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Stringy corrections to the Wilson loop in $\mathcal{N}=4$ super Yang-Mills theory

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Abstract: We study stringy fluctuations as a source for corrections to the Wilson loop as obtained from the superstring on $AdS_5 \times S^5/\mathcal{N}=4$ SYM correspondence. We give a formal expression in terms of determinants of two-dimensional operators for the leading order correction.

Keywords: Sigma Models, Strong Coupling Expansion, Superstrings and Heterotic Strings, Brane Dynamics in Gauge Theories.

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1. Introduction

The remarkable duality between type IIB string theory on $AdS_5 \times \mathcal{M}_5$, ($\mathcal{M}_5$ a compact manifold), and conformally invariant supersymmetric Yang-Mills theory at the boundary of the anti-de Sitter space, proposed in [1] (and elaborated in [2, 3]), provides a new approach to the large N limit of four-dimensional gauge theories. Many aspects of this non-trivial relation have been and indeed still are, the subject of numerous investigations. An example, that will be further studied in this letter, is the computation of the potential between a heavy quark-antiquark pair [4, 5]. In gauge theory this is conveniently done by evaluating the expectation value of the Wilson loop operator. In the AdS/CFT scheme, one considers a string worldsheet with boundary fixed at the loop one is interested in. The (exponential of the) type IIB string action with this boundary condition is then the expectation value of the Wilson loop operator.

In refs. [4, 5], the authors considered the case $\mathcal{M}_5 = S^5$ leading to the maximally supersymmetric $\mathcal{N} = 4$ Yang-Mills theory. The expectation value of the Wilson loop was calculated to the lowest order by evaluating the area of the string worldsheet. A macroscopic string is stretched between the quark and antiquark at the boundary of the anti-de Sitter space. The non-trivial metric of this space means that it is energetically favourable for the string to fall in the interior of the AdS space along a geodesic connecting the two points on the boundary. The string action is the area of the worldsheet in the induced metric.

The full superstring action in the background $AdS_5 \times S^5$ has meanwhile been constructed in [6], and further considered in [7]. This enables one to address the question of stringy corrections to the classical area of the worldsheet. In this letter we will consider the sigma model corrections at one loop by expanding the string action to quadratic order in fluctuation. We first recapitulate briefly the set up and calculation in [4, 5]. We then start with the superstring action in [6] and expand it around the classical background using the normal coordinates [8]. The issue of gauge fixing to actually evaluate the determinant is discussed next. We finally give the result in terms of determinants of two-dimensional second order differential operators and discuss the issue of UV divergences.

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2. Wilson loop in AdS/CFT correspondence

Let us consider a static configuration of a quark-antiquark pair separated by a distance L. The Wilson loop is a rectangular one the sides of which are parallel to the time and one of the space directions. The length of the temporal side T is taken to infinity. In the dual string description there is a macroscopic string with endpoints fixed on the quarks by Dirichlet boundary condition. If, for simplicity we assume that the string ends are at the same point on S^5, the minimum energy configuration is that for which the string is stretched along a geodesic in AdS_5. This follows from the fact that the classical string action (with the overall factor α' set to 1) is

$$S_{NG} = \frac{1}{2\pi} \int d^2\sigma \sqrt{-\det ||G_{MN} \partial_i x^M \partial_j x^N||}\,, \tag{2.1}$$

where the G_{MN} is the metric on $AdS_5 \times S^5$ given by

$$ds^2 = R^2 \left[u^2 \left(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right) + \frac{du^2}{u^2} + d\Omega_5^2 \right]. \tag{2.2}$$

The spaces AdS_5 and S^5 have the same value of radius $(4\pi g N)^{1/4}$. Note that we have rescaled $u \to R^2 u$ compared to [4] to make the sigma model loop counting parameter R^2/α' manifest.

It is convenient to adopt a 'static gauge' to describe the macroscopic string. Let X^M denote the classical values of the string coordinates. We set $X^0 = \tau$, $X^1 = \sigma$ and assume that all the other coordinates except $U = U(\sigma)$ are independent of the worldsheet coordinates (τ, σ). The radial coordinate U has a nontrivial dependence on σ only, which is implicitly given by [4]

$$\partial_\sigma U = \pm \frac{U^2}{U_0^2} \sqrt{U^4 - U_0^4}\,, \tag{2.3}$$

with a constant $U_0 = (2\pi)^{3/2}/\Gamma(1/4)^2 L$. ($U_0$ is the closest the string comes to the origin of the AdS_5 space, and once again we have rescaled it by R^2.) Due to the symmetry of the string configuration it will be sufficient to restrict to the range $0 \leq \sigma \leq L/2$ and take the postive root in (2.3).

With this choice of classical solution, the metric induced on the worldsheet h^{cl}_{ij} is given by

$$\frac{ds^2_{cl}}{R^2} = -U^2\, d\tau^2 + \frac{U^6}{U_0^4}\, d\sigma^2\,. \tag{2.4}$$

The energy of the quark pair is calculated by evaluating the area A_R of the string worldsheet using this induced metric, and is (after subtracting an infinite contribution of the quark mass [4])

$$E = -\frac{4\pi^2\sqrt{2g^2_{YM}N}}{\Gamma(1/4)^4 L}\,, \tag{2.5}$$

that is of the form of Coulomb law. The $1/L$ dependence of the energy is a consequence of conformal invariance, but the $g^2_{YM}N = gN$, is non-perturbative from the point of view of the gauge theory.

How is this result calculated in classical string theory, corrected? One possible source of correction from the change in geometry has been ruled out by a number of authors [9, 6, 7] who argued that the $AdS_5 \times S^5$ is an exact string background.[1] However, corrections can arise from taking into account the fluctuation of the string around the given classical configuration. For this we will need to start with the type IIB superstring action in the $AdS_5 \times S^5$ background. Let us briefly sketch our approach before we go into the details.

We are going to replace the classical saddle-point approximation of [4] by a functional integral over the fluctuations

$$W(C) = \int [\mathcal{D}\delta X][\mathcal{D}\delta\theta]\; e^{-S_{IIB}(X+\delta X,\delta\theta)}, \tag{2.6}$$

where $\delta X, \delta\theta$ denote quantum fluctuations of the bosonic and fermionic coordinates. It was observed in [6] is that the string action on $AdS_5 \times S^5$ with radii $\mp R$ is R^2 times the action on $AdS_5 \times S^5$ with unit radii. Therefore as expected $1/R^2$ plays the role of a loop expansion parameter. The superstring action of [6] in this background is a Green-Schwarz type action and is invariant under worldsheet diffeomorphism and a local fermionic kappa symmetry. Further there is no supersymmetry on the worldsheet. However we will find that, much as in the case of flat space, a gauge fixing condition for kappa symmetry makes worldsheet fermions out of the fermionic coordinates of the target space.

[1] This is no longer true for gauge theory at finite temperature [10, 11].

3. Superstring action in $AdS_5 \times S^5$ background

In ref. [6], Metsaev and Tseytlin have constructed an action for the IIB theory in the $AdS_5 \times S^5$ background. The action is defined as a covariant κ-symmetric two-dimensional sigma model on a supercoset appropriate for this background. Explicitly, the part of the lagrangean relevant to us here is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\sqrt{-h}h^{ij}\left(e_i^{\hat{a}} - i\bar{\theta}^I\hat{\gamma}^{\hat{a}}(D_i\theta)^I\right)\left(e_j^{\hat{a}} - i\bar{\theta}^J\hat{\gamma}^{\hat{a}}(D_j\theta)^J\right) - \\ &-i\epsilon^{ij}e_i^{\hat{a}}\left(\bar{\theta}^1\gamma^{\hat{a}}(D_j\theta)^1 - \bar{\theta}^2\gamma^{\hat{a}}(D_j\theta)^2\right), \qquad (3.1)\end{aligned}$$

where the notation used is same as in [6]. In particular h_{ij} is a two-dimensional metric, $e_i^{\hat{a}} = \left(e_\mu^a\partial_i x^\mu, e_{\mu'}^{a'}\partial_i y^{\mu'}\right)$ is defined in terms of vielbeins of AdS_5 and S^5, $\hat{\gamma}^{\hat{a}} = \left(\gamma^a, i\gamma^{a'}\right)$ satisfy the $SO(4,1)$ and $SO(5)$ Clifford algebras respectively and $\theta^I, I = 1,2$ label the two sets of spinors of these. The two-dimensional indices i,j run over 0 and 1, $(a,a') = (0,\ldots,4;5,\ldots 9)$ are (flat) tangent space indices for $AdS_5 \times S^5$, and similarly for the curved indices (μ,μ').

In order to compute the one loop contribution we will expand (3.1) to second order in the fluctuations around the classical solution of [4]. In this background the metric on the worldsheet h_{ij} is the one induced from the target space (2.4), and we will fix h_{ij} to this value (which we will call h_{ij}^{cl}) by exploiting worldsheet diffeomorphism. Notice that this differs from the standard practice of working with a flat (or conformally flat) worldsheet metric.

The classical solution is non-trivial only in the bosonic part along the AdS_5 directions. Consequently fluctuations in bosonic degrees of freedom along the AdS_5 and S^5 space, and fluctuations in the fermionic variables all decouple (to second order). Therefore one can study these independently and add up their contributions.

4. Fluctuations of the bosonic coordinates

4.1 The AdS_5 part

The metric in the AdS_5 space is

$$ds^2_{AdS_5} = u^2 dx^a dx^b \eta_{ab} + \frac{du^2}{u^2}. \qquad (4.1)$$

In the above $a,b = 0,\ldots,3$; however, in the following the label $a = 4$ will refer to the 'radial' coordinate u, and its quantum fluctuations. The classical solution has been reviewed in section 2. We now expand the action in terms of the normal coordinates using standard technology[8]. To quadratic order this leads to

$$\begin{aligned}\mathcal{L}_{AdS_5} = -\sqrt{-h_{cl}}\Big[&1 + \frac{1}{2}h_{cl}^{ij}\eta_{ab}D_i\xi^a D_j\xi^b + \eta_{ab}\xi^a\xi^b - \\ &- \frac{1}{2}h_{cl}^{ij}G_{\mu\lambda}G_{\nu\rho}(\partial_i X^\mu)(\partial_j X^\rho)E_a^\lambda E_b^\nu\xi^a\xi^b\Big], \qquad (4.2)\end{aligned}$$

where, $D_i\xi^a = \partial_i\xi^a + (\partial_i X^\mu)\omega^a_{\mu b}\xi^b$. In the above we have used the expression of the Riemann tensor in terms of the metric $R_{\mu\nu\rho\lambda} = -\left(G_{\mu\rho}G_{\nu\lambda} - G_{\mu\lambda}G_{\nu\rho}\right)$, in AdS space.

However eqn (4.2) is not the end of the story, as the variation of the metric h^{ij}_{cl} lead to nontrivial constraints

$$\mathcal{C}_{ij} \equiv E_{ij}(\xi) - \frac{1}{2}h^{cl}_{ij}\left(h^{kl}_{cl}E_{kl}(\xi)\right) \approx 0\,, \tag{4.3}$$

where $E_{ij}(\xi) = G_{\mu\nu}(x_{cl})\left(\partial_i x^\mu_{cl}E^\nu_a D_j\xi^a + \partial_j x^\nu_{cl}E^\mu_a D_i\xi^a\right)$. These constraints need to be taken into account. This can be done by following standard procedure. However it turns out to be easier to calculate the full induced metric (including fluctuations), which can then be shown to be equal to the sum of (4.2) plus Lagrange multipliers times the constraints (4.3).

In order to write the final form of the AdS_5 part of the lagrangean, let us introduce the following linear combinations

$$\begin{aligned}
\xi^{\parallel} &= \frac{U_0^2}{U^2}\xi^1 + \frac{\sqrt{U^4 - U_0^4}}{U^2}\xi^4 \\
\xi^{\perp} &= -\frac{\sqrt{U^4 - U_0^4}}{U^2}\xi^1 + \frac{U_0^2}{U^2}\xi^4\,.
\end{aligned} \tag{4.4}$$

These are the (normalized) fluctuations along the direction parallel (respectively perpendicular) to the classical string configuration. (That these indeed parametrize the fluctuations parallel and perpendicular to the string background is most evident in terms of the normal coordinates ξ^μ with curved indices.) In terms of these variables the quadratic part of the lagrangean takes the following simple form

$$\begin{aligned}
\mathcal{L}^{(2)}_{AdS_5} = -\frac{1}{2}\sqrt{-h_{cl}}\Big[& h^{ij}_{cl}\left(\partial_i\xi^\perp\partial_j\xi^\perp + \partial_i\xi^2\partial_j\xi^2 + \partial_i\xi^3\partial_j\xi^3\right) + \\
& + 2\left(1 - \frac{U_0^4}{U^4}\right)\left(\xi^\perp\right)^2 + 2\left(\xi^2\right)^2 + 2\left(\xi^3\right)^2\Big]\,.
\end{aligned} \tag{4.5}$$

In writing (4.5), we have ignored some total derivative terms. One observes that the fluctuations ξ^0 and $\xi^\parallel$ along the worldsheet have dropped out of the action. This is a consequence of the worldsheet diffeomorphism, which is completely fixed if we eliminate two redundant degrees of freedom by choosing

$$\xi^0 = \xi^\parallel = 0\,. \tag{4.6}$$

This gauge choice is analogous to the non-covariant light-cone gauge, and is consistent with the static gauge employed to write the classical solution.

Finally we notice that the covariant laplacean of the induced metric (2.4) with its canonical connection, $\Delta_{cl} = \frac{1}{\sqrt{h_{cl}}}\partial_i\left(\sqrt{h_{cl}}h^{ij}_{cl}\partial_j\right)$, appears in (4.5). For future use, let us rewrite (4.5) as

$$\mathcal{L}^{(2)}_{AdS_5} = \frac{1}{2}\sqrt{h_{cl}}\left[\sum_{a=2,3,\perp}\xi^a\Delta_{cl}\xi^a - 2\left(\xi^2\right)^2 - 2\left(\xi^3\right)^2 + (R^{(2)} - 4)\left(\xi^\perp\right)^2\right], \tag{4.7}$$

where, $R^{(2)} = 2(U^4 + U_0^4)/U^4$ is the scalar curvature of the two-dimensional induced metric. The fluctuations ξ^2 and ξ^3 in the transverse directions are seen to be massive, while $\xi^\perp$ moves in a potential. In addition, as $U \to \infty$ the fluctuations must be required to vanish as that is where the heavy quarks sit.

4.2 The S^5 part

We have assumed for simplicity a trivial background for the S^5 part. Both the quarks are at the same point on S^5, and this classical position is independent of the worldsheet coordinates (τ, σ). Let $\eta^{a'}$, $a' = 5, \ldots, 9$ be the normal coordinates denoting the quantum fluctuations on the sphere. These variables behave like the $\xi^{2,3}$ fluctuations in the AdS_5 space. The lagrangean relevant for the S^5 part is

$$\mathcal{L}^{(2)}_{S^5} = -\frac{1}{2}\sqrt{-h_{cl}}\; h^{ij}_{cl}\partial_i\eta^{a'}\partial_j\eta^{a'} = \frac{1}{2}\sqrt{h_{cl}}\;\eta^{a'}\Delta_{cl}\eta^{a'} \,. \tag{4.8}$$

The fluctuations are massless and the the second order operator is just the laplacean of the induced metric.

5. Fluctuations of the fermionic coordinates

Let us start by recalling the covariant derivative $(D_j\theta)^I$ appearing in (3.1):

$$\begin{aligned}(D_j\theta)^I = D^{IJ}_j\theta^J &= \left[\delta^{IJ}\left(\partial_j + \frac{1}{4}(\partial_j x^\mu_{cl})\omega^{ab}_\mu\gamma^{ab}\right) - \frac{i}{2}\epsilon^{IJ}(\partial_j x^\mu_{cl})e^a_\mu\gamma^a\right]\theta^J \\ &= \mathcal{D}_j\theta^I - \frac{i}{2}\epsilon^{IJ}(\partial_j x^\mu_{cl})e^a_\mu\gamma^a\theta^J \,. \end{aligned} \tag{5.1}$$

In [6] there are additional terms in the above definition, but in our context those vanish. The first term $\mathcal{D}_j\theta^I = \partial_j\theta^I + \frac{1}{4}(\partial_j x^\mu_{cl})\omega^{bc}_\mu\gamma^{bc}\theta^I$, is the standard covariant derivative on the fermions. The additional second term appears due to the non-trivial coupling to the RR 5-form field strength. Substituting above in (3.1), and using the properties of gamma matrices and the fermions given in [6], the fermionic part of the action in the background of the macroscopic string is compactly written as

$$\mathcal{L}_F = -\sqrt{-h_{cl}}\,(\,\bar\theta^1 \quad \bar\theta^2\,)\begin{pmatrix} 2ie^a_\mu(\partial_i x_{cl})^\mu\gamma^a\mathcal{P}^{ij}_-\mathcal{D}_j & 1-\mathcal{B} \\ -1-\mathcal{B} & 2ie^a_\mu(\partial_i x_{cl})^\mu\gamma^a\mathcal{P}^{ij}_+\mathcal{D}_j \end{pmatrix}\begin{pmatrix}\theta^1 \\ \theta^2\end{pmatrix}, \tag{5.2}$$

where $\mathcal{B} = \frac{1}{2\sqrt{-h_{cl}}}\epsilon^{ij}e^a_\mu e^b_\nu(\partial_i X^\mu)(\partial_j X^\nu)\gamma^{ab}$, and $\mathcal{P}^{ij}_\pm = \frac{1}{2}\left(h^{ij}_{cl} \pm \epsilon^{ij}/\sqrt{-h_{cl}}\right)$ are projection operators similar to the ones in flat space [12].

Let us define the following combination of gamma matrices

$$\begin{aligned}\gamma^\parallel &= \frac{U_0^2}{U^2}\gamma^1 + \frac{\sqrt{U^4 - U_0^4}}{U^2}\gamma^4 \,, \\ \gamma^\perp &= -\frac{\sqrt{U^4 - U_0^4}}{U^2}\gamma^1 + \frac{U_0^2}{U^2}\gamma^4 \,, \end{aligned} \tag{5.3}$$

in analogy with (4.4), and let $\gamma^{\pm} = \frac{1}{2}\left(\gamma^0 \pm \gamma^{\parallel}\right)$. In the background of the classical solution, the diagonal terms are simply $i\gamma^{\pm}\left(\frac{1}{U}\mathcal{D}_\tau \pm \frac{U_0^2}{U^3}\mathcal{D}_\sigma\right)$, and $\bar\theta^1\mathcal{B}\theta^2 = \bar\theta^2\mathcal{B}\theta^1 = -(2U^4/U_0^4)\bar\theta^1\gamma^{0\parallel}\theta^2$.

Now recall that the action (3.1) or (5.2) has a local fermionic gauge symmetry, the so called κ-symmetry, which has to be fixed so as to remove the redundant fermionic degrees of freedom. It turns out that a most convenient choice is to set

$$\gamma^-\theta^1 = 0\,, \qquad \gamma^+\theta^2 = 0\,. \tag{5.4}$$

With this choice, the lagrangean (5.2) simplifies to

$$\mathcal{L}_F = -\sqrt{-h_{cl}}\,(\,\bar\theta^1 \quad \bar\theta^2\,)\begin{pmatrix} i\gamma^+\bar D_+ & 2 \\ -2 & i\gamma^-\bar D_- \end{pmatrix}\begin{pmatrix}\theta^1 \\ \theta^2\end{pmatrix}, \tag{5.5}$$

where we have defined the two-dimensional covariant derivative with tangent space indices

$$\bar D_\pm = \frac{1}{U}\mathcal{D}_\tau \pm \frac{U_0^2}{U^3}\mathcal{D}_\sigma = \varepsilon_0^\tau D_\tau \pm \varepsilon_1^\sigma D_\sigma\,, \tag{5.6}$$

ε_0^τ and ε_1^σ being a set of two-dimensional (inverse) vielbeine of the classical induced metric (2.4).

The equations of motion that follow from this lagrangean are

$$\begin{aligned}
\gamma^+\left(\partial_+ + \frac{\sqrt{U^4-U_0^4}}{2U^2}\right)\theta^1 + \theta^2 &= 0\,,\\
\gamma^-\left(\partial_- - \frac{\sqrt{U^4-U_0^4}}{2U^2}\right)\theta^2 - \theta^1 &= 0\,,
\end{aligned} \tag{5.7}$$

where the derivatives are with respect to tangent space indices on the worldsheet, and their definitions are similar to (5.6) above. However the above form is somewhat deceptive as $\gamma^{\pm}$ depend on σ, and hence are not covariantly constant. This situation is remedied by exploiting the κ-symmetry fixing condition (5.4). After some straightforward manipulations, one arrives at

$$\begin{aligned}
i\gamma^0\nabla_+\theta^1 + \theta^2 &\equiv i\gamma^0\left(\partial_+ + \frac{\omega}{2} + A\right)\theta^1 + \theta^2 = 0\\
i\gamma^0\nabla_-\theta^2 - \theta^1 &\equiv i\gamma^0\left(\partial_- - \frac{\omega}{2} - A\right)\theta^2 - \theta^1 = 0\,,
\end{aligned} \tag{5.8}$$

where $\omega = \varepsilon_0^\tau\omega_\tau^{01}$ is the contribution from the spin connection and $A = \frac{U_0^2}{U^2}\gamma^{14}$ is an additional gauge connection.

Now with the help of the following definition for two-dimensional gamma-matrices

$$\rho^+ = \begin{pmatrix} 0 & 0 \\ \gamma^0 & 0\end{pmatrix}, \qquad \rho^- = \begin{pmatrix} 0 & \gamma^0 \\ 0 & 0\end{pmatrix} \tag{5.9}$$

the equations of motion are compactly expressed as

$$\left(i\rho^j \nabla_j + \rho^3\right) \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix} = 0\,, \tag{5.10}$$

with ρ^3 =diag($\mathbf{1}$,$-\mathbf{1}$), and $\theta = (\,\theta^1\theta^2\,)$ is a 'two component' spinor of the two-dimensional worldsheet. This is the AdS analogue of the well-known 'metamorphosis' of target-space spinors into world-sheet spinors [12].

Coming back to (5.8) now, we see that θ^1 (say) is completely determined in terms of θ^2, which should be treated as independent fermionic fields. Since each θ had, to start with, 16 components and the gauge condition fixing κ-symmetry (5.4) reduces these by half, the fermions have altogether eight on-shell degrees of freedom. This of course matches with those of the bosonic fields.

The coupled set of first order equations (5.8) can be traded for the second order equation

$$\left(-\Delta_F - \frac{1}{4}R^{(2)} + 1\right)\theta^2 = 0\,, \tag{5.11}$$

for θ^2 alone, and similarly for θ^1. In the above, the 4×4 matrix operator $\Delta_F = h^{ij}\nabla_i\nabla_j$ is the laplacean of the generalized covariant derivative (including gauge connection) (5.8) acting on the fermions.

It is natural to wonder how the equations of motion for the fluctuations, fermionic ((5.8) or (5.10)) and as well as bosonic (4.7), go over to their flat space limit as the radius R is taken to ∞. The action of ref. [6], after a Wigner-Iönoü contraction (rescaling by appropriate power of R followed by $R \to \infty$), has a flat space limit. We are however, studying the fluctuations in the background of a macroscopic string. It is not clear to us how, if at all, such a configuration in the $AdS_5 \times S^5$ geometry smoothly goes over to an analogous configuration in flat space.

6. Towards evaluation of the determinants

We are now in a position to perform the functional integration over the fluctuations, and give a formal expression for the one-loop result. Collecting (4.7), (4.8) and (5.11), we find the following expression for $W(C)$ in (2.6):

$$W(C) = e^{-A_R}\,\frac{\det\left(-\Delta_F - \frac{1}{4}R^{(2)} + 1\right)}{\det\left(-\Delta_{cl} + 2\right)\det^{1/2}\left(-\Delta_{cl} + 4 - R^{(2)}\right)\det^{5/2}\left(-\Delta_{cl}\right)}\,. \tag{6.1}$$

Recall that e^{-A_R} is the classical contribution, A_R being the (regulated) area of the worldsheet; and that the fermionic operator in the numerator is a 4×4 matrix operator. The formal determinants in (6.1) suffer from potential divergences and need to be regulated. Out of many ways to make sense of these, the heat kernel regularization is particularly convenient. There is a vast literature on this — we will use ref. [13], which gives an asymptotic expansion for (the logarithm of) such determinants as an infinite sum. The first few terms in the sum are (regularized) divergent contributions.

Let Λ be an ultraviolet cut-off. It is then easy to see (using the results of [13]), that the quadratic divergence $c_2\Lambda^2$ cancels between the bosons and fermions. In addition, since we have a worldsheet with boundary, there is a linear divergence $c_1\Lambda$, which cancels in the same way. Finally, the coefficient c_0 of the logarithmically divergent term $c_0 \ln \Lambda$, is given by the difference in the quadratic potential between bosons and fermions. We find that, contrary to naive expectation, this coefficient does not vanish. Specifically, we find that this term is proportional to

$$c_0 \sim \int d^2\sigma \sqrt{h_{cl}} R^{(2)} = 4T \int_{U_0}^{\infty} dU \frac{U^4 + U_0^4}{U^2\sqrt{U^4 - U_0^4}}\,, \tag{6.2}$$

where we have used the classical solution (2.3).

Notice that if we were working in the Neveu-Schwarz-Ramond formalism, divergence of the form (6.2), (modulo subtleties involving the boundary), would have signalled a non-zero β-function. And to restore conformal invariance of the sigma model, one will need to shift the dilaton. In the Green-Schwarz approach that we are working with, such a conclusion is far from obvious, as there is no correpondence between the β-function and equations of motion of the spacetime fields. Therefore one should be cautious of such an interpretation with its implication apparently at variance with the conformal invariance of the SYM theory. This point is worth further critical examination.

While we really do not know the full significance of the logarithmic divergence, let us nevertheless try to understand it in our context. To this end, we evaluate the coefficient c_0 by substituting the upper limit of U-integration by a cut-off U_{max}. This is not a new scale, but was already introduced to regularize the classical contribution [4]. Now we expand (6.2) in terms of the small parameter U_0/U_{max},

$$c_0 \sim \int d^2\sigma \sqrt{h_{cl}} R^{(2)} = 4TU_{max} \left(1 + \mathcal{O}\left(\frac{U_0^4}{U_{max}^4}\right)\right). \tag{6.3}$$

To leading order this does not depend on the separation between the quarks, and goes to only renormalize the (infinite) mass of the quarks.[2] The higher corrections vanish in the limit $U_{max} \to \infty$.

We recall that the $AdS_5 \times S^5$ background was the near horizon limit of N D3-branes. However, the set up to calculate the quark potential differs in one small, but important way. Here one starts with $N + 1$ D3-branes and takes one of them far away from the others. This introduces a scale, whose only effect in the near horizon limit is seen in the (infinite) mass of the heavy quarks. The logarithmic divergence at

[2] Since the mass dimension of U is one the UV cut-off is dimensionless. The divergence (6.3) can be absorbed in a redefinition of U_{max}.

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one-loop goes only to 'renormalize' this hidden scale. Therefore we conclude, (albeit with some caution), that the one loop regularization does not affect the potential energy between the quark-antiquark pair.

Some of the higher (non-divergent) terms in the infinite sum in the expression of the determinants can be read from ref. [13]. It would be nice to compute the determinants in closed form.

7. Conclusion

In this letter we study the effect of stringy fluctuations around the classical macroscopic string background that define expectation value of Wilson loop operator in the AdS/CFT framework. To this end we expand the Green-Schwarz type superstring action for the $AdS_5 \times S^5$ background [6] to second order in fluctuation around the classical solution in ref. [4]. Both reparametrization as well as the local fermionic κ-symmetry is fixed for this background leaving only physical degrees of freedom. We fix diffeomorphism not by the standard choice of a (conformally) flat metric on the worldsheet, but rather by fixing it to be the metric induced from the target space. Our κ-symmetry fixing condition likewise differs from that given in refs. [7], and is more suitable for the problem at hand. We comment on the evaluation of the determinants that are the result of functional integration. Surprisingly, we find that the divergent contributions do not completely cancel between the bose and fermi fields. Our understanding of this is admittedly somewhat tentative, and we leave this issue open for further exploration.

Note added. After we completed this work, the paper [14] appeared in the archive. This also studies stringy fluctuations affecting Wilson loop in AdS/CFT, but in the finite temperature case. Same applies to [15]. The techniques of the present paper have been used in [16] to study the fluctuations of membranes of M-theory in $AdS_7 \times S^4$ background. Some preliminary result of the present paper was reported in the Ahrenshoop Workshop in Buckow, Germany (September, 1998) by S.F. and in the String Workshop in Puri, India (December, 1998) by D.G.

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References

[1] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [`hep-th/9711200`].

[2] S. Gubser, I. Klebanov and A. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [`hep-th/9802109`].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [`hep-th/9802150`].

[4] J. Maldacena, *Wilson loops in large-N field theories*, *Phys. Rev. Lett.* **80** (1998) 4859 [`hep-th/9803002`].

[5] S.-J. Rey and J. Yee, *Macroscopic strings as heavy quarks in large N gauge theory and anti-de Sitter supergravity*, `hep-th/9803001`.

[6] R. Metsaev and A. Tseytlin, *Type IIB superstring action in* $AdS_5 \times S^5$ *background*, *Nucl. Phys.* **B 533** (1998) 109 [`hep-th/9805028`].

[7] R. Kallosh, J. Rahmfeld and A. Rajaraman, *Near horizon superspace*, *J. High Energy Phys.* **09** (1998) 002 [`hep-th/9805217`];
R. Kallosh, *Superconformal action in Killing gauge*, `hep-th/9807206`;
I. Pesando, *A* κ *fixed type IIB superstring action on* $AdS_5 \times S^5$, *J. High Energy Phys.* **11** (1998) 002 [`hep-th/9808020`];
R. Kallosh and J. Rahmfeld, *The GS string action on* $AdS_5 \times S^5$, *Phys. Lett.* **B 443** (1998) 143 [`hep-th/9808038`];
R. Kallosh and A. Tseytlin, *Simplifying superstring action on* $AdS_5 \times S^5$, *J. High Energy Phys.* **10** (1998) 016 [`hep-th/9808088`];
A. Rajaraman and M. Rozali, *On the quantization of the GS string on* $AdS_5 \times S^5$, `hep-th/9902046`.

[8] L. Alvarez-Gaumé, D. Freedman and S. Mukhi, *The background field method and the ultraviolet structure of the supersymmetric nonlinear sigma models*, *Ann. Phys. (NY)* **134** (1981) 85.

[9] T. Banks and M. Green, *Nonperturbative effects in* $AdS_5 \times S^5$ *string theory and* $D = 4$ *susy Yang-Mills*, *J. High Energy Phys.* **05** (1998) 002 [`hep-th/9804170`].

[10] S. Gubser, I. Klebanov and A. Tseytlin, *Coupling constant dependence in the thermodynamics of* $N = 4$ *supersymmetric Yang-Mills theory*, `hep-th/9805156`.

[11] J. Pawelczyk and S. Theisen, $AdS_5 \times S^5$ *metric at order* α'^3, `hep-th/9808126`.

[12] M. Green, J. Schwarz and E. Witten, *Superstring theory*, vol. I, Cambridge University Press, Cambridge 1987.

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[13] T. Branson, P. Gilkey and D. Vassilevich, *Vacuum expectation value asymptotics for second order differential operators on manifolds with boundary*, *J. Math. Phys.* **39** (1998) 1040 [hep-th/9702178];
P. Gilkey, *Invariance theory, the heat equation and the Atiyah-Singer index theorem* 2nd ed., CRC Press, Boca Raton 1995

[14] J. Greensite and P. Olesen, *Worldsheet fluctuations and heavy quark potential in the AdS/CFT approach*, *J. High Energy Phys.* **04** (1999) 001 [hep-th/9901057].

[15] S. Naik, talk in the Workshop on *String theory, gauge theories and quantum gravity*, Puri, India 1998 (unpublished); and private communications.

[16] S. Förste, *Membrany corrections to the string anti-string potential in M5-brane theory*, *J. High Energy Phys.* **05** (1999) 002 [hep-th/9902068].