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A Pure Spin-Connection Formulation of Gravity

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Errata

A pure spin-connection formulation of gravity

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An error was made in section 3 in deriving the pure spin-connection action (3.9) for general relativity in the presence of a cosmological constant. The error invalidates this action as well as the non-polynomial actions derived in section 4 for GR coupled to matter. (These include gauge fields and massive fields of any kind.) The existence of an error was conjectured by Peldán (1991), who obtained results by another method which did not agree with the results of our paper.

The error occurred as follows. Let us begin with the action (3.3):

$$\tfrac{1}{2}\int \operatorname{Tr}(X^{-1}M)+\mu(\operatorname{Tr}X-\Lambda). \tag{3.3}$$

Here μ is a Lagrange multiplier density of weight one, Λ is the cosmological constant, and M is the skew product of two spin-connection curvature 2-forms, $M := \varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu}{}^{AB}R_{\rho\sigma}{}^{CD}$. The variational equations arising from the μ and X variations are

$$\operatorname{Tr}X=\Lambda \tag{1}$$

$$X^2=\mu^{-1}M. \tag{2}$$

If (2) could be solved uniquely for $X=X(\mu, M)$, the result could be substituted back into the action (3.3) yielding

$$\tfrac{1}{2}\int \mu(2\operatorname{Tr}X(\mu, M)-\Lambda). \tag{3}$$

By use of the Cayley–Hamilton theorem, one *can* solve (2) for X, *provided (1) is assumed.* The solution is then given by

$$X=M(M-\tfrac{1}{2}\operatorname{Tr}M+\tfrac{1}{2}\mu\Lambda^2)((\mu\det M)^{1/2}+\mu\Lambda M)^{-1} \tag{4}$$

provided the trace of the right-hand side is Λ†. Substitution of (4) in (3) would therefore be justified if the μ variational equation for the action (3) implied $\operatorname{Tr}X=\Lambda$. The error is here: while this is indeed true for the action (3.3), it is *not* true for (3), since $X(\mu, M)$ in (4) is not homogeneous of degree $-\tfrac{1}{2}$ in μ when Λ is non-zero.

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Reference

Peldán P 1991 *Class. Quantum Grav.* **8** 1765

† Actually this solution is ambiguous due to the appearance of the square root term. In fact, it is only a solution for one of the two roots, as can be verified by a computation in which the inverse in (4) is evaluated using the cofactor formula.

