger}_{\mathbf{n},a})$, our hopping terms only involve one unitary link field. The difference comes about because the fermions in our formulation are scattered to both sites and links, and it suffices to have a single unitary link field to build a gauge invariant hopping term. However, for example, with adjoint fermions, the fermions are all residing on the sites and the simplest gauge invariant object requires two parallel transports.

The fermion scalar interaction can easily be incorporated from eq. (2.5). We obtain

$$S_{f-s} = \frac{1}{g^2} \sum_{\mathbf{n}} \mathrm{tr} \Big[\lambda_{\mathbf{n}} (U^{\dagger}_{a,\mathbf{n}-\hat{\mathbf{e}}_a} h_{a,\mathbf{n}-\hat{\mathbf{e}}_a} \psi_{a,\mathbf{n}-\hat{\mathbf{e}}_a} - \psi_{a,\mathbf{n}} U^{\dagger}_{a,\mathbf{n}} h_{a,\mathbf{n}}) + \\ + \xi_{ab,\mathbf{n}} (h_{a,\mathbf{n}} U_{a,\mathbf{n}} \psi_{b,\mathbf{n}+\hat{\mathbf{e}}_\mathbf{a}} - \psi_{b,\mathbf{n}} h_{a,\mathbf{n}+\hat{\mathbf{e}}_\mathbf{b}} U_{a,\mathbf{n}+\hat{\mathbf{e}}_\mathbf{b}}) \Big] , \qquad (2.16)$$

The fermion-scalar interaction part of the action can be regarded as decoration of the fermion hopping terms with scalars. The unitary link fields are needed to make gauge invariant fermion-scalar interaction terms since most of the fermions are residing on the links.

3. Observables and global symmetries

In pure gauge theories, the observables are the the gauge invariant Wilson loops, constructed out of an ordered product of the unitary link fields along the loop. In supersymmetric lattice gauge theories (constructed within the approach of this work), the link fields

	$U_{1,\mathbf{n}}$	$U^\dagger_{1,\mathbf{n}}$	$U_{2,\mathbf{n}}$	$\overline{U}^\dagger_{2,\mathbf{n}}$	$h_{1,\mathbf{n}}$	$h_{2,\mathbf{n}}$	$\lambda_{\mathbf{n}}$	$\xi_{12,\mathbf{n}}$	$\psi_{1,\mathbf{n}}$	$\psi_{2,\mathbf{n}}$
r_1	$+1$	-1	0	0	0	0	0	-1	1	0
r_2	0	0	$+1$	-1	0	0	0	-1	0	1
Y	0	0	0	0	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

Table 3: The $\mathrm{U}(1)^3$ symmetry charges of the lattice theory. The first two are associated with the center symmetry of the two dimensional lattice gauge theory and the Y is R-symmetry.

can be both fermions and unitary matrices.(Scalar are placed at the sites by the polar decomposition eq. (2.7).) Thus, the natural generalization of the Wilson loops are the gauge invariant loops, constructed out of an ordered product of group valued unitary and algebra valued fermionic link fields, also possibly decorated with arbitrary insertions of site fermions and scalars along the loop [26]. Obviously, a subset of observables are the usual Wilson loops. Bosonic and fermionic loops solely composed of fermions are also possible, such as $\mathrm{Tr}[\psi_{1,\mathbf{n}}\psi_{2,\mathbf{n}+\hat{\mathbf{e}}_1}\xi_{12,\mathbf{n}}]$, a fermionic triangular plaquette. The correlation function of such loops are also among observables.

The questions about the nature of these loops, such as "how do we classify them", or "what do these loops corresponds to" can be partially be answered by analyzing the global symmetries. For example, in pure gauge theories Wilson loops are neutral under the center of the gauge group and Polyakov loops transform in the one dimensional representation of the center. For the $\mathrm{U}(N)$ lattice gauge theory in d dimensions, the action possesses a $\mathrm{U}(1)^d$ symmetry associated with the invariance of the action under $U_{a,\mathbf{n}} \to e^{i\alpha_b\delta_{ab}}U_{a,\mathbf{n}}$, an independent $\mathrm{U}(1)$ rotations in each direction.[5] A particular $\mathrm{U}(1)$ charge of a Polyakov loop can be characterized by the the number of winding of the loop in that direction.

The supersymmetric lattice action that we examined in section 2 has a $\mathrm{U}(1)^3$ symmetry. The charges under the global $\mathrm{U}(1)^3$ symmetry are listed in table 3. (This table has the same content as table 2, but it has been rewritten in terms of new variables for convenience.) The $\mathrm{U}(1)^2$ associated with (r_1, r_2) are associated with the square of the center symmetry of the gauge group $\mathrm{U}(N)$ in the two dimensional lattice. The last $\mathrm{U}(1)$ was clear from the outset; it is the global R-symmetry.

Let us now consider the observables. Assume the lattice is compact in all directions, a discretized torus. Let us first consider the loops which do not wind around the lattice. All the closed loops of this type are neutral under $\mathrm{U}(1)^2$, associated with center symmetry. This can be easily read of from the table 3. However, it is not hard to see that only a subclass of observables are neutral under $\mathrm{U}(1)_R$. This is the case if the total number of $\lambda_{\mathbf{n}}$, and $\xi_{12,\mathbf{n}}$ (whose R charges are $\frac{1}{2}$) is exactly same as the total number of $\psi_{1,\mathbf{n}}$, and $\psi_{2,\mathbf{n}}$ (whose R charges are $-\frac{1}{2}$). The observables with a nonvanishing R-charge and transforming homogeneously under $\mathrm{U}(1)_R$ rotation have zero expectation value. (As long as the $\mathrm{U}(1)_R$ is not spontaneously broken.) This type of observable can be used to probe the spontaneous breaking of $\mathrm{U}(1)_R$ symmetry. (see, for example, the review [27] and [28]

[5] More precisely, consider a gauge theory with a gauge group G on a d dimensional torus. Let $C(G)$ denote the center of the gauge group G. Then, the Polyakov loops are charged under $C(G)^d$. In particular, for $\mathrm{U}(N)$ gauge group, the center is $\mathrm{U}(1)$ and the Polyakov loops are charged under $\mathrm{U}(1)^d$.

about the spontaneous breaking of R-symmetry, Z_{2N}, in $\mathcal{N} = 1$ SU(N) SYM theory on the four dimensional lattice.) In particular, the expectation value of any observable with an odd number of fermions is zero, which is fermion number conservation modulo two.

There are also observables, the counterparts of the Polyakov loops, which are charged under the center $\mathrm{U}(1)^2$ and possibly under $\mathrm{U}(1)_R$. In general, an observable is associated with a charge triplet, $(n_1, n_2, \frac{n_R}{2})$ under $\mathrm{U}(1)^3$. The n_1 (n_2) is the net number of windings in direction one (two). However, there is subtlety here, which does not show up in the pure gauge theory. In pure gauge theory, a Polyakov loop in a direction can be undone by traversing the same loop in the opposite direction, since the constituents are unitary matrices. In our case, since some of the link fields are algebra valued fermions, the back-tracking (Polyakov) loops do not necessarily cancel. The integer n_R is the net R-charge of the corresponding loop. Obviously, an observable which is charged under any one of the global symmetries has vanishing expectation value as long as the global symmetry associated with the given charge is not spontaneously broken.

4. Prospects

The technique we have described here can be applied to all the euclidean lattice constructions [1 − 3] for extended supersymmetric gauge theories listed in table 1. It is also possible to work with spatial lattices, [4] which are more suitable for hamiltonian formulation of supersymmetric lattice gauge theories.

It would be useful to investigate the strong coupling expansion for the observables (loops and the connected correlators of loops) in the large-N_c limit, where N_c is the number of colors. Or equivalently, one can examine the lattice regularized loop equations for the supersymmetric lattice gauge theories by generalizing [26] to the theories with massless fermions. It would be interesting to examine to what extend the large N_c reduction holds in the supersymmetric gauge theories [29, 30] and understand their large-N_c phase diagram.

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