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'Buddha's light' of cumulative particles*

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To the memory of Lyonya Kondratyuk, outstanding scientist and person

Abstract

We show analytically that in the cumulative particles production off nuclei multiple interactions leads to a glory-like backward-focusing effect. Employing the small phase space method, we arrived at a characteristic angular dependence of the production cross section $d\sigma \sim 1/\sqrt{\pi - \theta}$ near the strictly backward direction. This effect takes place for any number $n \geqslant 3$ of interactions of rescattered particles, either elastic or inelastic (with resonance excitations in intermediate states), when the final particle is produced near the corresponding kinematical boundary. In the final angles interval, including the value $\theta = \pi$, the angular dependence of the cumulative production cross section can have a crater-like (or funnel-like) form. Such a behaviour of the cross section near the backward direction is in qualitative agreement with some of the available data. Explanation of this effect and the angular dependence of the cross section near $\theta \sim \pi$ are presented for the first time

Keywords: nuclear glory phenomenon, azimuthal focusing, cumulative particles

1. Introduction

Intensive studies of the particles production processes in high-energy interactions of different projectiles with nuclei, in regions forbidden by kinematics for the interaction with a single free nucleon, began back in the 1970s mostly at JINR (Dubna) and ITEP (Moscow). Relatively simple experiments could provide information about such objects as fluctuations of the

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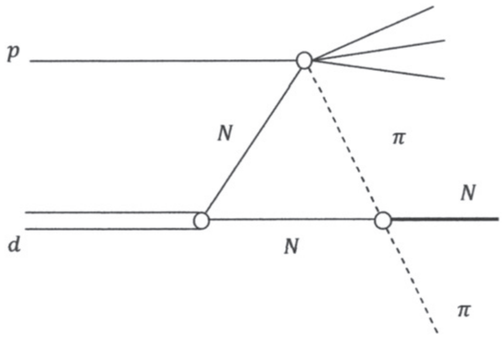

Figure 1. The simplest pion rescattering diagram which leads to the partial fill-up of the 'kinematically forbidden' region for the case of the cumulative pion production by protons on the deuteron [24].

nucleus density [1] or, discussed much later, few nucleon (or multiquark) clusters probably existing in nuclei. At JINR, such processes have been called 'cumulative production' [2, 3], whereas at ITEP the variety of properties of such reactions has been called 'nuclear scaling' [4–6] because certain universalities of these properties have been noted and confirmed somewhat later at much higher energy, with 400 GeV incident protons [7, 8] and 40 GeV/c incident pions, kaons and antiprotons [9, 10]. A new wave of interest in this exciting topic has recently appeared. New experiments have been performed in ITEP [11], aimed at defining the weight of multiquark configurations in the carbon nucleus[4].

The interpretation of these phenomena as being a manifestation of the internal structure of nuclei assumes that the secondary interactions, or, more generally, multiple interaction processes (MIP) do not play a crucial role in such production [12–18]. Generally, the role of secondary interactions in the particles production off nuclei is at least two-fold: they decrease the amount of produced particles in the regions, where it was large (it is, in particular, the screening phenomenon), and increase the production probability in regions where it was small; so, they smash out the whole production picture.

The development of the Glauber approach [19, 20] to the description of particles scattering off nuclei has been considered many years ago as remarkable progress in understanding the particles–nuclei interactions. Within the Glauber model, the amplitude of the particle–nucleus scattering is presented in terms of elementary particle–nucleons amplitudes and the nucleus wave function describing the nucleon distribution inside the nucleus. The Glauber screening correction for the total cross section of particle scattering off deuteron allows widely accepted, remarkably simple and transparent interpretation.

Gribov [21] explained nontrivial peculiarities of the space–time picture of such scattering processes and concluded that inelastic shadowing corrections play an important role at high enough energy and should be included into consideration[5].

In the case of large angle particle production, the background processes that mask the possible manifestations of nontrivial details of nuclear structure are subsequent multiple interactions with nucleons inside the nucleus leading to the particles emission in the 'kinematically forbidden' region. Leonid Kondratyuk was the first to note that rescattering of

[4] We do not pretend here to give a comprehensive review of numerous experiments on cumulative particles production.

[5] The pion double-charge exchange scattering is an interesting example of the reaction where the inelastic intermediate states give the dominant contribution at high enough energy [22, 23].

intermediate particles could lead to the final particles emission in 'kinematically forbidden' regions (KFR). The rigorous investigation of the double interaction process in the case of pion production off deuteron (see figure 1) has been made first by L Kondratyuk and V Kopeliovich in [24]. Later, the multiple interaction processes leading to nucleons production in KFR were investigated in [25] and in more detail in [26], where the magnitude of the cumulative protons production cross sections was estimated, as well.

M A Braun and V V Vechernin *et al* have made many interesting and important observations and have investigated processes leading to the particles emission in KFR [27–35], including the processes with resonances in the intermediate state [27, 28]. They found also that processes with pions in the intermediate state lead to the nucleons emission in KFR due to subsequent processes, such as $\pi N \to N \pi$ [31, 32]. Basic theoretical aspects of MIP leading to the cumulative particles emission and some review of the situation in this field up to 1985 have been presented in [36].

Several authors attempted the cascade calculations of cumulative particles production cross sections relying upon the available computing codes created previously [38–43]. The particles production cross section was found to be in reasonable agreement with the data. Different kinds of subprocesses play a role in these calculations, and certain work should be performed for detailed comparison. In calculations by the NOMAD Collaboration, the particles formation time has been considered as a parameter, and results near to the experimental observations have been obtained for this time equal to $\sim$2 Fm [42, 43]; see the discussion below.

While many authors have admitted the important role of the final state interactions (FSI), most of them did not discuss the active role of such interactions, i.e. their contribution to particles production in KFR; see e.g [44]. It has been stated in a number of papers that multiple interactions cannot describe the spectra of backwards-emitted particles. Such a statement, in fact, has no firm grounds because there were, so far, no reliable calculations of the MIP contributions to the cross sections and other observables in the cumulative particles production reactions. Moreover, such calculations are hardly possible because, as we argue in the current paper, necessary information about elementary interactions amplitudes is still lacking.

Several specific features of the MIP mechanism have previously been noted experimentally and discussed theoretically [26, 36, 37], among them the presence of the recoil nucleons, whose amount grows with the increasing energy of the cumulative particle; the possible large value of the cumulative baryons polarization; and some others; see [36]. The enhancement of the production cross section near the strictly backward direction has been detected in a number of experiments, first at JINR (Dubna) [45, 46] and somewhat later at ITEP (Moscow) [47, 48]. This glory-like effect, which can also be called the 'Buddha's light' of cumulative particles, has been shortly discussed previously in [26, 36]. More experimental evidence of this effect has appeared since that time [49, 50]. Here, we show analytically that the presence of the backward-focusing effect is an intrinsic property of the multiple interaction mechanism leading to the cumulative particles production. The detailed treatment of this effect is presented, including the angular dependence of the particles production cross section near the strictly backward direction. To our knowledge, the proof of the existence of the nuclear glory phenomenon was absent so far in the literature.

In the next section, the peculiarities of the kinematics of the processes in KFR will be recalled, while in section 3, the small phase space method of the MIP contributions calculation to the particles production cross section in KFR is described. In section 4 the focusing effect, similar to what is known in optics as glory phenomenon, is described in detail. The

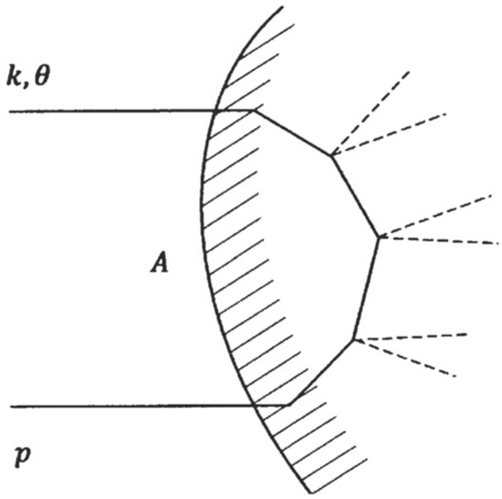

Figure 2. Schematical picture of the multiple interaction process within the nucleus A leading to the emission of the final particle with the momentum k at the angle θ relative to the projectile proton momentum. The binary reactions are assumed to take place in secondary interactions.

final section contains a discussion of problems and conclusions. Some mathematical aspects of the nuclear glory phenomenon are presented in the appendix.

2. Details of kinematics

When the particle with four-momentum $p_0 = (E_0, \vec{p}_0)$ interacts with the nucleus with the mass $m_t \simeq Am_N$, and the final particle of interest has the four-momentum $k_f = (\omega_f, \vec{k}_f)$ the basic kinematical relation is

$$\left(p_0 + p_t - k_f\right)^2 \geqslant M_f^2, \tag{2.1}$$

where M_f is the sum of the final particles masses, except the detected particle of interest. At large enough incident energy, $E_0 \gg M_f$, we obtain easily

$$\omega_f - zk_f \leqslant m_t, \tag{2.2}$$

which is the basic restriction for such processes. $z = \cos\theta < 0$ for particles produced in a backward hemisphere. The quantity $(\omega_f - zk_f)/m_N$ is called the cumulative number (more precisely, the integer part of this ratio plus one).

Let us recall some peculiarities of the multistep processes kinematics established first in [25, 26] and described in detail in [36]. It is very selective kinematics, essentially different from the kinematics of the forward scattering off nuclei when random walking of the particle is allowed in the plane perpendicular to the projectile momentum. Schematically, the multistep process is shown on figure 2.

Rescatterings. For light particles (photon, also π-meson), iteration of the Compton formula

$$\frac{1}{\omega_n} - \frac{1}{\omega_{n-1}} \simeq \frac{1}{m}\Big[1 - \cos\big(\theta_n\big)\Big] \tag{2.3}$$

allows to get the final energy in the form (here $m = m_N$ is the nucleon mass)

$$\frac{1}{\omega_N} - \frac{1}{\omega_0} = \frac{1}{m}\sum_{n=1}^{N}\Big[1 - \cos\big(\theta_n\big)\Big] \tag{2.4}$$

The maximal energy of the final particle is reached for the co-planar process when all scattering processes take place in the same plane and each angle equals to $\theta_k = \theta/N$. As a result, we obtain

$$\frac{1}{\omega_N^{\max}} - \frac{1}{\omega_0} = \frac{1}{m}N\left[1 - \cos\left(\theta/N\right)\right] \tag{2.5}$$

Already at $N > 2$ and for $\theta \leqslant \pi$ the $1/N$ expansion can be made (it is, in fact, the $1/N^2$ expansion):

$$1 - \cos\left(\theta/N\right) \simeq \theta^2\Big/2N^2\Big(1 - \theta^2\big/12N^2\Big) \tag{2.6}$$

and for large enough incident energy ω_0 we obtain

$$\omega_N^{\max} \simeq N\frac{2m}{\theta^2} + \frac{m}{6N}. \tag{2.7}$$

This expression works quite well beginning with $N = 2$. This means that the kinematically forbidden for interaction with single nucleon region is partly filled up due to elastic rescatterings. Remarkably, this rather simple property of rescattering processes has not been even mentioned in the pioneer papers [2–6][6].

In the case of the nucleon–nucleon scattering (scattering of particles with equal nonzero masses in the general case) it is convenient to introduce the factor

$$\zeta = \frac{p}{E + m}, \qquad 1 - \zeta^2 = \frac{2m}{E + m}, \tag{2.8}$$

where p and E are spatial momentum and total energy of the particle, respectively, with the mass m. When scattering takes place on the particle which is at rest in the laboratory frame, the ζ factor of scattered particle is multiplied by $\cos\theta$, where θ is the scattering angle in the laboratory frame. So, after N rescatterings we obtain the ζ factor

$$\zeta_N = \zeta_0 \cos\theta_1 \cos\theta_2 \dots \cos\theta_N. \tag{2.9}$$

As in the case of the small mass of rescattered particle, the maximal value of final ζ_N is obtained when all scattering angles are equal

$$\theta_1 = \theta_2 = \dots = \theta_N = \theta/N, \tag{2.10}$$

and the co-planar process takes place. So, we have

$$\zeta_N^{\max} = \zeta_0\left[\cos\left(\theta/N\right)\right]^N. \tag{2.11}$$

[6] This property was well known, however, to V M Lobashev, who observed experimentally that the energy of the photon after two-fold interaction can be substantially greater than the energy of the photon emitted at the same angle in a one-fold interaction.

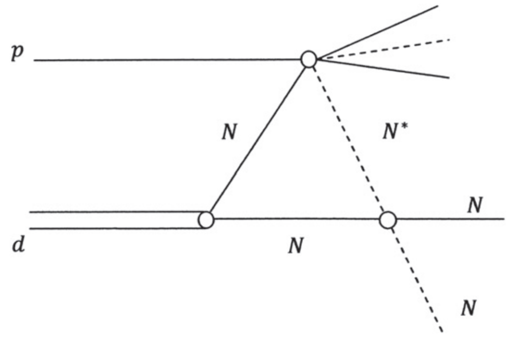

Figure 3. The diagram of the two-fold interaction process on the deuteron with the nucleon resonances (or Δ isobars) excitations in intermediate states.

The final momentum is from (2.11)

$$k^{\max} = 2m\frac{\zeta^{\max}}{1 - \left(\zeta^{\max}\right)^2} \tag{2.12}$$

Again, at large enough N and large incident energy ($\zeta_0 \to 1$), the $1/N^2$ expansion can be made at $k \gg m$, and we obtain the first terms of this expansion

$$k_N^{\max} \simeq N\frac{2m}{\theta^2} - \frac{m}{3N}, \tag{2.13}$$

which coincides at large N with previous result for the rescattering of light particles, but preasymptotic corrections are negative in this case and are twice greater[7].

The normal Fermi motion of nucleons inside the nucleus makes these boundaries wider [36]:

$$k_N^{\max} \simeq N\frac{2m}{\theta^2}\left[1 + \frac{p_F^{\max}}{2m}\left(\theta + \frac{1}{\theta}\right)\right], \tag{2.14}$$

where it is supposed that the final angle θ is large, $\theta \sim \pi$. For numerical estimates we took the step function for the distribution in the Fermi momenta of nucleons inside of nuclei, with $p_F^{\max}/m \simeq 0.27$, see [36] and references there. At large enough N normal Fermi motion makes the kinematical boundaries for MIP wider by about 40%.

There is a characteristic decrease (down-fall) in the cumulative particle production cross section due to simple rescatterings near the strictly backward direction. However, inelastic processes with excitations of intermediate particles, i.e. with intermediate resonances, are able to fill up the region at $\theta \sim \pi$.

Resonance excitations in intermediate states. The elastic rescatterings themselves are only the 'top of the iceberg'. Excitations of the rescattered particles, i.e. production of resonances in intermediate states which go over again into detected particles in subsequent interactions, provide the dominant contribution to the production cross section. The simplest examples of such processes may be $NN \to NN^* \to NN$, $\pi N \to \rho N \to \pi N$, etc. The important role of resonance excitations in intermediate states for cumulative particles production has been noted first by M Braun and V Vechernin [27] and somewhat later in [26]; see figures 3 and 4. At incident energy about few GeV, the dominant contribution into cumulative protons

[7] The preasymptotic corrections given by equations (2.7) and (2.13) are presented here for the first time.

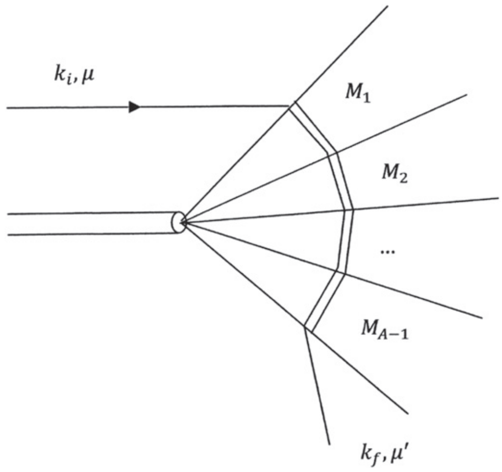

Figure 4. Schematical picture of the multiple interaction process with the resonance excitations in intermediate states. The resonances may either be different or the same in different intermediate states.

emission provide the processes with $\Delta(1232)$ excitation and reabsorption; see [36] and [40]. Experimentally the role of dynamical excitations in cumulative nucleon production at intermediate energies has been established in [51] and, at higher energy, in [52].

When the particles in intermediate states are slightly excited above their ground states, approximate estimates can be made. Such resonances could be $\Delta(1232)$ isobar, or $N^*(1470)$, $N^*(1520)$ etc. for nucleons, two-pion state or $\rho(770)$, etc. for incident pions, $K^*(880)$ for kaons. This case has been investigated previously with the result for the relative change (increase) of the final momentum k_f (equation (8) of [26])

$$\frac{\Delta k_f}{k_f} \simeq \frac{1}{N} \sum_{l=1}^{N-1} \frac{\Delta M_l^2}{k_l^2}, \tag{2.15}$$

or

$$\Delta k_f^2 \simeq \frac{2}{N^3} \sum_{l=1}^{N-1} l^2 \Delta M_l^2, \tag{2.16}$$

with $\Delta M_l^2 = M_l^2 - \mu^2$, k_l is the value of three-momentum in the lth intermediate state. This effect can be explained easily: the additional energy stored in the mass of the intermediate particle is transferred to the kinetic energy of the final (cumulative) particle.

The number of different processes for the N-fold MIP is $(N_R + 1)^{N-1}$, where N_R is the number of resonances making important contributions to the process of interest. The greatest kinematical advantage has the process with resonance production at the $(N - 1)$-th step of the whole process, with its subsequent de-excitation at the last step[8]. To calculate contributions of all these processes, one needs not only to know the cross sections and spin structure of the amplitudes $NN_1^* \to NN_2^*$ at the energies up to several GeV, but also must consider correctly

[8] In some cascade calculations, the important contribution to the cumulative nucleons production gives the process with production of pions of not high-energy with its subsequent absorption by a two-nucleon pair. This process can be, at least partly, part of the processes with resonance formation and reabsorption, because pions of moderate energies are produced mostly via resonance formation and decay to the nucleon and pion.

the possible interference between amplitudes of different processes. Such information is absent and will likely be unavailable in the near future.

To produce the final particle at the absolute boundary available for the nucleus as a whole, one needs to have the masses of intermediate resonances (or some particles system) of the order of incident energy, $s \sim E_0 m_A$.

In this extreme case

$$M_l^2(\max) \simeq s_A \frac{l}{A}\left(1 - \frac{l}{A}\right) \tag{2.17}$$

where $s_A \simeq 2AE_0 m$ ([26], appendix). Interaction with all A nucleons should take place, and the intermediate mass is maximal at $l \sim A/2$. For the deuteron the intermediate mass at the absolute boundary should be

$$M_1^D(\max) \sim s_D/4 \simeq E_0 m/2. \tag{2.18}$$

This case is of academic interest, only. Our aim is to show that the whole region of final particles momenta allowed for interaction with the nucleus as a compact object can be covered due to MIP, but the price for this are the extremely large masses of intermediate states.

What is most important is the following: at arbitrary high incident energy, the kinematics of all subsequent processes are defined by the momentum and the angle of the outgoing particle. In other words, for the nucleus fragmentation with particles emitted backwards with probably large, but limited to a few GeV, energies, the fragmentation of nucleon takes place in the first interaction act of the MIP, according to kinematics analyzed above. The slight dependence of the whole MIP on the incident projectile, hadron or lepton, follows from this observation, as it was noted long ago by Leksin *et al* [4–6].

The theory of elementary particles based on the S-matrix approach operates with so called |in> and <out| states as initial and final states of the process under consideration. It is assumed that there is enough time for the formation of the outgoing particles and the fields surrounding them. Usually, it is in complete correspondence with the experimental conditions, when the elementary interaction amplitude is studied by means of cross sections, polarization observables, etc. measurements.

The situation may be, however, quite different when the interaction of the projectile with nucleons inside the nucleus takes place. The role of the formation time in the interaction of the particle within some medium has been discussed long ago. One of the pioneer papers is the one by Landau and Pomeranchuk [53] where the electromagnetic processes of photon emission and pair production by electrons have been considered. Similar to the case of electromagnetic interactions, the hadron formation time is on the order of

$$\tau^{\text{form}} \sim 1/(\omega - k_z) \tag{2.20}$$

if the incident energy is large enough, where ω and k_z are the energy and the longitudinal momentum of the produced particle, respectively, and the axis z is defined by the momentum of the incident particle. When the particle is produced in the forward direction with large enough energy (momentum), the formation time becomes

$$\tau^{\text{form}} \sim \frac{2\omega}{\mu^2}, \tag{2.21}$$

where μ is the mass of the produced particle. So, formation time, or coherence length in forward direction, become very large for the energetic particle produced in the direction of the projectile momentum (see, e.g [54]. for review of the history of this problem and references. The nuclei fragmentation region has not been discussed in [54]).

As noted above, for the production of a particle on a target with the mass m_t at high enough incident energy, the inequality takes place:

$$\omega - k_z \leqslant m_t, \tag{2.22}$$

at the kinematical boundary, the equality takes place. As we have shown in this section, to produce a final particle beyond the kinematical boundary due to multiple interaction process, in the first interaction, the particle should be produced near the kinematical boundary, i.e.

$$\omega_1 - \cos\theta_1 k_1 \sim m_N, \tag{2.23}$$

therefore, the formation time of the first produced particle

$$\tau_1^{\mathrm{form}} \sim 1/(\omega_1 - \cos\theta_1 k_1) \sim 1/m \tag{2.24}$$

is necessarily small, and the whole production picture is of quasiclassical character. The interesting phenomena observed in the high-energy particles–nuclei interaction reactions and widely discussed in the literature [54], connected with the large formation time of the particles produced in a forward direction, do not take place in the cumulative production processes.

3. The small phase space method for the MIP probability calculations

This method, most adequate for analytical and semi-analytical calculations of the MIP probabilities, has been proposed in [26] and developed later in [36]. It is based on the fact that, according to what was established in [25, 26] and presented in the previous section on kinematical relations, there is a preferable plane of the whole MIP leading to the production of energetic particles at large angle θ, but not strictly backwards. Also, the angles of subsequent rescatterings are close to θ/N. Such kinematics have been called optimal, or basic kinematics. The deviations of real angles from the optimal values are small, and are defined mostly by the difference $k_N^{\max} - k$, where $k_N^{\max}(\theta)$ is the maximal possible momentum reachable for definite MIP, and k is the final momentum of the detected particle. $k_N^{\max}(\theta)$ should be calculated taking into account normal Fermi motion of nucleons inside the nucleus, and also resonances excitation–de-excitation in the intermediate state. Some high power of the difference $(k_N^{\max} - k)/k_N^{\max}$ enters the resulting probability.

Within the quasiclassical treatment adequate for our case, the probability product approximation is valid, and the starting expression for the inclusive cross section of the particle production at large angles contains the product of the elementary subprocess matrix elements squared; see, e.g.; equation (4.11) of [36].

After some evaluation, introducing differential cross sections of binary reactions $d\sigma_l/dt_l(s_l, t_l)$ instead of the matrix elements of binary reactions $M_l^2(s_l, t_l)$, we came to the formula for the production cross section due to the N-fold MIP [26, 36]

$$f_N\left(\vec{p}_0, \vec{k}\right) = \pi R_A^2 G_N\left(R_A, \theta\right) \int \frac{f_1\left(\vec{p}_0, \vec{k}_1\right)\left(k_1^0\right)^3 x_1^2 \mathrm{d}x_1 \mathrm{d}\Omega_1}{\sigma_1^{\mathrm{leav}}\omega_1}$$
$$\times \prod_{l=2}^{N}\left(\frac{\mathrm{d}\sigma_l(s_l, t_l)}{\mathrm{d}t_l}\right)\frac{\left(s_l - m^2 - \mu_l^2\right)^2 - 4\, m^2\mu_l^2}{4\pi m\sigma_l^{\mathrm{leav}}k_{l-1}}$$
$$\times \prod_{l=2}^{N-1}\frac{k_l^2 \mathrm{d}\Omega_l}{k_l\left(m + \omega_{l-1} - z_l\omega_l k_{l-1}\right)}\frac{1}{\omega_N'}\delta\left(m + \omega_{N-1} - \omega_N - \omega_N'\right). \tag{3.1}$$

Here, $z_l = \cos\theta_l$, σ_l^{leav} is the cross section defining the removal (or leaving) of the rescattered object at the corresponding section of the trajectory, it is smaller than the corresponding total cross section. $G_N(R_A, \theta)$ is the geometrical factor that enters the probability of the N-fold multiple interaction with the definite trajectory of the interacting particles (resonances) inside the nucleus. This trajectory is defined mostly by the final values of $\vec{k}$ $(k,\ \theta)$, according to the kinematical relations of the previous section. Inclusive cross section of the rescattered particle production in the first interaction is $\omega_1 \mathrm{d}^3\sigma_1/\mathrm{d}^3k_1 = f_1(\vec{p}_0, \vec{k}_1)$ and $\mathrm{d}^3k_1 = (k_1^0)^3 x_1^2 \mathrm{d}x_1$, $\omega_N = \omega$ —the energy of the observed particle.

To estimate the value of the cross section (3.1), one can extract the product of the cross sections out of the integral (3.1) near the optimal kinematics and multiply by the small phase space available for the whole MIP under consideration [25, 36]. Further details depend on the particular process. For the case of the light particle rescattering, π-meson for example, $\mu_l^2/m^2 \ll 1$, we have

$$\frac{1}{\omega_N{}'}\delta\left(m + \omega_{N-1} - \omega_N - \omega_N{}'\right) = \frac{1}{kk_{N-1}}\delta\left[\frac{m}{k} - \sum_{l=2}^{N}\left(1 - z_l\right) - \frac{1}{x_1}\left(\frac{m}{p_0} + 1 - z_1\right)\right]. \tag{3.2}$$

To get this relation, one should use the equality $\omega_N' = \sqrt{m^2 + k^2 + k_{N-1}^2 - - 2kk_{N-1}z_N}$ for the recoil nucleon energy and the well-known rules for manipulations with the δ-function. When the final angle θ is considerably different from π, there is a preferable plane near which the whole multiple interaction process takes place, and only processes near this plane contribute to the final output. At the angle $\theta = \pi$, strictly backwards, there is azimuthal symmetry and the processes from the whole interval of azimuthal angle $0 < \phi < 2\pi$ contribute to the final output (azimuthal focusing; see next section). A necessary step is to introduce azimuthal deviations from these optimal kinematics, φ_{k}, $k = 1,\ \ldots, N-1$; $\varphi_N = 0$ by definition of the plane of the process, $(\vec{p}_0, \vec{k})$. Polar deviations from the basic values, θ/N, are denoted as ϑ_{k}, obviously, $\sum_{k=1}^{N}\vartheta_k = 0$. The direction of the momentum $\vec{k}_l$ after lth interaction, $\vec{n}_l$, is defined by the azimuthal angle φ_1 and the polar angle $\theta_l = (l\theta/N) + \vartheta_1 + \ldots + \vartheta_l$, $\theta_N = \theta$.

Then, we obtain results by making the expansion in φ_1, ϑ_1 up to quadratic terms in these variables:

$$z_k = \left(\vec{n}_k\vec{n}_{k-1}\right) \simeq \cos(\theta/N)\left(1 - \vartheta_k^2/2\right) - \sin(\theta/N)\vartheta_k$$
$$+ \sin(k\theta/N)\sin[(k-1)\theta/N](\varphi_k - \varphi_{k-1})^2/2. \tag{3.3}$$

In the case of the rescattering of light particles, the sum enters the phase space of the process

$$\sum_{k=1}^{N}\left(1-\cos\vartheta_k\right)=N[1-\cos(\theta/N)]+\cos(\theta/N)\sum_{k=1}^{N}\Big[-\varphi_k^2\sin^2(k\theta/N)+ \\ +\frac{\varphi_k\varphi_{k-1}}{\cos(\theta/N)}\sin(k\theta/N)\sin((k-1)\theta/N)\Big]-\frac{\cos(\theta/N)}{2}\sum_{k=1}^{N}\vartheta_k^2 \quad (3.4)$$

To derive this equality, we used that $\varphi_N=\varphi_0=0$—by definition of the plane of the MIP, and the mentioned relation $\sum_{k=1}^{N}\vartheta_k=0$. We used also the identity, valid for $\varphi_N=\varphi_0=0$:

$$\frac{1}{2}\sum_{k=1}^{N}\left(\varphi_k^2+\varphi_{k-1}^2\right)\sin(k\theta/N)\sin[(k-1)\theta/N]=\cos(\theta/N)\sum_{k=1}^{N}\varphi_k^2\sin^2(k\theta/N). \quad (3.5)$$

It is possible to present the quadratic form in angular variables which enters (3.4) in the canonical form and to perform integration easily; see appendix B and equation (4.23) of [36], and also the appendix in this paper. As a result, we have the integral over angular variables of the following form:

$$I_N\left(\Delta_N^{\rm ext}\right)=\int\delta\left[\Delta_N^{\rm ext}-z_N^\theta\left(\sum_{k=1}^{N}\varphi_k^2-\varphi_k\varphi_{k-1}/z_N^\theta+\vartheta_k^2/2\right)\right]\prod_{l=1}^{N-1}{\rm d}\varphi_l{\rm d}\vartheta_l \\ =\frac{\left(\Delta_N^{\rm ext}\right)^{N-2}\left(\sqrt{2}\pi\right)^{N-1}}{J_N\left(z_N^\theta\right)\sqrt{N}(N-2)!\left(z_N^\theta\right)^{N-1}}, \quad (3.6)$$

$z_N^\theta=\cos(\theta/N)$. Since the element of a solid angle ${\rm d}\Omega_l=\sin(\theta\, l/N){\rm d}\vartheta_l{\rm d}\varphi_l$, we made here the substitution $\sin(\theta\, l/N){\rm d}\varphi_l\to{\rm d}\varphi_l$ and ${\rm d}\Omega_l\to{\rm d}\vartheta_l{\rm d}\varphi_l$, $z_N^\theta=\cos(\theta/N)$. The whole phase space is defined by the quantity

$$\Delta_N^{\rm ext}\simeq\frac{m}{k}-\frac{m}{p_0}-N\left(1-z_N^\theta\right)-\left(1-x_1\right)\frac{m}{p_0} \quad (3.7)$$

which depends on the effective distance of the final momentum (energy) from the kinematical boundary for the N-fold process. The Jacobian of the azimuthal variables transformation squared is

$$J_N^2(z)={\rm Det}\,||a_N||, \quad (3.8)$$

where the matrix $||a_N||$ defines the quadratic form $Q_N(z,\varphi_k)$, which enters the argument of the δ-function in equation (3.6):

$$Q_N(z,\varphi_k)=a_{kl}\varphi_k\varphi_l=\sum_{k=1}^{N}\varphi_k^2-\frac{\varphi_k\varphi_{k-1}}{z}. \quad (3.9)$$

For example,

$$Q_3(z,\varphi_k)=\varphi_1^2+\varphi_2^2-\varphi_1\varphi_2/z;\quad Q_4(z,\varphi_k)=\varphi_1^2+\varphi_2^2+\varphi_3^2 \\ -\left(\varphi_1\varphi_2+\varphi_2\varphi_3\right)/z; \quad (3.9a)$$

see the next section and the appendix.

The phase space of the process in 3.1, which depends strongly on $\Delta_N{}^{\rm ext}$, after integration over angular variables can be presented in the form

$$\Phi_N^{\text{pions}} = \frac{1}{\omega_N'}\delta\left(m + \omega_{N-1} - \omega_N - \omega_N'\right)\prod_{l=1}^{N} \mathrm{d}\Omega_l = \frac{I_N\left(\Delta_N^{\text{ext}}\right)}{kk_{N-1}}$$
$$= \frac{\left(\sqrt{2}\pi\right)^{N-1}\left(\Delta_N^{\text{ext}}\right)^{N-2}}{kk_{N-1}(N-2)!\sqrt{N}J_N\left(z_N^\theta\right)\left(z_N^\theta\right)^{N-1}} \tag{3.10}$$

The normal Fermi motion of target nucleons inside of the nucleus increases the phase space considerably [26, 36]:

$$\Delta_N^{\text{ext}} = \Delta_N^{\text{ext}}\Big|_{p_F=0} + \vec{p}_l^{\,F}\vec{r}_l/2m, \tag{3.11}$$

where $\vec{r}_l = 2m(\vec{k}_l - \vec{k}_{l-1})/k_l k_{l-1}$. A reasonable approximation is to take vectors $\vec{r}_l$ according to the optimal kinematics for the whole process, and the Fermi momenta distribution of nucleons inside of the nucleus in the form of the step function. Integration over the Fermi motion leads to increase of the power of Δ_N^{ext} and change of numerical coefficients in the expression for the phase space. Details can be found in [26, 36], but they are not important for our mostly qualitative treatment here.

For the case of the nucleons rescattering, there are some important differences from the light particle case, but the quadratic form that enters the angular phase space of the process is essentially the same, with an additional coefficient:

$$\Phi_N^{\text{nucleons}} = \frac{1}{k(m+\omega_{N-1})}\int \delta\left[\Delta_{N,\text{nucl}}^{\text{ext}} - \left(z_N^\theta\right)^N Q_N(\varphi_k) - \frac{\left(z_N^\theta\right)^{N-2}}{2}\sum_{l=1}^{N}\vartheta_l^{\,2}\right]\prod_{l=1}^{N}\mathrm{d}\Omega_l$$
$$= \left(\frac{\sqrt{2}\pi}{\zeta_0 z^{N-1}}\right)^{N-1}\frac{\left(\Delta_{N,\text{nucl}}^{\text{ext}}\right)^{N-2}}{(N-2)!\sqrt{N}J_N\left(z_N^\theta\right)}\frac{\left(1-\zeta_N^2\right)\left(1-\zeta_{N-1}^2\right)}{4m^2\zeta_N} \tag{3.12}$$

where

$$\Delta_{N,\text{nucl}}^{\text{ext}} = \zeta_N - \left(1-x_1\right)\zeta_N\frac{1-\zeta_1^2}{1+\zeta_1^2} - \frac{k}{m+\omega}, \tag{3.13}$$

with $\zeta_N = \zeta_0\left(z_N^\theta\right)^N$, $\zeta_1 = \zeta_0 z_N^\theta$. As in the case of the light particle rescattering, the normal Fermi motion of nucleons inside the nucleus can be taken into account.

4. The backward-focusing effect (Buddha's light of cumulative particles)

This is the sharp enhancement of the production cross section near the strictly backward direction, $\theta = \pi$. This effect was first noted experimentally in Dubna (incident protons, final particles pions, protons and deuterons) [45, 46] and somewhat later by Leksin's group at ITEP (incident protons of 7.5 GeV/c, emitted protons of 0.5 GeV/c) [47]. This striking effect was not well studied previously, both experimentally and theoretically. In the papers [26, 36] where the small phase space method has been developed, it was noted that this effect can appear due to multiple interaction processes (see p 122 of [36]). However, the consideration of this effect was not detailed enough; the explicit angular dependence of the cross section near the backward direction, $\theta = \pi$, has not been established, and estimates and comparisons with data have not been made[9].

The backward focusing effect has been observed and confirmed later in a number of papers for different projectiles and incident energies [48–50]. It seems to be difficult to explain the backward focusing effect as coming from interaction with dense few nucleon clusters existing inside the nucleus.

Mathematically, the focusing effect comes from the consideration of the small phase space of the whole multiple interaction process by the method described in the previous section and in [26, 36]. It takes place for any MIP, regardless the particular kind of particles or resonances in the intermediate states. As it was explained in section 2, when the angle of cumulative particle emission is large, but different from $\theta = \pi$, there is a preferred plane for the whole process. When the final angle $\theta = \pi$, then integration over one of azimuthal angles takes place for the whole interval $[0, 2\pi]$, which leads to a rapid increase of the resulting cross section when the final angle θ approaches π.

We show first that the azimuthal focusing takes place for any values of the polar scattering angles $\theta_k^{\rm opt}$. For arbitrary angles $\theta_{\rm k}$ the cosine of the angle between directions $\vec{n}_k$ and $\vec{n}_{k-1}$ is

$$z_k = \left(\vec{n}_k\vec{n}_{k-1}\right) \simeq \cos\left(\theta_k - \theta_{k-1}\right)\left(1 - \vartheta_k^2/2\right) - \sin\left(\theta_k - \theta_{k-1}\right)\vartheta_k + \sin\left(\theta_k\right)\sin\theta_{k-1}(\varphi_k - \varphi_{k-1})^2/2. \tag{4.1}$$

After substitution $\sin\theta_k\varphi_k \to \varphi_k$ we obtain

$$z_k = \left(\vec{n}_k\vec{n}_{k-1}\right) \simeq \cos\left(\theta_k - \theta_{k-1}\right)\left(1 - \vartheta_k^2/2\right) - \sin\left(\theta_k - \theta_{k-1}\right)\vartheta_k + \frac{s_{k-1}}{2s_k}\varphi_k^2 + \frac{s_k}{2s_{k-1}}\varphi_{k-1}^2 - \varphi_{k-1}\varphi_k, \tag{4.2}$$

where we introduced shorter notations $s_k = \sin\theta_k$.

It follows from equation (4.2) that in the general case of arbitrary polar angles $\theta_{\rm k}$, the quadratic form depending on the small azimuthal deviations $\varphi_{\rm k}$ which enters the sum $\sum_k(1 - z_k)$ for the N-fold process is

$$Q_N^{\rm gen}(\varphi_k, \varphi_l) = \frac{s_2}{s_1}\varphi_1^2 + \frac{s_1 + s_3}{s_2}\varphi_2^2 + \frac{s_2 + s_4}{s_3}\varphi_3^2 + ... + \frac{s_{N-2} + s_N}{s_{N-1}}\varphi_{N-1}^2 - 2\varphi_1\varphi_2 - 2\varphi_2\varphi_3 - ... - 2\varphi_{N-2}\varphi_{N-1} = ||a||^{\rm gen}\left(\theta_1, ..., \theta_{N-1}\right)_{kl}\varphi_k\varphi_l, \tag{4.3}$$

[9] One of the authors (VBK) discussed the cumulative (backward) particles production off nuclei with professor Ya A Smorodinsky who noted its analogy with known optical phenomenon—glory, or 'Buddha's light'. The glory effect has been mentioned by Leksin and collaborators [49]; however, it was not clear to the authors of [49], can it be related to cumulative production, or not. In the case of the optical (atmospheric) glory phenomenon, the light scatterings take place within droplets of water or another liquid. A variant of the atmospheric glory theory can be found in [55]. However, the optical glory is still not fully understood; the existing explanation is still incomplete; see, e.g. www.atoptics.co.uk/droplets/glofeat.htm. In nuclear physics, the glory-like phenomenon due to Coulomb interaction has been studied in [57] for the case of low energy antiprotons (energy up to a few KeV) interacting with heavy nuclei.

with $s_N = \sin\theta$. For example, for $N = 5$ we have the matrix

$$\|a\|^{\text{gen}}_{N=5}\left(\theta_1, \theta_2, \theta_3, \theta_4\right) = \begin{bmatrix} s_2/s_1 & -1 & 0 & 0 \\ -1 & (s_1+s_3)/s_2 & -1 & 0 \\ 0 & -1 & (s_2+s_4)/s_3 & -1 \\ 0 & 0 & -1 & (s_3+s_\theta)/s_4 \end{bmatrix}, \tag{4.4}$$

$s_\theta = s_5$, and generalization to arbitrary N is straightforward.

The determinant of this matrix can be easily calculated. It can be shown by induction that at arbitrary N

$$\text{Det}\left(\|a\|^{\text{gen}}_N\right) = \frac{s_\theta}{s_1}, \qquad s_\theta = s_N. \tag{4.5}$$

It follows from the generalized expression (4.4) for the matrix $\|a\|$ that

$$\text{Det}\,\|a\|^{\text{gen}}_{N+1}(\theta) = \frac{s_{N-1} + s_\theta}{s_N}\text{Det}\left(\|a\|^{\text{gen}}_N\right)\left(\theta_N\right) - \text{Det}\left(\|a\|^{\text{gen}}_{N-1}\right)\left(\theta_{N-1}\right), \tag{4.6}$$

where $\theta_{N+1} = \theta$. Since $\text{Det}\left(\|a\|^{\text{gen}}_N\right)(\theta_N) = s_N/s_1$ and $\text{Det}\left(\|a\|^{\text{gen}}_N\right)(\theta_{N-1}) = s_{N-1}/s_1$, we easily obtain

$$\text{Det}\,\|a\|^{\text{gen}}_{N+1}(\theta) = \left(\frac{s_{N-1} + s_\theta}{s_N}\right)\frac{s_N}{s_1} - \frac{s_{N-1}}{s_1} = \frac{s_\theta}{s_1}. \tag{4.7}$$

After integration with the delta-function containing the quadratic form over the small azimuthal deviations, we obtain

$$\begin{aligned} \delta\left(\Delta - \|a\|^{\text{gen}}_N\left(\theta_1,\dots,\theta_{N-1}\right)_{kl}\varphi_k\varphi_l\right)\mathrm{d}\varphi_1\dots\mathrm{d}\varphi_{N-1} &= \frac{\Delta^{(N-3)/2}}{\text{Det}\,\|a\|^{\text{gen}}_N\,(N-3)\,!!}(2\pi)^{(N-3)/2}c_{N-3} \\ &= \sqrt{\frac{s_1}{s_\theta}}\,\frac{\Delta^{(N-3)/2}}{(N-3)\,!!}(2\pi)^{(N-3)/2}c_{N-3}, \end{aligned} \tag{4.8}$$

$c_n = \pi$ for odd n, and $c_n = \sqrt{2\pi}$ for even n, and $N - 3 \geqslant 0$, see the appendix.

We obtain from the above expressions the characteristic angular dependence of the cumulative particles production cross section near $\theta = \pi$:

$$\mathrm{d}\sigma \sim \sqrt{\frac{s_1}{s_\theta}} \simeq \sqrt{\frac{s_1}{\pi - \theta}}, \tag{4.9}$$

since $\sin\theta \simeq \pi - \theta$ for $\pi - \theta \ll 1$.

This formula does not work at $\theta = \pi$, because integration over the azimuthal angle that defines the plane of the whole MIP takes place in the interval $(0, 2\pi)$. The result for the cross section is final, of course, as we show in detail for the case of optimal kinematics.

For the optimal kinematics with equal polar scattering angles $\theta_k = k\theta/N$ (see section 2), and the general quadratic form goes over into the quadratic form obtained in [36] with some coefficients:

$$Q^{\text{gen}} \to 2z^\theta_N Q\left(z^\theta_N, \varphi_k, \varphi_l\right), \qquad z^\theta_N = \cos\left(\theta/N\right), \tag{4.10}$$

and

$$\text{Det}\left(\|a\|^{\text{gen}}_N\right) = \left(2z^\theta_N\right)^{N-1}\text{Det}\left(\|a\|_N\right). \tag{4.10a}$$

It is convenient to present the quadratic form which enters the δ - function in (3.6) as

$$Q_N\left(z_N^\theta, \varphi_k, \varphi_l\right) = J_2^2\left(\varphi_1 - \frac{\varphi_2}{2zJ_2^2}\right)^2 + \frac{J_3^2}{J_2^2}\left(\varphi_2 - \frac{J_2^2\varphi_3}{2zJ_3^2}\right)^2 + \dots$$
$$\dots + \frac{J_{N-1}^2}{J_{N-2}^2}\left(\varphi_{N-2} - \frac{J_{N-2}^2\varphi_{N-1}}{2zJ_{N-1}^2}\right)^2 + \frac{J_N^2}{J_{N-1}^2}\varphi_{N-1}^2. \tag{4.11}$$

For the sake of brevity, we omitted here the dependence of all J_k^2 on their common argument z_N^θ. The recurrent relation

$$J_N^2(z) = J_{N-1}^2(z) - \frac{1}{4z^2}J_{N-2}^2(z) \tag{4.12}$$

can be obtained from (4.11), since, as it follows from (3.6) and (3.9)

$$Q_{N+1}(z, \varphi_k, \varphi_l) = Q_N(z, \varphi_k, \varphi_l) + \varphi_N^2 - \varphi_N\,\varphi_{N-l}/z \tag{4.13}$$

(recall that for the $N+1$-fold process $\varphi_{N+1} = 0$ by definition of the whole plane of the process). The proof of relation (4.12) is given in the appendix.

The following formula for $J_N^2(z_N^\theta)$ has been obtained from [36]:

$$\mathrm{Det}\left|\left|a_{kl}\right|\right| = J_N^2\left(z_N^\theta\right) = 1 + \sum_{m=1}^{m<N/2}\left(-\frac{1}{4\left(z_N^\theta\right)^2}\right)^m \frac{\prod_{k=1}^{m}(N-m-k)}{m!}$$
$$= 1 + \sum_{m=1}^{m<N/2}\left(-\frac{1}{4\left(z_N^\theta\right)^2}\right)^m C_{N-m-1}^m. \tag{4.14}$$

Recurrent relations for Jacobians with subsequent values of N and with the same argument as z:

$$J_{N+1}^2(z) = J_N^2(z) - \frac{1}{4z^2}J_{N-1}^2(z) = J_{N-1}^2(z)\left(1 - \frac{1}{4z^2}\right) - \frac{1}{4z^2}J_{N-2}^2(z) \tag{4.15}$$

can be continued easily to lower values of N and also used for calculations of J_N^2 at any N starting from two known values, $J_2^2(z) = 1$ and $J_3^2(z) = 1 - 1/(4z^2)$ (see the appendix). The equation (4.14) can be confirmed in this way.

The condition $J_N(\pi/N) = 0$ leads to the equation for z_N^π, the solution for which (one of all possible roots) provides the value of $\cos(\pi/N)$ in terms of radicals. The following expressions for these Jacobians take place [26, 36]

$$J_2^2(z) = 1; \qquad J_3^2(z) = 1 - \frac{1}{4z^2}; \qquad J_4^2(z) = 1 - \frac{1}{2z^2}, \tag{4.16}$$

$J_3(\pi/3) = J_3(z = 1/2) = 0$, $J_4(\pi/4) = J_4(z = 1/\sqrt{2}) = 0$. Let us give here less trivial examples. For $N = 5$

$$J_5^2 = 1 - \frac{3}{4z^2} + \frac{1}{16z^4}, \qquad \left(J_5^2\right)'_z = \frac{3}{2z^3} - \frac{1}{4z^5} \tag{4.17}$$

and one obtains $\cos^2(\pi/5) = (3 + \sqrt{5})/8$, $J_5(\pi/5) = 0$.

At $N = 6$

$$J_6^2 = 1 - \frac{1}{z^2} + \frac{3}{16z^4} = J_3^2\left(1 - \frac{3}{4z^2}\right), \qquad \left(J_6^2\right)'_z = \frac{2}{z^3} - \frac{3}{4z^5}. \tag{4.18}$$

see also equation (A.9). For $N = 7$

$$J_7^2 = 1 - \frac{5}{4z^2} + \frac{3}{8z^4} - \frac{1}{64z^6}, \qquad \left(J_7^2\right)'_z = \frac{5}{2z^3} - \frac{3}{2z^5} + \frac{3}{32z^7}. \tag{4.19}$$

$J_7(\pi/7) = 0$.

$$J_8^2 = 1 - \frac{3}{2z^2} + \frac{5}{8z^4} - \frac{1}{16z^6} = J_4^2\left(1 - \frac{1}{z^2} + \frac{1}{8z^4}\right),$$
$$\left(J_8^2\right)'_z = \frac{3}{z^3} - \frac{5}{2z^5} + \frac{3}{8z^7}; \tag{4.20}$$

see equation (A.9); $J_8(\pi/8) = 0$. For arbitrary N, J_N^2 is a polynomial in $1/4z^2$ of the power $\lfloor (N-1)/2 \rfloor$ (integer part of $(N-1)/2$); see equation (4.14). These equations can be obtained using the elementary mathematics methods as well; see in the appendix equations (A.14)–(A.16). The case $N = 2$ is a special one, because $J_2(z) = 1$ is a constant. In this case the two-fold process at $\theta = \pi$ (strictly backwards) has no advantage in comparison with the direct one (see equation (2.5)), if we consider the elastic rescatterings.

For particles emitted strictly backwards, the phase space has a different form, instead of $J_N(\theta/N)$ enters $J_{N-1}(\theta/N)$, which is different from zero at $\theta = \pi$, and we have instead of equation (3.6)

$$I_N(\varphi, \vartheta) = \int \delta\left[\Delta_N^{\rm ext} - z_N^\pi\left(\sum_{k=1}^{N}\varphi_k^2 - \varphi_k\varphi_{k-1}/z_N^\pi + \vartheta_k^2/2\right)\right]\left[\prod_{l=1}^{N-2} {\rm d}\varphi_l {\rm d}\vartheta_l\right] 2\pi {\rm d}\vartheta_{N-1}$$
$$= \frac{\left(\Delta_N^{\rm ext}\right)^{N-5/2}\left(2\sqrt{2}\pi\right)^{N-1}}{J_{N-1}\left(z_N^\pi\right)\sqrt{N}(2N-5)!!\left(z_N^\pi\right)^{N-3/2}}, \tag{4.21}$$

This follows from equation (4.11) where at $\theta = \pi$, the last term disappears, since $J_N(\pi/N) = 0$ and integration over ${\rm d}\varphi_{N-1}$ takes place over the whole 2π interval.

To illustrate the azimuthal focusing that takes place near $\theta = \pi$, the ratio of the phase spaces near the backward direction and strictly at $\theta = \pi$ is useful. The ratio of the observed cross sections in the interval of several degrees slightly depends on the elementary cross sections and is defined mainly by this ratio of phase spaces. It is

$$R_N(\theta) = \frac{\Phi(z)}{\Phi(\theta = \pi)} = \sqrt{\frac{\Delta_N^{\rm ext}}{z_N^\pi}}\frac{(2n-5)!!}{2^{N-1}(N-2)!}\frac{J_{N-1}\left(z_N^\pi\right)}{\sin(\pi/N)J_N\left(z_N^\theta\right)} \tag{4.22}$$

Near $\theta = \pi$, we use that

$$J_N\left(z_N^\theta\right) \simeq \sqrt{\frac{\pi - \theta}{N}\left[J_N^2\right]'\left(z_N^\pi\right)\sin\frac{\pi}{N}} \tag{4.23}$$

and thus we get

$$R_N(\theta) = C_N\sqrt{\frac{\Delta_N^{\rm ext}}{\pi - \theta}} \tag{4.24}$$

with

$$C_N = \frac{J_{N-1}\left(z_N^\pi\right)\sqrt{N}}{\left[\left(J_N^2\right)'\left(z_N^\pi\right)\right]^{1/2}[\sin(\pi/N)]^{3/2}} \frac{(2N-5)\,!!}{\sqrt{z_N^\pi}(N-2)\,!2^{N-1}} \tag{4.25}$$

We also need the values of $J_{N-1}[\pi/N]$ to estimate the behaviour of the cross section near $\theta = \pi$, which are given in table 1. Integration over variable x_1 leads to multiplication C_N by factor $(2N-3)/(2N-2)$, i.e. it makes it smaller, increasing the effect under consideration.

According to equation (4.15), the differential cross section of the cumulative particle production increases with increasing angle θ. At the critical value

$$\theta^{\rm crit} \simeq \pi - C_N^2 \Delta_N^{\rm ext}, \qquad \epsilon^{\rm crit} = \pi - \theta^{\rm crit} \simeq C_N^2 \Delta^{\rm ext} \tag{4.26}$$

it becomes equal to the cross section at $\theta = \pi$, which is proportional to equation (4.12), and may slightly increase further with increasing θ. But near $\theta = \pi$ it should decrease, to become again $d\sigma/d\Omega|_{\theta=\pi}$ at $\theta = \pi$. So, the differential cross section has a crater-like (or funnel-like) form near the backward direction. We do not provide here the detailed description of the cross section in the transition region between $\theta^{\rm crit}$ and $\theta = \pi$: this is technically a rather complicated problem, and not so important for us now.

Characteristic values of $\Delta^{\rm ext}$ are defined by kinematical boundaries described in section 2, equations (2.7), (2.13), and we obtain easily

$$\Delta_{\rm typical}^{\rm ext} \sim \theta^2/2N(N+1) < \pi^2/2[N(N+1)], \tag{4.27}$$

so it is not greater than ~0.5 for $N = 3$ and decreases rapidly with increasing N. Therefore, the values of $\epsilon^{\rm crit}$ may be quite small, about several degrees.

Inclusion of resonance excitation in one (or several) intermediate states leads to the increase of the quantity $\Delta_N^{\rm ext}$ according to formulas of section 2, and to the increase of the phase space of the whole MIP, but the effect of azimuthal focusing persists. Quite similar results can be obtained for the case of nucleons, only some technical details are different; see section 3. The inclusion of the normal Fermi motion of nucleons inside the nucleus increases the values of $\Delta_{\rm ext}^N$, but the numerical coefficient in C_N becomes smaller. The behaviour given by equation (4.15) is in good agreement with the available data, so the value of the constants C_N is not important for our semiquantiatative treatment. The comparison of the observed behaviour with predicted behaviour according to the simple law $d\sigma \sim A + B/\sqrt{\pi - \theta}$ is presented in figures 5, 6 and 7.

We selected several examples where qualitative agreement of data with predicted behaviour takes place. In figure 5 the inclusive cross section of the production of positive pions by projectile protons with momentum 8.9 GeV/c is presented for pions with momentum 0.5 GeV/c (*Pb* as a target) and for pions with momentum 0.3 GeV/c (*He* as a target) [46]. In figure 6, angular distributions of secondary protons with kinetic energy between 0.06 and 0.24 GeV emitted from the Pb nucleus are presented in arbitrary units. The momentum of the projectile protons is 4.5 GeV/c [49]. In figure 7 angular distributions of secondary pions with kinetic energy greater 0.14 GeV emitted from the Pb nucleus, are presented, also in arbitrary units. The momentum of the projectile protons is 4.5 GeV/c. Data are taken from figure 5 of paper [49].

There are other data where the glory-like effect is clearly seen. In many other cases, the flat behaviour of the differential cross section near $\theta \sim \pi$ takes place, but it was probably not sufficient angular resolution to detect the enhancement of the cross section near $\theta = \pi$. In some experiments, the deviation of the final angle from 180° is large; therefore, further measurements near $\theta = \pi$ are desirable, also for kaons and hyperons as cumulative particles.

Table 1. Numerical values of the quantities which enter the particles production cross section near backward direction, $\theta = \pi$. Here, $z_N^\pi = \cos(\pi/N)$.

N	$(J_N^2(z_N^\pi))'$	$\sin(\pi/N)$	$\left[(J_N^2(z_N^\pi))' \sin^3(\pi/N)\right]^{1/2}$	$J_{N-1}[z_N^\pi]$	C_N
3	4	0.866	1.612	1	0.38
4	2.83	0.707	0.999	0.707	0.32
5	2.11	0.588	0.655	0.486	0.29
6	1.540	0.5	0.438	0.333	0.27
7	1.087	0.434	0.298	0.229	0.26

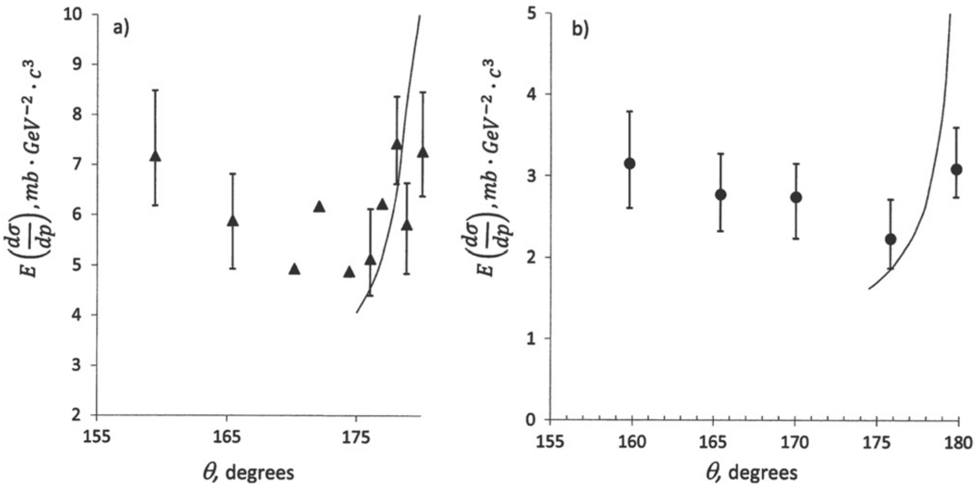

Figure 5. The angular dependence of the inclusive cross section of the production of positive pions by projectile protons with momentum 8.9 GeV/c. (a) pions with momentum 0.5 GeV/c emitted from Pb nucleus. The error bars at some points have not been clearly indicated in the original paper; (b) pions with momentum 0.3 GeV/c emitted from He nucleus. The data are taken from figure 18 of the paper [46].

5. Discussion and conclusions

The nature of the cumulative particles is complicated and not well understood so far. There are different possible sources of their origin, including the color forces [56], one of them are the multiple collisions inside the nucleus, i.e. elastic or inelastic rescatterings. We have shown that the enhancement of the particles production cross section off nuclei near the backward direction, the glory-like backward focusing effect, is a natural property of the multiple interaction mechanism for cumulative particles production. It takes place for any multiplicity of the process, $N \geqslant 3$, when the momentum of the emitted particle is close to the corresponding kinematical boundary. The universal dependence of the cross section, $d\sigma \sim 1/\sqrt{\pi - \theta}$ near the final angle $\theta \sim \pi$, takes place regardless the multiplicity of the process. This statement by itself is quite rigorous and presented for the first time in the literature. The competition of the processes of different multiplicities can make this effect difficult for observation in some cases. Currently we can speak only about qualitative, and in some cases semiquantitative, agreement with data. It is not clear yet how the transition to strictly backward direction proceeds. The angular distribution of emitted particles near $\theta = \pi$

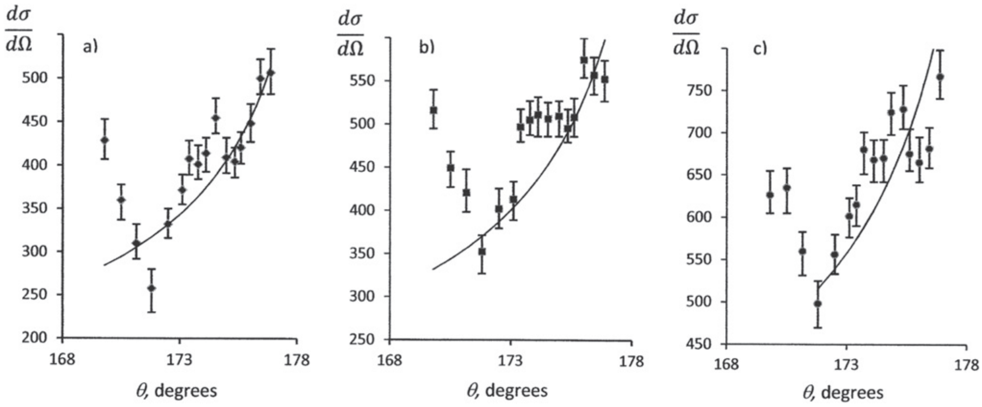

Figure 6. Angular distributions of secondary protons with kinetic energy between 0.06 and 0.24 GeV emitted from the Pb nucleus, in arbitrary units. The momentum of the projectile protons is 4.5 GeV/c. (a) The energy of emitted protons in the interval 0.11 – 0.24 GeV; (b) the energy interval 0.08 – 0.11 GeV; (c) the energy interval 0.06 – 0.08 GeV. Data obtained by G A Leksin group at ITEP, taken from figure 3 of paper [49].

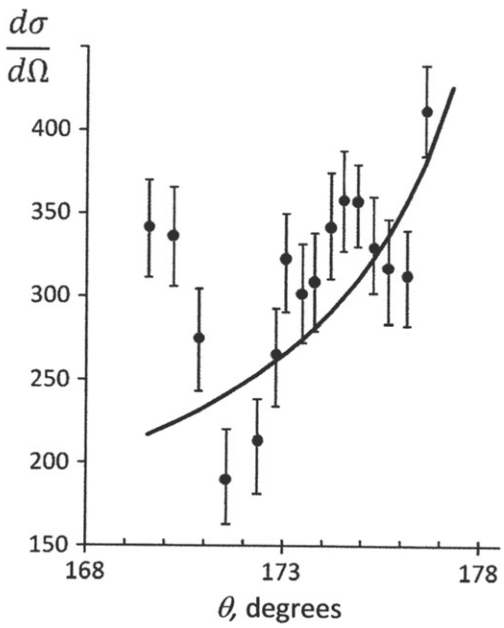

Figure 7. Angular distributions of secondary pions with kinetic energy greater 0.14 GeV emitted from the Pb nucleus, in arbitrary units. The momentum of the projectile protons is 4.5 GeV/c. Data obtained by G A Leksin group at ITEP, taken from figure 5 of paper [49].

can have a narrow dip, i.e. it may be of a crater (funnel)-like form. Further studies are necessary for better understanding.

This effect, observed in a number of experiments at JINR and ITEP, is a clear manifestation of the fact that multiple interactions make important contribution to the cumulative particles production probability, although it does not exclude the contribution of interaction of the projectile with few-nucleon, or multiquark clusters possibly existing in nuclei. We have proved the existence of the azimuthal focusing for arbitrary polar angles (rescattering of the light particles) and for the case of the optimal (basic) configuration of the MIP, also for nucleon rescattering. Investigation of other possible variants of the optimal kinematical configurations, besides those considered in the present paper may be of interest, but obviously, the azimuthal focusing, discussed e.g. in [55] for the optical glory phenomenon, takes place for any kind of MIP; only some technical details are different.

It would be important to detect the focusing effect for different types of produced particles, baryons and mesons. This effect can be considered as a 'smoking gun' of the MIP mechanism. If this nuclear glory-like phenomenon is observed for all kinds of cumulative particles, its universality would be a strong argument in favor of importance of MIP. Reactions where such an effect is not observed would provide more chances for revealing nontrivial peculiarities of nuclear structure. The role of the multiple interaction processes leading to the large-angle particles production off nuclei is certainly still underestimated by many authors, theoreticians and experimentalists. Further efforts are necessary to settle this extremely difficult and important challenge of disentangling between the nontrivial effects of the nuclear structure and the MIP contributions.

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Appendix

Here, we present for the reader's convenience some formulas and relations that have been used in sections 3 and 4.

$$I_n(\Delta) = \int \delta\left(\Delta - x_1^2 - \ldots - x_n^2\right) \mathrm{d}x_1 \ldots \mathrm{d}x_n = \pi \frac{(2\pi)^{(n-2)/2}}{(n-2)\,!!} \Delta^{(n-2)/2} \tag{A.1}$$

for integer even n.

$$I_n(\Delta)_n = \int \delta\left(\Delta - x_1^2 - \ldots - x_n^2\right) \mathrm{d}x_1 \ldots \mathrm{d}x_n = \frac{(2\pi)^{(n-1)/2}}{(n-2)\,!!} \Delta^{(n-2)/2} \tag{A.2}$$

for integer odd n. Relations

$$\int_0^\pi \sin^{2m}\theta \; \mathrm{d}\theta = \pi \frac{(2m-1)\,!!}{(2m)\,!!}; \qquad \int_0^\pi \sin^{2m-1}\theta \; \mathrm{d}\theta = 2\frac{(2m-2)\,!!}{(2m-1)\,!!}, \tag{A.3}$$

m—integer, allow to check (A.1) and (A.2) easily.

The equality takes place

$$\int\delta\left(\Delta - x_1^2 - \ldots - x_n^2\right)\delta(x_1 + x_2 + \ldots + x_n)\mathrm{d}x_1\ldots\mathrm{d}x_{n-1}\mathrm{d}x_n = \frac{1}{\sqrt{n}}I_{n-1}(\Delta) \tag{A.4}$$

More generally, for any quadratic form in variables x_k, $k = 1,\ldots n$ after diagonalization we obtain

$$\int\delta\left(\Delta - a_{kl}x_kx_l\right)\mathrm{d}x_1\,\ldots\,\mathrm{d}x_n = \int\delta\left(\Delta - {x_1}'^2 - \ldots - {x_n}'^2\right)\frac{\mathrm{d}x_1'\ldots\mathrm{d}x_n'}{\sqrt{\det\|a\|}}$$

$$= \frac{1}{\sqrt{\det\|a\|}}I_n(\Delta). \tag{A.5}$$

Let t be the transformation matrix that brings our quadratic form to the canonical form:

$$\tilde{t}\ a\ t = \mathcal{I}, \tag{A.6}$$

where $\mathcal{I}$ is the unit matrix $n \times n$, and $\tilde{t}_{kl} = t_{lk}$. Then the equality takes place for the Jacobian of this transformation

$$(\det\|t\|)^{-2} = J_a^2(z) = \det\|a\|, \qquad (\det\|t\|)^{-1} = J_a(z) = \sqrt{\det\|a\|}. \tag{A.7}$$

To obtain the relation (4.12), we write first the recurrent relation for the quadratic form

$$Q_{N+1}(z, \varphi_k, \varphi_l) = Q_N(\varphi_k, \varphi_l) + \varphi_N^2 - \varphi_N\,\varphi_{N-l}/z, \tag{A.8}$$

then rewrite it similar to equation (4.11) and write down the equality for the last several terms

$$\frac{J_N^2}{J_{N-1}^2}\varphi_{N-1}^2 + \varphi_N^2 - \frac{\varphi_N\varphi_{N-1}}{z} = \frac{J_N^2}{J_{N-1}^2}\left(\varphi_{N-1} - \frac{J_{N-1}^2}{J_N^2}\frac{\varphi_N}{2z}\right)^2 + \frac{J_{N+1}^2}{J_N^2}\varphi_N^2. \tag{A.9}$$

From equality of coefficients before φ_N^2 in the left and right sides we obtain

$$1 = \frac{J_{N-1}^2}{4z^2J_N^2} + \frac{J_{N+1}^2}{J_N^2} \tag{A.10}$$

and equation (4.12) follows immediately.

The relation can be obtained from equation (4.12)

$$J_N^2(z) = J_{N-k}^2(z)J_{k+1}^2(z) - \frac{1}{4z^2}J_{N-k-1}^2(z)J_k^2(z) \tag{A.11}$$

which, at $N = 2m$, $k = m$ (m is the integer), leads to the remarkable relation

$$J_{2m}^2(z) = J_m^2(z)\left(J_{m+1}^2(z) - \frac{1}{4z^2}J_{m-1}^2(z)\right). \tag{A.12}$$

Relation (A.12) can be verified easily for J_4^2, J_6^2 and J_8^2, see section 4. It follows from (A.10) that at $N = 2m$ not only $J_N(\pi/N) = 0$, but also $J_N(2\pi/N) = 0$, which has a quite simple explanation.

For the odd values of N another useful factorization property takes place:

$$J_{2m+1}^2(z) = \left(J_{m+1}^2(z)\right)^2 - \frac{1}{4z^2}\left(J_m^2(z)\right)^2$$

$$= \left(J_{m+1}^2(z) - \frac{1}{2z}J_m^2(z)\right)\left(J_{m+1}^2(z) + \tfrac{1}{2z}J_m^2(z)\right), \tag{A.13}$$

which can be easily verified for J_7^2 and J_5^2 given in section 4.

The polynomials J_N^2 and equations for $z_N^\pi = \cos(\pi/N)$ can be obtained in more conventional way. There is an obvious equality

$$[\exp(\mathrm{i}\pi/N)]^N = \exp(\mathrm{i}\pi) = -1 \tag{A.14}$$

It can be written in the form

$$[\cos(\pi/N) + \mathrm{i}\sin(\pi/N)]^N = -1, \tag{A.15}$$

or separately for the real and imaginary parts

$$\mathrm{Re}\left\{[\cos(\pi/N) + \mathrm{i}\sin(\pi/N)]^N\right\} = -1, \qquad \mathrm{Im}\left\{[\cos(\pi/N) + \mathrm{i}\sin(\pi/N)]^N\right\} = 0. \tag{A.16}$$

The polynomials in $z_N^\pi = \cos(\pi/N)$ which are obtained in the left side of (A.16) coincide with polynomials obtained in section 4. However, some further efforts are necessary to get recurrent relations A.9 and A.10.

The following useful relations can be verified:

$$\left(2z_N^\theta\right)^{N-1} J_N^2\left(z_N^\theta\right) \sin\frac{\theta}{N} = \sin\theta. \tag{A.17}$$

These relations provide the link between the general case considered at the beginning of section 4 and the particular case of the optimal kinematics with all scattering angles equal to θ/N.

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