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The ambient dose equivalent at flight altitudes: a fit to a large set of data using a Bayesian approach

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Abstract

The problem of finding a simple, generally applicable description of worldwide measured ambient dose equivalent rates at aviation altitudes between 8 and 12 km is difficult to solve due to the large variety of functional forms and parametrisations that are possible. We present an approach that uses Bayesian statistics and Monte Carlo methods to fit mathematical models to a large set of data and to compare the different models. About 2500 data points measured in the periods 1997–1999 and 2003–2006 were used. Since the data cover wide ranges of barometric altitude, vertical cut-off rigidity and phases in the solar cycle 23, we developed functions which depend on these three variables. Whereas the dependence on the vertical cut-off rigidity is described by an exponential, the dependences on barometric altitude and solar activity may be approximated by linear functions in the ranges under consideration. Therefore, a simple Taylor expansion was used to define different models and to investigate the relevance of the different expansion coefficients. With the method presented here, it is possible to obtain probability distributions for each expansion coefficient and thus to extract reliable uncertainties even for the dose rate evaluated. The resulting function agrees well with new measurements made at fixed geographic positions and during long haul flights covering a wide range of latitudes.

1. Introduction

Over the last decade, an extensive worldwide net of dose and dose rate measurements at flight altitudes has become available. Several groups have performed measurements on board aircraft using different dosemeter systems. A comprehensive overview of measurements performed up to 2003 is given in a report published by the European Commission [1] in 2004. In Europe, these efforts were initiated in response to the directive 96/29/EURATOM [2] which requires the dose assessment of occupationally exposed persons even when they are only exposed to natural sources of ionising radiation. As a consequence of this directive, the question of dose assessment for air crew working in the radiation field of secondary cosmic radiation at altitudes around 10–12 km became an important issue in Europe.

In Germany, the directive 96/29/EURATOM was implemented by a revision of the Radiation Protection Ordinance in 2001; see in particular section 103 which describes the radiation protection of air crew members. Based on the recommendation of the German Radiation Protection Commission [3], doses for air crew may be assessed by calculations according to the corresponding flight routes. But any software used for this purpose must be approved by the national aviation authority, i.e. the Luftfahrt-Bundesamt (LBA). For the approval, two criteria must be fulfilled: (1) the software packages must be tested according to common rules for software evaluation (non-functional testing) and (2) the calculated routes doses or dose rates must be distributed around measured values within an interval of $\pm 30\%$ (functional testing). It is especially for the latter purpose that the Physikalisch-Technische Bundesanstalt (PTB) maintains several dosemeter systems to measure the ambient dose equivalent at aviation altitudes. Only the quantity ambient dose equivalent $H^*(10)$ or its rate is used for functional testing. Therefore, any instrument used must be able to measure the dose or dose rate in terms of $H^*(10)$ and the instrument must be traceable to a national standard as, for example, provided by PTB.

In this paper we describe a method to fit a mathematical model to the complete data set measured by PTB using all relevant influence parameters such as altitude, vertical cut-off rigidity and neutron monitor rate. The aim is to have an easy to handle functional description of the worldwide dose rate distribution at flight altitudes and at different solar activity. Since we approach the problem with Bayesian statistical methods, we have tools available to investigate and compare different competing mathematical models that may be used to describe the data and we can evaluate an uncertainty for each dose rate value calculated with the final model function. Based on the mathematical model that was optimal according to our calculations, the computer code 'Flight DOSe calculator" (FDOScalc) was developed and tested with recently measured data at constant flight conditions and on flights to Cape Town (South Africa) and Bangkok (Thailand).

Throughout this work *altitude* represents the barometric altitude expressed in terms of flight level FL. Constant flight level means that the aircraft flies at an isobar. In a standard atmosphere, the altitude in units of flight level can be transformed into units of metre via the conversion coefficient 30.48 m/FL.

2. Summary of the PTB data

From 1997 to 1999, a combination of an ionisation chamber and a modified moderator-type neutron area monitor with increased sensitivity to the high-energy neutron component was used for measurements on 39 flight legs from Frankfurt (Germany) to North America, South America and Asia [4]. Both readings were added and then compared to a tissue equivalent proportional counter (TEPC) leading to an 'in-field calibration factor' of (1.0 ± 0.1). This method has the advantage that dose rate values with a relative statistical uncertainty of less than $\pm 10\%$ can be obtained in small time intervals, i.e. 15 min, owing to the counting statistics of the ionisation chamber and the rem counter. A set of 892 data points on flight levels between FL 230 and FL 390 were measured. Since these measurements were part of the ACREM project [5] supported by the European Commission, the data set is referred to as the ACREM data.

Due to the fact that the entire detector equipment for the ACREM measurements required a substantial amount of space, the measurements were carried out on board a Lufthansa

Cargo Boeing 747-200. An improved cabin baggage sized system based on a TEPC which could perform measurements on passenger flights was therefore developed during the ACREM project. This system, named πDOS (PTB Inflight DOsimetry System) [6], was installed on board a Lufthansa Airbus A340 from December 2003 to September 2004 [7] as part of the DOSMAX project supported by the European Commission [8]. The flight routes covered during that time period always departed from Frankfurt with destinations mainly in North America, although some flights to South America, the Middle East and Asia were also available for measurements. Since the equator was never crossed during that period, additional measurements were made on 10 single flights to Cape Town (South Africa), Male (Maldives) and Windhoek (Namibia) up to November 2006. On these flights πDOS was carried aboard by a passenger and handled as cabin baggage. From all of these flights a total of 1537 data points between FL 250 and FL 415 were measured on 265 flight legs. Because a TEPC has a rather low neutron detection efficiency, a minimum of one hour measuring time was required to obtain a reasonable statistical uncertainty of about $\pm 15\%$ at flight altitudes.

3. Bayesian approach

Before discussing in some detail the models that we considered, we give a brief overview of Bayesian parameter estimation [9, 10] and the use of the deviance information criterion (DIC) for model comparison [10, 11].

Bayesian statistics is the application of the rules of probability theory to problems of inference. The main formula that is needed is Bayes' theorem, which can be written in the form

$$P(\theta|d, I) \propto P(d|\theta, I)P(\theta|I). \tag{1}$$

We omitted an overall normalisation factor, but this factor is easy to recover using the condition $\int d\theta\, P(\theta|d, I) = 1$. In equation (1), $P(A|B)$ denotes the (conditional) probability that A is true when B is true (i.e., the probability of A given B). We use θ for the parameters of the model and d for the data. I is standard notation to denote any background information that is relevant to the problem, $P(d|\theta, I)$ is the likelihood function, and $P(\theta|I)$ and $P(\theta|d, I)$ are the prior and posterior probabilities of the parameters, respectively. The conditioning on I is sometimes omitted in calculations to simplify notation.

The goal of the analysis is to calculate the posterior $P(\theta|d, I)$, which is the probability of the parameters given the measured data. Everything that we need to know is in $P(\theta|d, I)$: probabilities of individual parameters can be obtained by marginalisation of the posterior, while estimates of the parameter values and their uncertainties can be estimated from the median and width of these marginal distributions, respectively. To calculate the posterior using equation (1), we need to determine the likelihood function and we need to choose appropriate priors.

The likelihood function is the probability that we would have measured a particular ambient dose equivalent rate $\dot{H}_{\text{meas}}$ when the parameters take values θ. In our case, we have assumed that the data has uncertainties that are normally distributed, which corresponds to the likelihood function

$$P(d|\theta, I) \propto \sqrt{\tau}\exp\{-\tau(\dot{H}_{\text{meas}} - \dot{H}_{\text{mod}})^2/2\}, \tag{2}$$

where $\dot{H}_{\text{mod}}$ is the dose rate according to the particular model, and therefore a function of the parameters. In our analysis, we treat the precision $\tau \equiv \sigma^{-2}$ as an unknown parameter, and we let the data determine its value. The only assumption that we made regarding the precision is that it depends weakly on $\dot{H}_{\text{mod}}$ according to $\tau \propto \dot{H}_{\text{mod}}^{-1}$ (this assumption is approximately valid because $\dot{H}_{\text{mod}} \sim \dot{H}_{\text{meas}}$, the data is proportional to the count rate measured in the detector, and the count rate was high enough for counting statistics to be modelled by a normal distribution).

The prior distributions represent what we know about the value of a parameter before analysing the data. Although it would have been possible to use informative priors, we used standard non-informative priors for the analysis whenever possible, which is a conservative assumption. The precision τ was assigned a gamma distribution and all the other parameters were assigned constant distributions. For physical reasons (see section 4), we have introduced a cut-off that restricts the range of some of the parameters.

In addition to estimating optimal values of parameters for different models, we were also interested in comparing models. To do this, we calculated the deviance information criterion (DIC). This is a Bayesian method of model comparison that uses a criterion based on a trade-off between the fit of the data to the model and the corresponding complexity of the model. Models with smaller DIC are better supported by the data. Differences of more than 10 might rule out the model with the higher DIC, and differences between 5 and 10 are considered substantial. For more information on DIC, see [10, 11].

All calculations were carried out using WinBUGS [12], a software package for Bayesian estimation using Markov Chain Monte Carlo (MCMC) which was developed at the MRC Biostatistics Unit in Cambridge, UK.

4. Testing different models

In previous articles [4, 7], it was shown that relatively simple relations could be used to describe the measured dose rate as a function of the altitude, geomagnetic latitude, and flux of secondary neutrons at ground level. Since the neutron monitor rate is a direct measure of the flux of primary cosmic ray particles into the atmosphere, it can be used to evaluate the change in the dose rate at flight altitudes owing to changes in the solar activity. For the monitoring of the secondary neutron flux at ground level, the neutron monitor at Oulu (Finland) [13] was selected.

During the ACREM measurements, i.e. during the beginning of the solar cycle 23, the daily averaged Oulu monitor count rates N_{NM} were between 6200 and 6500 min^{-1}. The πDOS measurements were made after the solar maximum during the decreasing phase of solar activity with monitor count rates ranging between 5700 and 6000 min^{-1}. Thus, the effect of solar activity could be taken into account. The shortcoming of this method (described in detail in [7]) is that the functional description of the data was established at a fixed altitude of FL 350 and in the polar region, i.e. geomagnetic latitude $B_{\mathrm{m}} > 58°$, and in the equator region, i.e. at $B_{\mathrm{m}} < 30°$.

The dependence on the geomagnetic latitude B_{m} is not the best parameter to describe the influence of the earth's magnetic field. A better parameter is the vertical cut-off rigidity r_{c}, which describes the minimum rigidity required for a charged particle to enter the magnetic field and to reach a certain altitude. As it was shown in [4] and [7], the measured data corrected to a common altitude of FL 350 follow smoothly a simple function:

$$\dot{H}_{\mathrm{mod}} = \dot{H}_0 + \dot{H}_1 \mathrm{e}^{-(r_{\mathrm{c}}/r_0)^2}. \tag{3}$$

In such a formulation the ambient dose equivalent rate in the polar region, i.e. where $r_{\mathrm{c}} \approx 0$, is given by

$$\dot{H}_{\mathrm{polar}} \approx \dot{H}_0 + \dot{H}_1, \tag{4}$$

and in the equator region, i.e. where $r_{\mathrm{c}} > r_0 = 6.26$ GV, it is given by

$$\dot{H}_{\mathrm{equ}} \approx \dot{H}_0. \tag{5}$$

We now generalise equation (3) by introducing dependences on altitude h, vertical cut-off rigidity r_{c} and Oulu neutron monitor count rate N_{NM} for $\dot{H}_0$ and $\dot{H}_1$. In addition, the two parameters in the exponential function are not set to fixed values but are determined by the data (compare equations (6)–(8) to equation (3)). The effective vertical cut-off rigidity r_c was computed for the geographic GPS positions at an altitude of 20 km using the MAGNETOCOSMIC code [14–16] with the IGRF magnetic field parameter IGRF-10 [17] and using the Tsyganenko-89 model of the magnetospheric magnetic field for 1998 and 2004. Here and in the following the effective vertical cut-off rigidities were evaluated at a geomagnetic activity described by the planetary k_p-index with $k_p = 2$.

To generalise equation (3), we carried out a Taylor expansion of $\dot{H}_0$ and $\dot{H}_1$ about fixed values of h and N_{NM} and kept the lower order terms. This allows an easy comparison with the previous analysis given in [7]. We describe here three of the models which we investigated. They differ only in the choice of terms that are kept. Model 1 is the simplest one and it includes only linear terms in h and N_{NM} in the expansion of $\dot{H}_0$ and $\dot{H}_1$:

$$\begin{aligned} &\dot{H}_{\mathrm{mod}-1} = \dot{H}_0 + \dot{H}_1 \mathrm{e}^{-(r_c/c)^d} \\ &\dot{H}_0 = a_{00} + a_{01}(h - h^0) f_{\mathrm{km}} + b_{00}(N_{\mathrm{NM}} - N^0_{\mathrm{NM}}) \\ &\dot{H}_1 = a_{10} + a_{11}(h - h^0) f_{\mathrm{km}} + b_{10}(N_{\mathrm{NM}} - N^0_{\mathrm{NM}}). \end{aligned} \tag{6}$$

For the Taylor expansion, we set $h^0 = \mathrm{FL150}$ and $N^0_{\mathrm{NM}} = 4500\ \mathrm{min}^{-1}$, respectively. The analysis with model 1 gave the same result as in [7] as far as the parameter b_{00} is concerned, that is, that in the equator region the dependence of N_{NM} is negligible. This was then implemented in model 2:

$$\begin{aligned} &\dot{H}_{\mathrm{mod}-2} = \dot{H}_0 + \dot{H}_1 \mathrm{e}^{-(r_c/c)^d} \\ &\dot{H}_0 = a_{00} + a_{01}(h - h^0) f_{\mathrm{km}} \\ &\dot{H}_1 = a_{10} + b_{10}(N_{\mathrm{NM}} - N^0_{\mathrm{NM}}) + (a_{11} + b_{11}(N_{\mathrm{NM}} - N^0_{\mathrm{NM}}))(h - h^0) f_{\mathrm{km}}. \end{aligned} \tag{7}$$

Unlike model 1 which only considers linear terms, the equation for $\dot{H}_1$ includes now an additional term: the mixed term of the variables h and N_{NM}. Such an approach now considers that the altitude dependence might be different at different phases of the solar cycle. Model 3 is similar to model 2, but it includes more higher order terms of the Taylor expansion:

$$\begin{aligned} &\dot{H}_{\mathrm{mod}-3} = \dot{H}_0 + \dot{H}_1 \mathrm{e}^{-(r_c/c)^d} \\ &\dot{H}_0 = a_{00} + a_{01}(h - h^0) f_{\mathrm{km}} \\ &\dot{H}_1 = a_{10} + b_{10}(N_{\mathrm{NM}} - N^0_{\mathrm{NM}}) + c_{10}(N_{\mathrm{NM}} - N^0_{\mathrm{NM}})^2 \\ &\qquad + (a_{11} + b_{11}(N_{\mathrm{NM}} - N^0_{\mathrm{NM}}) + c_{11}(N_{\mathrm{NM}} - N^0_{\mathrm{NM}})^2)(h - h^0) f_{\mathrm{km}}. \end{aligned} \tag{8}$$

All models in equations (6)–(8) have in common that the parameters c and d enter in the same way. The factor $f_{\mathrm{km}} = 30.48 \times 10^{-3}$ km/FL converts the altitude given in flight levels into an altitude given in kilometre.

We evaluated other models which were similar to these (i.e., they differed only in the terms of the Taylor expansion that were kept), but we omit the details since models 1–3 are sufficient to illustrate the approach that we followed. To decide on the optimal model we examined the convergence of the MCMC algorithm that is part of the WinBUGS software, we compared posterior probabilities of the parameters, we looked at correlations, and we took into consideration the DIC criterion. This allowed us to evaluate whether a model had more parameters than the data could determine, whether there were not enough parameters to provide an adequate fit to the data, and, more generally, whether an increase in complexity was justified by the data or not. Our conclusion is that model 3 is the favoured model within the class of models that we considered. Table 1 shows optimal values for the individual parameters for each of the three models (these are derived from the median of the marginalised posterior) and the results of the evaluation of the DIC. For model 3, we also provide the 95% credible intervals. Plots of the posterior probability densities of the parameters are given in figure 1. One can see

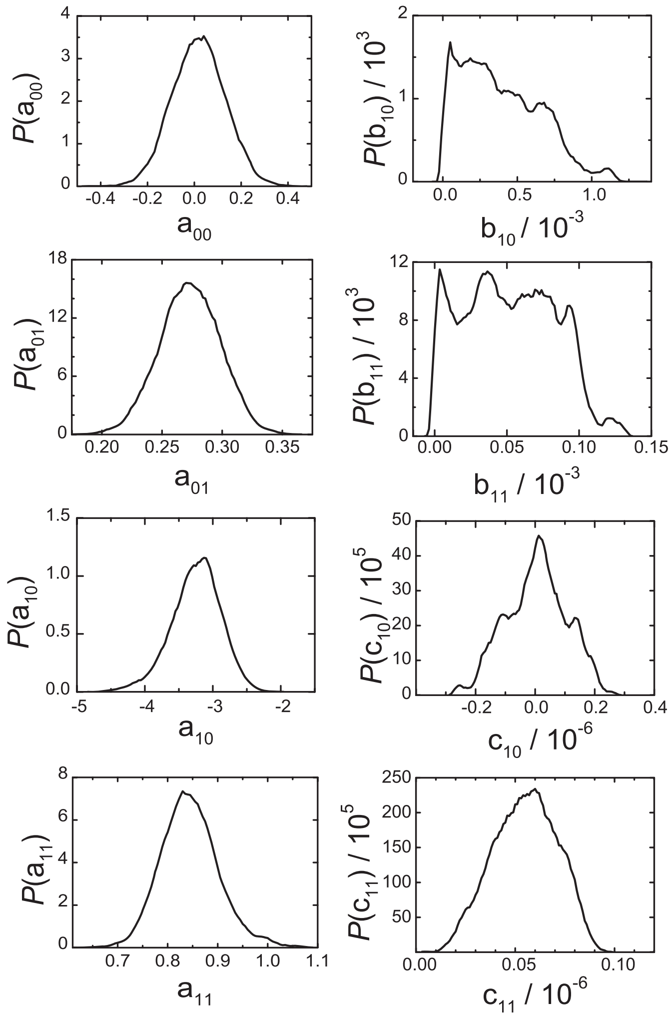

Figure 1. Probability densities of the parameters of model 3.

from the plots that most densities are approximately symmetric. In the case of the parameters b_{10} and b_{11}, lower limits were set when defining the priors and this leads to corresponding lower limits for the posteriors. These lower limits were introduced to ensure that the ambient dose equivalent rate is a monotonically increasing function of N_{NM} in an appropriate range.

Figure 2 shows how the measurements are distributed with respect to the values $\dot{H}_{\mathrm{mod-3}}$ predicted by model 3. The spread, as estimated from the width at half maximum of the distribution, agrees with the estimate of the relative statistical uncertainty of the data which is in the order of $\pm 13\%$.

5. Comparison with experimental data

The development of a code to calculate the ambient dose equivalent rate at a local point at a given flight altitude is straightforward using the mathematical formulae described in section 4. Model 3 as defined by equation (8) is implemented into the code FDOScalc. Only the vertical

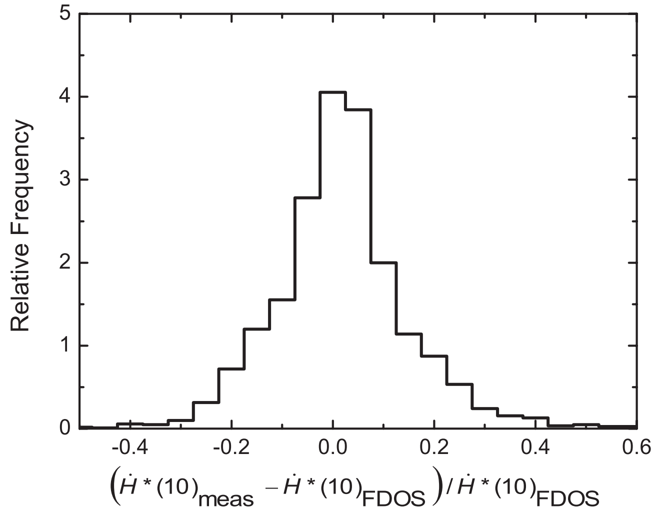

Figure 2. Distribution of relative differences of the measured data $\dot{H}^*(10)_{\text{meas}}$ with respect to values predicted by model 3 as implemented in the code FDOScalc.

Table 1. Results of the fitting procedure using Bayesian parameter estimation and the three models from equations (6)–(8). For model 3, we also provide the 95% credible interval.

Parameter	Model 1	Model 2	Model 3	95% credible interval for model 3
a_{00} (μSv h^{-1})	−0.094	0.029	0.012	(−0.209, 0.239)
a_{01} (μSv h^{-1} km^{-1})	0.275	0.267	0.272	(0.219, 0.322)
a_{10} (μSv h^{-1})	−5.299	−2.976	−3.374	(−4.524, −2.666)
a_{11} (μSv h^{-1} km^{-1})	1.080	0.659	0.852	(0.737, 1.025)
b_{00} (μSv h^{-1} min)	3.997×10^{-5}			
b_{10} (μSv h^{-1} min)	1.612×10^{-3}	2.081×10^{-4}	4.357×10^{-4}	$(0.204, 10.69) \times 10^{-4}$
b_{11} (μSv h^{-1} min km^{-1})		2.545×10^{-4}	5.266×10^{-5}	$(0.176, 11.94) \times 10^{-5}$
c_{10} (μSv h^{-1} min^2)			0.069×10^{-7}	$(-1.793, 2.474) \times 10^{-7}$
c_{11} (μSv h^{-1} min^2 km^{-1})			4.730×10^{-8}	$(1.007, 8.075) \times 10^{-8}$
c (GV)	7.474	7.503	7.464	(7.118, 7.884)
d	1.614	1.613	1.625	(1.526, 1.730)
DIC	4241	4232	4228	

cut-off rigidity, the barometric altitude and the neutron monitor counts rate of the Oulu station in Finland are required to run the code. The geographic coordinates are usually obtained during the measurements by a GPS system.

New measurements were performed at constant flight conditions in the south of Germany (November 6, 2007) and in the south of Norway (November 7, 2007) as part of the CARAMEL project organised by the German Aerospace Center (DLR) [18]. The carrier used was a Dassault Falcon 20 of DLR which in principal allows measurements at flight altitudes of up to FL 420.

The measurements by πDOS are used as reference for the ambient dose equivalent rate at the four positions. The results of the measurements and the values computed with FDOScalc are summarised in table 2. Due to recent improvements in our approach, these values are slightly different from the ones published in [18]. In figure 3 the results are plotted as a function of the vertical cut-off rigidity r_c. For comparison the results of a similar intercomparison, i.e. the

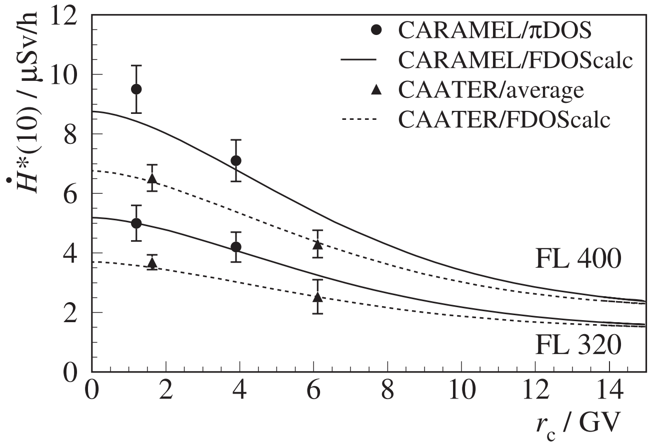

Figure 3. The data measured πDOS during the CARAMEL project [18] and the averaged results of the CAATER project [20] are plotted together with the calculation using FDOScalc. Also plotted are the total uncertainties of the measurements.

Table 2. Results of the πDOS measurements during the CARAMEL project in 2007. Given are the type A uncertainties evaluated according to GUM [19]. The values calculated with FDOScalc are based on the positions in [18].

Pos. (number)	B_m (deg)	r_c (GV)	Altitude (FL)	$\dot{H}^*(10)_{meas}$ (μSv h^{-1})	$\dot{H}^*(10)_{FDOS}$ (μSv h^{-1})
1	60	1.2	400	9.5 ± 0.8	8.4
2	60	1.2	320	5.0 ± 0.6	5.0
3	49	3.9	400	7.1 ± 0.7	6.8
4	49	3.9	320	4.2 ± 0.5	4.1

CAATER project [20], are also plotted in figure 3. Since the positions were not exactly on the same longitude, the calculation with FDOScalc uses a mean longitude of 10.0°E and 9.3°E for the CAATER and CARAMEL results, respectively.

The excellent overall agreement between FDOScalc and both the CAATER results from 2003 and the new CARAMEL results from 2007 provides evidence of the validity of the approach described in this paper. The effect of solar activity is clearly visible in figure 3. During the CAATER measurements, the Oulu neutron monitor count rate was about 5712 min^{-1}, i.e. during high solar activity, and during the CARAMEL measurements around 6671 min^{-1}, i.e. during low solar activity. This leads to the large differences of the dose rate especially at high latitudes, i.e. small values for r_c, and high altitudes. Although the neutron monitor count rate during the CARAMEL measurements slightly exceeds the maximum value in our data set, the calculations show very good agreement with these new measurements.

Since the CAATER and CARAMEL results were obtained far away from the equator region and only close to the same longitude at around 10°E, two additional inflight measurements are used for validation: (1) from Frankfurt to Cape Town (South Africa) in November 2007 and (2) from Dusseldorf to Bangkok (Thailand) in September 2008. These flights cover a wider range of longitudes and latitudes which leads to a wide range of vertical cut-off rigidities of up to 17 GV. The results of these πDOS measurements are compared to FDOScalc in figures 4 and 5.

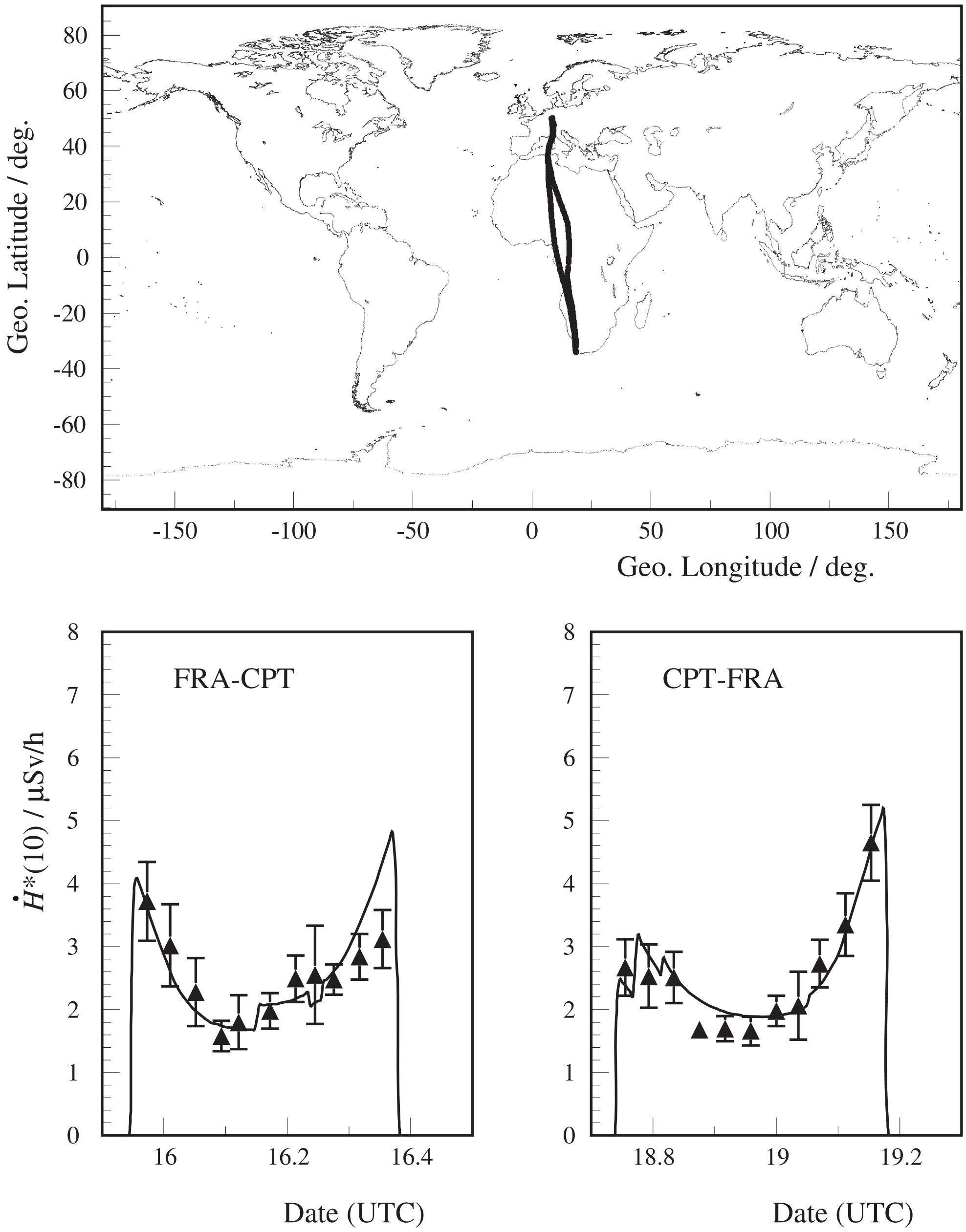

Figure 4. Measured and calculated dose rate profiles for the flight from Frankfurt (FRA) to Cape Town (CPT) and back in November 2007. The coast lines were obtained from [21]. The bottom plots show the measurements using πDOS with their type A uncertainty. The solid line is the calculation with FDOScalc.

6. Uncertainties for FDOScalc

As mentioned before, the precision τ in equation (2) was treated as an unknown parameter. The standard deviation of the data, calculated using $\sigma = \tau^{-1/2}$, was on average about 13%, which agrees well with the value expected from the statistical uncertainty of the data. Since the

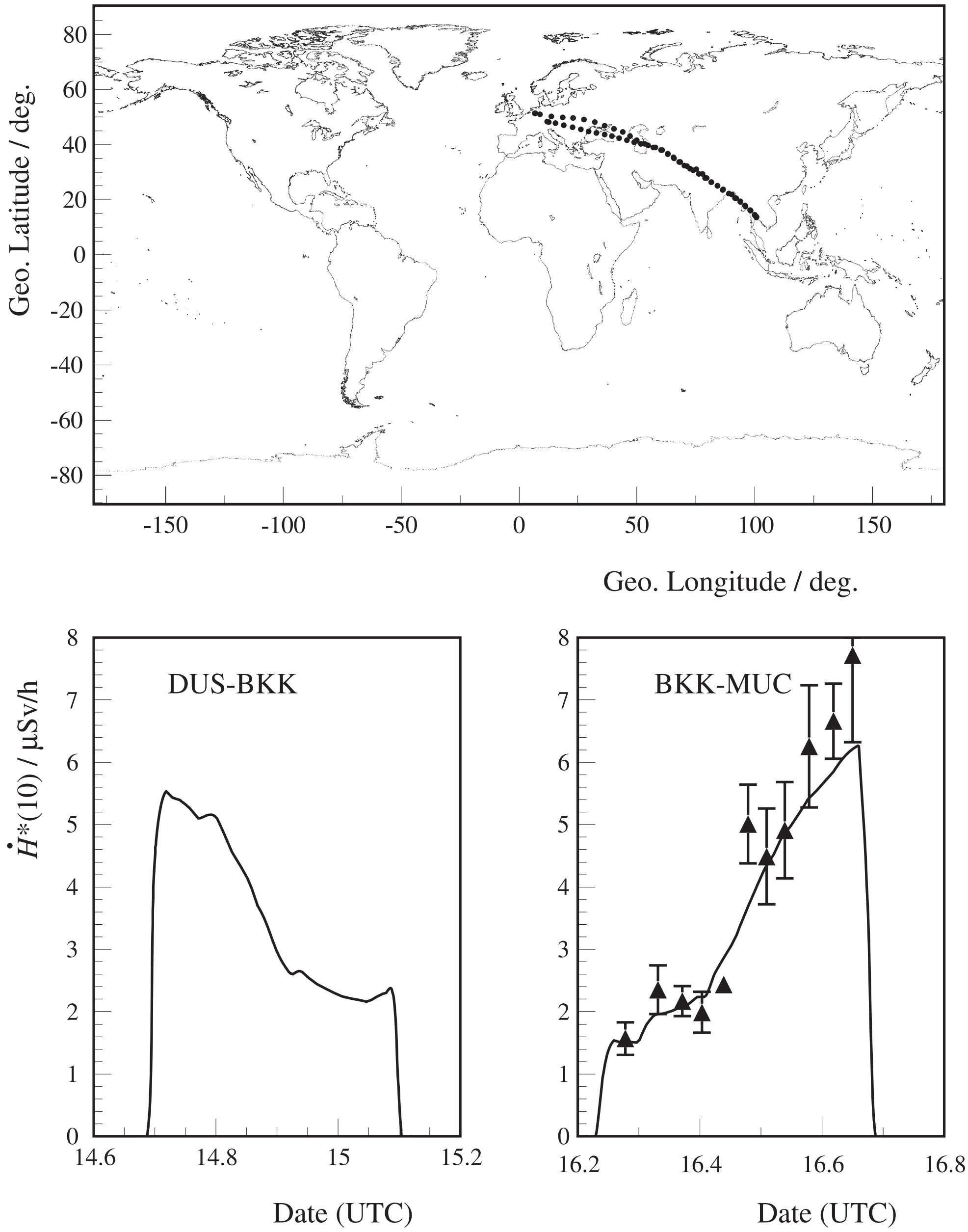

Figure 5. Same as in figure 4 but for flights from Dusseldorf (DUS) to Bangkok (BKK) and back to Munich (MUC) in September 2008. No data could be measured on the flight to Bangkok due to technical problems of πDOS.

dose rate function $\dot{H}_{\mathrm{mod}-3}$ was estimated using approximately 2500 data points, one expects an uncertainty $\Delta\dot{H}$ which is substantially lower.

It is straightforward to evaluate a posterior for the dose rate function $\dot{H}_{\mathrm{mod}-3}$ and from this posterior to estimate the uncertainty $\Delta\dot{H}$ using a Bayesian approach. When this is done, we find on average an uncertainty of the order of $\pm1\%$. The analysis shows that the model works well for the data that were used to test it, and hence that this estimate of uncertainty should be realistic for the range of parameters covered by the data. However, the uncertainty $\Delta\dot{H}$ may be higher if we extrapolate and use the model for ranges of parameter values that lie far away from the values considered in the analysis. In this case one might need to include higher order terms in the Taylor expansion which we have neglected here.

7. Summary

We have developed an approach based on Bayesian statistics to analyse a total of 2429 measurements of ambient dose equivalent rate at aviation altitudes. The final result is a fit to the data using a function that depends on three parameters: barometric altitude, vertical cut-off rigidity, and the count rate of the neutron monitor in Oulu (Finland).

We considered different mathematical models and computed their corresponding DIC to decide which of the mathematical functions provided the best description of the data. In all of these models, the dependence on the vertical cut-off rigidity is described using an exponential function while the dependence on the remaining parameters is given in terms of a Taylor expansion. The DIC was used to investigate the relative importance of the different coefficients. The result of the analysis is therefore a function that is similar to the model described in the simpler analyses presented in [4, 7].

The solar activity is included in the description by means of the count rate of the neutron monitor station on Oulu (Finland). Thus, we have selected a variable which only reflects the global changes of the cosmic ray flux into the atmosphere. This assumption may not be valid for solar activity phases leading to count rates that are outside of the ranges given here and dose rates calculated during a so-called ground level enhancement (GLE) may not be estimated correctly by our model.

Nevertheless, the model may be used to calculate the ambient dose equivalent rate during the normal phases of solar activity at all locations worldwide, as long as the parameter ranges fall within the values of the data (altitude: FL 230–FL 415; NM Oulu count rate: 5700–6500 min^{-1}; vertical cut-off rigidity r_c: 0.0–17.5 GV). The Bayesian analysis leads to estimates of the uncertainty of the calculated dose rate of the order of $\pm 1\%$, which directly reflects the number and quality of the measured data.

The function described with model 3 was incorporated into the program FDOScalc. The vertical cut-off rigidities are calculated from the input data, i.e. latitude, longitude and altitude. The date is used to provide the algorithm with the daily averaged count rate of the neutron monitor station in Oulu (Finland). Comparison with new data taken in 2007 and 2008 supports the validity of our approach. Furthermore, in a recent code comparison organised by the EURADOS association [22], FDOScalc showed an excellent agreement with other codes [23]. Since FDOScalc is based on dose rate data which are traceable to the national standards maintained at PTB, it may be argued that FDOScalc can be used to validate other measurements and codes as far as dose evaluation during normal solar and magnetic conditions is concerned.

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