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An out-of-plane linear motion measurement system based on optical beam deflection

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Abstract
Measurement of out-of-plane linear motion with high precision and bandwidth is indispensable for development of precision motion stages and for dynamic characterization of mechanical structures. This paper presents an optical beam deflection (OBD) based system for measurement of out-of-plane linear motion for fully reflective samples. The system also achieves nearly zero cross-sensitivity to angular motion, and a large working distance. The sensitivities to linear and angular motion are analytically obtained and employed to optimize the system design. The optimal shot-noise limited resolution is shown to be less than one angstrom over a bandwidth in excess of 1 kHz. Subsequently, the system is experimentally realized and the sensitivities to out-of-plane motions are calibrated using a novel strategy. The linear sensitivity is found to be in agreement with theory. The angular sensitivity is shown to be over 7.5-times smaller than that of conventional OBD. Finally, the measurement system is employed to measure the transient response of a piezo-positioner, and, with the aid of an open-loop controller, reduce the settling time by about 90%. It is also employed to operate the positioner in closed-loop and demonstrate significant minimization of hysteresis and positioning error.

Keywords: linear motion measurement, optical beam deflection, precision motion control

(Some figures may appear in colour only in the online journal)

1. Introduction
Measurement of out-of-plane linear motion of an object is of central concern to precision engineering. Measurement of linear motion with high precision enables obtaining the displacements of precision motion stages, and subsequently controlling them. Therefore, motion measurement systems are integral components of micro- and nano-positioning stages [1, 2], such as magnetically levitated stages and piezo-actuated flexure stages, that find applications in semiconductor manufacturing and scanning probe microscopy [3, 4]. Likewise, measurement of linear motion with high bandwidth can be employed to characterize the quasi-static and dynamic response of mechanical structures, such as micro-electro-mechanical-systems (MEMS). In particular, the frequency response, mode-shapes and transient responses of the structures can be determined [5–7]. These, in turn, can be employed to obtain accurate models of the structures or to perform structural health monitoring. Thus, in view of the widespread requirement for out-of-plane precision motion measurement, it is desirable to develop measurement systems that are simple but yet, achieve high accuracy and speed. It is also desirable that they be easily adapted for use in a variety of different applications.

Out-of-plane linear motion can be measured either by employing contact type sensors, such as capacitive sensors, linear variable differential transformer (LVDT) and strain gages [8–10], or by means of non-contact type optical sensors. Between the two, the non-contact type sensors are preferred for measurement of motion of MEMS devices since they avoid issues of sensor integration and do not result in mass loading. Optical sensors for out-of-plane displacement can be based either on interferometry [11, 12] or on focus-error detection [13, 14]. Interferometry-based motion measurement is generally performed using optical heterodyne detection, and is the basis for the construction of the laser Doppler vibrometer (LDV) [11, 12].
LDV enables measurement of out-of-plane displacement with a measurement resolution of up to 0.01 nm and range up to 10 cm [12]. However, this technique requires relatively sophisticated optics and high-bandwidth electronics that together render it relatively complex and expensive. Thus, it is primarily employed for characterization of dynamics and generally not for precision motion control. In contrast, the focus error detection technique employs simpler electronics. Further, in a typical system such as the double wedge focus error detection system, the achieved measurement range has been reported to be ±5 μm with a resolution of 0.01 μm [13, 14]. However, owing to the high numerical aperture of the objective lens, the working distance of this system is limited to just a few millimeters. This imposes spatial constraints that limit its applicability.

Optical beam deflection (OBD) [15] is a measurement technique that enables measurement of out-of-plane angular motion of fully reflective samples, by sensing the resulting deflection of a beam of light reflected from the sample surface. Conventionally, the reflected beam is captured on a quadrant photodetector, and the displacement of the spot along two directions is sensed [16]. Thus, OBD possesses significantly simpler optics and electronics compared to other techniques, but yet, achieves sub-micro-radian angular measurement resolution and a measurement bandwidth of over 1 MHz [17]. It is therefore widely employed in scanning probe microscopy to measure the deflection of the micro-cantilever probe, and subsequently control it with sub-angstrom precision [18, 19]. The high resolution, speed and simplicity of construction render OBD an ideal technique for precision motion measurement. Conventionally, however, OBD possesses poor sensitivity to out-of-plane displacement since this does not change the direction of the reflected beam. Further, if a conventional OBD system is employed to measure linear motion, it would possess unacceptably high cross-sensitivity to angular motion, and thereby be of limited practical use.

This paper presents a measurement system based on OBD that enables measurement of out-of-plane displacement with high sensitivity, while ensuring that its cross-sensitivity to angular motion is negligible and ideally zero. The working distance of the system can be designed to be large, thereby minimizing spatial constraints especially when measuring motion of macro-scale objects. The sensitivities of the system to linear and angular motion are theoretically analyzed and employed to optimize its design. Assuming that the resolution is limited only by photodetector shot noise, the system is demonstrated to theoretically achieve sub-angstrom resolution over a bandwidth in excess of 1 kHz. Subsequently, the system is experimentally realized. The linear sensitivity and the angular cross-sensitivity are experimentally calibrated. The experimentally calibrated sensitivities are shown to be in agreement with the theory. The system is employed to characterize the dynamic response of a piezo-actuated positioner and to improve its open-loop response. Finally, the measurement system is employed to implement feedback control of the positioner and significantly minimize the effects of hysteresis and positioning error.

The rest of the paper is divided as follows: section 2 discusses the design, analysis and optimization of the system, and the analysis of its measurement resolution. Section 3 discusses the development and calibration of the measurement system. Subsequently, the application of the measurement system for measurement and control of out-of-plane motion of a piezo-positioner is discussed. Conclusion is presented in section 4.

2. Design and analysis of the measurement system

Section 2.1 discusses the principles of operation of the measurement system, and derives the sensitivity to linear motion and the cross-sensitivity to angular motion. Section 2.2 discusses optimization of the measurement system, and derives the shot noise-limited resolution under the optimal conditions.

2.1. Principle of the measurement system

Figure 1(a) shows a schematic of the proposed measurement system. A laser beam of wavelength $\lambda$ is focused at a point $O$, on a flat, reflective sample whose out-of-plane motion is to be measured. The angle of incidence on the sample surface is $\theta$. The reflected light is passed through a convex lens of focal length $f$ placed at a distance $f_0$ ($> f$) from $O$. A convex mirror of radius of curvature $r_o$ is placed behind the lens such that the focal point of the mirror is coincident with the image $I$ of the point $O$ formed by the convex lens. Finally, the light reflected from the convex mirror is captured by a split photodetector of diameter $D_t$ placed at a distance $f_t$ from the mirror.

Figure 1(b) illustrates the principle of measurement: an out-of-plane displacement $\Delta z$ changes the location from which light is reflected from the sample, thereby changing the position of the image $I$. This, in turn, changes the location at which the laser beam strikes the convex mirror. Due to the spatial variation in the slope of the convex mirror, the beam striking at the new location is reflected in a different direction compared to the original beam. This causes a displacement of the laser spot on the photodetector. Thus, the convex mirror ‘transforms’ the lateral displacement of the laser beam to a change in its direction of propagation. Figure 1(c) illustrates the principle employed to minimize cross-sensitivity to angular motion: an out-of-plane angular motion $\Delta \theta$ of the sample changes the direction of the beam relative to the optic axis of the lens but does not change the position of the image $I$. Since the focal point of the convex mirror is chosen to coincide with $I$, the beam reflected from the mirror does not suffer any change in direction. Consequently, the laser spot shifts by a negligible amount on the photodetector, thereby achieving low sensitivity to angular motion. In addition, it is worth noting that the cross-sensitivity of the system to in-plane motion of a flat sample is zero, since this does not change either the position or the direction of the reflected beam. Thus, the proposed design results in a system that is sensitive only to out-of-plane displacement. It is also worth noting that if the surface roughness of the sample is comparable to the wavelength $\lambda$, the resulting speckle field leads to a spatially random...
intensity distribution within the reflected spot on the detector. As a consequence, the dynamic range of measurement gets restricted by the characteristic spacing between speckles and not by the size of the actual spot [20]. For larger displacements, the resulting random change in intensity distribution on the detector reduces the signal-to-noise ratio. Speckle patterns can be avoided by employing an optical source that has short coherence length, such as a superluminescent diode.

To derive the sensitivity of the measurement system to the displacement \( \Delta z \), it is noted that this displacement causes the virtual image \( O' \) of the point \( O \) in the reflective surface of the sample to shift by \( 2\Delta z \) (figure 2(b)). Thus, the image \( O' \) shifts normal to the optic axis of the convex lens by an amount \( 2\Delta z \sin \theta \). If the magnification of the lens is \( M \), i.e. \( fR - f = M \), the resulting displacement of the image \( I \) normal to the optic-axis of the lens is given by \( \Delta x = 2M \sin \theta \Delta z \). The resulting movement \( \Delta x' \) of the beam on the surface of the convex mirror is obtained to be

\[
\Delta x' = 2M \frac{\sin \theta}{\cos \phi} \Delta z, \tag{1}
\]

where, \( \phi \) represents the angle of incidence of the beam on the convex mirror.

Assuming that the radius of the convex mirror is much greater than the spot diameter on its surface, its surface can be approximated to the paraboloid \( z = -(x^2 + y^2)/2r_c \) in the vicinity of the spot. Thus, any displacement \( \Delta x' \) of the beam along the X-axis of the paraboloid causes a change in angle of reflection \( \Delta \phi \) by the amount

\[
\Delta \phi = -2 \frac{\partial^2 z}{\partial x^2} \Delta x' = 2 \frac{\Delta x'}{r_c}. \tag{2}
\]

The shift in the spot causes it to illuminate the right half of the photodetector more than the left half. If \( V_i (i = A, B) \) are the voltage outputs of each of the halves, the signal \( \Delta V = V_A - V_B \) is proportional to the difference between the

**Figure 1.** Schematics showing (a) the construction of the proposed measurement system (b) strategy employed to achieve high sensitivity to out-of-plane linear motion (c) strategy employed to achieve low sensitivity to out-of-plane angular motion.

**Figure 2.** Graphs showing (a) the dependence of normalized sensitivity \( S/k_{ph} \) on the ratio \( r_c/M \). The other parameters were assumed to be \( \theta = \phi = 45^\circ \) and \( d_0 = 30 \mu m \), (b) the dependence of \( S_{max}/k_{ph} \) on \( d_0 \) and (c) the dependence of \( r_c/M_{opt} \) on \( d_0 \).
optical power \(\Delta P\) incident on the left and right halves of the detector. Assuming that the reflected beam possesses a power \(P\) and a Gaussian intensity distribution with spot size of diameter \(a\) on the detector, the corresponding difference in power \(\Delta P\) is given by [21]

\[
\Delta P \approx 4\sqrt{\frac{P}{\pi}} a \text{erf}\left(\frac{\sqrt{2} D_0}{a}\right)\Delta a.
\]  

(4)

From equations (1)–(4), the output voltage \(\Delta V\) is related to \(\Delta z\) as

\[
\Delta V = \frac{16\sqrt{2}}{\sqrt{\pi}} k_{ph} PM \text{erf}\left(\frac{\sqrt{2} D_0}{a}\right) \frac{\ell_d \sin \theta}{ar \cos \phi} \Delta z = S \Delta z,
\]  

(5)

where, \(S\) represents the sensitivity to displacement measurement and \(k_{ph}\) is the light-to-voltage conversion gain of the detection circuit, given by \(k_{ph} = \eta R_W G\), where \(\eta\) is the spectral responsivity of the detector, \(R_W\) is the current-to-voltage \((I-V)\) conversion gain, and \(G\) is the gain of the amplifier cascaded with the \(I-V\) converter.

It is worth noting that although, in principle, the sensitivity of the system to angular motion is zero, in practice, the difficulty in positioning the convex mirror precisely at the desired position relative to the point \(I\) results in a small cross-sensitivity. If the offset between the point \(I\) and its actual position is \(\delta x\), the beam reflected from the convex mirror experiences an angular deflection \(\Delta \phi\) given by

\[
\Delta \phi = \frac{2}{Mr_c} \delta x \Delta \theta.
\]  

(6)

For this case, the cross-sensitivity \(\delta S_\theta\) of the measurement system, obtained by using equations (3), (4) and (6) is given by

\[
\delta S_\theta = 8\sqrt{2} \frac{k_{ph} P M}{\sqrt{\pi}} \text{erf}\left(\frac{\sqrt{2} D_0}{a}\right) \frac{\ell_d}{r M} \delta x.
\]  

(7)

Thus, it is seen that the cross-sensitivity can be reduced by minimizing the offset \(\delta x\), and enhancing the magnification \(M\).

2.2. Optimization of the measurement system

Equation (5) reveals that the sensitivity of the system can be ensured to be high by choosing the angles \(\theta\) and \(\phi\) to be large and the power \(P\) to be high. To clearly reveal the dependence of \(S\) on the other parameters, equation (5) is rewritten in terms of the divergence \(\theta_0\) of the beam reflected from the convex mirror by noting that \(a/\ell_d \approx 2\theta_0\). This is given by

\[
S = \frac{8\sqrt{2}}{\sqrt{\pi}} k_{ph} P M \text{erf}\left(\frac{\bar{D}_0}{\sqrt{2} \theta_0}\right) \frac{\sin \theta}{\theta_0 \cos \phi},
\]  

(8)

where, \(\bar{D}_d = D_d/\ell_d\). From Gaussian optics [22], the divergence \(\theta_0\) of the reflected laser beam that is focused to a spot of diameter \(d\) on a convex mirror of radius \(r\) can be derived to be

\[
\theta_0 = \frac{a}{2\ell_d} = \frac{2\lambda \sqrt{1 + \pi^2 d^4/4\lambda^2 r_c^2}}{\pi d}.
\]  

(9)

Further, the diameter \(d\) is related to the focused spot diameter \(d_0\) on the sample as

\[
d = Md_0.
\]  

(10)

From equations (9) and (10), it is seen that in the limit \(d^2 \gg \lambda r\), the angle of divergence is dependent on \(r/\lambda M\) and is given by \(\theta_0 \approx Md_0/r\). Further, from equation (8), it is seen that in this limit, the sensitivity \(S\) is also dependent on \(r/\lambda M\).

Thus, figure 2(a) plots the dependence of the normalized sensitivity \(S k_{ph} P\) on \(r/\lambda M\), and reveals that there exists an optimal value \(r/\lambda M_\text{opt}\) for which the sensitivity is maximized. For larger values of \(r/\lambda M\), the sensitivity is reduced due to lesser change \(\Delta \phi\) upon sample displacement. For smaller values of \(r/\lambda M\), the excessive divergence of the beam causes the spot diameter \(a\) to exceed the diameter of the detector, and thus reduces the sensitivity owing to loss of optical power. Figures 2(b) and (c) study the dependence of the maximum normalized sensitivity \(S_\text{max}/k_{ph} P\) and the optimal value of \(r/\lambda M\) on the diameter \(d_0\) of the incident beam, and show that to achieve higher sensitivity, it is desirable to choose a smaller diameter \(d_0\) for the spot incident on the sample, and a corresponding smaller \(r/\lambda M\).

Assuming that, for a particular spot diameter \(d_0\) on the sample, the measurement resolution is limited by the photodiode shot noise of density \(v_n = G R_0 \sqrt{2 e} P\), where \(e\) is the electronic charge, the displacement noise density is given by \(\Delta z_n = v_n/S_\text{max}\). For typical numerical values \(P = 1\) mW, \(\eta = 0.4\) A W\(^{-1}\), \(d_1 = 30\) \(\mu\)m, and \(\bar{D}_d = 1/4\), the displacement noise density is obtained to be \(\Delta z_n \sim 1.4 \times 10^{-13}\) m/\((\text{Hz})\)^\(^{-1}\) while the optimal value of \(r/\lambda M_\text{opt}\) obtained from figure 2(a), is 0.35 mm. Thus, theoretically, it is possible to achieve sub-angstrom measurement resolution over a bandwidth in excess of 1 kHz.

3. Development and evaluation of the measurement system

Section 3.1 discusses the experimental realization of the measurement system. Section 3.2 discusses the calibration of the sensitivity to linear motion and the cross-sensitivity to angular motion. Section 3.3 discusses the use of the measurement system to measure the transient response of a piezo-positioner and to improve its settling time. It also discusses feedback control of the positioner to minimize hysteresis and positioning error.

3.1. Development of the measurement system

In order to develop the measurement system, light beam from a laser diode (wavelength 633 nm, power 5 mW) was first expanded by a factor of 4 and focused on the sample by a convex lens of focal length 5 cm. The resulting spot size \(d_0\) was measured to be in the range 30 – 40 \(\mu\)m. The angle of incidence on the sample was chosen to be \(\theta = 45^\circ\). In order to achieve adequate working distance, the focal length of the lens that collected the reflected light from the sample was
chosen to be $f = 5 \text{ cm}$. A reflective steel sphere of radius $r_c = 500 \mu\text{m}$ (Material: SS 1.4034(AISI 420c), Grade 10, Nanoball GmbH) was chosen as the convex mirror. It is worth noting that an identical measurement system can also be realized by replacing the convex mirror by a convex lens of same focal length, placed at the same distance from the detector. However, this was not adopted owing to the greater difficulty in procuring lens of such small focal length. Based on the spot diameter $d_0$ and the chosen $r_c$, the optimal magnification $M$, obtained from figure 2(c) was in the range 1–1.4. Accordingly, the lens was placed at a distance of $d_O = 8.5 \text{ cm}$ from the point of incidence $O$ on the sample so that the magnification achieved in practice, viz., $M = 1.43$, was close to the theoretical optimum. The corresponding image was formed at the point $I$ located at a distance $d_I = 12 \text{ cm}$ behind the lens. The convex mirror was placed close to the image $I$ such that its focus was approximately coincident with $I$ and its local surface was tilted at $\phi = 45^\circ$ to the laser beam incident on it. The reflected beam was captured by a photodetector (S5991-01, Hamamatsu) placed at a distance $d_d = 4 \text{ cm}$ from the convex mirror.

Figure 3(a) shows the photograph of the developed system. The output of the photodetector was passed through a $I-V$ converter of gain $R_{IV} = 470 \Omega$, amplified further and finally acquired by a real-time controller (DS1103, dSPACE) (figure 3(b)). In addition to data acquisition, the real-time controller enabled generating the necessary control commands to calibrate the measurement system and to actuate stages. These control commands were implemented using MATLAB® SIMULINK®.

3.2. Calibration of the measurement system

3.2.1. Calibration of sensitivity to linear motion. Figure 4(a) shows the schematic of the set-up employed to calibrate the linear sensitivity of the measurement system. The measurement laser beam was focused on a flat silvered polymer surface (DF2000 MA, 3M) mounted on a piezo-based nano-positioner (NanoCube P-611.3S, PI GmbH). The nano-positioner could be operated either in open-loop, or, by employing the in-built strain-gage attached to the piezo-positioner. The amplification gain was set to $G = 10$. Graphs showing (c) the measurement range of the system and (d) the comparison between the measurement after inversion of nonlinearity and the output of the strain-gage attached to the piezo-positioner. The scale on the right shows the error between the two. In (c) and (d), the amplifier gain was set to $G = 1$. The real-time controller was operated at 16 kHz update rate.

In order to determine the measurement range of the system, the gain of the measurement circuit was reduced by a factor of 10 to prevent the electronics from saturating, and the output of the measurement system was recorded in response to a large displacement input from the piezo-positioner. Figure 4(c) plots the response of the system and shows that the measurement range is about $\pm 9 \mu\text{m}$. However, the response is significantly smaller owing primarily to the non-Gaussian intensity distribution within the light spot incident on the detector.

Figure 3. (a) Photograph of the optical setup of the measurement system. The inset shows a magnified image of the convex mirror. (b) Schematic showing the block diagrams of the electronics employed for signal conditioning and data acquisition.

Figure 4. (a) Schematic of the experimental setup for calibration of sensitivity to out-of-plane displacement. Graph showing (b) the response of the measurement system to the applied calibrating input. The amplification gain was set to $G = 10$. Graphs showing (c) the measurement range of the system and (d) the comparison between the measurement after inversion of nonlinearity and the output of the strain-gage attached to the piezo-positioner.
nonlinear beyond ±6 μm since for motion of this range, the displacement of the spot is comparable to the spot diameter on the detector. Consequently, the relationship between the output voltage and the spot displacement is nonlinear [19]. In order to enhance the linear range, the nonlinear behavior was inverted. To invert the nonlinearity, a mathematical equation \( \Delta V = f(\Delta z) \) was obtained for the experimentally calibrated dependence of \( \Delta V \) on \( \Delta z \) using polynomial least-square fit. This was done by means of a ninth order polynomial in the range \(-9 \mu m \leq \Delta z \leq 8.64 \mu m \) where \( \Delta V \) increased monotonically with \( \Delta z \). In subsequent experiments, the displacement \( \Delta z \) corresponding to a measured deflection \( \Delta V \) was estimated as \( \Delta z = f^{-1}(\Delta V) \). To compute \( f^{-1} \), the look-up table block in MATLAB® SIMULINK™ was employed, wherein \( N \) equally spaced displacement values \( \Delta z_i (i = 1 \ldots N) \) within the measurement range were chosen as the output argument to the table while the input argument was defined to be \( \Delta V_i = f(\Delta z_i) \). For measured values \( \Delta V = \Delta V_i \) the block employed interpolation to estimate the corresponding \( \Delta z \). In practice \( N \) was chosen to be 1000. The resulting compensation enabled measurement of motion in a linear manner up to ±8.83 μm. The accuracy of the measurements was verified by comparing it with the pre-calibrated output of the strain-gage integrated with the piezo-positioner (figure 4(d)). It is seen from the figure that the error is small everywhere within the range, with the average being about 0.32 nm, and the maximum error is about 0.306 μm. The large error occurs only near the end of the range, i.e. \( 8.5 \mu m \leq \Delta z \leq 8.64 \mu m \), and is attributed to the imperfect fit of the function \( \Delta V = f(\Delta z) \) to the calibration data in this range.

3.2.2. Calibration of cross-sensitivity to angular motion To calibrate the angular cross-sensitivity of the system, a magnetically actuated micro-cantilever beam (Octosensor, Micromotive GmbH) was mounted on the piezo-positioner and the measurement laser was focused at the end of the beam (figure 5(a)). The micro-cantilever carried a permanent magnetic bead of moment \( m \) that was rigidly attached at the end of the cantilever behind the measurement location, and magnetized along the Z-axis. By applying external magnetic field \( B \) along the X-axis, a torque \( m \times B \) was exerted along the longitudinal (Y-) axis of the cantilever. This enabled twisting the cantilever so that the end of the cantilever rotated by an amount \( \alpha \) given by \( \alpha = mB/k_z \), where \( k_z \) represents the torsional stiffness of the micro-cantilever. In practice, the magnetic moment was chosen to be a permanent magnet micro-bead (MQP-S-11–9, Magnequench) of diameter 55 μm and magnetic moment \( m = 1.6 \times 10^{-8} \text{A} \cdot \text{m}^2 \) (figure 5(b)). A magnetic field of about 1G was applied, thereby causing the beam to twist about its longitudinal axis by 1.4 mrad. To verify that the actuation results substantially in angular motion but not out-of-plane displacement, the deformation profile of the micro-cantilever was measured by means of a laser Doppler vibrometer. Figure 5(c) plots the observed deformation profile and shows that the micro-cantilever undergoes negligible out-of-plane displacement, of just 8 nm, for an angular change of 1 mrad. Subsequently, the micro-cantilever was employed to provide angular change and record the response of the measurement system. The measured sensitivity was found to be 63 mV mrad\(^{-1}\) when the differential amplifier gain \( G \) was chosen to be \( G = 1 \). The corresponding linear motion sensitivity, obtained by following the procedure described in section 3.2.1, was found to be 650 mV μm\(^{-1}\). For a conventional OBD system, whose detector is placed at the same distance \( z_d = 4 \text{ cm} \) from the sample, the theoretical angular sensitivity for the same incident power on the detector was found to be 470 mV mrad\(^{-1}\), while the linear sensitivity, obtained from equation (4) by assuming that the spot displaces on the detector by \( \Delta a = 2 \Delta z \sin \theta \), was 7.8 mV μm\(^{-1}\). Thus, compared to conventional OBD system, it is seen that the measurement system possesses over 83-times greater sensitivity to linear motion and about 7.4-times lesser sensitivity to cross-axis angular motion.

3.3. Measurement and control of a piezo-positioner

In order to demonstrate the applicability of the measurement system, it was employed to characterize and subsequently improve the open-loop dynamics of the piezo-positioner. This was done by first identifying the response of the piezo-positioner by providing a step voltage input \( \Delta V_{\text{TR}} \), and subsequently cascading it with an open-loop controller \( G_s(s) \) that canceled the undesired dynamics of the open-loop system.
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and improved the response (figure 6(a)). Figure 6(b) plots the step response of the system without the controller and shows that it exhibits underdamped oscillations predominantly at $f_1 = 70$ Hz and $f_2 = 240$ Hz, and possesses a settling time of $140$ ms. In order to cancel the underdamped dynamics and improve the settling time, the controller $G_c(s)$ was chosen to be

$$ G_c(s) = \frac{s^2\omega_1^2 + 2\xi_1\omega_1s + 1}{s\omega_1 + 1} \left[ \frac{s^2\omega_2^2 + 2\xi_2\omega_2s + 1}{s\omega_2 + 1} \right], \quad (11) $$

where, $\omega_1 = 2\pi f_1$, $\xi_1 = 0.11$, $\omega_2 = 2\pi f_2$, and $\xi_2 = 0.12$.

Figure 6(b) also plots the step response of the system with the open-loop controller, and shows that the settling time is now reduced to $14.7$ ms, i.e. by a factor of $9.5$ compared to the previous case, and the overshoots are entirely eliminated. This highlights the ability of the measurement system to measure the transient response of motion stages and to aid in their improvement.

The measured displacement of the piezo-positioner was then employed to control the piezo-actuator in closed loop (figure 7(a)). An integral controller of gain $K_I = 10$ was employed to control the motion of the piezo-positioner. Figure 7(b) plots the step response of the piezo-positioner and shows that the closed-loop time-constant is about $100$ ms.

Subsequently, a slow triangular waveform was provided to actuate the piezo-positioner, and its response was recorded both in open-loop and in closed-loop. Figure 7(c) plots the input-output relationship for the two cases and shows that, in open-loop, the response exhibits significant hysteresis, of over $0.32 \mu$m and the response does not follow the provided reference. In contrast, the closed-loop response reveals reduces the hysteresis to $0.1 \mu$m, i.e. by a factor of three, and demonstrates nearly perfect tracking of the provided reference. The hysteresis can be reduced further by employing higher gain for the controller.

4. Conclusion

This paper presented the development and evaluation of a measurement system, based on OBD, to measure out-of-plane linear motion of fully reflective samples. The design of the system ensured high sensitivity to out-of-plane displacement, low cross-sensitivity to angular motion, high bandwidth and large working distance. The sensitivity of the system was theoretically analyzed and subsequently employed to optimize its geometry. The analysis also revealed that the shot-noise limited resolution of the system is less than an angstrom over a bandwidth in excess of $1$ kHz. The optimally designed measurement system was experimentally realized and calibrated. The calibrated result demonstrated a linear response with a range of about $\pm 6 \mu$m and sensitivity comparable to that predicted theoretically, and over 80 times greater than that of conventional OBD. Likewise, experimentally calibrated angular sensitivity was found to be significantly lesser than that of conventional OBD. Finally, the measurement system was employed to compensate for the undesired open-loop dynamics of a piezo-positioner and improve its settling time, and as part of a closed-loop system to minimize hysteretic effect and improve positioning accuracy. By virtue of its high displacement sensitivity, low cross-sensitivities and large working distance, the proposed system can be employed to measure and control out-of-plane motion of precision motion stages. Likewise, by virtue of its high bandwidth and small spot size, it can also be employed to characterize the dynamics of MEMS devices.
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References