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The compression and bearing capacity of cohesive layers

possible to find u_r and u_θ by substitution in equation (62) since the terms containing them have vanished, but instead we must solve equations (5), and (6) substituting the solutions for Δ_s and W_s. It is important that the various constants which appear in the final solutions should be correctly related, and for this it is necessary to substitute them in equations (5) and (6) and then back in equation (62).

For the force on the sphere, we may again use equation (79):

$$F = -\frac{4}{3}\pi a^2[c_{11}f'(a) - 2c_{44}g'(a)]$$

From equations (93) and (94) we find

$$f'(a) = -\frac{3}{1+2\mu^2}\left(\frac{u_0}{a}\right) \tag{95}$$

and

$$g'(a) = \frac{3\mu^2}{1+2\mu^2}\left(\frac{u_0}{a}\right) \tag{96}$$

Therefore

$$F = 12\pi a u_0\left(\frac{c_{11}}{1+2\mu^2}\right) \tag{97}$$

Now

$$c_{11} = E\left[\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}\right] \tag{98}$$

and

$$1+2\mu^2 = \frac{5-6\sigma}{1-2\sigma} \tag{99}$$

where E is the Young's modulus and σ is the Poisson's ratio for the medium.

Substituting equations (98) and (99) in equation (97), we find:

$$F = 12\pi a E u_0\left[\frac{1-\sigma}{(1+\sigma)(5-6\sigma)}\right] \tag{100}$$

From Timoshenko can be derived:

$$F = \frac{3\pi^2 a E u_0}{2(\pi+4)(1-\sigma^2)}$$

The following table gives the ratio F/aEu_0 for the various Poisson's ratios in the two cases:

Ratio F/aEu_0 for various Poisson's ratios

σ	$\frac{F}{aEu_0}$ (infinite solid)	$\frac{F}{aEu_0}$ (semi-infinite solid)
0	7·54	2·07$_5$
0·1	7·00	2·09$_5$
0·2	6·62	2·16$_0$
0·3	6·34	2·28$_0$
0·4	6·22	2·47$_0$
0·5	6·29	2·76$_5$

As one would expect, the value for the infinite solid is rather greater than double that for the semi-infinite case, the additional shearing stress across the equatorial plane in the former case adding to the stiffness of the system; this effect becomes more marked as the rigidity of the surrounding medium increases (i.e. as Poisson's ratio falls).

The compression and bearing capacity of cohesive layers*

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A theoretical study of the compression of blocks and cylinders of purely cohesive material between rough plates shows that the yield pressure of the blocks and cylinders increases with the roughness of the plates and with decreasing height of the specimen. Compression tests of blocks and cylinders of Plasticine and clay between perfectly rough plates are found to be in good agreement with the theory. The above analysis has been extended to obtain an estimate of the yield pressure of footings on cohesive layers; theoretically the bearing capacity should increase with the adhesion of the material on the footing and base and with decreasing thickness of the layer. The results of some loading tests on circular and strip model footings on Plasticine layers with a perfectly rough base conform with the theory.

1. INTRODUCTION

It has been known for some time that the yield pressure of a uniform material is only unique if the test specimens have a certain minimum height in relation to their width. As the height of the specimens is reduced below this minimum, the yield pressure is apparently increased. A theory to explain this phenomenon can readily be developed for purely cohesive materials as indicated in the first part of this paper, which also presents additional experimental evidence. Similarly, it has been found that the yield pressure of a footing on a layer of uniform material is only unique if the layer has a certain minimum thickness in relation to the width of the loaded area. As the thickness of the layer is decreased below this minimum, the yield pressure is increased. A theoretical method for estimating the bearing capacity of purely cohesive layers is outlined in the second part of the paper and compared with the results of loading tests on model footings. The effect of the rate of strain on the stresses in the material is outside the scope of this paper.

2. THEORY OF COMPRESSION

When a block of cohesive material is compressed between parallel rigid plates which are wider than the block, Prandtl[1] indicated that the stresses in it gradually change from the elastic to the plastic beginning at the edges of the block. As the load increases, the plastic zones spread to the centre of the block and then outwards until failure occurs by a relatively large compression and an outward displacement of the material. The average pressure on the block to produce such plastic deformation will be called the yield pressure of the block, and the corresponding zones of plastic equilibrium in the material (according to Prandtl) are shown in Fig. 1(*a*) for a long block which is wide compared to its height and is compressed between perfectly rough plates. In each quadrant of the block there is a central zone *ABC*, which remains in an elastic state of equilibrium and acts as part of the plate. Adjacent to these zones are a cycloidal shear zone *ACDE* and a zone of transition *DEG*: they are followed by a radial shear zone *EFG* at the edge of each plate and an outer zone *FGH*, which remains in an elastic state. For a narrow block the various shear zones coalesce as indicated in Fig. 1(*b*).

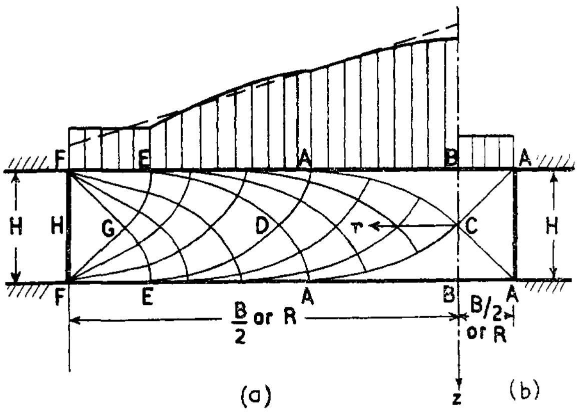

Fig. 1. Plastic zones and contact pressure for block between perfectly rough plates

(*a*) Wide block. (*b*) Narrow block.

Distribution of contact pressure:

Actual ————
Approximate — — — —
Centre line — · — —

The plastic equilibrium in the different zones can be established from the boundary conditions, starting at the free surface. For a plastic-rigid material whose shearing strength s is constant and independent of load, i.e

$$s = c \tag{1}$$

where c = unit cohesion, Prandtl[1] has shown that at some distance from the free edges of a long block of width B and height H [Fig 1(*a*)] the contact pressure on perfectly rough plates increases uniformly towards the centre; and on the simplifying assumption that the cycloidal shear zone extends throughout the block, the yield pressure is

$$p = c[(B/2H) + \pi/2] \tag{2}$$

This solution has been extended by Sokolovsky[2] to the compression of a long block by partially rough plates with an adhesion $c_a = mc$, where m is the degree of mobilization of shearing strength on the plates $(0 \leqslant m \leqslant 1)$. On the same assumptions as above the contact pressure increases as before and

$$p = c\left[\frac{mB}{2H} + \frac{\sin^{-1} m}{m} + \sqrt{(1 - m^2)}\right] \tag{3}$$

The more exact shear zones near the free edges of the block lead to a greater contact pressure there, which is practically balanced by a smaller contact pressure on the central zones because of zero shearing stress above the vertical centre line of the block [Fig. 1(*a*)]. That the yield pressure is almost unaffected has been shown by numerical step-by-step computations of some typical blocks between rough[2,3] and smooth[4] plates.

When a cylindrical specimen is compressed as above, plastic flow of the material occurs in both horizontal and vertical (radial) planes. Normal to the radial planes act hoop stresses, which in accordance with Coulomb–Mohr's theory of failure are assumed equal to the minor principal stresses. On that basis and the simplifying assumption that in radial planes the shear zones are identical to those in transverse planes of a corresponding long block [Fig. 1(*a*)], a solution of the problem is given in the Appendix This shows that at some distance from the perimeter of a cylinder of diameter $2R$ and height H [Fig 1(*a*)] the contact pressure on the plates increases uniformly towards the centre as for a long block; on the assumption that the cycloidal shear zone extends throughout the cylinder, the yield pressure is

$$p = c\left[\frac{2mR}{3H} + \frac{\sin^{-1} m}{m} + \sqrt{(1 - m^2)}\right] \tag{4}$$

Comparison of equations (3) and (4) indicates that the compressive strength of a thin cylinder between rough plates is up to one-third less than that of a corresponding long block. The more exact shear zones near the perimeter and centre of the cylinder would lead to a distribution of contact pressure which is similar to that indicated for a long block [Fig 1(*a*)] and gives a somewhat greater yield pressure, especially for high cylinders.

If a similar pressure distribution ("stress roof") holds also for blocks of other shapes in plan, an approximate estimate of their yield pressures can readily be obtained from the average contact pressure in plan. Thus, for a rectangular block of length L, width B, and height H,

$$p = c\left[\frac{mB}{2H}\left(1 - \frac{B}{3L}\right) + \frac{\sin^{-1} m}{m} + \sqrt{(1 - m^2)}\right] \tag{5}$$
$$= cM_c$$

where M_c is the yield pressure factor, which depends on the

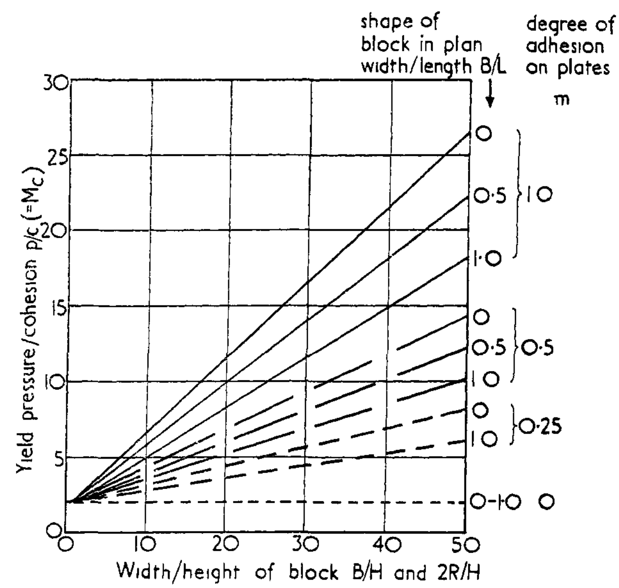

Fig. 2. Yield pressure of blocks and cylinders

Prisms and cylinders are given by $B/L = 1 \cdot 0$.

shape of the block and the adhesion on the plates. For a long block equation (5) becomes identical with (3), and for a block of square plan (5) gives the same compressive strength as for a cylinder, equation (4).

The results of this analysis are given in Fig 2 for blocks of various shapes and show that the yield pressure should be directly proportional to the width/height ratio B/H (or $2R/H$) of the blocks and approximately also to the degree of adhesion m on the plates. For narrow blocks and high cylinders the more exact shear zones have been used in the estimates, which for $B/H \leqslant 1$ give in all cases the compressive strength of the material of $2c$, for any m. The yield pressure of blocks between perfectly smooth plates is also $2c$ and is independent of the ratios B/H and B/L.

3. COMPRESSION TESTS

Apart from some exploratory investigations[5] test results have been published[6, 7] for short cylinders of plastic materials, such as Plasticine and unvulcanized rubber, compressed in a parallel plate plastometer, similar tests have been carried out on metals.[8] No attempt was made in these tests to ensure that the plates were sufficiently rough to develop the full shearing strength of the material nor was the adhesion on the plates determined independently. The test results cannot, therefore, be analysed adequately

The above shortcomings were avoided by Jurgenson[9] who carried out compression (squeezing) tests on thin rectangular clay samples between rough solid plates in a special box. Two of the short sides of the material were restrained so that plastic flow took place in two directions only and the conditions represented plane strain as far as possible. The shearing strength deduced from the experimental results on the basis of equation (1) agreed well with the value determined from standard compression and direct shearing tests on the clay. The same applied to similar tests on thin blocks of lead.[10]

Although the previous compression tests on rectangular blocks seemed to confirm the corresponding analysis, the tests were made on short restrained specimens so that long blocks were only simulated. Nor were any test results on short cylinders available hitherto with a known amount of adhesion on the plates. An extensive series of compression tests with full adhesion has therefore been carried out at the Building Research Station on rectangular blocks of various shapes in plan up to a long strip and on short cylinders to provide some check of the present theory While most of this investigation was made on firm Plasticine for convenience, some additional tests were carried out on an undisturbed soft clay from a site where a large circular foundation had tilted, partly on account of plastic flow of the clay.[11] For these materials and the standard testing procedure in all experiments (see below) the effect of time on the stresses is thought to be small. Before each series of tests the Plasticine was thoroughly mixed; the samples and standard compression cylinders were prepared and tested at the same temperature to minimize temperature effects.

The experiments were carried out in a compression apparatus with a ring-type load gauge. Except in a few exploratory tests, the material was compressed between parallel rigid plates which had been roughened or covered with sand paper to ensure full mobilization of the shearing strength on the plates. Usually a load sufficient to cause appreciable plastic flow was applied, the vertical deformation was observed by a dial gauge at various time intervals until equilibrium was reached after about 15 min, which has been taken as standard throughout the present series of tests. In a few cases the material was compressed at a constant rate of deformation of 0·06 in/min.

The specimens were rectangular Plasticine blocks of various shapes and from 1 0 to 8·5 in long, 0 3 to 1·0 in wide and 0 05 to 0 3 in high, which were compressed to final width/length ratios B/L varying from 0 1 to 1 0 and width/height ratios B/H from 5 to about 50. In addition Plasticine and clay cylinders from 1·5 to 3 0 in diameter and 0 04 to 3 2 in high were compressed to final diameter/height ratios $2R/H$ from 0 5 to about 60. The shearing strength of the Plasticine was 12 lb/in^2 on the average and was obtained independently from standard compression tests on 1·5 in diameter and 3 in high cylinders tested at the same rate of deformation as that used in the main experiments. The shearing strength of the clay determined in a similar manner ranged from 3 3 to 7 6 lb/in^2 depending on the water content of the particular sample.

The measured yield pressure for appreciable plastic flow of the rectangular blocks is shown in Fig. 3 for the three main final shapes, i.e. strip, short rectangle and square, and the results for the cylinders are shown in Fig. 4. In spite of some scatter of the results, which for the Plasticine is probably largely due to slight temperature variations and for the clay is due to natural variability of the soil, the yield pressure is in each case found to be directly proportional to the final width/height ratio of the specimen and in agreement with the theoretical estimates for perfectly rough plates; the observed yield pressures do not depend on the initial shape of the specimen The experimental results for a strip ($B/L = 0\ 1$) are somewhat lower than those given by the theory for a long block, which appear, however, to be approached (Fig. 3).

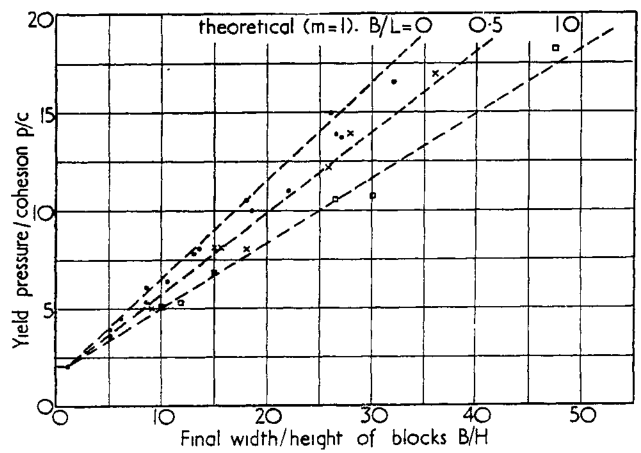

Fig. 3. Results of compression tests on rectangular blocks

Experimental results:

Average final shape:

Strip ($B/L = 0\ 1$) •

Rectangle ($B/L = 0·6$) ×

Square ($B/L = 1·0$) □

Theoretical results — — — — —

The very few exploratory tests on cylinders between fairly smooth (brass) plates indicate that according to the theory only about 60% of the shearing strength of the material was mobilized along the plates (Fig. 4), this shows the importance in parallel plate plastometers of providing perfectly rough

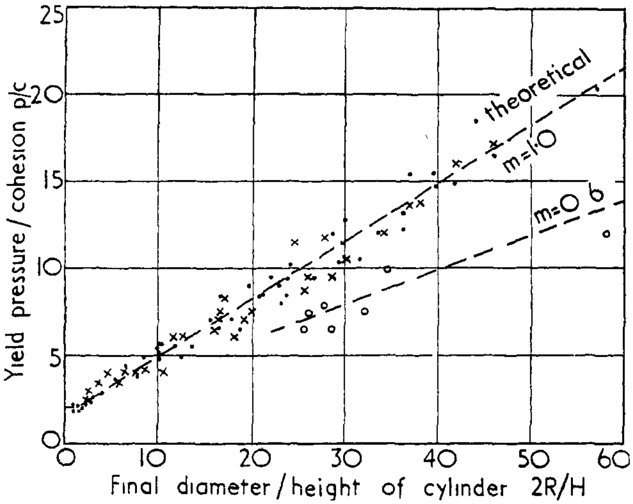

Fig. 4 Results of compression tests on cylinders

Experimental results

Material	*Plates*	*Symbol*
Plasticine	perfectly rough	•
Plasticine	brass (smooth)	○
Soft clay	perfectly rough	×
Theoretical results		— — — — —

plates or of determining the degree of adhesion independently before the shearing strength can be deduced.

Further support for the theory is obtained from some additional tests on short cylinders compressed at a constant rate of deformation. If the shearing stress along the plates is calculated from equation (3) for various diameter/height ratios of the specimen and is plotted against the corresponding vertical strains, Fig. 5(*a*), it is found that these stress-strain relations are similar to and in reasonable agreement with that of the standard compression test. The stress-strain curve of the latter test shows that for the particular material and method of testing there was very little work-hardening within the range of strains investigated.

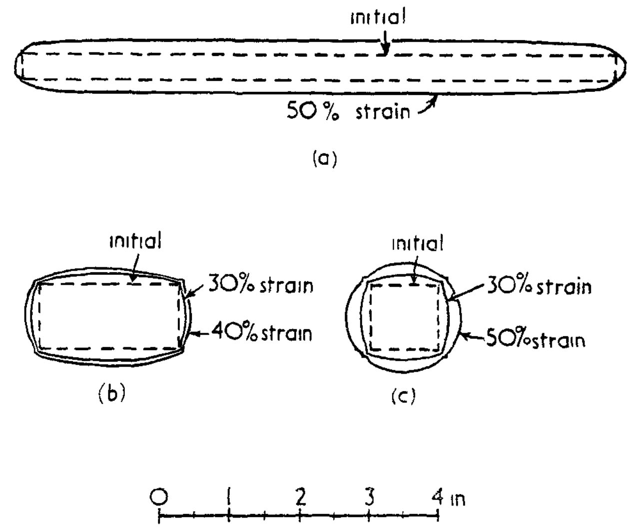

Fig. 6 Plan views of typical blocks before and after compression

(*a*) Strip. (*b*) Rectangle. (*c*) Square.

Examination of the specimens after compression showed that for long blocks plastic flow in plan occurred at right angles to the long centre line while the material of shorter rectangular blocks flowed also towards the short sides (Fig. 6). Square specimens exhibited practically radial flow so that for

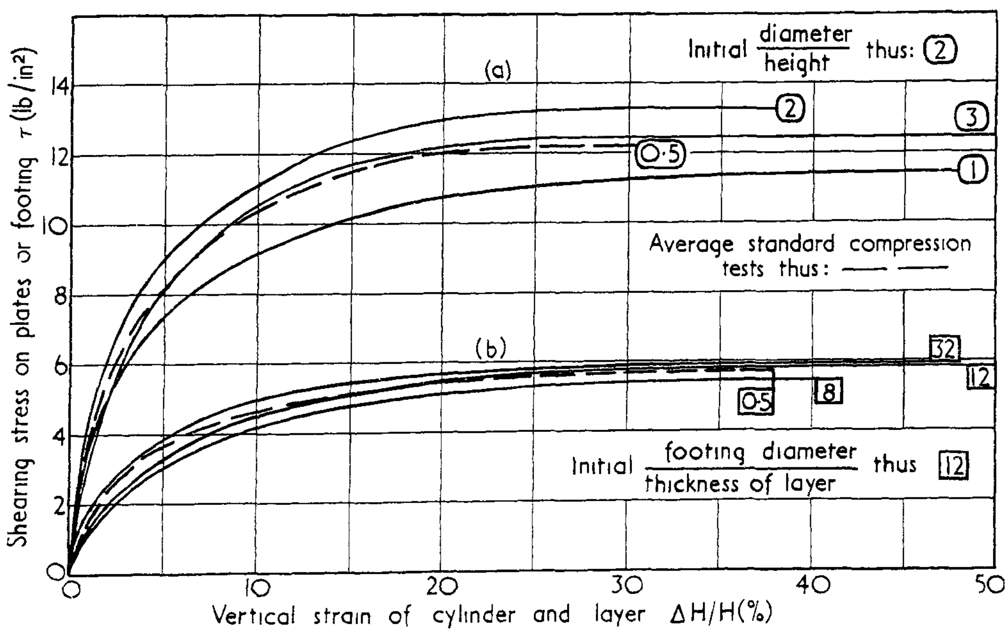

Fig. 5. Relation between shearing stress and strain of cylinders and thin layers

(*a*) Tests on thin cylinders (*b*) Tests on circular footings on thin layers greater strains the specimens assumed a circular shape, and for short cylinders the flow was radial, as would be expected. In general, the material flowed to the nearest side of the specimen as assumed in the theory. The vertical sides of the specimens showed a slight barrelling due to resistance of the material along the rough plates. The initial and final volumes of the Plasticine specimens were found to be sensibly the same, but it was impossible to avoid some consolidation of the clay, especially with thin specimens. In the latter case the shearing strength of the consolidated material was used.

4. THEORY OF BEARING CAPACITY

When a footing on a layer of cohesive material underlain by a rigid base stratum is loaded, the stresses in the material near the footing change from elastic to plastic, like those in the loaded block considered earlier. The material beneath the footing is displaced outwards, and outside the loaded area the movement is outwards and upwards towards the surface of the layer. The average pressure on the footing to produce appreciable plastic deformation of the material will be called the yield pressure of the footing, by analogy with the loaded block. The corresponding zones of plastic equilibrium in the material are shown in Fig. 7(*a*) for a shallow strip footing extends throughout the layer immediately below the footing, the yield pressure is

$$q = c\left(\frac{B}{2H} + N_c - 1\right) + \gamma D \tag{6}$$

from equation (2) and bearing capacity theory,[12] where γ = weight per unit volume of material and N_c is the bearing capacity factor for a strip footing on a layer of great thickness; this factor depends mainly on the depth of the footing and was calculated previously.[12]

Similarly, for a circular footing of diameter $2R$ loaded as above, it may be assumed as before that the shear zones in radial planes are identical with those in transverse planes of a corresponding strip footing, Fig 7(*a*) On that basis the distribution of the contact pressure is similar in both cases as the contribution from the hoop stresses is small and

$$q = c\left(\frac{2R}{3H} + N_{cr} - 1\right) + \gamma D \tag{7}$$

where N_{cr} is the bearing capacity factor for a circular footing on a layer of great thickness.[12]

The more exact shear zones under the footing, as in the case of blocks, lead to a greater contact pressure near the edges, which is, however, only partly offset by a smaller contact

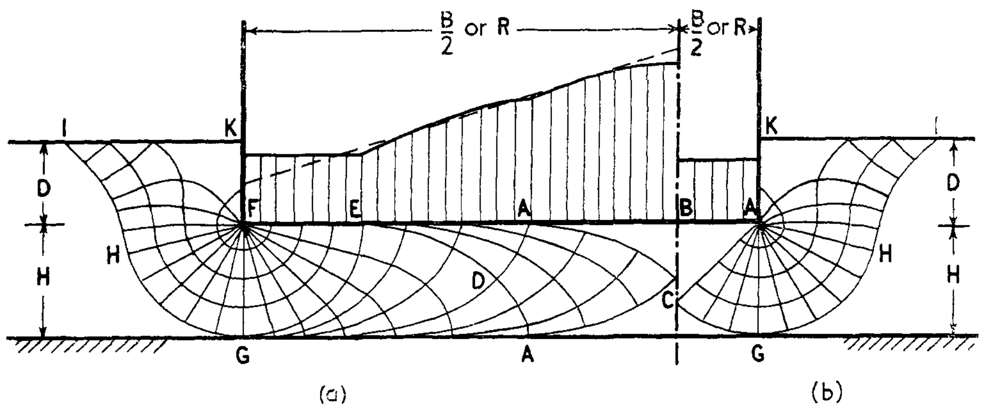

Fig. 7. Plastic zones and contact pressure for perfectly rough footing on layer with perfectly rough base

(*a*) Wide footing. (*b*) Narrow footing.

Distribution of contact pressure
Actual ————
Approximate — — — —
Centre line — · — —

which is wide compared with the thickness of the layer, both footing and base stratum being perfectly rough. Below the footing the zones are the same as for a wide block under compression (Section 2) except near the edge of the footing where only one radial shear zone *EFG* occurs and extends to the base stratum. This zone is continuous with a radial shear zone *FGH* and followed by a mixed shear zone *FHIK* outside the footing where the zones are the same as for a layer of great thickness.[12] For a narrow footing the various shear zones coalesce as indicated in Fig 7(*b*).

The plastic equilibrium in the zones can be established as before. At some distance from the edges of a strip footing of width B and depth D on a purely cohesive layer of thickness H (beneath the footing), Fig. 7(*a*), the contact pressure on a perfectly rough footing increases uniformly towards the centre as before; and on the assumption that the cycloidal shear zone pressure on the central zones as indicated in Fig. 7(*a*). It is, therefore, more convenient to express the yield pressure by the relation

$$q = cN_{cq} + \gamma D \tag{8}$$

where N_{cq} is the yield pressure factor and is a function of M_c (Section 2) and N_c; this factor depends on the depth and shape of the footing as well as the thickness of the layer.

The results of this analysis are given in Fig 8 for strip and circular footings with a rough base and a perfectly smooth shaft; for a footing with a perfectly rough shaft the yield pressure factor N_{cq} is greater by about 0·5[12] and the skin friction on the shaft has to be added to obtain the total failing load. Fig. 8 shows that the theoretical yield pressure of a footing on a thin layer is directly proportional to the ratio of footing width/thickness of layer B/H (or $2R/H$) and increases with footing depth. Beyond a depth of about three times the thickness of the layer, the yield pressure should remain independent of depth and increase only with B/H (or $2R/H$). For thick layers the more exact shear zones have been used in the estimates, which for B/H (or $2R/H$) $\leqslant \sqrt{2}$ give the bearing capacity of a footing in a homogeneous material of great depth. This value also gives the lower limit of the yield pressure for a footing and base stratum with a trace of adhesion ($m \simeq 0$) and is sensibly independent of B/H (or $2R/H$), Fig 8. The yield pressure is almost directly proportional to the degree of adhesion m (similar to Fig. 2) and can, therefore, readily be interpolated between the above limits.

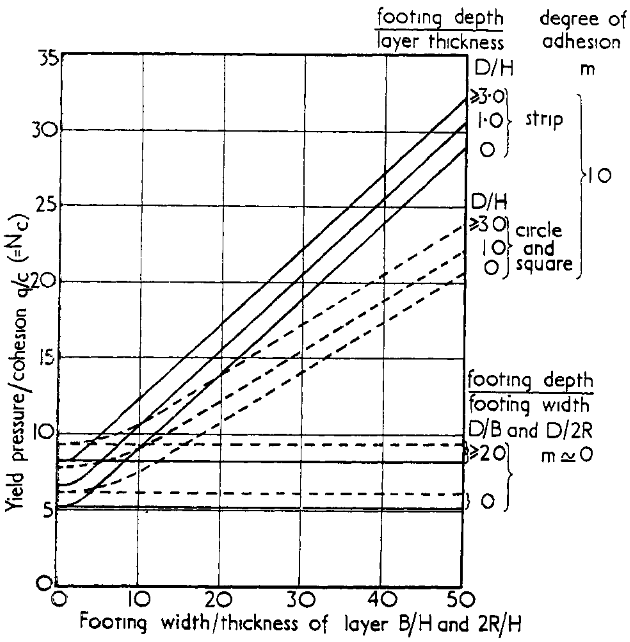

Fig. 8. Yield pressure of footings on layers
Strip footing ($B/L = 0$) ————
Circular or square footing ($B/L = 1$) — — — — — —

It is of interest to note that for thick layers the yield pressure of a rough circular footing is up to one-fifth greater than that of a corresponding strip footing on account of the hoop stresses in the material. For thin layers, however, the yield pressure of a circular footing is up to one-third less than that of a strip footing since the hoop stresses are then relatively small and more than offset by the smaller effect of the shearing resistance on the footing and base stratum. The yield pressures of rectangular footings of length L and width B on thin layers can be estimated by interpolation between the curves for strip and circular footings (Fig. 8) in direct proportion to B/L (similar to Fig. 2), which was also found to be sufficiently accurate for thick layers.[12]

5. LOADING TESTS

To provide a check of the present theory for full adhesion, some loading tests were carried out on model circular and a few strip footings resting on Plasticine layers of various thicknesses. The circular footings were of 1 35 in diameter and the strip footings were 0·5 in wide by 5 0 in long, on layers from 0·1 to 3·0 in thick and supported by a rigid base; both footing and base were covered with emery paper to ensure full mobilization of the shearing strength. The experiments were carried out in a compression apparatus as before (Section 2); the load was applied in small increments, each acting until the settlement of the footing was sensibly complete. The final ratios of footing width (or diameter) to the thickness of the layer below the footing (B/H or $2R/H$) varied from 0 5 to about 60. The overall rate of deformation in the experiments was 0·1 in/min. The material used for these tests was considerably softer than for the previous series, and had a shearing strength of 5 8 lb/in^2.

The experimental yield pressure of the footings for appreciable plastic flow of the material is shown in Fig. 9. For circular footings on thin layers it increases in direct proportion to the ratio of footing width/final thickness of layer, and the results are in fair agreement with the theoretical estimates for perfectly rough footings and base. The results of the limited number of tests on strip footings are consistent with the theory. All estimates were based on footings below the surface of the layer on account of the relatively great penetration required for mobilization of the shearing strength and the simultaneous rise of the material around the footings.

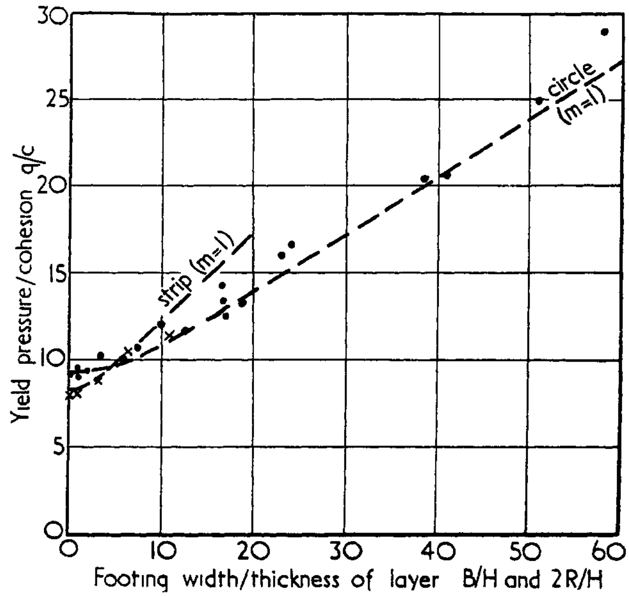

Fig 9. Results of loading tests on footings on layers
Experimental results.
Strip footing ($L/B = 10$) ×
Circular footing •
Theoretical results — — — — —

A further check of the theory is obtained from the loading tests on footings on very thin layers when the footing depth is fairly constant during the test. If the shearing stress along the footing and base is calculated from equation (7) for various ratios of footing width (or diameter) to thickness of the layer below the footing and plotted against the corresponding vertical strains in the layer, Fig. 5(b), it is found that these stress-strain relations are similar to and agree well with that of the standard compression test.

Examination of the specimens after testing showed that for thicker layers the material heaved slightly around the footing to a distance of about the width or diameter independent of the thickness of the layer. For thin layers the heave was well defined, extended around the whole footing to a distance of about twice the original thickness of the layer, and was independent of the footing width, Fig. 10. The volume of the

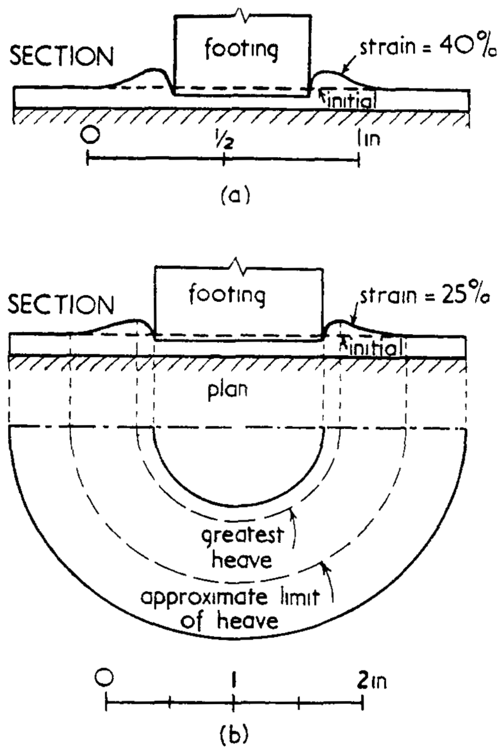

Fig. 10. Typical thin layers before and after test
(*a*) Strip footing. (*b*) Circular footing.
Centre line — — —

deformed material outside the footing was found to be of the same order of magnitude as that displaced below the footing so that the deformations occurred without noticeable volume change. The above results would support the mechanism and extent of failure assumed in the theory.

6. CONCLUSION

The previous theory of the compression of a long block of purely cohesive material has been extended to blocks of rectangular plan and cylinders compressed between parallel rigid plates of any degree of adhesion. The theoretical results show that the yield pressure of thin blocks and cylinders should increase rapidly with the adhesion of the material on the plates and with decreasing height of the specimen. Compression tests on blocks and cylinders of Plasticine and clay between perfectly rough plates were found to be in reasonable agreement with the theoretical yield pressure and mechanism of failure. The experiments also suggest that the vertical stress-strain relations of the blocks and cylinders are the same as in the standard compression tests on the material.

The above analysis has been extended on the basis of bearing capacity theory to strip, rectangular and circular footings at any depth in a purely cohesive layer resting on a rigid base stratum. The theoretical results show that the yield pressure should increase rapidly with the adhesion of the material on the footing and base and with decreasing thickness of the layer below the footing. Loading tests on rough circular and a few strip model footings on Plasticine layers with a perfectly rough base are in fair agreement with the theoretical yield pressure and mechanism of failure. The vertical stress-strain relations of very thin layers are again similar to those of standard compression tests.

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APPENDIX

Compression of cylinder between partially rough plates

On the simplifying assumptions made in Section 2 and at some distance from the perimeter of a thin cylinder of diameter $2R$ and height H compressed between parallel rigid plates with an adhesion mc, the governing equation of equilibrium of the radial stress σ_r, vertical stress σ_z and shearing stress τ is[5] for cylindrical co-ordinates (r, z), Fig. 1(*a*)

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau}{\partial z} = 0 \qquad (a)$$

the plasticity yield criterion is[1]

$$\sigma_z - \sigma_r = 2\sqrt{(c^2 - \tau^2)} \qquad (b)$$

and the shearing stress (as for a long block) is

$$\tau = 2mcz/H \qquad (c)$$

Substituting equations (*b*) and (*c*) into equation (*a*) and integrating, the direct stresses are

$$\left.\begin{aligned} \sigma_r &= 2mc(R - r)/H - 2c\sqrt{[1 - (2mz/H)^2]} - A \\ \sigma_z &= 2mc(R - r)/H + A \end{aligned}\right\} \qquad (d)$$

where $$A = \left[\frac{\sin^{-1} m}{m} + \sqrt{(1 - m^2)}\right]c$$

determined from the boundary condition of $\int_0^{H/2} \sigma_r dz = 0$ for $r = R$.

If the cycloidal shear zone, which can be shown to satisfy equations (*d*), extends throughout the cylinder, the contact pressure on the plates (σ_z for $z = \pm H/2$) increases uniformly with distance from the perimeter of the cylinder and the average value of this pressure gives the yield pressure

$$p = c\left[\frac{2mR}{3H} + \frac{\sin^{-1} m}{m} + \sqrt{(1 - m^2)}\right] \qquad (e)$$

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