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To cite this article: Amin Bibo and Mohammed F Daqaq 2015 Smart Mater. Struct. 24 094006

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An Analytical Framework for the Design and Comparative Analysis of Galloping Energy Harvesters under Quasi-Steady Aerodynamics

Amin Bibo and Mohammed F. Daqaq

Nonlinear Vibrations and Energy Harvesting Laboratory (NOVEHL)
College of Engineering and Science
Clemson University, Clemson, SC 29634
email: mdaqaq@clemson.edu

Abstract

This paper presents a generalized formulation, analysis, and optimization of energy harvesters subjected to galloping and base excitations. The harvester consists of a cantilever beam with a bluff body attached at the free end. A nondimensional lumped-parameter model which accounts for the combined loading and different electro-mechanical transduction mechanisms is presented. The aerodynamic loading is modeled using the quasi-steady assumption with polynomial approximation. A nonlinear analysis is carried out and an approximate analytical solution is obtained. A dimensional analysis is performed to identify the important parameters that affect the system’s response. The analysis of the response is divided into two parts. The first treats a harvester subjected to only galloping excitations. It is shown that, for a given shape of the bluff body and under quasi-steady flow conditions, the harvester’s dimensionless response can be described by a single universal curve irrespective to the geometric, mechanical, and electrical design parameters of the harvester. In the second part, a harvester under concurrent galloping and base excitations is analyzed. It is shown that, the total output power depends on three dimensionless loading parameters; wind speed, base excitation amplitude, and excitation frequency. The response curves of the harvester are generated in terms of the loading parameters. These curves can serve as a complete design guide for scaling and optimizing the performance of galloping-based harvesters.

1 Introduction

Flow energy harvesters (FEHs) are devices that channel energy from a moving fluid to a mechanical oscillator by coupling the dynamic forces culminating from the motion of the fluid past the oscillator to its natural modes of vibration. As a result of this coupling, the oscillator undergoes large amplitude oscillations which can be transformed into electricity using an electromechanical transduction mechanism, e.g., piezoelectric, electromagnetic, or electrostatic. Flow energy harvesters offer a simpler design alternative to traditional
rotary type generators with a basic premise of providing a scalable, easy to fabricate, and yet efficient energy scavenging mechanism.

In general, the efficacy of a FEH mechanism depends on the portion of the kinetic energy of the flow which is converted into elastic energy, and subsequently transformed into electricity via the electromechanical coupling mechanism. The nature and strength of this coupling varies according to the fluid-structure interaction mechanism utilized in the harvester’s design. Researchers have proposed several designs with different excitation mechanisms such as vortex-induced vibrations or wake-galloping [1–3], galloping [4, 5], flutter [6–9], and combined aeroelastic and base excitations [10–12].

From a performance perspective, the attachment of the airfoil or prismatic structure to the flexible structure as in the flutter and galloping approaches increases the aeroelastic coupling due to the fact that the dynamic loads on the tip body are transmitted directly to the beam. This improves the fluid-elastic conversion efficiency by two orders of magnitude when compared to the vortex-induced vibrations as reported in [13]. Additionally, for flutter and galloping-based harvesters, the onset of instability which results in steady-state oscillations is a consequence of a dynamic bifurcation, and is hence, followed by a monotonic increase in the output power with the wind speed. On the other hand, wake-galloping oscillators respond to the flow due to resonant interactions between the beam and the vortices shedding from the trailing edge of the bluff body.

As such, large amplitude oscillations are only attainable for a small frequency bandwidth around the natural frequency of the beam. Those oscillations are also of the same order of magnitude as the characteristic length of the bluff body [14]. As such, they are orders of magnitude smaller than the oscillations resulting from the galloping instability.

For all the aforementioned reasons, galloping and flutter are preferred over vortex induced vibrations for energy harvesting purposes. While no previous comparative studies have been performed, its the authors’ opinion that galloping is more suitable than flutter because galloping-based harvester is not sensitive to the coupling between the pitch and plunge modes which can shift the flutter speed into much higher wind velocities or lead to static divergence. Not to mention that, in the case of piezoelectric transduction, torsional mode oscillations of a flutter-based device cannot be easily captured and converted into electricity. It is also worth mentioning that depending on the size and the shape of the prismatic body and the associated Strouhal’s number, a harvester may experience vortex-induced and galloping oscillations separately or in combination.

The open literature contains a large number of examples describing the design and characterization of FEHs incorporating the galloping mechanism [13, 15–20]. Results have been presented for different prismatic bodies, geometric, and material properties of the oscillator, as well as different transduction mechanisms and circuit designs. However, as of today, there is no clear understanding of the relative performance of these devices, or which combination of design parameters yield the optimal performance of the harvester for a given flow conditions. This paper aims to fill this void by presenting a generalized formulation, analysis, and optimization of galloping energy harvesters. We hope that the results provided here will provide additional
insights towards designing more efficient galloping flow energy harvesters. To achieve this goal, the rest of the paper is organized as follows: In Section 2, a lumped-parameter mathematical model representing the dynamics of a harvester subjected to galloping and external base excitations is presented. In Section 3, the method of multiple scales is utilized to obtain an approximate analytical solution of the system. The solution is then used to analyze the response of the harvester in the absence of base excitations in Section 4, and perform design parameter study in section 5. In Section 6, the response of the harvester is analyzed in the presence of base excitations. Finally, the important conclusions of this work are presented in Section 7.

2 Mathematical Formulation

Figure 1: A schematic diagram of the single-mode flow energy harvester.

We consider a linear electromechanical oscillator in cross flow as depicted in Fig. 1 (a). The mechanical oscillator consists of a bluff body of mass, $M$, supported by a flexible structure of effective stiffness, $K$, and effective viscous damping coefficient, $C$. The oscillator is coupled to an electric circuit through either a first-order $RC$ circuit representing a piezoelectric transduction mechanism or a first-order $RL$ circuit representing an electromagnetic transduction as shown in Fig. 1 (b) and (c), respectively. Here, $C_p$ is the capacitance of the piezoelectric element, $L$ is the inductance of the harvesting coil, and $\theta$ is the a linear electromechanical coupling coefficient. In addition to aerodynamic forces, $F_y$, corresponding to wind speed, $U$, the harvester is subjected to a base excitation, $y_b$. Due to the combined loading, the mass oscillates in the cross flow direction with an absolute displacement, $y$. These oscillations produce an electric output, $r$, across an electric load, $R$. The electric output, $r$, represents the voltage in piezoelectric energy harvesters and the current in electromagnetic ones. The load, $R$, is the parallel equivalent of the piezoelectric resistance, $R_p$, and the load resistance, $R_l$, for piezoelectric transduction, and the series equivalent of the load and coil resistance, $R_c$, for the electromagnetic transduction. The equations governing the motion of the combined...
The system can be written as
\[ M\ddot{y} + C\dot{y} + Ky - \theta r = F_y(y, \dot{y}) + Ky_b + C\dot{y}_b, \quad (1) \]
\[ C_p\ddot{r} + \frac{r}{R} + \theta (\dot{y} - \dot{y}_b) = 0, \quad \text{piezoelectric}, \]
\[ L\dot{r} + Rr + \theta (\dot{y} - \dot{y}_b) = 0, \quad \text{electromagnetic}, \quad (2) \]
where the dot represents a derivative with respect to time, \( t \).

Following the experimental validated model of Parkinson [21], the vertical component of the aerodynamic force, \( F_y \), acting on the bluff body is modeled using a quasi-steady assumption with a \( m \)th-order polynomial approximation in \( \dot{y} \) such that
\[ F_y = \frac{1}{2} \rho U^2 L_t D_t C_{\dot{y}}, \quad (3) \]
\[ C_{\dot{y}} = \sum_{n: \text{odd}}^m A_n \left( \frac{\dot{y}}{U} \right)^n + \sum_{n: \text{even}}^m A_n \left( \frac{\dot{y}}{U} \right)^n |\dot{y}|, \quad n \geq 1, \quad (4) \]
where \( \rho \) is the air density, \( L_t \) and \( D_t \) are, respectively, the length and the cross-flow dimension of the bluff body, and the unknown coefficients, \( A_n \), are dependent on the general geometry and aspect ratio of the bluff body. These are usually obtained empirically from normal aerodynamic force measurements on a static bluff body at different angles of attack [21].

### 2.1 Non-dimensional Model

To obtain a dimensionless form of Equations (1) and (2), we introduce the following non-dimensional parameters
\[ \tilde{y} = \frac{y}{D_t}, \quad \tilde{y}_b = \frac{y_b}{D_t}, \quad \mu = \frac{\rho L_t D_t^2}{4M}, \quad \tilde{U} = \frac{U}{\omega_n D_t}, \]
\[ \tilde{r} = \frac{C_p}{\theta D_t} r, \quad \kappa = \frac{\theta^2}{K C_p}, \quad \alpha = \frac{1}{RC_p \omega_n}, \quad \text{piezoelectric} \]
\[ \tilde{r} = \frac{L}{\theta D_t} r, \quad \kappa = \frac{\theta^2}{KL}, \quad \alpha = \frac{R}{L \omega_n}, \quad \text{electromagnetic} \]
where \( \tilde{y} \), \( \tilde{y}_b \), and \( \tilde{r} \) represent the dimensionless transverse displacement, base displacement, and electric quantity, respectively, \( \mu \) is the flow to harvester mass ratio, \( \tilde{U} \) is the reduced wind speed, \( \kappa \) is the dimensionless electromechanical coupling, \( \alpha \) is the mechanical to electrical time-constant ratio. The natural frequency of the harvester at short-circuit conditions is given by \( \omega_n = \sqrt{K/M} \) and is used to introduce the non-dimensional time as \( \tilde{t} = \omega_n t \); whereas the mechanical damping ratio, \( \zeta_m \), is defined by \( C = 2\zeta_m M \omega_n \). In terms of the non-dimensional parameters, Equations (1) and (2) can be rewritten as
\[ \ddot{\tilde{y}} + 2\zeta_m \dot{\tilde{y}} + \tilde{y} - \kappa \tilde{r} = 2\mu \tilde{U}^2 C_{\dot{y}} + \tilde{y}_b + 2\zeta_m \dot{\tilde{y}}_b, \quad (5) \]
\[ \ddot{\tilde{r}} + \alpha \dot{\tilde{r}} + (\dot{\tilde{y}} - \tilde{y}_b) = 0. \quad (6) \]
Here, the prime denotes a derivative with respect to non-dimensional time, $\bar{t}$ and the non-dimensional lateral force coefficient is given by

$$C_{\bar{y}} = \sum_{n: \text{odd}} A_n \left( \frac{\bar{y}'}{\bar{U}} \right)^n + \sum_{n: \text{even}} A_n \left( \frac{\bar{y}'}{\bar{y}'} \right)^n \quad n \geq 1. \quad (7)$$

### 2.2 Model Assumptions

The general model presented here invokes several assumptions on the fluid-structural interaction problem that are worth mentioning:

1. The quasi-steady assumption: This is a very common assumption that simplifies the modeling of the fluid interactions with the bluff body \[14\]. Quasi-steadiness essentially implies that the motion of the bluff body is too slow compared to the motion of the fluid such that vertical force stays constant for a given angle of attack. This assumption requires that \( U / (\omega_n D_t) \geq 10 \)

2. The effect of added mass and fluid damping is neglected. This is a valid assumption for low density and low viscosity fluids such as air.

Additionally, the restoring force is assumed to be a linear function of the displacement. This implies that the geometric nonlinearities in the structure can be neglected which is an accurate assumption for sufficiently small deflections.

### 3 Approximate Analytical Solution

In an attempt to understand the dynamics described by Equations (5) and (6), we present an approximate analytical solution of these equations utilizing the method of multiple scales \[22\]. Towards that end, the time dependence is expanded into fast and slow time scales in the form $T_0 = \bar{t}$ and $T_1 = \epsilon \bar{t}$, respectively, where $\epsilon$ is a scaling parameter. Using the new time scales the time derivatives can be expressed as

$$\frac{d}{dt} = D_0 + \epsilon D_1 + O(\epsilon^2), \quad \frac{d^2}{dt^2} = D_0^2 + 2\epsilon D_0 D_1 + O(\epsilon^2), \quad (8)$$

where $D_n = \frac{\partial}{\partial T_n}$. Furthermore, we expand $\bar{y}$ and $\bar{r}$ in the following forms:

$$\bar{y}(\bar{t}) = \bar{y}_0(T_0, T_1) + \epsilon \bar{y}_1(T_0, T_1) + O(\epsilon^2),$$

$$\bar{r}(\bar{t}) = \bar{r}_0(T_0, T_1) + \epsilon \bar{r}_1(T_0, T_1) + O(\epsilon^2), \quad (9)$$

The constant coefficients in Equation (5) are also scaled such that the effect of viscous damping, electromechanical coupling, and aerodynamic forcing appear at the same order of the perturbation problem. In other words, we let

$$\zeta_m = \epsilon \zeta_m, \quad \kappa = \epsilon \kappa, \quad \text{and} \quad \mu = \epsilon \mu. \quad (10)$$
Furthermore, we assume the base displacement to be sinusoidal of the form:

\[ \ddot{y}_b = \epsilon |\ddot{y}_b| \cos(\Omega t). \]  

(11)

where \( |\ddot{y}_b| \) and \( \Omega \) are its amplitude and frequency, respectively. Since the influence of the base excitations is dominant near the natural frequency of the harvester, only its primary-resonant influence is analyzed; that is, the excitation frequency is assumed to be close to the natural frequency of the harvester. Therefore, a detuning parameter, \( \sigma \), is introduced to describe the nearness of these two frequencies by letting \( \Omega = (1 + \epsilon \sigma) \).

This yields

\[ \ddot{y}_b = \epsilon |\ddot{y}_b| \cos(T_0 + \sigma T_1). \]  

(12)

Substituting the time scaling, its derivatives, and the scaled parameters back into Equations (5) and (6) then collecting terms of equal powers of \( \epsilon \) yields \( O(\epsilon^0) \):

\[
D_0^2 \ddot{y}_0 + \ddot{y}_0 = 0, \\
\]  

(13)

\[
D_0 \ddot{r}_0 + \alpha \ddot{r}_0 = -D_0 \ddot{y}_0, \\
\]  

(14)

\( O(\epsilon^1) \):

\[
D_0^2 \ddot{y}_1 + \ddot{y}_1 = -2D_0 D_1 \ddot{y}_0 - 2\zeta_n D_0 \ddot{y}_0 + \kappa \ddot{r}_0 + 2\mu \ddot{U}^2 C_{\ddot{y}_0} \ddot{y}_0 + |\ddot{y}_b| \cos(T_0 + \sigma T_1), \\
\]  

(15)

\[
D_0 \ddot{r}_1 + \alpha \ddot{r}_1 = -D_1 \ddot{r}_0 - D_1 \ddot{y}_0 - D_0 \ddot{y}_1 - |\ddot{y}_b| \sin(T_0 + \sigma T_1), \\
\]  

(16)

where

\[
C_{\ddot{y}_0} = \sum_{n: \text{odd}} A_n \left( \frac{D_0 \ddot{y}_0}{U} \right)^n + \sum_{n: \text{even}} A_n \left( \frac{D_0 \ddot{y}_0}{U} \right)^n \frac{D_0 \ddot{y}_0}{|D_0 \ddot{y}_0|}, \quad n \geq 1. \\
\]  

(17)

The solution of the zeroth-order perturbation problem can be written as

\[
\ddot{y}_0 = a(T_1) \cos \phi, \\
\ddot{r}_0 = \gamma a(T_1) \sin [\phi - \psi], \quad \gamma = \frac{1}{\sqrt{1 + \alpha^2}}, \quad \psi = \sin^{-1} \gamma, \\
\]  

(18)

where \( \phi = [T_0 + \beta(T_1)] \); while \( a(T_1) \) and \( \beta(T_1) \) are, respectively, slowly varying amplitude and phase functions to be determined at the next step. Substituting Equations (18) and (19) into Equation (15) and
expressed as a polynomial function of $\zeta$ or $\phi$ components is given by

$$a' = -\frac{1}{\pi} \int_0^{2\pi} \left\{ \zeta_m a \sin \phi + \frac{\kappa}{2} \gamma a \sin [\phi - \psi] + \frac{|\bar{y}_b|}{2} \cos [\phi - \beta + \sigma T_1] \right\} \sin \phi d\phi$$

$$-\frac{\mu U^2}{\pi} \sum_{n: \text{even}} A_n \left( \frac{a}{U} \right)^n \int_0^{2\pi} \sin^n \phi \frac{1}{|\sin \phi|} \sin \phi d\phi$$

$$-\frac{\mu U^2}{\pi} \sum_{n: \text{odd}} A_n \left( \frac{a}{U} \right)^n \int_0^{2\pi} \sin^n \phi \sin \phi d\phi,$$

(20)

$$a\beta' = -\frac{1}{\pi} \int_0^{2\pi} \left\{ \zeta_m a \sin \phi + \frac{\kappa}{2} \gamma a \sin [\phi - \psi] + \frac{|\bar{y}_b|}{2} \cos [\phi - \beta + \sigma T_1] \right\} \cos \phi d\phi$$

$$-\frac{\mu U^2}{\pi} \sum_{n: \text{even}} A_n \left( \frac{a}{U} \right)^n \int_0^{2\pi} \sin^n \phi \frac{1}{|\sin \phi|} \cos \phi d\phi$$

$$-\frac{\mu U^2}{\pi} \sum_{n: \text{odd}} A_n \left( \frac{a}{U} \right)^n \int_0^{2\pi} \sin^n \phi \cos \phi d\phi.$$

(21)

Using De Moivre’s formula, Euler’s formula and binomial theorem [23], one can write

$$\sin^n \phi = \frac{1}{2^n} \left( \frac{n}{2} \right) + \frac{\bar{n} - 1}{2^n} \sum_{k=0}^{\bar{n}-1} (-1)^n \left( \frac{n}{k} \right) \cos \left[ (n - 2k) \phi \right], \quad \text{if } n \text{ is even}$$

(22)

$$\sin^n \phi = \frac{\bar{n} - 1}{2^n} \sum_{k=0}^{\bar{n}-1} (-1)^{\bar{n}-k} \left( \frac{n}{k} \right) \sin \left[ (n - 2k) \phi \right], \quad \text{if } n \text{ is odd}.$$  

(23)

Carrying out the integration in Equations (20) and (21), we obtain

$$a' = -\zeta_T a + \mu U^2 C_a + \frac{|\bar{y}_b|}{2} \sin \delta,$$

(24)

$$a\delta' = \left( \sigma - \frac{\zeta_c}{\alpha} \right) a + \frac{|\bar{y}_b|}{2} \cos \delta,$$

(25)

where $\delta = \sigma T_1 - \beta$. The parameter $\zeta_T = \zeta_m + \zeta_c$ represents the total damping in the system, in which the electrical damping component is given by $\zeta_c = \alpha \kappa / [2(1 + \alpha^2)]$. The lateral force coefficient, for $n \geq 1$, is expressed as a polynomial function of $a$ as

$$C_a = \left[ \sum_{n: \text{even}} \frac{4A_n}{\pi} \left( \frac{a}{2U} \right)^n \sum_{k=0}^{n/2} (-1)^{\bar{n}-k} \left( \frac{n+1}{k} \right) \right] + \sum_{n: \text{odd}} A_n \left( \frac{a}{2U} \right)^n \left( \frac{n+1}{2^n} \right).$$

(26)

3.1 Asymptotic Response

At steady-state, the fixed points of Equations (24) and (25), $a_0$ and $\delta_0$, respectively, correspond to the steady-state amplitude and phase of the periodic solutions of the original Equations, i.e. Equations (5) and (6).

Therefore, the periodic non-dimensional displacement and voltage can be written, to the first approximation, as

$$\bar{y} = a_0 \cos (\Omega t - \delta_0) + O(\epsilon), \quad \bar{v} = \gamma a_0 \sin [\Omega t - (\delta_0 + \psi)] + O(\epsilon).$$

(27)
where the fixed points, \( a_0 \) and \( \delta_0 \), are obtained by setting the time derivatives in Equations (24) and (25) to zero, i.e. \( a' = \delta' = 0 \). Squaring and adding the resulting expressions yields the following nonlinear flow-frequency-response equation:

\[
(\Omega^* a^*)^2 + \left[ U^*^2 C_{a^*} - a^* \right]^2 = |\bar{y}_b|^2,
\]

(28)

where

\[
\Omega^* = \frac{\sigma - \zeta_\alpha/\zeta_T}{\zeta_T}, \quad a^* = \frac{a\mu}{\zeta_T}, \quad U^* = \frac{\bar{U}\mu}{\zeta_T}, \quad |\bar{y}_b|^2 = \frac{|\bar{y}_b|\mu}{2\zeta_T^2},
\]

\[
C_{a^*} = \left[ \sum_{n: \text{even}} 4A_n \left( \frac{a^*}{2U^*} \right)^n \sum_{k=0}^{n/2} \left( \frac{-1}{n+1-2k} \right)^n \frac{n+1}{k} \right] + \sum_{n: \text{odd}} A_n \left( \frac{a^*}{2U^*} \right)^n \frac{n+1}{2n+1}.
\]

### 3.2 Stability Analysis

Not all the resulting fixed points are stable resulting in stable periodic responses of the harvester. Therefore, we assess that stability of the resulting asymptotic solutions by evaluating the eigenvalues of the Jacobian matrix associated with Equations (24) and (25), which is given by

\[
J = \begin{bmatrix}
\zeta_T \left( U^{*2} \frac{dC_{a^*}}{da^*} - 1 \right) & \zeta_T^2 |\bar{y}_b|^2 \cos (\zeta_T \delta^*) \\
-|\bar{y}_b|^2 \cos (\zeta_T \delta^*) & -\zeta_T |\bar{y}_b|^2 \sin (\zeta_T \delta^*)
\end{bmatrix}.
\]

The eigenvalues of the Jacobian matrix, \( \lambda_i \), are then obtained by taking the determinant of the Jacobian matrix which yields the following characteristic equation:

\[
\lambda_i^2 - \zeta_T \left[ U^{*2} \left( \frac{C_{a^*}}{a^*} + \frac{dC_{a^*}}{da^*} \right) - 2 \right] \lambda_i + \zeta_T^2 \left[ U^{*2} \left( \frac{C_{a^*}}{a^*} - 1 \right) \left( U^{*2} \frac{dC_{a^*}}{da^*} - 1 \right) \right] = 0.
\]

(29)

By inspecting Equation (29), it can be noted that stable fixed points, and thereby, stable periodic solutions exist when

\[
U^{*2} \left[ \frac{C_{a^*}}{a^*} + \frac{dC_{a^*}}{da^*} \right] < 2, \quad \left[ \Omega^{*2} + \left( \frac{U^{*2} C_{a^*}}{a^*} - 1 \right) \left( U^{*2} \frac{dC_{a^*}}{da^*} - 1 \right) \right] > 0.
\]

### 3.3 Numerical Validation

To validate the asymptotic analytical solutions, the results of the perturbation analysis are compared to a numerical integration of the equations of motion. Two sets of results are presented here to validate the solution with and without the base excitation term as depicted in Fig. 2 (a) and (b), respectively.

Results presented in Fig. 2 are obtained for a harvester with a bluff body of a trapezoidal cross-section (cross-stream rear to front face ratio 3/4, depth to front face ratio 1) whose normal aerodynamic force can be presented using a seventh order polynomial with coefficients, obtained experimentally by Lau et al [24], given as \( A_1 = 2.79, A_2 = 0, A_3 = -84.5, A_4 = 0, A_5 = 1.2388 \times 10^3, A_6 = 0, \) and \( A_7 = -4.994 \times 10^3 \). For the combined loading case, Fig. 2 (b), the dimensionless base displacement parameter is set to \(|\bar{y}_b|^2 = 4.35 \times 10^{-2}\) with \( \sigma = 0 \).
Figure 2: Variation of the dimensionless amplitude of the response with the dimensionless wind speed: (a) without base excitation and (b) with base excitation. Lines represent analytical results: (solid) for stable solutions, (dash-dot) for quasi-periodic solutions and (dash) for saddles. Markers represent numerical results for the periodic responses only: (circle) for forward sweep and (plus) for backward sweep.

The results in Fig. 2 show excellent agreement between the analytical and numerical solutions with and without base excitations and for both branches of solutions generated by forward and backward sweeps of wind speed. This demonstrates the accuracy of the analytical approximation and its ability to predict the periodic responses of the harvester and the various bifurcations occurring in the parameters space. Specifically, it can be clearly seen that for the case considered here, and in the absence of the base excitation, the fixed points undergo a supercritical Hopf bifurcation near \( U^* = 0.38 \). As a result, the static solution loses stability giving way to a dynamic periodic solution whose amplitude increases with the wind speed. Near \( U^* = 0.82 \) the dynamic solution undergoes a cyclic fold bifurcation and the response jumps to a larger-orbit period solution. Further increase in \( U^* \) causes a smooth increase in the harvester’s output following the large orbit branch of solutions.

4 Harvester’s Response in the Absence of Base Excitations

When the harvester is subjected to aerodynamics forces only, i.e., \( |\bar{y}_b|^* = 0 \), the response equation reduces to

\[
U^*2 \frac{C_{a^*}}{a^*} - 1 = 0, \tag{31}
\]

with stable limit-cycle solutions existing when

\[
U^*2 \frac{dC_{a^*}}{da^*} - 1 < 0. \tag{32}
\]

Equation (31) and (32) can be solved together for the amplitude and stability of the periodic responses of the harvester. It is evident that these equations contain only the aerodynamic constants, \( A_n \), characterizing the
aerodynamic force which depends only on the cross section of the bluff body. All other geometric, mechanical,
and electrical properties of the harvester are contained within the parameter, \( U^* \). This leads to the important
collection that the response of all galloping harvesters having the same aerodynamic constants (bluff body)
can be described by a universal curve in the plane \( U^* \times a^* \) irrespective of the other design parameters.

Figure 3 shows examples of normal force coefficient plots and the corresponding universal response curves
of the harvester. To obtain the universal response curve, static wind tunnel tests are first conducted to
characterize the cross-section by constructing the normal force coefficient versus angle of attack curve. This
curve is then approximated in the form of a polynomial function of \( \alpha_0 = \dot{y}/U \) using curve fitting. Once the
empirical coefficients, \( A_n \), are obtained, Equation (31) is solved to generate the universal response curve in
the plane \( U^* \times a^* \). The stability of the solutions is then assessed by utilizing Equation (32).

Depending on the sign and magnitude of the coefficients \( A_n \), which are different for different bluff bodies,
four different possibilities for the universal response of the harvester arise as demonstrated in Fig 3. The
first and most common scenario occurs when \( A_1 = \frac{dC_F y}{d\alpha_0} \bigg|_{\alpha_0=0} > 0 \) and the \( C_F y \) curve is concave up. In this
case, as shown in Fig. 3 (a), as \( U^* \) is increased, a smooth transition from no steady-state oscillations, \( a^* = 0 \),
to periodic oscillations occurs near \( U_0^* \) which represents a supercritical Hopf bifurcation in the parameter
space.

The second case occurs when \( A_1 > 0 \) and the \( C_F y \) curve is concave downward. In this scenario, a subcritical
Hopf bifurcation occurs near \( U_0^* \) which results in a sudden jump in the response from no steady-state
oscillations to large amplitude oscillations as shown in Fig. 3 (b). The third possibility occurs when \( A_1 > 0 \)
with an inflection point in the \( C_F y \) curve. In this case, oscillation hysteresis due to a cyclic-fold bifurcation
can occur in the response curve as demonstrated in the interval between \( U_1 \) and \( U_2 \) in Fig. 3 (c). When this
hysteresis occurs, depending on the actual values of aerodynamic coefficients \( A_n \), the separation between the
bifurcation points is directly proportional to the ratio \( \zeta_T/\mu [25] \). Finally, a fourth possibility can arise when
\( A_1 \leq 0 \) which represents the case of a hard oscillator. In this case, two branches of periodic oscillations are
born at \( U_0^* \), as shown in Fig. 3 (d). By checking their stability, it is found that the lower branch is unstable
while the upper branch is stable. In this situation, the harvester can oscillate only if large initial conditions
are applied. The responses associated with the four scenarios can be further understood by inspecting the
phase portraits of the response for different intervals as shown in Fig. 3.

4.1 The Universal Curve

In the previous section, we noted the presence of a universal curve for galloping oscillators. This is a curve
in the \( U^* \times a^* \) plane which is only sensitive to the geometry of the bluff body, but is otherwise invariant
under any changes in the design parameters. The universal curve was initially identified by Novak [25] in
1969 for galloping oscillators. Here, we show that this universal relation can be extend for galloping energy
harvesters even in the presence of base excitations. We also show that this curve is an invaluable tool which
allows a simple and direct comparative analysis of the performance of galloping energy harvesters. To that
end, we carried an experimental study wherein a galloping energy harvester with a square bluff body is considered. Using static wind tunnel tests, we characterized the normal lift coefficient and found that it can be accurately represented by a third order polynomial in \( \dot{y} \) as following:

\[
C_{F_y} = A_1 \left( \frac{\dot{y}}{U} \right) + A_3 \left( \frac{\dot{y}}{U} \right)^3 ,
\]

with the aerodynamic constants, \( A_1 = 2.5 \), and \( A_3 = -70 \). In this scenario, Equation (31) can be solved to obtain the steady-state amplitudes of the transverse displacement, electric quantity, and harvested power, \( P \) as

\[
\frac{|y|}{y_0} = \frac{|r|}{r_0} = \frac{2}{\sqrt{3}} \sqrt{\frac{U*}{A_3} (A_1 U^* - 1)} , \quad \frac{|P|}{P_0} = \left( \frac{|r|}{r_0} \right)^2 ,
\]

where the corresponding dimensionless quantities are, respectively, given by

\[
y_0 = \frac{\zeta_T D_t}{\mu} , \quad r_0 = \frac{\theta y_0}{C_p \sqrt{1 + \alpha^2}} \text{ (piezoelectric)},
\]

\[
r_0 = \frac{\theta y_0}{L \sqrt{1 + \alpha^2}} \text{ (electromagnetic)}, \quad P_0 = \frac{r_0^2}{R}.
\]
Equation 34 is used to generate the universal curve for the non-dimensional displacement, electric quantity, and power for all galloping harvesters having the same square-sectioned bluff body as shown in Fig. 4. The figure clearly depicts the galloping speed of the harvester and the monotonic increase of the response amplitude with the reduced velocity within the typical range considered.

Next, to verify that this curve is actually universal for all energy harvesters with the same bluff body, experiments with five harvesters of different design parameters and a fixed bluff body are conducted. To change the other parameters of the harvester including its stiffness and damping, two beams of different materials, namely, Steel and Aluminum, are considered. For the Steel beam, three different lengths are used yielding frequencies in a range between 3.1 Hz and 4.1 Hz. The mechanical damping ratio is identified experimentally under short circuit conditions using the logarithmic decrement method and is found to vary between 0.0039 and 0.0043. For the Aluminum beam, two different beam cuts are used with natural frequencies of 3.44 Hz and 4.04 Hz. The estimated damping ratio is found to remain constant at 0.0034. The wind speed is increased incrementally and the steady-state amplitude of the beam tip deflection is measured. The experimental results are then projected into the $U^* \times \left| \frac{y}{y_0} \right|$ plane as shown in Fig. 5 clearly indicating that the data collapse nicely onto effectively a single universal curve for all configurations.

In view of Equation 34 and Fig. 5, it can be concluded that the displacement measurements of a single beam, without any electromechanical transduction components, can be used to estimate the aerodynamic force coefficients and construct the universal response curve of any cross-sectional shape to avoid the static wind tunnel test measurements. This curve, which would be applicable to all harvesters incorporating the same bluff body, can then be used not only to predict the displacement response, but also the voltage and output power for different scales, material properties, and circuit components provided that the quasi-steady
Assumption is valid and all inherent flow conditions remain the same.

As shown in Fig. 6, the universal curve also permits comparing performance of different bluff bodies by simply inspecting variation of $y/y_0$ versus $U^*$ without the need to carry out experiments to determine the actual output voltage and power. For instance, by comparing the curves of Fig. 6, it can be directly concluded that there exists a set of mechanical and electrical design parameters for which an energy harvester with a square-sectioned bluff body will always outperform the ones with a D-shaped and triangular sections even when these are optimally designed. Similarly, it can be also concluded that, for larger wind speeds, a harvester with a 53° isosceles-triangular section bluff body can always be designed to outperform the D-shaped one if both were to be designed using the optimal parameters; whereas, the D-shaped section can always outperform the triangular one at the lower wind speeds provided that both are designed to operate optimally.

5 Influence of the Design Parameters

The aforementioned analytical analysis can now be used to study the influence of the design parameters for any bluff body. The higher the order of the polynomial used to describe the lift coefficient, the more difficult it is to present the results in a succinct form. Here, we present the results for the special case of a cubic symmetric lift curve where only $A_1$ and $A_3$ do not vanish. This lift curve can be used to represent a square-sectioned bluff body.
Figure 6: (Color online) Experimental universal response curves of galloping harvesters with different bluff bodies. Squares for a square section, circles for D-shaped section, and triangles for a 53\degree isosceles-triangular section. Solid lines represent theoretical predictions. In all cases, the bluff body is oriented with the flat surface facing the wind. The maximum turbulence intensity is 5%.

5.1 Electric Parameters

The influence of the electric parameters can be analyzed by investigating the electrical damping which contains the time constant ratio, $\alpha$, and the electromechanical coupling, $\kappa$. The time constant ratio captures the influence of the electric load, $R$, while $\kappa$ represents the strength of coupling between the mechanical and electrical subsystems. To study the influence of the electrical damping on the response of the harvester, it is more convenient to rewrite the power expression of Equation (34) in the following form:

$$\frac{|P|}{P_0^*} = \frac{8}{3A_3} U_* \left( \frac{\zeta_e}{\zeta_m} \right) \left[ A_1 U_* - \left( \frac{\zeta_e}{\zeta_m} \right) - 1 \right],$$  \hspace{1cm} (35)

where $P_0^* = MD_i^2 (\zeta_m \omega_n)^3 / \mu^2$, and $U_* = \mu \bar{U} / \zeta_m$. The reduced cut-in wind speed of the harvester, $U_c$, is obtained by setting the right-hand side of the equation to zero, which yields $U_c = \frac{1}{A_1} \left[ 1 + \left( \frac{\zeta_e}{\zeta_m} \right) \right]$. This implies that, for a given design of the harvester, the minimum cut-in wind speed can be attained when $\zeta_e = 0$, i.e. at short or open circuit. However, at these extreme conditions, the output power of the harvester approaches zero. As the electric-to-mechanical damping ratio is increased, either by increasing the electromechanical coupling or as $\alpha$ approaches one as shown in Fig. 7 (b), the cut-in wind speed increases linearly as depicted in Fig. 7 (a).

The optimal electric-to-mechanical damping ratio which maximizes the harvested output power, can be determined by setting the derivative of Equation (35) with respect to $\zeta_e / \zeta_m$ to zero. This yields $\left( \frac{\zeta_e}{\zeta_m} \right)_{opt}^{opt} = \frac{1}{2} (A_1 U_* - 1)$ at which the maximum output power is $\frac{|P|_{opt}}{P_0^*} = \frac{2}{3A_3} U_* [A_1 U_* - 1]^2$. As shown in Fig. 7 (c), the optimal value of $\zeta_e / \zeta_m$ varies linearly with wind speed whereas the corresponding maximum power
Figure 7: Variation of (a) the cut-in wind speed with the electric-to-mechanical damping ratio (b) the cut-in wind speed with time constant ratio and electromechanical coupling-to-mechanical damping ratio and (c) the optimal electric-to-mechanical damping ratio and the corresponding dimensionless maximum output power with wind speed. Results in (c) are obtained for square-sectioned bluff body with $A_1 = 2.5$ and $A_3 = 70$.

varies quadratically with it. From a practical perspective however, adaptively changing the electric load to optimize the electric damping ratio as the wind speed is varied is very hard to achieve because it requires complex circuit conditioning components.

5.1.1 Optimal Electric Load

The electric load of the harvesting circuit represents one important parameter that is usually optimized to enhance the flow of energy from the environment. Optimizing the output power with respect to the electric load is achieved by substituting $\zeta_e = \alpha \kappa / \left[2(1 + \alpha^2)\right]$ back into the optimal electric-to-mechanical damping ratio and solving for $\alpha_{\text{opt}}$. Analyzing the resulting solutions reveals that the ratio of the electromechanical coupling to the mechanical damping separates the optimization results into two distinctive regions:

When $\left(\frac{\kappa}{\zeta_m}\right) < 2(A_1 U_\star - 1)$, the optimal load resistance embedded within the optimal time constant ratio,
\[ \alpha_{opt}, \text{ and the corresponding maximum output power are given by} \]

\[ \frac{|P^*|}{P_0^*} = \frac{\alpha_{opt} = 1}{3A_3} \left( \frac{\kappa}{\zeta_m} \right)^2 U_* \left[ A_1 U_* - \frac{1}{4} \left( \frac{\kappa}{\zeta_m} \right) - 1 \right]. \]

The previous equations indicate that, for small values of \( \kappa/\zeta_m \), i.e. small coupling to mechanical damping ratio, the output power exhibits a single maximum. This maximum always occurs at the same optimal resistive load corresponding to \( \alpha_{opt} = 1 \), i.e. \( R^* = 1/(C_p\omega_n) \) (piezoelectric) and \( R^* = L\omega_n \) (electromagnetic). This represents traditional linear impedance matching.

On the other hand, when \( \frac{\kappa}{\zeta_m} > 2 (A_1 U_* - 1) \), the optimal load and the maximum output power are given by

\[ \frac{1}{\alpha_{opt}} = \left( \frac{\kappa}{\zeta_m} \right) \pm \sqrt{\left( \frac{\kappa}{\zeta_m} \right)^2 - 4 (A_1 U_* - 1)^2}, \quad \frac{|P^*|}{P_0^*} = \frac{2}{3A_3} U_* [A_1 U_* - 1]^2. \]

Here, the results of the optimization yield two different values for the optimal load with both values providing the same maximum output power. Figure 8 (a) and (b) provide further insights into these optimization results. As \( \kappa/\zeta_m \) is increased, i.e. the electromechanical coupling is increased beyond the critical value \( 2 (A_1 U_* - 1) \), the global maximum value of the power becomes a local minimum, as represented by the dashed-lines in Fig. 8 (a), and two new optimal loads branch out. The value of these two optimal electric loads yield exactly similar values for the maximum output power. As such, the harvesting circuit can be designed with two modes of operation; the high voltage/low current mode or the high current/low voltage mode by utilizing the small or the large optimal loads, respectively.
5.1.2 Optimal Electromechanical Coupling

As shown in Fig. 8 (b), for a given \( U_* \), the maximum output power increases with \( \kappa/\zeta_m \) up to the critical value of 2 \( (A_1 U_* - 1) \). This optimal coupling-to-damping ratio represents the optimal value beyond which the maximum attainable output power saturates and cannot be increased even if the electromechanical coupling is increased. This seemingly counterintuitive results can be explained by realizing that the electromechanical coupling acts as electric damping which, when increased significantly, shifts the cut-in flow speed into higher values as shown in Fig. 7 (a); thereby reducing the net energy transferred from the flow to the harvester.

5.2 Efficiency Estimation at the Optimal Conditions

The total aero-electro-mechanical efficiency of the harvester can be defined as the ratio of the generated electric power to the total input power available in a steady flow. The harvested power at the optimal electric design parameters is given by \(|P^*|\) in Equation (37), while the total input wind power can be defined as \( P_{in} = \frac{1}{2} \rho A_f U^3 \), where \( A_f \) is the frontal area of the harvester in operation which can be related to tip deflection via \( A_f = 2(D_t/2 + |y|)L_t \). After simplification, the total conversion efficiency of the harvester can be written as

\[
\eta = \frac{(A_1 U_* - 1)^2}{A_3 U_*^2 \left[ 2\sqrt{6} \left( \frac{\zeta_m}{\mu} \right) \sqrt{\frac{U_*}{A_3}} (A_1 U_* - 1) + 3 \right]}. \tag{38}
\]

For given aerodynamic coefficients, \( A_1 \), and, \( A_3 \), which represent the shape of the bluff body and its ability to extract energy from the flow, i.e. its aeroelastic conversion efficiency, Equation (38) can be used to predict the total efficiency of the harvester in terms of two dimensionless parameters only; the wind speed \( U_* \) and the mechanical damping to mass ratio \( \zeta_m/\mu \). Figure 9 depicts the efficiency of a harvester with a square-sectioned bluff body with \( A_1 = 2.5 \) and \( A_3 = 70 \). The figure shows that, for a given \( \zeta_m/\mu \), there is an optimal reduced wind speed, \( \bar{U} \), at which maximum conversion efficiency is achieved. Furthermore, by inspecting the variation of the efficiency for a given \( \bar{U} \), one realizes that the efficiency increases significantly as the ratio \( \zeta_m/\mu \) decreases. This can be achieved by minimizing the mechanical damping in the system and/or by increasing the size of the bluff body while simultaneously reducing its mass.

6 Response in the Presence of Base Excitations:

In the presence of base excitations, the response equation, Equation (28), is governed by only three dimensionless loading parameters; the flow \( U^* \), the base displacement \( |\bar{y}_b|^* \), and its frequency \( \Omega^* \). As such, the universal response of all harvesters of a given bluff body shape can be generated in the 4-dimensional parameter space \((\alpha^* \times U^* \times |\bar{y}_b|^* \times \Omega^*)\) allowing the design of efficient harvesters subjected to concurrent loading. Two cases are discussed; the first studies the performance of the harvester under combined loading when the wind speed parameter is below the cut-in wind speed associated with the galloping instability,
i.e. $U^* < 1/A_1$, while the second case studies the performance for wind speeds above the galloping speed $U^* \geq 1/A_1$.

### 6.1 Response below the cut-in wind speed:

When the velocity of the flow is below the galloping speed, i.e. $U^* < 1/A_1$, the self-sustained oscillations cannot be excited and the harvester’s response only contains the frequency of excitation, $\Omega$. As such, the response is always periodic with a frequency matching the excitation frequency. However, for a given wind speed, the amplitude of the harvester’s response $a^*$, and, hence the output power can either be amplified or reduced depending on the sign of the aerodynamic damping represented by the polynomial $(-C_{a*})$. More specifically, for small oscillations’ amplitude, $a^*$, the positive low-order terms will dominate the negative high-order ones making the effective aerodynamic damping, $C_{a*}$, negative. This, in turn, reduces the total damping in the system and causes response amplification. On the other hand, for larger amplitudes, $a^*$, the aerodynamic damping is positive which increases the total damping and causes response reduction.

To demonstrate the influence of the aerodynamic damping on the response of the harvester, a third-order polynomial expansion of $C_{a*}$ is considered, i.e. $C_{a*} = A_1 \left( \frac{a^*}{\mu} \right)^3 + \frac{3}{4} A_3 \left( \frac{a^*}{\mu} \right)^3$. Studying the sign of $C_{a*}$ reveals that, for a given wind speed, there is a critical base displacement, $|\bar{y}_b|_{cr}$ at which $C_{a*} = 0$. At this critical value, the response of the harvester under base excitations is not influenced by the aerodynamic damping. For the case of $\Omega^* = 0$, this critical base displacement can be expressed as $|\bar{y}_b|_{cr} = \sqrt{\frac{4 A_1}{3 A_3}} U^*$. For further demonstration, a harvester of a square-sectioned bluff body with $A_1 = 2.5$ and $A_3 = -70$ is considered. If the harvester is subjected to flow velocity equals half of the cut-in wind speed, i.e. $U^* =$
Figure 10: Variation of the critical base excitation loading term with wind speed $U^*$. (solid-line) for square-sectioned bluff body $(A_1 = 2.5, A_3 = -70)$ and (dashed-line) for bluff body with trapezoidal section $(A_1 = 2.79, A_3 = -84.5, A_5 = 1.2388 \times 10^3, A_7 = -4.994 \times 10^3)$.

$1/(2A_1) = 0.2$, the corresponding critical value of the base displacement is found to be $|\bar{y}_b|_{cr} = 4.36 \times 10^{-2}$, Fig. 10. Below this value the harvester’s response is amplified due to the aerodynamics loading and vise versa.

Figure 11 depicts the frequency-response curves of the harvester for different values of the base displacement. Results are presented for both loading scenarios, i.e. base excitation only (dashed-line) and combined loading with $U^* = 1/(2A_1)$ (solid-line). Results clearly demonstrate that, for small values of $|\bar{y}_b|*$, a harvester produces more power under the combined loading as compared to its output from vibratory excitations only. As the base excitation level is increased, the power amplification decreases until it approaches zero near $|\bar{y}_b|* = |\bar{y}_b|_{cr}$. As the base excitation is increased further, response deamplification occurs. As a result, the harvester produces more power under vibratory excitations only as shown in Fig. 11 (b). It should be noted that the critical base displacement can also be defined for more complex bluff bodies requiring higher order polynomial expansion for the lift force. Such curve has been generated for a a bluff body with a trapezoidal-section, as shown by the dashed-line in Fig. 10.

Figure 12 shows different frequency response curves obtained at a fixed base excitation, when $|\bar{y}_b|* < |\bar{y}_b|_{cr}$, and different wind speeds below the cut-in wind speed. As the wind speed is increased, the amplitude of the harvester’s response increases from shifting the critical base displacement into higher values as shown in Fig. 10. When $U^* = 0$, the peak response at $\Omega^* \approx 0$ is simply given by $a^* = |\bar{y}_b|^*$. This means that the peak dimensionless response equals the dimensionless base displacement input. Moreover, in the case of a third order expansion of the aerodynamic coefficient, the peak response equation is given by

$$\frac{3}{4U^*}A_3a^3 + [1 - A_1U^*]a^* = |\bar{y}_b|^*.$$ (39)

Hence, one can define the amplification factor of the response under the combined loading with respect to
Figure 11: Frequency response curves for $U^* = 0.5/A_1 = 0.2$ and different base excitations $|\bar{y}_b|^*$. (a) below the critical excitation $|\bar{y}_b|_{cr}, |\bar{y}_b|^* = 0.01, 0.02, 0.03, \text{and} 0.0436$. (b) above the critical excitation $|\bar{y}_b|^* = 0.0436, 0.06, \text{and} 0.08$. Solid-line represents response from combined excitations and dashed-line represent response due to base excitation. Results are obtained for square-sectioned bluff body ($A_1 = 2.5, A_3 = -70$).

that from base excitation only at the peak frequency as $f = a^*/|\bar{y}_b|^*$. Substituting $f$ in Equation (39), the peak amplification factor equation can be given by

$$
\frac{3}{4} A_3 \frac{|\bar{y}_b|^*}{U^*} f^3 + [1 - A_1 U^*] f = 1. \tag{40}
$$

Equation (40) is used to study the variation of the peak amplification factor with the input base displacement and wind speed as shown in Fig. 13. The contour lines in the figure represent all combinations of wind speed and base displacement that yield the same amplification factor with the line $f = 1$ representing the critical base displacement as function of wind speed. Figure 13 also serves as a tool to predict the peak response
and the associated harvested power resulting from the combined loading for wind speeds below the cut-in wind speed by simply using $a^* = f \times |\bar{y}_b|^*$. At the cut-in wind speed, $U^* = 1/A_1$, Equation (40) can be solved for the peak amplification factor as $f = \sqrt{\frac{4}{-3A_1A_3|\bar{y}_b|^*}}$. This factor can reach up to 4.65 for the case shown in Fig. 12 indicating that the response of the harvester under base excitation can be amplified 4.65 times when the harvester is subjected to wind loading corresponding to the cut-in wind speed.
Figure 14: (a) Frequency response curves for different base displacements $|\bar{y}_b|^\ast$: 0.01, 0.025, 0.05, 0.1 and fixed wind speed $U^\ast = 1.5/A_1$. Solid lines represent stable periodic solutions, dash-dot lines for unstable quasiperiodic solutions, and dashed line for saddle points. (b) The associated RMS value of the response: circles for $|\bar{y}_b|^\ast = 0.1$ and dashed-line for $|\bar{y}_b|^\ast = 0$. Results are obtained for a harvester with square-sectioned bluff body ($A_1 = 2.5$, $A_3 = -70$).

6.2 Response above the cut-in wind speed:

When the harvester is excited at its base and the wind speed is above the cut-in wind speed, $U^\ast > 1/A_1$, the response contains the excitation frequency, $\Omega$, and the limit-cycle oscillation frequency, $\omega_n$. Consequently, the response can be periodic or quasiperiodic in time depending on the stability of the fixed points, $a^\ast$. If $a^\ast$ is stable then the solution is certainly periodic in time. Otherwise, the response can either undergo a secondary Hopf bifurcation which introduces additional frequencies to the dynamics leading to quasi-periodic responses, or other types of bifurcation that are of lesser importance to the present analysis (e.g. symmetry breaking, cyclic fold, transcritical, etc.). The stability of the resulting solutions can be easily ascertained by the condition given in Equation (30) which depends on the three dimensionless parameters $U^\ast$, $|\bar{y}_b|^\ast$, and $\Omega^\ast$.

Towards investigating the influence of these three parameters on the output of the harvester, the analytical approximation is used to generate the universal frequency-response curves of the harvester at $U^\ast = 1.5/A_1$ and different base displacements as depicted in Fig. 14 (a). Here, stable solutions are presented by solid lines while unstable solutions are presented by dash-dotted lines for the quasi-periodic solutions, and by dashed lines for unstable, physically unrealizable periodic orbits. Figure 14 (a) demonstrates that, at a given $|\bar{y}_b|^\ast$ and for small $|\Omega^\ast|$, the response is always periodic where the free-oscillation component of the response is entrained by the forced component. This, in turn, results in a synchronized periodic response with the response frequency matching the excitation frequency. On the other hand, when $|\Omega^\ast|$ is large, the periodic response loses stability via a secondary Hopf bifurcation, yielding quasiperiodic responses on either side of the symmetric frequency response curve.
Figure 15: (a) Universal response curves as function of base displacement $|\bar{y}_b|^*$ and different wind speeds $U^* = 2/A_1$, $2.5/A_1$, $3/A_1$ for $\Omega^* = 0$. Solid lines represent stable periodic solutions, dash-dot lines for unstable quasiperiodic solutions, and dashed line for saddle points. (b) Contours of the peak amplification factor as function of base displacement $|\bar{y}_b|^*$ and wind speed $U^*$. Results are obtained for a harvester with square-sectioned bluff body ($A_1 = 2.5$, $A_3 = 70$).

Figure 14 (a) also demonstrates that for small values of $|\bar{y}_b|^*$, three branches of solution can coexist. The lower branch represents quasi-periodic responses that extend over the whole range of frequencies, while the higher amplitude solutions represent a branch of stable periodic orbits (upper branch) and a branch of unstable periodic orbits (lower branch) which collide and destruct each other in two cyclic fold bifurcations on either side of $\Omega^* = 0$. As $|\bar{y}_b|^*$ increases, only a single branch of solutions exists for all values of $\Omega^*$. This branch is periodic near $\Omega^* = 0$ but becomes quasi-periodic as $|\Omega^*|$ becomes large. Evidently, the bandwidth of frequencies associated with periodic solutions increases with $|\bar{y}_b|^*$.

From a performance perspective, it should be noted that the total harvested average power will be maximum at resonance, $\Omega^* = 0$, and minimum near the frequency where a transition from stable to unstable response occurs. This can be seen by inspecting the associated RMS value of the response as depicted by circles in Fig. 14 (b). Far away from resonance, the average power corresponding to the quasiperiodic response approaches that resulting from aerodynamic loading only as presented by the $|\bar{y}_b|^* = 0$ dashed-line in Fig. 14 (b).

The analytical approximation can also be used to study the peak response at resonance, $\Omega^* = 0$, and generate the universal response curves for different $U^*$ and $|\bar{y}_b|^*$ as shown in Fig. 15 (a). It is evident that, for the considered range of parameters, the harvester’s response under the combined loading increases with $U^*$ and $|\bar{y}_b|^*$. To measure the effective improvement in performance of the integrated harvester above the cut-in wind speed, a new peak amplification factor can be defined as the ratio between the response under combined loading and the response from galloping excitation only. For the third-order expansion case, these are given by Equation (39) and Equation (34), respectively. Figure. 15 (b) depicts contours of the peak
amplification factor as function of $U^*$ and $|\bar{y}_b|^*$. For a given combination of loading conditions, the response of the harvester can be estimated by multiplying the corresponding amplification factor with the response resulting from galloping which is given by the top-axis for each wind speed.

7 Conclusion

In this effort, we developed an analytical framework to predict and optimize the performance characteristics of galloping energy harvesters. A nondimensional mathematical model describing the dynamics of the harvester when subjected to both galloping and base excitations is presented. The aerodynamic loading is modeled using the quasi-steady assumption. The method of multiple scales is used to obtain analytical expressions governing the steady state response of the harvester. These expressions are then utilized to investigate the response of the harvester in two cases. The first case studies the performance in the absence of base excitations. It is shown that, for a given shape of the bluff body and under quasi-steady flow conditions, the harvesters dimensionless response can be described by a single universal curve irrespective to the geometric, mechanical, and electrical design parameters of the harvester. The universal relationship is validated experimentally and used through optimization analysis to obtain the design parameters that minimize the cut-in wind speed and provide maximum output power. In the second case, the harvester’s response is studied under concurrent loading. It is shown that, the performance of the harvester is governed by three dimensionless loading parameters. The influence of the these parameters on the response is investigated for wind speeds below and above the cut-in wind speed. Results show that, below the cut-in wind speed, there is a critical base-displacement value beyond which response reduction occurs when compared to the response resulting from base-excited harvester. Response curves are also generated for wind speeds above the cut-in wind speed which show that the response can be periodic or quasiperiodic with the peak power obtained near resonance.

Acknowledgement

This material is based upon work supported by the National Science Foundation under CAREER Grant No. 1055419. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.

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