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Corrigendum: A variational perspective on controllability (2010 *Inverse Problems* 26 015004)

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Lemma 2.1 within this paper is correct. However, its applications in sections 3–5 are not. This requires a justification that the constant in the statement of the lemma can be taken independently of the sequence. This may be hard or even impossible to achieve. Depending on the initial and final data taken as traces of some specific u_0, it could be noted that there are no uniformly bounded minimizing sequences for the error. Though the functional analytical setting used in the paper is different from the universally accepted one, the author believed that this was permitted in order to have equivalence between controllability and unique continuation. That is definitely wrong because the existence of minimizers for the error does not hold in general, even if the analytical setting is different.

What is shown in the paper is that the unique continuation property is equivalent to the fact that the only possible critical value of the error is zero, and so critical points of the error can only occur at solutions of the controllability problem. In particular, the numerical scheme that stems from this perspective, and is briefly described in the final section of the paper, can never get stuck in local minima but will steadily decrease the error to its infimum. If the successive values of this error, as the iterations proceed, become closer and closer to zero, we are indeed approximating a minimizing sequence of the error, and of the controllability problem. The practical performance of this algorithm has already been tested in several scenarios [1–5], and it performs reasonably well.

All of the results in that initial paper must be restated accordingly. We just state the main two such results for the heat and wave equations. Note that the setting for the heat equation is described at the beginning of section 4. In particular, the 'corrector' v is defined in (4.1), and the error functional $E(u)$ is precisely the (square of the) size of the corrector in the appropriate norm.

Theorem 4.1. *Let the time $T > 0$ be given, and $u^{(0)} \in H^1((0, T) \times (0, 1))$ furnishing initial and final data $u_0(x) = u^{(0)}(0, x)$, $u_T(x) = u^{(0)}(T, x)$, and complying with $u^{(0)}(t, 0) = 0$ for all times. If the only function $v \in L^2((0, T); H^1_0(0, 1))$ verifying, in a weak sense,*

$$v_t + v_{xx} = 0 \text{ in } (0, T) \times (0, 1), \qquad v_x(t, 1) = 0 \text{ in } (0, T),$$

is the trivial function $v \equiv 0$, then the only possible critical value for E is zero.

The proof of this theorem is fully contained in the proof of the original result. The mistake is precisely the statement about the existence of minimizer which is *not* valid in general.

For the wave equation, we would consider $X = H^{2,1}_{t,x}((0,T)\times(0,1))$, and H the closure in X of the class of smooth, compactly supported functions in $(0,T)\times(0,2)$. If $u^{(0)}$ provides desired initial, final, and boundary conditions, then the error functional E is defined exactly as in section 5. Theorem 5.1 should then be restated as follows.

Theorem 5.1. *Let the time $T > 0$ be given, and $u^{(0)} \in X$ furnishing appropriate initial, final, and boundary data. If the only function $v \in L^2((0,T); H^1_0(0,1))$ verifying, in a weak sense,*

$$-v_{tt} + v_{xx} = 0 \text{ in } (0,T)\times(0,1), \qquad v_x(t,1) = 0 \text{ in } (0,T),$$

is the trivial function $v \equiv 0$, then the only possible critical value for E is zero.

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