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On the effective interactions of a light gravitino with matter fermions*

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Abstract: If the gravitino is light and all the other supersymmetric particles are heavy, we can consider the effective theory describing the interactions of its goldstino components with ordinary matter. To discuss the model-dependence of these interactions, we take the simple case of spontaneously broken supersymmetry and only two chiral superfields, associated with the goldstino and a massless matter fermion. We derive the four-point effective coupling involving two matter fermions and two goldstinos, by explicit integration of the heavy spin-0 degrees of freedom in the low-energy limit. Surprisingly, our result is not equivalent to the usual non-linear realization of supersymmetry, where a pair of goldstinos couples to the energy-momentum tensor of the matter fields. We solve the puzzle by enlarging the non-linear realization to include a second independent invariant coupling, and we show that there are no other independent couplings of this type up to this order in the low-energy expansion. We conclude by commenting on the interpretation of our results and on their possible phenomenological implications.

Keywords: Spontaneous Symmetry Breaking, Supersymmetry Breaking, Supergravity Models.

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Contents

1 Introduction

It is quite plausible that the theory of fundamental interactions lying beyond the Standard Model has a spontaneously broken $N = 1$ space-time supersymmetry (for reviews and references, see e.g. [1]). However, the dynamical origin of the energy scales controlling supersymmetry breaking is still obscure, and different possibilities can be legitimately considered. In this paper, following the general strategy outlined in [2], we concentrate on the possibility that the gravitino mass $m_{3/2}$ is much smaller than all the other supersymmetry-breaking mass splittings. In this case, the $\pm 1/2$ helicity components of the gravitino, corresponding to the would-be goldstino $\tilde{G}$, have effective couplings with the various matter and gauge superfields much stronger than the gravitational ones. Exploiting the supersymmetric version of the equivalence theorem [3], in a suitable energy range we can neglect gravitational interactions and define a (non-renormalizable) effective theory with spontaneously broken global supersymmetry.

In this general framework, we analyze the low-energy amplitudes involving two goldstinos and two matter fermions. According to the low-energy theorems for goldstino interactions [4], such amplitudes are controlled by the energy-momentum tensor $T_{\mu\nu}$ of the matter system. Indeed, explicit non-linear realizations of the supersymmetry algebra have been built [5, 6], and they precisely reproduce the behaviour prescribed by the low-energy theorems. In the present note, we follow an alternative procedure [2], starting from a theory where supersymmetry is linearly realized, although spontaneously broken, and the building blocks are all the superfields containing the light degrees of freedom. Restricting ourselves to energies smaller than the supersymmetry-breaking mass splittings, we solve the equations of motion for the heavy superpartners in the low-energy limit, and derive an effective theory involving only the goldstino and the light Standard Model particles, where supersymmetry is non-linearly realized. We finally compare the results obtained via this explicit procedure with those obtained by direct construction of the non-linear lagrangian, on the basis of the transformation properties of the goldstino and the matter fields.

A similar program has already been successfully implemented in a number of cases. In the simple case of a single chiral superfield, the effective low-energy four-goldstino coupling was computed [7], and the result can be shown to be physically equivalent to the non-linear realization of [5], in the sense that they give rise to the same on-shell scattering amplitudes. More recently, we computed the effective low-energy coupling involving two photons and two goldstinos [2]. Our result can be shown to be physically equivalent, in the same sense as before, to the non-linear realization of [6], where goldstino bilinears couple to the canonical energy-momentum tensor of matter and gauge fields.

In this paper, we discuss an interesting feature that emerges when we consider the effective low-energy coupling involving two goldstinos and two matter fermions. To make the case as clear and simple as possible, we consider only one massless left-handed matter fermion, we turn off gauge interactions and we impose a global $U(1)$ symmetry associated with matter conservation[1]. In contrast with the previous cases, the outcome of our calculation turns out to be physically inequivalent to the non-linear realization of [6]. To solve the puzzle, we go back to the superfield construction of non-linear realizations for goldstinos and matter fermions. We show that we can add to the invariant lagrangian, associated with the non-linear realization of [6], a second independent invariant, which contributes to the four-fermion interaction under consideration. The terms of this additional invariant containing two goldstinos cannot be expressed in terms of the energy-momentum tensor of the matter fermion. We also show that the most general form for the amplitude under consideration can indeed be parametrized, to this order in the low-energy expansion, in terms of only two supersymmetric invariants. After some comments on the interpretation of our results and on the open problems, we conclude with some anticipations [8] on the possible phenomenological implications.

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2 Calculational framework

As announced in the introduction, we consider an $N = 1$ globally supersymmetric theory containing only two chiral superfields. One of them will describe the goldstino $\tilde{G}$ and its complex spin-0 partner $z \equiv (S + iP)/\sqrt{2}$. The other one will describe a massless left-handed matter fermion f and its complex spin-0 partner $\tilde{f}$. According to the standard formalism [9], and neglecting for the moment higher-derivative terms, the lagrangian is completely specified in terms of a superpotential w and a Kähler potential K. To have

[1] With the given fermion content this symmetry is anomalous, but we can introduce a third chiral superfield, associated with a left-handed antimatter fermion f^c, that cancels the anomaly without affecting any of the following considerations. Also the other assumptions can be eventually relaxed, with no impact on our main result.

spontaneous supersymmetry breaking, and to consistently identify $\tilde{G}$ with the goldstino, we assume that, at the minimum of the scalar potential,

$$\langle F^0 \rangle \neq 0 \,, \qquad \langle F^1 \rangle = 0 \,, \tag{2.1}$$

where F^0 and F^1 denote the auxiliary fields associated with the goldstino and with the matter fermion, respectively. It will not be restrictive to assume that $\langle z \rangle = 0$. We shall also assume that $\langle \tilde{f} \rangle = 0$, consistently with an unbroken global $U(1)$ symmetry associated with matter conservation.

We proceed by expanding the defining functions of the theory around the vacuum, in order to identify the terms contributing to the effective four-fermion interaction involving two matter fermions and two goldstinos. Without loss of generality, we can write:

$$w = \hat{w}(z) + \ldots \,, \qquad K = \hat{K}(z,\bar{z}) + \tilde{K}(z,\bar{z}) \, |\tilde{f}|^2 + \ldots \,, \tag{2.2}$$

where the dots denote terms that are not relevant for our considerations. Taking into account eqs. (2.1) and (2.2), the mass spectrum of the model can be easily derived from standard formulae [9]. The goldstino and the matter fermion remain massless, whilst all the spin-0 particles acquire in general non-vanishing masses, proportional to $\langle F^0 \rangle$ and expressed in terms of w, K and their derivatives, evaluated on the vacuum. Moreover, even in the presence of non-renormalizable interactions, the expansion of the lagrangian in (canonically normalized) component fields can be rearranged in such a way that all the terms relevant for our calculation are expressed in terms of the mass parameters $(m_S^2, m_P^2, \tilde{m}_f^2)$, associated with the spin-0 partners of the goldstino and of the matter fermion, and the scale F of supersymmetry breaking, without explicit reference to w and K:

$$\begin{aligned}
\mathcal{L} \;&=\; \frac{1}{2}\left[(\partial^\mu S)(\partial_\mu S) - m_S^2 S^2\right] + \frac{1}{2}\left[(\partial^\mu P)(\partial_\mu P) - m_P^2 P^2\right] + (\partial^\mu \tilde{f})^*(\partial_\mu \tilde{f}) - \tilde{m}_f^2 |\tilde{f}|^2 + \\
&+\; i\overline{\tilde{G}}\bar{\sigma}^\mu \partial_\mu \tilde{G} + i\overline{f}\bar{\sigma}^\mu \partial_\mu f - \frac{1}{2\sqrt{2}F}[(m_S^2 S + i m_P^2 P)\tilde{G}\tilde{G} + (m_S^2 S - i m_P^2 P)\overline{\tilde{G}}\overline{\tilde{G}}] - \\
&-\; \frac{\tilde{m}_f^2}{F}(\tilde{f}^* \, \tilde{G} f + \tilde{f} \, \overline{\tilde{G}}\overline{f}) - \frac{\tilde{m}_f^2}{F^2} \, \tilde{G} f \, \overline{\tilde{G}}\overline{f} + \ldots \,.
\end{aligned} \tag{2.3}$$

In eq. (2.3), we have used two-component spinors with the conventions of [2]. The parameter $F \equiv < \overline{w}_{\bar{z}} (K_{\bar{z}z})^{-1/2} >$ (lower indices denote derivatives) defines the supersymmetry-breaking scale and has the dimension of a mass squared. For simplicity, we have assumed F to be real. We recall that, in our flat space-time, F is linked to the gravitino mass $m_{3/2}$ by the universal relation $F^2 = 3 \, m_{3/2}^2 \, M_{\rm P}^2$, where $M_{\rm P} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the Planck mass. Finally, the dots in eq. (2.3) stand for terms that do not contribute to the four-fermion amplitudes of interest[2].

[2]There are interaction terms proportional to $< \tilde{K}_z >$ and $< \tilde{K}_{\bar{z}} >$, not explicitly listed here, that are in principle relevant. An explicit computation shows that their total contribution vanishes. This is in agreement with the possibility of choosing normal coordinates [10], where such terms are absent.

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Starting from the lagrangian of eq. (2.3), we take the limit of a heavy spin-0 spectrum, with $(m_S, m_P, \tilde{m}_f)$ much larger than the typical energy of the scattering processes we would like to study. In this case, we can build an effective lagrangian for the light fields by integrating out the heavy states. As discussed in detail in [2], the crucial property of such an effective lagrangian will be its dependence on the supersymmetry-breaking scale F only, without any further reference to the supersymmetry-breaking masses $(m_S, m_P, \tilde{m}_f)$.

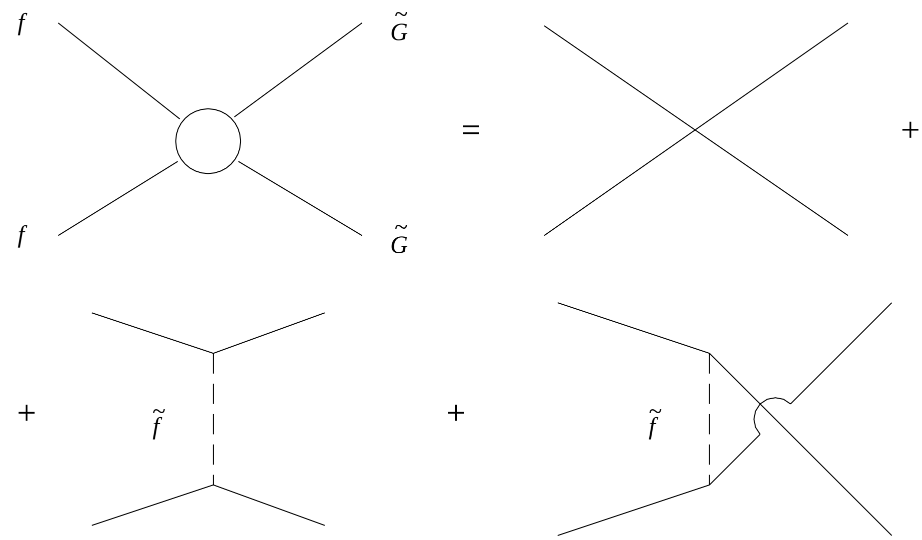

Figure 1: Diagrammatic origin of the four-fermion operator of eq. (2.4).

This property is the result of subtle cancellations among the different diagrams shown in fig. 1, corresponding to the contact term in the last line of eq. (2.3) and to $\tilde{f}$ exchange, and agrees with general results [3–6] concerning low-energy goldstino interactions. Focussing only on the terms relevant for our calculation, we obtain a local interaction term involving two matter fermions and two goldstinos, of the form

$$\mathcal{L}_{eff} = \frac{1}{F^2}[\partial_\mu(f\tilde{G})][\partial^\mu(\overline{f}\overline{\tilde{G}})] + \dots \,. \tag{2.4}$$

An alternative derivation of $\mathcal{L}_{eff}$ is possible, following a technique introduced in [11]. Denoting by ϕ and ϕ_f the superfields associated with the goldstino and with the matter fermion, respectively, we can impose the supersymmetric constraints $\phi^2 = 0$ a and $\phi\,\phi_f = 0$, and solve for the fermionic components imposing eq. (2.1). The result coincides with eq. (2.4).

3 A puzzling result

Could we have derived the effective interaction of eq. (2.4) from the non-linear realizations of the supersymmetry algebra that have been proposed up to now in the literature? To address this question, we recall that the non-linear realization of [5, 6] prescribes an effective interaction of the form

$$\mathcal{L}'_{eff} = \frac{i}{2F^2}[\tilde{G}\sigma^\mu\partial^\nu\overline{\tilde{G}} - (\partial^\nu\tilde{G})\sigma^\mu\overline{\tilde{G}}]\,T_{\nu\mu} + \dots \,, \tag{3.1}$$

where $T_{\nu\mu}$ is the canonical energy-momentum tensor of the matter fermions,

$$T_{\nu\mu} = i\overline{f}\overline{\sigma}_\nu \partial_\mu f + \dots \,, \tag{3.2}$$

and the dots stand for terms that do not contribute to the on-shell scattering amplitudes with two matter fermions and two goldstinos. Combining (3.1) with (3.2), we obtain:

$$\mathcal{L}'_{eff} = -\frac{1}{F^2}(\tilde{G}\sigma^\mu \partial^\nu \overline{\tilde{G}})(\overline{f}\overline{\sigma}_\nu \partial_\mu f) + \dots \,, \tag{3.3}$$

which looks very different from (2.4).

To check that (2.4) and (3.3) are really inequivalent, we concentrate on the scattering amplitudes for the process[3]

$$f\,\overline{f} \longrightarrow \tilde{G}\tilde{G} \,, \tag{3.4}$$

even if $f\tilde{G} \to f\tilde{G}$, $\overline{f}\tilde{G} \to \overline{f}\tilde{G}$ or $\tilde{G}\tilde{G} \to f\overline{f}$ would be equally good processes for this purpose. We denote by (p_1, p_2, q_1, q_2) the four-momenta of the incoming fermion and antifermion and of the two outgoing goldstinos, respectively. Notice that the only helicity configurations that can contribute to the process are, in the same order of the momenta and in obvious notation, (L, R, L, R) and (L, R, R, L).

On the one hand, from the effective lagrangian of eq. (2.4) we obtain the amplitudes:

$$a(L,R,L,R) = -\frac{(1+\cos\theta)^2 s^2}{4F^2} \,, \quad a(L,R,R,L) = \frac{(1-\cos\theta)^2 s^2}{4F^2} \,, \tag{3.5}$$

where $\sqrt{s}$ and θ are the total energy and the scattering angle in the centre-of-mass frame, leading to a total cross-section

$$\sigma(f\bar{f} \to \tilde{G}\tilde{G}) = \frac{s^3}{80\pi F^4} \,. \tag{3.6}$$

On the other hand, from the effective lagrangian of eq. (3.3) we obtain:

$$a'(L,R,L,R) = \frac{\sin^2\theta s^2}{4F^2} \,, \quad a'(L,R,R,L) = -\frac{\sin^2\theta s^2}{4F^2} \,, \tag{3.7}$$

leading to a total cross-section

$$\sigma'(f\bar{f} \to \tilde{G}\tilde{G}) = \frac{s^3}{480\pi F^4} \,. \tag{3.8}$$

We conclude that the two effective interactions (2.4) and (3.3) lead to the same energy dependence, but to different angular dependences and total cross-sections. Surprisingly, the two approaches seem to give physically different results[4].

[3] This process was already considered by Fayet [12], who gave the correct scaling law of the cross-section with respect to the gravitino mass and to the centre-of-mass energy in the low-energy limit.

[4] The above results can be easily extended to Dirac fermions, upon introduction of a second Weyl spinor f^c. For example, the total unpolarized cross section $\sigma(e^+e^- \to \tilde{G}\tilde{G})$ inferred from (3.6) would read $s^3/(160\pi F^4)$ and that from (3.8) $s^3/(960\pi F^4)$. Incidentally, we observe that both results are in disagreement with a previous computation [13], which found $\sigma(\tilde{G}\tilde{G} \to e^+e^-) = s^3/(20\pi F^4)$, corresponding to $\sigma(e^+e^- \to \tilde{G}\tilde{G}) = s^3/(40\pi F^4)$.

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4 Solution of the puzzle

To understand the origin of the discrepancy, we go back to the superfield construction of the non-linear realization of [6]. This is given in terms of the superfield

$$\Lambda_\alpha(x,\theta,\overline{\theta}) \equiv \exp(\theta Q+\overline{\theta Q})\,\tilde{G}_\alpha(x) = \tilde{G}_\alpha+\sqrt{2}F\theta_\alpha+\frac{i}{\sqrt{2}F}(\tilde{G}\sigma^\mu\overline{\theta}-\theta\sigma^\mu\overline{\tilde{G}})\partial_\mu\tilde{G}_\alpha+\ldots\,, \quad (4.1)$$

whose lowest component is the goldstino $\tilde{G}$, and a superfield

$$E_\alpha(x,\theta,\overline{\theta}) \equiv \exp(\theta Q+\overline{\theta Q})\,f_\alpha(x) = f_\alpha+\frac{i}{\sqrt{2}F}(\tilde{G}\sigma^\mu\overline{\theta}-\theta\sigma^\mu\overline{\tilde{G}})\partial_\mu f_\alpha+\ldots\,, \quad (4.2)$$

whose lowest component is the matter fermion f. In the simple case under consideration, the non-linear realization of [6] can be introduced via the supersymmetric lagrangian

$$\frac{1}{4F^4}\int d^4\theta\;\Lambda^2\overline{\Lambda}^2\; i\overline{E}\overline{\sigma}^\mu\partial_\mu E\,, \quad (4.3)$$

which leads precisely to the result of eq. (3.3), as can be easily verified by an explicit computation.

The crucial question is now the following: are there other independent invariants, besides (4.3), that can contribute to the effective interaction under consideration? The answer is positive, since a second invariant can be constructed:

$$\frac{\alpha}{F^2}\int d^4\theta\;\Lambda E\;\overline{\Lambda E}\,, \quad (4.4)$$

where α is an arbitrary dimensionless coefficient. The new invariant (4.4) gives, among other things, the following contribution to the four-fermion effective interaction under consideration:

$$\delta\mathcal{L}'_{eff} = \frac{\alpha}{4F^2}(\tilde{G}\sigma^\mu\partial^\nu\overline{f})(\overline{\tilde{G}}\overline{\sigma}_\nu\partial_\mu f)+\ldots\,, \quad (4.5)$$

where the dots stand for terms not contributing to the on-shell process under consideration. From the contact interaction displayed in eq. (4.5) we obtain the following non-vanishing amplitudes:

$$\delta a'(L,R,L,R) = \alpha\frac{(1+\cos\theta)s^2}{8F^2}\,, \qquad \delta a'(L,R,R,L) = -\alpha\frac{(1-\cos\theta)s^2}{8F^2}\,. \quad (4.6)$$

Since we have found a second invariant contributing to the process, we may wonder whether an appropriate linear combination of the two invariants can reproduce the result of eq. (3.5). Indeed, it is immediate to check that, with the special choice $\alpha=-4$, the combination $\mathcal{L}'_{eff}+\delta\mathcal{L}'_{eff}$ reproduces the scattering amplitudes obtained from $\mathcal{L}_{eff}$.

As a first comment on the interpretation of our results, we would like to stress that there is no reason to believe that the result of eq. (2.4) is more fundamental than the standard result of eq. (3.3). The important fact to realize is that, since two independent invariants can be constructed, both of which contribute to the effective four-fermion coupling under consideration, there is an ambiguity in the effective theory description, parametrized by the coefficient α in eq. (4.5). At the level of the linear realization, this ambiguity is contained in the coefficients of higher-derivative operators, which are not included in the standard Kähler formulation of eq. (2.2). Notice also that the new term (4.5) scales with F exactly as the term (3.3), which provides the coupling with $T_{\nu\mu}$. They both contain two derivatives and give rise to amplitudes with the same energy behaviour. Therefore, in the low-energy expansion of an underlying fundamental theory, they are on equal footing. Moreover, the new supersymmetric invariant (4.4) gives rise only to terms containing at least two goldstinos, without modifying the free matter fermion lagrangian.

Also, our results may admit a geometrical interpretation[5]. Using the equations of motion and some Fierz identities, we can rewrite the contribution (4.5) to the effective lagrangian as

$$\delta\mathcal{L}'_{eff} = \frac{\alpha}{8F^2}(i\,\epsilon^{\mu\nu\rho\lambda} - \eta^{\mu\nu}\eta^{\rho\lambda})[(\partial_\nu\tilde{G})\sigma_\rho(\partial_\mu\overline{\tilde{G}})](f\sigma_\lambda\overline{f}) = \frac{\alpha}{8F^2}(S^\lambda + T^\lambda)(f\sigma_\lambda\overline{f})\,, \quad (4.7)$$

where

$$S^\lambda \equiv i\,\epsilon^{\mu\nu\rho\lambda}(\partial_\nu\tilde{G})\sigma_\rho(\partial_\mu\overline{\tilde{G}})\,, \qquad T^\lambda \equiv -\eta^{\mu\nu}(\partial_\nu\tilde{G})\sigma^\lambda(\partial_\mu\overline{\tilde{G}})\,, \quad (4.8)$$

which suggests a possible coupling of the matter current to a non-trivial torsion term for the goldstino manifold.

5 General discussion

Are (4.3) and (4.4) the only independent invariants that contribute to the effective four-fermion coupling under consideration, or are there others? To answer this question, we look for all the local supersymmetric operators that respect the $U(1)$ global symmetry associated with matter conservation, and contribute to physical amplitudes with two goldstinos and two matter fermions that grow at most as s^2. Such operators have dimension $d \leq 4$, where the counting takes into account an overall factor $1/F^2$, necessarily associated with the two goldstinos. We do not consider operators with $d > 4$ because the corresponding amplitudes are suppressed by further powers of energy. Since we will use the superfields as building blocks, we recall that the matter superfield E has $d = 3/2$. For the goldstino, it is convenient to consider the rescaled superfield $\Lambda/\sqrt{2}F$, which has $d = -1/2$. In this way, the goldstino field $\tilde{G}$ always appears in the combination $(\tilde{G}/\sqrt{2}F)$. Throughout this section we will use units such that $\sqrt{2}F = 1$: the appropriate powers of F can be recovered at the end, simply by counting the goldstino fields. Finally, the integration measure $d^4\theta$ has $d = 2$, and an additional unit is associated with each explicit space-time derivative acting on the superfields.

The lowest-dimensional operator containing two matter-fermion and two goldstino component fields is a $d = 2$ four-fermion term of the kind $f\tilde{G}\,\overline{f}\overline{\tilde{G}}/F^2$. Is this allowed by supersymmetry? In terms of superfields, all the operators considered here contain precisely one matter superfield E and one conjugate matter superfield $\overline{E}$. In the absence of explicit space-time derivatives, the $d = 2$ invariants require six goldstino superfields.

[5]We thank S. Ferrara for discussions and suggestions on this point.

Such operators vanish identically because of the Grassmann algebra, which allows no more than four goldstino superfields. For each explicit space-time derivative, two additional goldstino superfields are needed to keep the overall dimension constant, and the previous argument still applies. Therefore no local $d = 2$ invariant is allowed by supersymmetry.

Moving to $d = 3$, the only independent operator without explicit space-time derivatives and (Pauli) σ-matrices is $E\Lambda\ \overline{E\Lambda}\ \Lambda^2$, up to an overall hermitean conjugation. However, this operator vanishes because of the Grassmann algebra. The result is unchanged if different Lorentz structures are considered, with any number of σ-matrices and $\epsilon_{\mu\nu\rho\lambda}$ tensors inserted. Adding explicit space-time derivatives requires the inclusion of additional goldstino superfields, and the Grassmann algebra forces the corresponding operators to vanish. No $d = 3$ invariant is permitted [6].

We are left with the $d = 4$ invariants. First, we consider the case of no explicit space-time derivatives. If σ-matrices are also excluded, then the only possibility is the new invariant $E\Lambda\ \overline{E\Lambda}$ of eq. (4.4). Moreover, it is not difficult to see that, thanks to well-known properties of the σ-matrices, expressions involving an arbitrary number of σ's and $\epsilon_{\mu\nu\rho\lambda}$ tensors always reduce to the invariant of eq. (4.4).

When one space-time derivative is added, the independent invariants containing only one σ are, up to integration by parts and hermitean conjugation:

$$\begin{aligned}
S_1 &= (\partial_\mu\Lambda)\sigma^\mu\overline{\Lambda}\ E\Lambda\ \overline{E\Lambda}\,, \\
S_2 &= \Lambda\sigma^\mu\overline{\Lambda}\ E(\partial_\mu\Lambda)\ \overline{E\Lambda}\,, \\
S_3 &= (\partial_\mu\Lambda)\sigma^\mu\overline{E}\ E\Lambda\ \overline{\Lambda}^2\,, \\
S_4 &= \Lambda\sigma^\mu\overline{E}\ E(\partial_\mu\Lambda)\ \overline{\Lambda}^2\,, \\
S_5 &= \Lambda\sigma^\mu\overline{E}\ E\Lambda\ \overline{\Lambda}(\partial_\mu\overline{\Lambda})\,, \\
S_6 &= E\sigma^\mu\overline{E}\ \Lambda(\partial_\mu\Lambda)\ \overline{\Lambda}^2\,, \\
S_7 &= \Lambda\sigma^\mu\overline{\Lambda}\ \Lambda(\partial_\mu E)\ \overline{E\Lambda}\,, \\
S_8 &= \Lambda\sigma^\mu\overline{E}\ \Lambda(\partial_\mu E)\ \overline{\Lambda}^2\,, \\
S_9 &= (\partial_\mu E)\sigma^\mu\overline{\Lambda}\ \Lambda^2\ \overline{E\Lambda}\,, \\
S_{10} &= (\partial_\mu E)\sigma^\mu\overline{E}\ \Lambda^2\ \overline{\Lambda}^2\,.
\end{aligned} \tag{5.1}$$

The invariants $S_1, \ldots, S_6$ do not produce terms without goldstino fields. We have explicitly evaluated the terms containing two matter fermions and two goldstinos, making use of integration by parts and of the equations of motion. The terms generated by S_5 and S_6 vanish. Those produced by S_1 and S_3 coincide, up to overall factors, with the operator of eq. (4.5). The terms coming from S_2 and S_4 are proportional to $(f\partial_\mu\tilde{G})(\overline{f}\partial^\mu\overline{\tilde{G}})$. The contributions of this four-fermion interaction to the helicity amplitudes for $f\overline{f} \to \tilde{G}\tilde{G}$ are however identical, up to overall factors, to those induced by the operator (4.5). Therefore, the inclusion of the invariants $S_1, \ldots, S_6$ merely amounts to a redefinition of the parameter α in the amplitudes of eq. (4.6).

[6] Of course, by releasing the requirement of matter conservation or by adding additional matter superfields, $d = 3$ invariants are allowed. They contain mass terms for the matter particles.

The invariants $S_7, \ldots, S_{10}$ give rise also to a term proportional to the matter-fermion kinetic term in the lagrangian. In particular, S_{10} is the invariant that occurs for a massless fermion according to the prescription of refs. [5, 6], and that was already discussed in the previous section [see eq. (4.3)]. We have explicitly expanded the invariants $S_7, \ldots, S_9$ up to terms containing two goldstinos. Then we have evaluated, for each invariant, the contributions to the helicity amplitudes for the process $f\overline{f} \to \tilde{G}\tilde{G}$. Once the normalization of the kinetic term for the matter fermion is properly taken into account, such contributions are exactly the same as those originated from the invariant S_{10}, despite the occurrence, in the intermediate steps of the computations, of new four-fermion operators. Therefore, any combination of $S_7, \ldots, S_{10}$, such that the matter kinetic term in the lagrangian is canonically normalized, gives rise to the physical amplitudes given in eq. (3.7), with no free parameters. This exhausts the case of one space-time derivative and one σ-matrix. All the invariants obtained by adding σ-matrices and $\epsilon_{\mu\nu\rho\lambda}$ tensors can be reduced to the invariants $S_1, \ldots, S_{10}$ by using properties of the σ-matrices.

The next case involves two space-time derivatives acting on the superfields. The independent invariants with no σ's are, up to integration by parts and hermitean conjugation:

$$\begin{aligned} S_{11} &= E(\partial_\mu \Lambda)\ \overline{E}(\partial^\mu \overline{\Lambda})\ \Lambda^2\ \overline{\Lambda}^2\,, \\ S_{12} &= E(\partial_\mu \Lambda)\ \overline{E\Lambda}\ \Lambda^2\ \overline{\Lambda}(\partial^\mu \overline{\Lambda})\,, \\ S_{13} &= E\Lambda\ \overline{E\Lambda}\ \Lambda(\partial_\mu \Lambda)\ \overline{\Lambda}(\partial^\mu \overline{\Lambda})\,. \end{aligned} \tag{5.2}$$

They produce an interaction of the type $(f\partial_\mu \tilde{G})(\overline{f}\partial^\mu \overline{\tilde{G}})$, as in the case of the invariants S_2, S_4. As we have seen, this does not affect the parametrization of the physical amplitudes provided by eq. (4.6). New invariants can be obtained by adding two σ-matrices. We have checked that the corresponding physical amplitudes are still given by eq. (4.6). More σ's and $\epsilon_{\mu\nu\rho\lambda}$ tensors do not generate independent invariants.

Finally, having more than two derivatives requires more than six goldstino superfields and the Grassmann algebra does not allow to build non-vanishing combinations.

In conclusion, assuming matter conservation, the most general amplitudes for processes involving two goldstinos $\tilde{G}$ and two massless matter fermions f can be parametrized in terms of only two supersymmetric invariants. The first one, eq. (4.3), is normalized by the requirement of providing a canonical kinetic energy for the matter system. The second one, eq. (4.4), brings a free parameter α in the expression of the amplitudes. No additional invariant is required, at least when only two goldstinos are present. This restricts the form of the helicity amplitudes. For instance, the general amplitudes for the process $f\overline{f} \to \tilde{G}\tilde{G}$ are just the sum of eqs. (3.7) and (4.6):

$$a_{GEN}(L,R,L,R) = \frac{1}{F^2}\left(tu - \frac{\alpha}{4}su\right)\,, \qquad a_{GEN}(L,R,R,L) = \frac{1}{F^2}\left(-tu + \frac{\alpha}{4}st\right)\,, \tag{5.3}$$

where (s,t,u) are the usual Mandelstam variables $t = -(s/2)(1-\cos\theta)$, $u = -(s/2)(1+\cos\theta)$, and the corresponding total cross-section is

$$\sigma_{GEN}(f\bar{f} \to \tilde{G}\tilde{G}) = \frac{(8 + 10\alpha + 5\alpha^2)s^3}{3840\pi F^4}\,. \tag{5.4}$$

Notice that the cross-section (5.4) is minimized for $\alpha = -1$, with $\sigma_{min} = s^3/(1280\pi F^4)$.

6 Comments and outlook

We conclude with some remarks on the interpretation, the possible extensions and the phenomenological implications of our results.

It would be interesting to see how our results can be interpreted within the framework of supersymmetric current algebra, which was successfully used for the first derivations of supersymmetric low-energy theorems [4]. We see a suggestive analogy with the textbook case of pion-nucleon scattering (see, e.g., section 19.5 of [14]), where the effective lagrangian consists of two independent terms, one completely controlled by the broken $SU(2) \times SU(2)$ symmetry and the other one containing the axial coupling g_A as an arbitrary parameter.

It would be also interesting to generalize our framework by including gauge interactions, and make contact with the recent results of [15]. At the level of local four-fermion operators, the arguments of the previous section are not affected by the presence of gauge interactions[7]. However, non-local four-fermion operators can in principle be generated by photon exchange, and this considerably complicates the discussion. We leave this to future investigations [8]. Since the process $e^+e^- \to \tilde{G}\tilde{G}$ may be used to extract a lower bound on the gravitino mass from supernova cooling (for recent discussions, see [2, 15, 16]), we expect a further clarification of this important phenomenological issue.

When extended to observable processes and realistic models, our results have other important phenomenological implications. Consider for example the reaction $f\overline{f} \to \tilde{G}\tilde{G}\gamma$, which probably gives the best signature of a very light gravitino at high-energy colliders, if all the other supersymmetric particles are above threshold. Also in this case, the explicit integration of the heavy superpartners gives results [8] that differ from those obtained [17] from the non-linear realization of [6]. In our opinion, it would be important to provide our experimental colleagues with a general framework to search for a superlight gravitino in a model-independent way, and we hope to develop this point soon.

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