LETTER

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Quantum self-gravitating collapsing matter in a quantum geometry

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Abstract

The problem of how space–time responds to gravitating quantum matter in full quantum gravity has been one of the main questions that any program of quantization of gravity should address. Here we analyze this issue by considering the quantization of a collapsing null shell coupled to spherically symmetric loop quantum gravity. We show that the constraint algebra of canonical gravity is Abelian both classically and when quantized using loop quantum gravity techniques. The Hamiltonian constraint is well defined and suitable Dirac observables characterizing the problem were identified at the quantum level. We can write the metric as a parameterized Dirac observable at the quantum level and study the physics of the collapsing shell and black hole formation. We show how the singularity inside the black hole is eliminated by loop quantum gravity and how the shell can traverse it. The construction is compatible with a scenario in which the shell tunnels into a baby universe inside the black hole or one in which it could emerge through a white hole.

Keywords: self gravitating shells, loop quantum gravity, canonical quantization, spin networks

There has been recent progress in loop quantum gravity with spherical symmetry. The key ingredient is the realization that through suitable combinations and rescalings the constraint algebra can be made a Lie algebra, both at a classical level and when quantized using loop quantum gravity techniques [1]. In vacuum the theory can be solved in closed form including finding the space of physical states annihilated by the constraints and suitable self-adjoint Dirac observables (quantities that commute with all the constraints) characterizing the physics. The singularity that is present inside black holes in the classical theory is replaced in the

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quantum theory with a region with large curvatures, large fluctuations of the curvature and high sensitivity to Planck scale microstructure, so it cannot be well approximated by a semiclassical geometry. The theory is well defined there and eventually one reaches a region inside the black hole where space–time is again semiclassical. This analysis has also been generalized to the electrovac case [2]. Test (non-backreacting) shells were also studied in this framework [3]. In this letter we would like to analyze the case of a self-gravitating shell. This is significant since it is the first full quantum treatment of a midisuperspace model including quantum matter in loop quantum gravity. Studies of collapsing shells have been carried out in the past, but either using semiclassical [4] or reduced phase space methods [5, 6]. Whereas in both cases one is led to regular solutions where the collapsing shell bounces and comes out, there is complete disagreement on the total lifetime of the process. Similar disparities also arise in the more recent ‘effective-geometry’ approaches [7, 8]. A fully quantum gravity treatment is required to settle the question of black hole lifetime. The present work represents a first step in this direction. It also provides the starting point for studying the backreaction of Hawking radiation, which cannot be properly studied with the previously existing time-independent solutions [9].

In a previous paper [3] we studied the motion of quantized null shells in a fixed quantum geometry background. In this work we would like to consider the full backreaction of the shells on the quantum geometry. We start from the Hamiltonian considered in our previous manuscript. The context is spherically symmetric gravity written in terms of Ashtekar variables. There are two pairs of canonical variables $E^\alpha$, $K_\alpha$ and $E^\psi$, $K_\psi$, that are related to the traditional canonical variables in spherical symmetry $dx^2 = N^2 dx^2 + R^2 d\Omega^2$ by $\Lambda = E^\alpha/\sqrt{|\psi|}$, $P_\lambda = -\sqrt{|\psi|} K_\alpha$, $R = \sqrt{|\psi|}$ and $P_\psi = -2\sqrt{|\psi|} K_\psi E^\psi/\sqrt{|\psi|}$ where $P_\lambda$, $P_\psi$ are the momenta canonically conjugate to $\Lambda$ and $R$ respectively, $x$ is the radial coordinate and $d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$. We will take the Immirzi parameter and Newton’s constant to be unity.

The total Hamiltonian (after a rescaling and an integration by parts) is given by

$$H_T = \int dx \left[ -N^r \left( -\sqrt{|\psi|} (1 + K_\psi^3) + \frac{(E^\psi)^2 \sqrt{|\psi|}}{4(E^\psi)^2} + F(r)p \Theta(x - r) + 2M \right) 
+ N^a [(E^\psi) K_\psi + E^\psi K_\psi' - p \delta(x - r)] + 2N_a M + N_r [F(r)p + 2M] \right],$$

with $\Theta$ the Heaviside function, $r$ the position of the shell and $p$ its canonical momentum.

Following Louko et al [10] we took the coordinates $x$, $r$ to have the $(-\infty, \infty)$ range and assumed that the variables have the usual fall-offs in asymptotic radial coordinates. We are including the usual boundary term at spatial infinity with $N_i$ the lapse there. The lapse at the other end of the manifold. We are allowing the possibility of a pre-existing black hole of mass $M$. Also, $F(r) = \sqrt{|\psi|} \{ \eta (E^\psi)^{\eta} (E^\psi)^{\eta} + 2K_\psi (E^\psi)^{\eta} \} |_{\psi = 0}$. $N$ is a rescaled lapse, $N = N_{\text{orig}} E^\psi / (E^\psi)^{\eta}$ with $N_{\text{orig}}$ the original unrescaled lapse and the shift is also changed $N^r = N^r_{\text{orig}} + 2N_{\text{orig}} K_\psi \sqrt{|\psi|} (E^\psi)^{\eta}/(E^\psi)^{\eta}$ with prime denoting derivative with respect to the radial coordinate $x$. The parameter $\eta = \pm 1$ is the sign of the momentum $p$, depending on it one will have shells that are either outgoing or ingoing if one is outside the black hole.

Notice that $m \equiv F(r)p/2$ is a Dirac observable, which can be verified by direct calculation of its Poisson brackets with the constraints. Also, taking into account the falloff of the gravitational variables at spatial infinity the constraint indicates that the ADM mass of the space–time is $M + m$ (if there is no pre-existing black hole in the space–time and all the mass is provided by the shell we have $M = 0$). The lapse at infinity $N_i$ may be written as usual [11] in terms of the proper time at infinity $t$ as $N_i = \dot{t}$. It can also be verified that the Hamiltonian
constraint for gravity coupled to a shell has an Abelian algebra with itself. One can also check that the Poisson brackets with the diffeomorphism constraint are the usual ones.

There also exists a Dirac observable,
\[ \hat{V} = \int_0^\infty dy \left[ -2F^{-1}(y) + \eta(1 + 2(M + m)/y) \right] + \tau - \eta \left[ r + 2(M + m)\ln(r/(2(M + m))) \right], \]
and it can be checked that it has vanishing Poisson brackets with all the constraints and is canonically conjugate to \( m \). The observable \( V \) is associated with the Eddington–Finkelstein coordinate \( v \) of an observer at \( \text{scri} \) from which the shell is incoming or exiting.

For the quantization, we choose the same Hilbert space as in the test shell case, which is a direct product of the Hilbert space of vacuum spherically symmetric gravity and \( L^2 \)-functions for the shell. As in previous papers for the gravitational part we consider linear combinations of products of cylindrical functions of the form
\[ T_{g,\tilde{g},l}(K, K_p) = \prod_{v \in G} \exp \left( \frac{i}{2} \int_{x} dx \ K_v(x) \right) \times \prod_{x \in G} \exp \left( \frac{i}{2} \mu_j K_p(x_j) \right), \] (2)
where the label \( k \) is the valence associated with the edge \( e \), and \( \mu_j \) is the valence associated with the vertex \( x \) (usually called ‘coloring’).

We adopt a representation for the point holonomies as quasi-periodic functions (for an alternative choice see [12]), so that the labels \( \mu_j \) belong to a countable subset of the real line with equally displaced points. We take \( \mu_j = 2\rho (l_j + \delta_j) \) with \( l_j \) an integer, \( \delta_j \) between 0 and 1 and \( \rho \) is the polymerization parameter. The kinematical Hilbert space associated with a given graph \( g \) is \( \mathcal{H}_{\text{kin}}^{M} = \mathcal{H}_{\text{kin}}^{M} \otimes \mathcal{H}_{\text{kin}}^{\text{sh}} \otimes \left( \bigotimes_{j} \ell_j^2 \otimes \ell_j^2 \right) \), where \( \ell_j^2 \) and \( \ell_j^2 \) denote the space of square summable functions of \( k_j \) and \( \mu_j \), respectively, \( \mathcal{H}_{\text{kin}}^{M} \) is the Hilbert space of square summable functions of the ADM mass and \( \mathcal{H}_{\text{kin}}^{\text{sh}} \) is the Hilbert space associated with the shell variables, square integrable functions of \( r \).

The full Hilbert space is equipped with the inner product
\[ \langle \psi | \psi' \rangle = \frac{1}{2} \sum_{i} \left( \theta_i \psi_i F(r) \theta_i + \bar{F}(r) \theta_i \psi_i \theta_i \right), \] (3)
where \( \theta_i \) and \( X_i \) are operators acting on the wavefunction of the shell and defined as
\[ \theta_i \psi(r) = \int_0^r dx \hat{\theta}(x + \epsilon - r) \psi(r), \] (4)
\[ \bar{\theta}_i \psi(r) = \int_0^r dx \hat{\bar{\theta}}(x + \epsilon - r) \psi(r), \] (5)
\[ X_i = \frac{\epsilon_i}{2} (\delta_i \hat{\bar{p}} + \hat{p} \delta_i), \quad \hat{p} \psi(r) = -i \partial_r \psi(r), \] (6)
and $\epsilon_j$ is the spacing of the vertices $\epsilon_j = x_{j+1} - x_j$. The gravitational and shell geometrical parts of the Hamiltonian are

$$H^g = \hat{H}_g(x_j) = \hat{b}_j(-1 - \hat{K}^2_{x j}(x_j) + \hat{\alpha}^2_j [1/E^z]^2(x_j) + 2\hat{M}),$$

$$F_j = \epsilon_j^{-1}F(x_j) = 2\hat{b}_j\epsilon_j^{-1}(\hat{a}_j [1/E^z]^2(x_j) + [\hat{K}_{x j}/E^z](x_j)).$$

They can all be written in terms of the elementary operators

$$\widehat{K}^2_{x j}(x_j) = \frac{\sin(\rho K\varphi(x_j))}{\rho} \hat{E}^z(x_j) \frac{\sin(\rho K\varphi(x_j))}{\rho} \hat{E}^z(x_j)^{-1},$$

$$[\hat{K}_{x j}/E^z](x_j) = \frac{\sin(\rho K\varphi(x_j))}{\rho} \cos(\rho K\varphi(x_j)) \hat{E}^z(x_j)^{-1},$$

$$[1/E^z]^2(x_j) = \cos(\rho K\varphi(x_j)) \hat{E}^z(x_j)^{-1} \cos(\rho K\varphi(x_j)) \hat{E}^z(x_j)^{-1},$$

$$\hat{a}_j = \frac{\eta}{2\epsilon_j}(\hat{E}^z(x_j) - \hat{E}^z(x_{j-1})), \quad \hat{b}_n = \sqrt{\hat{E}^z(x_j)},$$

where as usual we have polymerized the $K\varphi$ variable but also introduced a holonomy correction in $E^z$ such that the change is equivalent to a canonical transformation at the classical level. There one has that $\{K\varphi(x), E^z(y)\} = \delta(x - y)$ and also that $\left\{\frac{\sin(\rho K\varphi(x))}{\rho}, \cos(\rho K\varphi(x))^{-1} E^z(y)\right\} = \delta(x - y)$. So we replace $K\varphi$ by the sines and $E^z$ by itself times the inverse cosine in all expression, that is the canonical transformation. This polymerization preserves the constraint algebra and leads to the original classical theory when $\rho \to 0$. Interestingly it can also be extended to matter fields, like scalar fields, and it preserves the constraint algebra. In a rather lengthy but straightforward calculation it can be shown that for the case of the shell the constraint has an Abelian commutator algebra with itself also at the quantum level.

Unlike the static vacuum case, we do not at the moment know how to find in closed form the space of physical states associated with the constraints we defined. However, with the constraints represented by well-defined operators, we can recognize the quantum observables of the model $\hat{O}(z(x))$, $\hat{M}$, $\hat{r}$, $\hat{F}p$ and $\hat{V}$ ($z(x)$ is a function of the radial variable that takes values in the interval $[-1, 1]$ that enters the definition of $\hat{E}^z$, we give more details below, for additional details see [12]). The former is present in the vacuum theory already and corresponds to the diffeomorphism invariant content of $\hat{E}^z$, or in other words that diffeomorphisms in one-dimension cannot change the order of the vertices of the spin network. The middle ones are the ADM mass and its canonically conjugate momentum, the time at infinity. The two latter ones are associated with the shell. They have the commutator, $[\hat{F}p, \hat{V}] = 2i\hbar$. In terms of them we would like to express dynamical variables of the problem as parameterized Dirac observables (evolving constants of the motion) and study the physics in the quantum regime. It should be emphasized that we do not have at the moment an explicit self-adjoint polymer implementation of the ADM observables associated with the shell. However, we can adopt a partial reduced quantization for the system by adopting a standard representation for them together with the loop quantized observable $\hat{O}(z(x))$. Given the algebra of quantum–Dirac observables, we can characterize the eigenstates of the complete set of observables and define parameterized Dirac observables based on them.

Following similar steps to the ones we carried out in the vacuum case [13] leads to explicit expressions for the components of the metric as functions of the Dirac observables and classical (functional) parameters $K\varphi$ and $z(x)$. We are interested in studying the whole
space–time. Although one can choose any coordinates for this purpose, we adopt here the horizon penetrating Eddington–Finkelstein ones \[10\]. At the classical level one considers that $E^x$ is time independent, what fixes $N^0 = 0$, and a relationship between $K^j$ and $E^x$ given by

$$
K^j = \frac{R_5 [\Theta(x) - \Theta(-x)]}{\sqrt{|E^x|} \sqrt{1 + \frac{R_5}{|E^x|}}},
$$

where

$$
R_5 = 2M + 2m[\Theta(x)\Theta(\sqrt{|E^x|} + (t - V)) + \Theta(-x)\Theta(\sqrt{|E^x|} - (t - V))].
$$

We will concentrate from now on in the case without a pre-existing black hole, i.e. $M = 0$. The expression for $K^j$ reduces to ordinary Eddington–Finkelstein outside the shell in $x > 0$ and satisfies $K^j = \{K^j, H_T\}$. Besides, one can check that $N = 1/2$ and

$$
E^x = \left(\frac{E^x}{2}\right)^2 \sqrt{1 + \frac{R_5}{|E^x|}}.
$$

These expressions take values for $x < 0$ that allow to extend the parameterized observables to the region beyond the singularity. For instance, the resulting expressions for the metric are (for reasons of space we give only one component, the others are similar in nature)

$$
g_{xx} = \frac{R_5 (E^x)^2}{2 |E^x|} [\Theta(x) - \Theta(-x)].
$$

So the metric components are represented as parameterized Dirac observables that depend on two parameters, one of them functional, $z(x)$, and $t$. Notice that the resulting metric is time dependent. To understand this time dependence we recall that in Eddington–Finkelstein coordinates time is related to the position of the shell by $t = x_r = V$ with $x_r$ the radius of the shell, $E^x(x_r) = x_r^2$, and $V$ constant along the shell. The resulting picture of the geometry is that of an ingoing shell that forms a black hole traversing its event horizon, going towards $x = 0$ and continuing beyond to the region of $x < 0$ as we observed in the case of test shells \[3\]. Let us comment that the previous gauge choice extends the kind of gauge fixings usually adopted where $K^j$ was a $c$-number. We now consider a choice that also depends on the observables. The conservation of the gauge fixing ensures the consistency of the choice.

The above physical description of the metric is classical, based on looking at the expressions of the metric as classical parametrized Dirac observables. Let us discuss how this vision can be implemented in detail in the quantum theory. In this case all previous expressions can be promoted to quantum operators by means of the usual quantization rules, i.e. replacing the classical observables by their quantum analogs (including $O$). Then, the operator associated with $E^x$ on a given physical state $|\rho_{km}, \ell\rangle$ is

$$
\hat{E}^x(s_j) [\hat{k}, m] = \hat{O}(z(x_j)) [\hat{k}, m] \equiv \hat{\ell}_{\text{Planck}}^2 k_{\text{Int}(z(x_j))} [\hat{k}, m],
$$

where $\text{Int}$ refers to the integer part, $\ell_{\text{Planck}}^2 = h$ and $z(x)$ is an arbitrary function taking values in $[-1, 1]$ whose choice determines the radial coordinate chosen, $n = 2\nu + 1$ is the number of vertices of the considered physical state $|\hat{k}, m\rangle$ (we work in a finite domain of length $L$ to avoid dealing with asymptotic issues involving spin nets). We must recall that $s_j$ is the radial coordinate of the $j$th point in the spin network. In order to approximate semiclassical space–times it is convenient to consider $n$ to be very large as well as superpositions of states of different eigenvalues of $\hat{m}$. We then form a wavepacket centered in $m_0$ of width $\sigma_m$. Although there are infinitely many possible choices for the sequence $\hat{k}$, we will choose a simple one that leads to a good semiclassical behavior. We will place the vertices of the spin network at radial
positions $x_i = (i + 1)\Delta$ and $\Delta$ a spacing bounded below by the quantization of areas $\Delta > \hbar \ell_{\text{Planck}}/(2\sqrt{\nu})$. We have for $i \geq 0$ that $x_i \in [\Delta, L] \text{ with } L = \Delta(v + 1)$. Also, $z(x) = x/L$ and $k_i = \text{Int}(x_i^2/\hbar^2 \ell_{\text{Planck}})$. In addition $(E^+)^i = \hbar \ell_{\text{Planck}}(k_i - k_{i-1})/(2\hbar) \sim (2i + 1)\Delta$. On the other hand, for $i < 0$ we have that $k_i = \text{Int}(i^2\Delta^2/\hbar^2 \ell_{\text{Planck}})$ and $(E^0)^i = \hbar \ell_{\text{Planck}}(k_0 - k_{-1})/\Delta = 2\Delta$ and it is non-vanishing for all $i$s where $k_i$, i.e. $E^x$, changes sign at $x = 0$. The allowed spin networks need to exclude the value $k_0 = 0$ in order to ensure that the description is singularity free. The fact that both the Hamiltonian constraint and the observables are independent of $K_x$ (conjugate to $E^x$) ensures that this condition is a consistent restriction on the physical Hilbert space (see for instance [1]). Notice that $K_x$ is only present in the diffeomorphism constraint that does not change the sequence of values $\hat{k}$. With these assumptions the result of the quantum construction is essentially a discretization of the above classical expressions of the metric on a lattice determined by a given spin network. They are extremely well approximated by their classical counterparts in regions away from where the classical singularity used to be. Close to it, such expressions are sensitive to the lattice chosen. In addition, in order to improve the semiclassical behavior in the regions far away from the singularity, one can also consider superpositions of spin networks with different values of $\hat{k}$. This will imply that there are states providing a description of the region close to where the singularity was where there will be large fluctuations of the metric and therefore the singularity is replaced by a highly quantum but regular region that is not well approximated by a semiclassical space–time.

Let us now consider a state representing a collapsing shell. If we start the evolution at a given asymptotic past time, the shell will form a black hole (once the shell reaches its Schwarzschild radius) and eventually it will reach the high quantum regime. Due to the particular topology we choose (with two separate asymptotic regions), the shell will transverse to the other asymptotically flat region and emerge there leaving behind a white hole geometry. This differs from the results of minisuperspace treatments, e.g. [5]. Because we are not including fields, and therefore Hawking radiation, one ends up with a black hole that resembles the eternal black hole we discussed in the past [1], with an interior region (which can be viewed as a ‘baby universe’) beyond where the singularity was that is disconnected from the exterior. Let us notice that we have chosen from the very beginning a particular topology. Other choices are available (for instance a space–time with one asymptotic region) but they have not been carefully analyzed yet. Loop quantum gravity, and its application to this particular model, is not presently able to deal with dynamical topology change. This is why each scenario must be treated separately. It is likely that in any of these situations we could approximate large portions of such space–times with the results of this paper, and we cannot disregard the emergence of a shell from the highly quantum region through a white hole that has been conjectured in scenarios such as those of [4, 5, 7, 8, 14–16].

We have already set up the framework for studying back reaction of Hawking radiation in this context [9]. What prevented us from carrying out the explicit calculation was the lack of a dynamical collapse model for the background. The ultraviolet finite terms of the stress energy tensor diverge if one studies the backreaction on an eternal black hole, as it involves an infinite amount of Hawking radiation. This paper provides such a dynamical collapse model. Nevertheless, one could try to investigate the evaporation process for small black holes, and see if the model admits an exact treatment for the study of the information loss paradox. It therefore opens the possibility of studying the complete evaporation of a black hole in loop quantum gravity. It also opens the possibility of studying other types of matter collapse in spherical symmetry, like scalar fields, which exhibit a rich classical dynamics [17, 18].
Summarizing, we have provided a detailed model for quantum collapse of matter and formation of a black hole in a quantum geometry. It provides insights into possible scenarios of black hole formation and evaporation.

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