Physical (ir)relevance of ambiguities to Lorentz and CPT violation in QED

Received: *February 23, 2001*, Accepted: *April 26, 2001*

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Abstract: We discuss the regularization and renormalization of QED with Lorentz and CPT violation, and argue that the coefficient of the Chern-Simons term is an independent parameter not determined by gauge invariance. We also study these issues in a model with spontaneous breaking of Lorentz and CPT symmetries and find an explicit relation with the ABJ anomaly. This explains the observed convergence of the induced Chern-Simons term.

Keywords: Space-Time Symmetries, Discrete and Finite Symmetries.

JHEP04(2001)032

An effective description of the Standard Model incorporating possible Lorentz and CPT non-invariant terms has been developed by Colladay and Kostelecký [1]–[5] and by Coleman and Glashow [6, 7]. One particular Lorentz and CPT breaking term involves the photon and has the Chern-Simons form

$$\mathcal{L}^{\rm CS} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\kappa_\mu A_\nu F_{\rho\sigma}\,, \tag{1}$$

where κ_μ is a constant vector that produces a privileged direction in space-time. $\mathcal{L}^{\rm CS}$ is not gauge invariant but the corresponding term in the action is. This Chern-Simons term has been well studied in the literature [8]–[13] and it has been shown that a timelike κ_μ produces instabilities and has problems with causality or unitarity. A consistent quantization of the theory seems possible, nevertheless, for a spacelike κ_μ [13]. On the other hand, astrophysical observations put very stringent experimental limits on this parameter, indicating a vanishing κ_μ [14]. There has been some recent interest and controversy [15]–[30] about the possibility that this Chern-Simons term be induced by radiative corrections arising from a CPT and Lorentz violating term in the fermionic sector:

$$\mathcal{L}^b = b_\mu \bar\psi\gamma_5\gamma^\mu\psi\,, \tag{2}$$

with b_μ a constant vector. If this were the case, it seems that the strong bounds on κ_μ would translate into strong bounds for those non-invariant terms. The mentioned controversy arises from the fact that the calculations give an ambiguous finite radiative correction. Throughout this paper, "ambiguous" means regularization dependent. Some papers claim that a particular method gives the correct result [22] and others argue that gauge invariance requires a vanishing induced term [7, 26, 28]. The purpose of this letter is to try to clarify the situation and to discuss whether (and to what extent) a Lorentz and CPT violating fermion sector is excluded by the astrophysical limits on κ_μ. First we argue that the radiative correction to the Chern-Simons term is not fixed by gauge invariance but by normalization conditions. Then we study a simple generalization of this theory in which the induced term is directly connected to the ABJ anomaly [31, 32].

The correction to $\mathcal{L}^{\rm CS}$ in the effective action can be extracted from the zero momentum limit of the parity-odd part of the vacuum polarization:

$$(\Delta\kappa)_\mu = -\frac{1}{2}b_\mu K(0)\,, \tag{3}$$

where

$$\Pi^{\mu\nu}_{\rm odd}(p) = \epsilon^{\mu\nu\rho\sigma} b_\rho p_\sigma K(p)\,. \tag{4}$$

$\Pi^{\mu\nu}_{\rm odd}(p)$ is superficially linearly divergent by power counting. We assume that some well-defined regularization method is used. One degree of divergence is carried away by the external momentum, so $K(p)$ is at most logarithmically divergent. At one loop it turns out that the divergent part in the integrand is proportional to $(k_\mu k_\nu -$

$1/4\eta_{\mu\nu}k^2)/k^6$, where k is the loop momentum. Hence, the integral is finite but regularization dependent. The structure $x_\mu x_\nu/x^2$ that appears in non-perturbative methods [20, 21, 22, 24, 29] is its position space counterpart. It is clear then that the radiative correction $(\Delta\kappa)_\mu$ is regularization dependent and (at least to one loop) finite. The ambiguity has been described in [18] as resulting from the non invariance under chiral transformations of the measure in the path integral. We note that the papers that claim to obtain a unique (and "correct") result by the application of a particular method are implicitly using a regularization that fixes the ambiguity. It remains to see whether some symmetry (namely, gauge invariance) determines the value of $(\Delta\kappa)_\mu$. We would also like to understand why the correction is finite at one loop.

Gauge invariance is implemented at the quantum level by the Ward identities.[1] The relevant ones for our problem read

$$p_\mu \Pi^{\mu\nu}_{\rm odd}(p) = p_\nu \Pi^{\mu\nu}_{\rm odd}(p) = 0\,. \tag{5}$$

These identities are automatically fulfilled for any $K(p)$ due to the presence of the antisymmetric tensor. So it seems that gauge invariance has nothing to say here. This is actually what one would expect since the integrated $\mathcal{L}^{\rm CS}$ is gauge invariant and hence allowed. However, Coleman and Glashow argued that the combination of gauge invariance and analiticity forbids the correction $(\Delta\kappa)_\mu$ [7]. The argument (adapted to the present problem) goes as follows: consider the 1PI correlation function $\Gamma_{\mu\nu}(p,q)$ of two photons and one insertion of the interaction $\mathcal{L}^b$ and allow for the two photons to carry different momenta (p and q, respectively). The insertion must then carry momentum $p+q$. Differentiating the Ward identities analogous to (5) with respect to p with fixed q and using analiticity at $p=0$ one learns that $\Gamma_{\mu\nu}(p,q)$ has at least one factor of p. The same reasoning applied to q shows that it also has at least one factor of q. Thus $\Gamma_{\mu\nu}(p,q)$ is $O(pq)$ and in the limit $q\to p$ (i.e. the limit in which b_μ is a constant) we find that it is $O(p^2)$ and then $K(0)=0$. The assumption of analiticity requires that there are no massless charged fermions (as is the case in nature). Coleman and Glashow showed that the result also holds in the presence of internal photons in multiloop calculations. Furthermore, the same argument can be applied to the case in which there are more than one CPT violating insertions (in this case, the diagram is power-counting superficially convergent).

Even though this argument is perfectly correct, it rests on one implicit requirement: gauge invariance is imposed for non-constant b_μ, i.e. when b_μ is promoted to a field $B_\mu(x)$. It is no wonder that this prevents the induction of a Chern-Simons term proportional to b_μ, since $B\wedge A\wedge dA$ is not gauge invariant. This strong requirement

[1] Strictly, the Ward identities need only be imposed for the renormalized amplitudes. However, we separate the discussion of the regularized and renormalized theory in order to compare with the literature.

is not necessary in the original formulation of the theory, in which b_μ is a constant. There, the only relevant Ward identities are (14), which do not constrain the value of $(\Delta\kappa)_\mu$. Actually, the same argument at the classical level would imply $\kappa_\mu = 0$, but it is clear that $\kappa_\mu \neq 0$ is allowed by gauge invariance of the action [16]. Coleman and Glashow explicitly assumed gauge invariance at the lagrangian level, but their argument is nevertheless useful for our problem. Indeed, it shows that

- $(\Delta\kappa)_\mu = 0$ to all orders in e (the electromagnetic coupling) and b_μ in regularization methods that preserve gauge invariance for non-constant b_μ and in which $\Pi^\mu_{\rm odd}(p) = \lim_{q\to -p}\Gamma^{\mu\nu}(p,q)$. These include dimensional regularization (as shown explicitly in [26]) and Pauli-Villars.[2]

- The vacuum polarization in any other regularization procedure can only differ, at one loop, by finite local terms. Therefore $(\Delta\kappa)_\mu$ is one-loop finite in any regularization [26].

- The only possible non-vanishing contribution at one loop is linear in b_μ. Indeed, higher order contributions are power-counting finite. Since the Coleman-Glashow limit necesarily coincides with the constant b_μ result for these convergent diagrams, they must vanish.

The fact that the radiative correction to $\mathcal{L}^{\rm CS}$ — a renormalizable term allowed by the symmetries — is regularization dependent is not surprising. The only unusual phenomenon here is that it is finite (at least at one loop). The reason for this will become clearer below, when we discuss the relation of this term with the ABJ anomaly.

To understand the significance of an ambiguous finite correction[3] we need to go beyond simple regularization and discuss the renormalization of the theory. The renormalization of QED with the CPT-violating terms $\mathcal{L}^{\rm CS}$ and $\mathcal{L}^b$ has been studied in detail by Bonneau in [26]. However, Bonneau considers a modification of the theory with sources for the CPT-violating terms. As we have discussed, this is too restrictive and leads to Ward identities that, combined with analiticity, forbid the correction $(\Delta\kappa)_\mu$. We have argued that this is not necessarily so in the theory $\mathcal{L}^{\rm QED} + \mathcal{L}^{\rm CS} + \mathcal{L}^b$. To renormalize this theory we must add the counterterms allowed by the symmetries. In particular, we have

$$\mathcal{L}^{\rm CS}_R = \frac{1}{2}(\kappa_\mu + (\delta\kappa)_\mu)\epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma}\,, \tag{6}$$

[2]Note that there are methods preserving gauge invariance and giving a non-vanishing $(\Delta\kappa)_\mu$. One example is differential regularization, in which certain scales are fixed by hand in such a way that the Ward identities are satisfied [33]. Since one only needs to impose gauge invariance in the original theory, any value can be obtained [17]. The result in constrained differential renormalization [34, 35] is not fixed either [36].

[3]Other similar examples of finite ambiguous radiative corrections are discussed in [37]. An example of a finite high-energy model of Lorentz and CPT violation leading to a determined $(\Delta\kappa)_\mu$ was provided in [19].

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where κ_μ is now a renormalized parameter. The renormalized Ward identities have the form (5), with $\Pi^{\mu\nu}(p)$ the renormalized vacuum polarization. They do not fix the local part of $\Pi^{\mu\nu}_{\rm odd}(p)$ nor the coefficient of the Chern-Simons like term in the effective action, which reads

$$\kappa_\mu^{\rm eff} = (\kappa + \Delta\kappa + \delta\kappa)_\mu \,, \tag{7}$$

The counterterms are determined when a particular consistent regularization method is used — this fixes $(\Delta\kappa)_\mu$ — and adequate normalization conditions are imposed. We follow [26] and choose the on-shell condition

$$\begin{aligned} \kappa_\mu &= -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\left[\frac{\partial}{\partial p_\sigma}\langle A^\nu(p)A^\rho(-p)\rangle\right]_{p=0} \\ &= \kappa_\mu^{\rm eff} \,. \end{aligned} \tag{8}$$

The renormalized 1PI correlation function in this equation includes the tree level contribution. For any regularization scheme, the normalization condition (8) implies

$$(\delta\kappa)_\mu = -(\Delta\kappa)_\mu \,. \tag{9}$$

The counterterm is one-loop finite. The renormalized parameter κ_μ is directly related to an observable and can only be determined by the experiment. We see that any regularization method is physically equivalent. In practice, a method like dimensional regularization is convenient because it allows to set $(\delta\kappa)_\mu = 0$ to all orders [26]. In the language of multiplicative renormalization the parameter κ_μ is then renormalized like $(\kappa_0)_\mu = Z_A^{-1}\kappa_\mu$, where $(\kappa_0)_\mu$ is a bare coupling and $Z_A^{1/2}$ is the wave function renormalization of the photon.

We are now in a position to answer the initial question about possible restrictions on b_μ. The observations indicate a vanishing or extremely small $\kappa_\mu^{\rm eff}$. According to (8) we should set $\kappa_\mu \approx 0$. Since κ_μ is an independent parameter it can be set to zero for any value of b_μ (and any regularization). Therefore, from the point of view of perturbative renormalization theory we find no constraint on b_μ. To constrain b_μ we also need to measure $\langle A^\nu(p)A^\rho(-p)\rangle_{\rm odd}$ off shell. There is a problem of naturalness though: if CPT and Lorentz symmetries are broken, it is not explained why we should find an (almost) vanishing value for κ_μ. Even if the breaking of CPT and Lorentz is suppressed, one needs a large amount of fine-tuning to obtain values of κ_μ and b_μ that differ by many orders of magnitude. Of course, these conclusions do not depend on the normalization condition (8). To illustrate this well-known fact, suppose that, at some order in perturbation theory, $(\Delta\kappa)_\mu = \beta b_\mu = \textit{finite}$ in a given regularization and choose as normalization conditions minimal substraction, i.e. substract only the divergences. Then, $(\delta\kappa)_\mu = 0$ and $\kappa_\mu^{\rm eff} = \kappa_\mu + \beta b_\mu$. The experiment tells us that $\kappa_\mu = -\beta b_\mu$. Although the value of the renormalized parameter κ_μ is different, the physical predictions are unchanged. Again, we see that there is a fine-tuning.

It is possible that the fine-tuning is explained by a particular mechanism of symmetry breaking. Following Colladay and Kostelecký, we assume that CPT and Lorentz are spontaneously broken. This occurs when fields transforming non-trivially under these symmetries acquire non-vanishing vacuum expectation values (vevs). Sponteanous symmetry breaking has the advantage of preserving observer Lorentz invariance [1]–[5], [38]. We need to know more details about the high energy theory. The simplest situation is that both b_μ and κ_μ are generated as the vev of an axial vector field B_μ [38]. We can write

$$B_\mu(x) = B_\mu^{(1)}(x) + B_\mu^{(2)}(x) = \rho(x)\partial_\mu\sigma(x) + \partial_\mu\phi(x)\,. \tag{10}$$

We have argued above that gauge invariance forbids a term $B \wedge A \wedge dA$ in the effective action. Therefore, as long as $B_\mu(x)$ is treated as a single entity, no κ_μ can be generated. But one could also renormalize ϕ and $B^{(1)}$ independently. The term $d\phi \wedge A \wedge dA$ is gauge invariant up to a total derivative. Hence it is allowed in the classical action and radiative corrections can in principle contribute to it. Upon spontaneous symmetry breaking $\langle \partial_\mu\phi\rangle$ = constant, non-vanishing b_μ and κ_μ would be generated. Similarly, one can also consider a model in which a gauge-singlet real pseudoscalar ϕ (an axion) acquires a non-constant vev of this kind.[4] In the following we study the radiative corrections for this model and then take $\langle\phi\rangle \propto b \cdot x$. We can also regard this procedure as a gauge-invariant modification of the Coleman-Glashow argument: we promote b_μ to a non-constant field in such a way that an induced gauge-invariant Chern-Simons like term is allowed, and then take the limit of constant b_μ.

Both motivations lead us to study the Lorentz and CPT invariant effective theory

$$\mathcal{L}^\phi \supset \bar\psi\left(i\not{D} - m + \frac{\tilde b}{\Lambda}\gamma_5\ \not{\partial}\phi + ic\gamma_5\phi + \frac{d}{\Lambda}\phi^2\right)\psi - \tilde\kappa\frac{1}{2\Lambda}\phi F^{\mu\nu}\tilde F_{\mu\nu}\,, \tag{11}$$

where $\tilde F_{\mu\nu} = 1/2\,\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, Λ is some large scale and the last term is gauge invariant and equal to $\frac{\tilde\kappa}{2\Lambda}\epsilon^{\mu\nu\rho\sigma}\partial_\mu\phi A_\nu F_{\rho\sigma}$ up to a total derivative. The axion ϕ is a gauge singlet and it can be external or dynamical. We shall interpret this theory in two different ways: as a particular model leading to CPT breaking and as an intermediate step to study the original theory $\mathcal{L}^{\rm QED} + \mathcal{L}^{\rm CS} + \mathcal{L}^b$, which is recovered for $c = d = 0$ and $\langle\phi\rangle = \frac{\Lambda}{\tilde b} b \cdot x$. We shall work at order $1/\Lambda$ and, unless otherwise specified, we assume $m \neq 0$.

We are interested in the radiative correction to the coefficient of the last term in the effective action, which is given by

$$\Delta\tilde\kappa = -\frac{\Lambda}{2}C(0,0)\,, \tag{12}$$

[4] This mechanism was considered in detail in [11, 9], where it was shown that a spacelike non-zero $\langle\partial_\mu\phi\rangle$ can be dynamically generated by radiative corrections.

where

$$\Gamma^{\mu\nu}(p,q) = \epsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma C(p,q)\,. \tag{13}$$

Here, $\Gamma^{\mu\nu}(p,q)$ is the radiative correction to the 1PI correlation function of one axion and two photons, $\langle A^\mu(p) A^\nu(q)\phi(-p-q)\rangle$, and all momenta are incoming. The form (13) is dictated by Lorentz covariance. The Ward identities

$$p_\mu \Gamma^{\mu\nu}(p,q) = q_\nu \Gamma^{\mu\nu} = 0\,. \tag{14}$$

are automatically satisfied. Note that the application of Coleman-Glashow argument tells us that $\Gamma^{\mu\nu}(p,q) \sim O(pq)$, which in this case is obvious from the structure (13). The corresponding correction to the effective action can be nevertheless non vanishing because both momenta correspond to the derivatives in the interaction of the generalized Chern-Simons term.

From power counting and Lorentz covariance, the contribution to $\Gamma^{\mu\nu}$ linear in c is superficially convergent. A simple calculation gives

$$(\Delta\tilde{\kappa})_c = \frac{ce^2\Lambda}{8\pi^2 m} \tag{15}$$

at one loop. The contribution proportional to $\tilde{b}$ is at most (superficially) linearly divergent by power-counting, but again the degree of divergence is lowered. A formal one-loop calculation without any explicit regularization gives

$$\begin{aligned}
\left[\frac{\partial}{\partial p_\rho}\frac{\partial}{\partial q_\sigma}\Gamma_b^{\mu\nu}(p,q)\right]_{p=q=0} &= \left[\frac{\partial}{\partial p_\rho}\frac{\partial}{\partial q_\sigma}\left(2\frac{\tilde{b}}{\Lambda}e^2\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{tr}\left\{\gamma^\mu\frac{1}{\not{k}-m}\gamma^\nu\frac{1}{\not{k}-\not{q}-m}\times\right.\right.\right.\\
&\qquad\left.\left.\left.\times(\not{p}+\not{q})\gamma_5\frac{1}{\not{k}+\not{q}-m}\right\}\right)\right]_{p=q=0}\\
&= 16i\frac{\tilde{b}}{\Lambda}e^2\epsilon^{\mu\nu\rho\alpha}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{4k^\sigma k_\alpha-(k^2-m^2)\delta^\sigma_\alpha}{(k^2-m^2)^3}\\
&= -\frac{\tilde{b}e^2}{2\pi^2\Lambda}\epsilon^{\mu\nu\rho\sigma}\,.
\end{aligned} \tag{16}$$

The factor 2 in the second line corresponds to the contributions of the two possible momentum flows, which are identical after a shift in k_μ. In the last line we have used symmetric integration:

$$\int \mathrm{d}^4k k^\sigma k_\alpha f(k^2) = \frac{1}{4}\delta^\sigma_\alpha\int\mathrm{d}^4k k^2 f(k^2)\,. \tag{17}$$

Hence we find $(\Delta\tilde{\kappa})_{\tilde{b}} = \tilde{b}e^2/4\pi^2$. Taking $\phi(x) = \frac{\Lambda}{\tilde{b}} b\cdot x$ and $c=0$ we would arrive at $(\Delta\kappa)_\mu = e^2/4\pi^2\, b_\mu$. However, some of the identites we have used in (16) are not well defined for divergent integrals. A rigorous calculation requires the use of some regularization procedure and the result can differ from the one above. In particular, (17) does not hold in general. Moreover, depending on what regularization one uses, there may be additional contributions coming from the shift in k or from the Dirac algebra. We note in passing that if we add to the result above the contribution of the momentum shift, following [16], we obtain $(\Delta\kappa)_\mu = 3e^2/16\pi^2 b_\mu$. This is the result found in non-perturbative calculations in b_μ that respect the identity (17) [16, 20, 21].

In order to discuss the regularization dependence and the possible restrictions of gauge invariance it is useful to write the contribution linear in $\tilde{b}$ in terms of the chiral triangle:

$$\Gamma^{\mu\nu}_{\tilde{b}}(p,q) = \frac{\tilde{b}e^2}{\Lambda}(p_\lambda + q_\lambda)V^{\mu\nu\lambda}(p,q)\,. \tag{18}$$

$V^{\mu\nu\lambda}(p,q) = \langle J^\mu(p)J^\nu(q)J_5^\lambda(-p-q)\rangle$ is the correlator of one chiral current $J_5^\lambda = \bar{\psi}\gamma_5\gamma^\lambda\psi$ and two vector currents $J^\mu = \bar{\psi}\gamma^\mu\psi$. The theory at hand has an invariance under chiral transformations of the fermion fields (not acting on ϕ) which is broken explicitly by the fermion mass term and also by the ABJ anomaly. This leads to the anomalous Ward identity

$$(p_\lambda + q_\lambda)V^{\mu\nu\lambda}(p,q) = 2miV^{\mu\nu}(p,q) + \epsilon^{\mu\nu\rho\sigma}p_\rho q_\sigma\,\mathcal{A}\,, \tag{19}$$

where $V^{\mu\nu}(p,q) = \langle J^\mu(p)J^\nu(q)J_5(-p-q)\rangle$, with $J_5 = \bar{\psi}\gamma_5\psi$, and $\mathcal{A}$ is the anomaly, which is independent of p,q (it is a delta function in position space). $V^{\mu\nu}(p,q) = \epsilon^{\mu\nu\rho\sigma}p_\rho q_\sigma V(p,q)$ is finite, regulator independent and essentially equal to the contribution to $\Gamma^{\mu\nu}$ linear in c. $\mathcal{A}$ is also finite but it is regularization dependent. However, if the Ward identities for the vector currents,

$$p_\mu V^{\mu\nu\lambda}(p,q) = q_\nu V^{\mu\nu\lambda}(p,q) = 0\,, \tag{20}$$

are enforced, the value of the chiral anomaly is fixed to be $\mathcal{A} = \frac{1}{2\pi^2}$. From (13), (18) and (19) we find

$$C_{\tilde{b}}(p,q) = \frac{\tilde{b}e^2}{\Lambda}(2mV(p,q) + \mathcal{A}) \tag{21}$$

and from (12) and (15),

$$(\Delta\tilde{\kappa})_b = \tilde{b}e^2\left(\frac{1}{4\pi^2} - \frac{1}{2}\mathcal{A}\right). \tag{22}$$

Taking $\langle\phi\rangle = \frac{\Lambda}{\tilde{b}}b\cdot x$ and $c=0$ we obtain

$$(\Delta\kappa)_\mu = e^2 b_\mu\left(\frac{1}{4\pi^2} - \frac{1}{2}\mathcal{A}\right). \tag{23}$$

If the equations (20) are satisfied we find $\Delta\tilde{\kappa} = 0$. This can be understood in the following way [39]. Identity (19) and analiticity in p and q imply that

$$V^{\mu\nu\lambda}(p,q) = \frac{1}{2}(2mV(0,0) + \mathcal{A})\epsilon^{\mu\nu\lambda\rho}(p_\rho - q_\rho)(1 + O(p) + O(q))\,. \tag{24}$$

The identities (20) then require $2mV(0,0) + \mathcal{A} = 0$. In many applications, the vector

Ward identities (20) correspond to gauge invariance and must be preserved. One might then think that gauge invariance determines $\Delta\tilde{\kappa} = 0$ and, for the original problem, $(\Delta\kappa)_\mu = 0$. This is not right because in this case gauge invariance does not imply (20) but only

$$p_\mu(p_\lambda + q_\lambda)V^{\mu\nu\lambda}(p,q) = q_\nu(p_\lambda + q_\lambda)V^{\mu\nu\lambda}(p,q) = 0\,. \tag{25}$$

Of course, the identities (25) are nothing but the Ward identities (14), and are satisfied for any value of $\mathcal{A}$. Since in our model the photon does not couple to any chiral current, there is no fundamental reason for imposing (20) in this case. Therefore $(\Delta\kappa)_\mu$ is also regularization dependent in our method. Equation (23) shows explicitly the relation between the induced Chern-Simons term and the chiral anomaly in perturbation theory, and explains several facts observed in the literature:

- The radiative correction is finite to one-loop because it is the sum of a finite amplitude and the chiral anomaly. The anomaly accounts for the ambiguity.

- The induced term vanishes at one loop in regularization methods that respect the Ward identities (14), like dimensional regularization or Pauli-Villars. We stress again that this does not mean that gauge invariance requires a vanishing correction. In order to extend this result to higher loops one would need to show that analiticity is not lost, as in the proof of Coleman-Glashow theorem.

- $(\Delta\kappa)_\mu$, though independent of m for $m \neq 0$, is discontinuos at $m = 0$ (in perturbation theory about $b_\mu = 0$). Indeed, in the massless case $V(p,q) = 0$ for dimensional reasons (and Lorentz covariance), so the first term of (23) is absent and the radiative correction is purely local. The difference between the massive and massless case for any fixed regularization is

$$(\Delta\kappa)^m_\mu - (\Delta\kappa)^0_\mu = \frac{e^2}{4\pi^2} b_\mu\,, \tag{26}$$

  which agrees with the calculation in [20]. (There it was shown that the exact result for a spacelike b_μ is continuous in m at $m = 0$; the equation above remains valid if $|b^2| < m^2$.) In perturbation theory the origin of the discontinuity is a potential infrared divergence (that also cancels out) when $m = 0$ and $p = q = 0$. Note that for $m = 0$ dimensional regularization gives a non-vanishing result. This does not contradict our argument above nor Coleman-Glashow theorem: the assumption of analiticity is invalidated by the presence of massless fermions.

- Higher orders in b_μ would arise from correlation functions of two photons and more than one axion. The generalization of (18) for these correlators can be studied in a similar way. The corresponding diagrams are superficially convergent and analiticity in the external momenta can be used to show that higher orders in b_μ do not contribute. In the massless case this is straightforward by dimensional analysis.

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The same connection with the anomaly was established in the path integral formalism in [18]. That method can also be employed in the axion model: one can eliminate the term proportional to $\tilde{b}$ by a local chiral transformation of the fermion fields with the parameter of the transformation proportional to the axion. This redefines c, d and $\tilde{\kappa}$. The redefinition of $\tilde{\kappa}$, arising from the Fujikawa jacobian, obviously corresponds to the anomaly in (19), whereas the radiative correction induced by the new part of c accounts for the first term on the r.h.s. of (19).

Let us consider now the renormalization of the theory (11). It is important to note that we are in a decoupling scenario [40, 41], so in perturbation theory there are two expansion parameters: e and Λ. We want renormalization to preserve the expansion in Λ. The radiative correction has the form

$$\frac{\Delta\tilde{\kappa}}{\Lambda} = cO(e^2) + \tilde{b}O(e^2)\frac{1}{\Lambda} + O\left(e^2\frac{1}{\Lambda^2}\right). \tag{27}$$

While the correction $\tilde{b}O(e^2)$ can be absorbed by a counterterm $\delta\tilde{\kappa}$, no counterterm with the adequate power in Λ can absorb the correction linear in c. Hence, there is a definite finite contribution given, at order ce^2, by (15). This is analogous to the case of $g-2$ in QED, which is not an independent parameter but a prediction of the theory (non-renormalizable operators describing new physics can only give small corrections to the QED value). With appropiate normalization conditions,[5]

$$\frac{\tilde{\kappa}^{\rm eff}}{\Lambda} = \frac{ce^2}{8\pi^2 m} + \frac{\tilde{\kappa}}{\Lambda} + \text{higher orders}\,. \tag{28}$$

If $\partial_\mu\phi$ gets a vev we recover the original theory $\mathcal{L}^{\rm QED} + \mathcal{L}^{\rm CS} + \mathcal{L}^{b}$ plus extra terms proportional to c and d and those describing the excitations about this vacuum. (28) translates into an analogous equation for $\kappa_\mu^{\rm eff}$. In particular, for non-zero c, a Chern-Simons term not suppressed by Λ is induced. The strong limits on $\kappa_\mu^{\rm eff}$ then imply strong limits on c. In fact, not only Lorentz and CPT but also translational invariance is broken in the fermion sector unless $c = d = 0$. We would have $c = d = 0$ automatically if the axion were a Goldstone boson of some spontaneously broken symmetry, as in the axion solution of the strong CP problem [42, 43, 44]. On the other hand the pure photon sector puts no bounds on $\tilde{b}$, but $b_\mu \gg \kappa_\mu$ requires a fine-tuning of the high-energy parameters, $\tilde{b} \gg \tilde{\kappa}$.

Finally, let us consider again the model in which b_μ is the vev of a vector boson and use the decomposition (10). We have just studied the behaviour of $B^{(2)}$. On the other hand, the term $B^{(1)}\wedge A\wedge dA$ in the effective action is forbidden by gauge invariance. If $\langle\phi\rangle = 0$ but $B^{(1)}$ gets a non-vanishing (small) vev, we obtain a theory with $\kappa_\mu = 0$ to all orders and $b_\mu \neq 0$. This would account for a possible sizable CPT and

[5]The normalization condition for $\tilde{\kappa}$ should be imposed for the contribution of order $1/\Lambda$ to the corresponding correlation function.

Lorentz violation in the fermion sector compatible with the limits from the absence of birefringence of the light. Observe that we have not used anomaly cancelation in the fundamental theory as in [1] (although B_μ could be in particular an extra gauge field). We will not try here to propose a realistic field-theoretical model of CPT and Lorentz violation. As a matter of fact, stability and causality at all scales seem to require non-local physics [38]. We should also mention here an interesting alternative to spontaneous Lorentz and CPT breaking. In [45], Klinkhamer has shown that quantum effects can generate a non-zero $(\Delta\kappa)_\mu$ in a classically Lorentz and CPT invariant chiral gauge theory when at least one dimension is compact. This CPT anomaly arises when a non-perturbative gauge anomaly is cancelled, similarly to the case of the parity anomaly in certain three-dimensional gauge theories. In this scenario the size of the Chern-Simons term is a definite prediction of the theory, fixed by gauge invariance and the classical Lorentz and CPT symmetries. Furthermore, this term is naturaly very small, as it is inversely proportional to the size of the universe.

Let us summarize our conclusions. $\mathcal{L}^{\rm CS}$ is a local renormalizable term allowed by the symmetries of the theory once Lorentz and CPT have been broken. Therefore its coefficient in the effective action is an independent parameter and can be determined only by comparison with the experiment. At this level the observed smallness of this coefficient is compatible with a larger CPT violation in the fermion sector. This entails, however, a fine-tuning that can be explained by particular field theory models. Using the theory (11) we have also obtained a relation between the induced CPT violating Chern-Simons term and the ABJ anomaly which accounts for the (one-loop) convergence of the radiative corrections.

Acknowledgments

It is a pleasure to thank R. Jackiw for enlightening discussions. I thank MECD for financial support.

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