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Experimental investigation of the transient phase of the Lorentz force response to the time-dependent velocity at finite magnetic Reynolds number

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Abstract

The working principle of Lorentz force velocimetry (LFV) is based on a linear dependence between measured force and velocity. We consider a case when a violation of that linear law takes place in order to take this effect into account for LFV. The response of the Lorentz force to a time-dependent velocity of solid conducting rods is experimentally studied. Solid conductors were chosen due to the fact that at a limited length of imposed magnetic field the end effects of secondary field generation are identical both in liquid and solid conductors. Thus one can simulate clearly a distortion of the imposed magnetic field in the case of non-stationary fluid flows. The magnetic Reynolds number $Re_m$ based on advection time was of the order of unity. It is demonstrated that magnetic field advection effects lead to a Lorentz force difference in low-$Re_m$ and finite-$Re_m$ cases. The induced magnetic field measurements and experimental estimation of the eddy current density are reported.

Keywords: Lorentz force velocimetry, magnetohydrodynamics, induced magnetic field

(Some figures may appear in colour only in the online journal)

1. Introduction

Lorentz force velocimetry (LFV) is a non-contact method giving a possibility to measure the velocity or mass flux of a conducting fluid [1–3]. This method is very promising for industrial applications because one does not need to immerse a probe in a hot metal since measurements are non-intrusive [4]. The key point in LFV is that force depends linearly on velocity, and this dependence is kept until a flow is stationary. If the flow is non-stationary, an imposed magnetic field can be perturbed by the flow. As a measure of these perturbations one can choose the magnetic Reynolds number $Re_m = \mu \sigma V L$, based on typical values of velocity $V$, electrical conductivity $\sigma$, and characteristic flow size $L$. Usually in a laboratory or industrial conditions $Re_m < 0.1$ (low-$Re_m$ case), i.e. the induced magnetic field is not high enough to change significantly the imposed field distribution because the diffusion and advection times of the magnetic field differ considerably. This means that if the device is calibrated, a conductor velocity can be obtained straightforwardly. However, in a non-stationary case during flow acceleration or deceleration $Re_m$ becomes of the order of unity (finite-$Re_m$ case) and the linear dependence between force and velocity fails. It can happen even at typical flow velocities. If the velocity changes rapidly, eddy currents and a corresponding induced magnetic field $b_i$ become high enough to alter the imposed magnetic field $B_0$. Usually $b_i$ has all three components and one of them counteracts the applied field $B_0$ so that this feedback from the flow results in a relative decrease in the Lorentz force. That leads to a measurement error because the Lorentz force responds differently to the same velocity change in cases when $B_0$ is perturbed. Errors in velocity measurements in metallurgical applications might
lead to an excessive or insufficient amount of an alloy ingredient, and consequently to undesirable properties of metal slabs. Therefore, it is important to know how to control liquid metal flow at any velocity value.

Application of LFV for non-stationary motions has not been studied sufficiently until now. Moreover, the effects of a finite magnetic Reynolds number are of fundamental importance since they play a key role in a non-stationary dynamo [5]. The aim of this work is to reach a finite magnetic Reynolds number when the effects of magnetic field diffusion and advection are commensurable at moderate velocities of a solid conductor and to estimate experimentally the measuring error. Also, we study here the link between the induced magnetic field intensity and a corresponding change in Lorentz force. In sections 2 and 3 we define the problem pointing to the key governing parameters and describe our experimental setup. Then in section 4 we present the Lorentz force measurements and discuss the effects of the induced magnetic field.

2. Problem definition

The considered problem is shown in figure 1. A solid electrically conducting rod which moves through a transverse magnetic field $B_0$ with a time-dependent velocity $V(t)$ is considered. Initially the rod is placed at rest into a non-homogeneous magnetic field $B_0$ created by two permanent magnets. At $t = t_0$ the rod starts to move, which gives rise, according to Ohm’s law, to eddy currents $j$, which are generated in the fringing area of the magnetic field and closed in areas before and behind the magnets where the magnetic field is absent (figure 2(a)). Eddy currents which carry an additional magnetic field $b_i$ interact with the applied field $B_0$. This interaction results
in the Lorentz force \( \bar{F}_L = \int j \times (\bar{B}_0 + \bar{b}_i) \, dV \), which on the one hand acts on the magnets, and on the other hand causes the rod to brake. The magnetic field \( \bar{b}_i \) carried by eddy currents can be strong enough to deform the initial distribution of \( \bar{B}_0 \) so that the magnetic field lines are bent. We note that although the evolution concerns the induced field only, the total field \( \bar{B}_0 + \bar{b}_i \) contributes to the Lorentz force.

The aim is to study the Lorentz force response \( \bar{F}_L(t) \) to the velocity input \( V(t) \) at different magnetic Reynolds numbers \( Re_m \), which have finite values due to a rapid change in the conductor velocity (figure 2(b)). The rod was accelerated within the advection time \( t \approx 80 \text{ ms} \), which was constant, and a condition \( Re_m \approx 1 \) was always fulfilled because \( t \) has an order of diffusion time \( t_{diff} = \mu \sigma D^2 \).

The evolution of the total magnetic field is divided into two phases. The first phase is transient. It starts at \( t = t_0 + \Delta t \) and is characterized by the non-dimensional magnetic Reynolds number \( Re_m^* \) based on the advection time \( \tau \) [6]:

\[
Re_m^* = \frac{\partial \bar{B} / \partial t}{\lambda \Delta \bar{B}} \sim \frac{D^2}{\lambda \tau},
\]

(1)

where \( \lambda = 1/\mu \sigma \) is the magnetic diffusivity, \( \mu = 4\pi \times 10^{-7} \text{ H m}^{-1} \) is the magnetic permeability, \( \sigma \) is the electrical conductivity, and \( D \) stands for a characteristic length. Within this phase, magnetic field lines are dragged with the conductor until the equilibrium between magnetic field advection and diffusion is achieved. When this happens, the second phase starts. This phase is stationary \( (t \gg t_0) \), and is described by the magnetic Reynolds number \( Re_{m}^V \) based on the conductor velocity \( V \) [6]:

\[
Re_{m}^V = \frac{V \times (\bar{V} \times \bar{B})}{\lambda \Delta \bar{B}} \sim \mu \sigma V D.
\]

These two magnetic Reynolds numbers need not necessarily have the same order of magnitude. A ratio \( Re_{m}^*/Re_{m}^V \) is determined by two key values: a time of velocity change and a final value of that velocity.

We briefly note the eddy currents which circulate in the conductor. Since the size of the magnets is finite, there are areas where the imposed magnetic field is homogeneous and non-homogeneous. A conductor’s motion through the area of a non-homogeneous magnetic field leads to the eddy currents \( j_1 \) and \( j_2 \) [7]. The induced magnetic field created by these currents opposes or enhances the external field \( \bar{B}_0 \) (figure 2(a)) resulting in the formation of \( \bar{B} \)-lines bending (figure 1(b)). These end-currents contribute to the Lorentz force, whose response to a time-dependent velocity is also non-stationary (figure 2(b)). In fluid flow, there are additional currents in the uniform area of \( \bar{B}_0 \), whose circulation plane is normal to a velocity vector [8]. But since we consider a solid rod, in the zone of a uniform magnetic field a potential difference is generated only because there are no boundary layers to let currents be closed through them. The charges are distributed through an extremely thin layer on the conductor surface. They constantly leak from the area of uniform magnetic field to a fringing area, but this leakage is always compensated by the induced electric field \( \bar{V} \times \bar{B} \).

3. Experimental setup

The experimental setup (figure 3(a)) consists of two thick aluminum plates with a piezoelectric force sensor PCB 208C01 mounted between them. On the top plate there is a magnetic Halbach array [9], which creates a constant transverse magnetic field in the range from 0 to 1 T depending on the distance.

Figure 3. Experimental setup: (a) a solid electrically conducting rod moves through a transverse magnetic field created by a linear Halbach array. A piezoelectric force sensor is used to measure the Lorentz force acting on the magnet system and the top plate; (b) an array of Hall sensors is mounted between the rod and magnets to measure the induced magnetic field on the background of the imposed \( \bar{B}_0 \). A relative position of the sensors is marked with black dots, and the dotted line represents the position of the magnets. The \( Z \)-axis is parallel to the axis of the rod.
between the magnets. A hole 20 cm in diameter is made in the center of the plates so that a thick massive conducting aluminum or copper rod 4 cm to 8 cm in diameter (table 1) can easily pass through. An array of seven Hall sensors CYTHS124 is installed in the area between the magnets and the rod for the induced magnetic field measurements. Due to a specific arrangement of the magnetization vectors a field distribution has four zones with sharp gradients leading to a higher Lorenz force amplitude. The structure of the magnetic field and the positions of the Hall sensors relative to the magnets are shown in figure 3 (b) (the black dots denote sensors, the dotted line the magnets). The sensors are mounted equidistantly, two of them are outside the initial field, and five are where \( B_0 \) was applied. A three-phase synchronous motor is used to rotate the spindle, which is connected through the cantilever to the rod. This mechanical system converts the spindle rotation velocity to the rod linear velocity. For the detailed description of the drive mechanism see [10]. The rotation velocity of the spindle is controlled by a computer with a 1 kHz frequency so that the rod can be accelerated up to 130 mm s\(^{-1}\) within 80 ms (figure 4).

Each measurement is performed by an 8-channel data logger Graphtec GL 900 with a sampling frequency 50 kHz.

### 4. Results

#### 4.1. Lorentz force measurement

The velocity signal \( V(t) \) of the rod was received directly from the motor which drives the rod. We performed measurements at five different velocity signals (figure 4(a)) having the acceleration time always at about 80 ms. The Lorentz force \( F_L(t) \) as a response (figure 4(b)) to these velocity inputs was measured at different magnetic Reynolds numbers \( \text{Re}_m \) for the copper and aluminum rods, and the force maxima versus the corresponding velocity peaks were plotted (figure 5(a)). As expected, there is a linear dependence between \( F_{\text{max}} \) and \( V_{\text{max}} \) because \( \text{Re}_m \sim N \) (e.g. for Cu rod with \( \sigma = 59 \text{ MS m}^{-1} \) and \( D = 80 \text{ mm} \) we have \( \text{Re}_m = 0.06 \)), and therefore:

\[
F_{\text{max}} \sim \sigma V_{\text{max}} B_0 D^2 l_1.
\]

where \( l_1 \) is the magnet length.

The presence of the transient phase in the response results in the fact that the measured Lorentz force at finite \( \text{Re}_m \) is smaller than the force obtained at the same velocity at low \( \text{Re}_m \) (figure 5(b)). The data in the low- \( \text{Re}_m \) case were obtained in exactly the same way as for the finite- \( \text{Re}_m \) case, but the acceleration of the rod was small enough to have \( \text{Re}_m \ll 1 \). The difference \( \Delta F = F_{\text{ln}} - F_{\text{max}} \sim 1 \) is explained by the interaction between \( B_0 \) and the induced magnetic field \( b_i \). Apart from a general perturbation of \( B_0 \), the magnetic field lines are dragged with the conductor, leading to a decrease in the product \( V \times B_0 \), which is responsible for the eddy current generation.

Figure 6(a) shows that the non-dimensional Lorentz force \( F^* \) decreases as the number \( \tilde{Lu} \) becomes higher. \( F^* \) is defined as the ratio \( F_l/F_{\text{max}} \), where \( F_{\text{max}} \) is taken from (3), and the parameter \( \tilde{Lu} \) is an analog of the Lundquist number in MHD. This represents a ratio between the diffusion time \( t_{\text{diff}} \) and

<table>
<thead>
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<th>Diameter ( D ) (mm)</th>
<th>( D^* )</th>
<th>( \text{Re}_m )</th>
</tr>
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<tr>
<td>40.17</td>
<td>0.61</td>
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<td>50.08</td>
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<td>40.1</td>
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<tr>
<td>50.05</td>
<td>0.66</td>
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</tr>
<tr>
<td>60.1</td>
<td>0.70</td>
<td>3.97</td>
</tr>
</tbody>
</table>
dissipation time $t_{\text{dissip}}$ that a magnetic field needs to convert mechanical energy into Joule heat [8]:

$$\tilde{L}a^2 = \left(\alpha B_0 L \sqrt{\mu / \rho}\right)^2 \frac{t_{\text{dissip}}}{t_{\text{diff}}} \quad \text{(4)}$$

This points to the idea that by means of the Lorentz force it is possible to measure the energy which is dissipated inside a conductor and estimate a value of the eddy currents [11]:

$$\frac{d}{dt} \int \rho V^2 d\Omega = -\frac{1}{\sigma} \int \hat{j}^2 d\Omega \sim F_L \cdot V \quad \text{(5)}$$

Taking $F_L \sim 1 \text{ N}$, $V \sim 80 \text{ mm s}^{-1}$, $D \sim 50 \text{ mm}$, magnet height $l_2 \sim 30 \text{ mm}$ and $\Omega = l_2 \pi D^2/4$, the estimation gives $j \sim 10^5 \text{ A m}^{-2}$. We also note that a higher $Re_m$ leads to a smaller time response of the system. The response is measured by the non-dimensional reaction time $T_{\text{rem}}$ (figure 2(b)), which shows how fast the Lorentz force rises from 0 to 98% of its asymptotic value. It was shown that $T_{\text{rem}}$ strongly decreases as a function of $Re_m$ (figure 6(b)). This stems from a general concept of the frozen state of the magnetic field lines in a conductor [11], i.e. an increase in $Re_m$ makes the system ‘conductor–magnetic field’ more stiff, and as a consequence the non-dimensional saturation time $T_{\text{rem}}$ becomes less.

Concluding this section, we would like to note that there is an excellent agreement between the measured Lorentz force and the force that was obtained analytically in [12] (figure 7). The theory is based on a 1D-model of a conducting plate moving through a homogeneous magnetic field with a time-dependent velocity. The problem is mathematically equivalent to the heat diffusion problem because the advection term in the magnetic field induction equation vanishes. We note that under the 1D-approach, the Lorentz force dynamics is completely governed by $Re_m$, excluding the possibility of studying the finite-$Re_m$ effects. Although the time response of the Lorentz force can be calculated thereby, its theoretically predicted amplitude needs further experimental verification.

### 4.2. Induced magnetic field measurement

Lorentz force dynamics is based on the magnetic field evolution inside and outside a conductor. Therefore, it is important to understand how the conductor changes the initial distribution of $B_0$ (figure 3(b)). Actually, a perturbation of an imposed magnetic field by a flow motion was already observed in [13, 14], but this phenomenon has never been considered as a reason for a Lorentz force measurement error. In order to study this relationship we used a rectangular copper bar ($5 \times 10$ cm) instead of rods. In that case the surface of the conductor is flat, and therefore we can put magnets closer to it, which improves the signal to noise ratio. Using Hall sensors we measured a normal component of $B(t)$ at seven points along the traveling direction of the bar whose $V_{\text{max}} \approx 80 \text{ mm s}^{-1}$. Before the onset of the motion there is no current generation and consequently $\partial B / \partial t \equiv 0$ (figure 8(a)). However, as soon as the bar starts to move, eddy currents ensue, giving rise to the sweeping of the applied magnetic field (figure 8(b)). We observed that $B_0$ increases ‘downstream’ of the rod motion and decreases ‘upstream’ of the rod motion according to figure 2(a). As for the oscillations of $\partial B / \partial t$, they result from the vibrations of the whole construction when the rod is accelerated rapidly along the Z-axis. We note that $\partial B / \partial t$ stops rising at $t \sim t_{\text{diff}}$ (figure 9). By that time the magnetic field has already diffused into a material, so that all non-stationary processes tend to vanish.

Having measured $\partial B / \partial t$ we can verify the current estimation:

$$j = \sigma v = -\frac{\sigma}{l_2} \frac{d\Phi}{dt} \quad \text{(6)}$$

$$\Phi = \int B dS, \quad \text{(7)}$$

where $l_2$ is the thickness of the magnet, and $S$ is the cross-section area of the bar. Combining equations (6) and (7), and having $\partial h / \partial t \sim 5 \text{ mT s}^{-1}, l_2 \sim 30 \text{ mm}$ and $S \sim 5 \text{ cm} \times 10 \text{ cm}$, we obtain $j \sim 5 \times 10^4 \text{ A m}^{-2}$, which justifies the estimation approach (5).
Generally, an important question can arise here: is it possible to predict a drop in the Lorentz force having measured a change in the applied magnetic field outside a conductor? The answer is yes. Taking into account equation (3), and assuming the change of magnetic field 

\[ B = B_0 - b_i \]

we obtain:

\[ F = F_0 \left( 1 - \frac{b_i}{B_0} \right)^2, \]

where \( F_0 \) corresponds to the unperturbed \( B_0 \). But for that we need to know the ratio \( b_i/B_0 \) inside a conductor. This ratio can be measured implicitly only. Assuming that \( b_i \) is carried by a current loop with a radius \( R \), and this loop circulates in the conductor at the distance \( z \) from a Hall sensor that measures \( b_i \), the ratio between the fields inside and outside the conductor is [15]:

\[ \frac{b_{in}}{b_{out}} = \left( 1 + \frac{z}{R} \right)^{3/2}. \]  

(9)

In our case the distance \( z \) is no more than 5 mm, and the radius \( R \) is equal to a characteristic length of the \( B_0 \) decay along the traveling direction of the bar (figure 2). As a rule, this length has the order of magnitude of the gap between the magnets, which in our case is equal to 6 cm, which means:

\[ b_{in} \approx b_{out}. \]  

(10)

Since the initial distribution of \( B_0 \) is well-known, we can estimate the Lorentz force drop according to equation (8). For a copper rod 5 cm in diameter the estimation gives \( \Delta F \approx 0.2 \) N, which is the same order of magnitude as the measured values (figure 5(b)).

5. Conclusions

To sum up, when Lorentz force velocimetry is used to measure a flow velocity, the measurement error can be high in the case of high-speed or non-stationary flow. For example, in the considered case the error has the same order of magnitude as the measured values (figure 5). This problem has been studied in a model experiment where the difference in Lorentz force amplitude at low \( Re_m^\ast \) and finite \( Re_m^\ast \) was measured. The experimental setup comprises moving Al and Cu conducting solid rods (\( \sigma = 20 \) MS m\(^{-1}\) and 59 MS m\(^{-1}\) correspondingly), which experienced an acceleration of up to 1.6 m s\(^{-2}\) within 80 ms. The rods were initially placed in a transverse non-homogeneous magnetic field of up to 1 T. Due to a rapid velocity change, a finite value of \( Re_m^\ast \) was reached so that diffusion and advection of the magnetic field became comparable, and the linear dependence between the Lorentz force and velocity was violated. This should be taken into consideration when evaluating the measurement error. The studied effects in solid conductors are similar to some extent to the effects in...
a conducting fluid, but the details are different. For instance, in fluid flow there are also electric currents induced in the homogeneous part of the magnetic field in a plane normal to the flow direction. These currents close through the Hartmann layers and layers which are parallel to the field. However, the main effects described in this article for solid conductors at finite magnetic Reynolds numbers take place in both cases and can be taken into account while measuring the velocities of non-stationary flows, because fringing effects contribute to the Lorentz force many times more than the effects in the area of the homogeneous magnetic field.

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