Communications in Theoretical Physics

New Form of Kerr–Newman Solution and Its Hawking Radiation via Tunneling

To cite this article: Jiang Qing-Quan *et al* 2006 *Commun. Theor. Phys.* **46** 1001

Commun. Theor. Phys. (Beijing, China) **46** (2006) pp. 1001–1004

Vol. 46, No. 6, December 15, 2006

New Form of Kerr–Newman Solution and Its Hawking Radiation via Tunneling*

JIANG Qing-Quan,[1] WU Shuang-Qing,[1] YANG Shu-Zheng,[2] and CHEN De-You[2]

[1]Department of Physics, Central China Normal University, Wuhan 430079, China

[2]Institute of Theoretical Physics, China West Normal University, Nanchong 637002, China

(Received January 16, 2006)

Abstract *Parikh–Wilzcek's recent work, which treats the Hawking radiation as semi-classical tunneling process from the event horizon of static Schwarzshild and Reissner–Nordström black holes, indicates that the factually radiant spectrum deviates from the precisely thermal spectrum after taking the self-gravitation interaction into account. In this paper, we extend Parikh–Wilzcek's work to research the Hawking radiation via tunneling from new form of rotating Kerr–Newman solution and obtain a corrected radiant spectrum, which is related to the change of Bekenstein–Hawking entropy, and is not pure thermal, but is consistent with the underlying unitary theory. Meanwhile, we point out that the information conservation is only suitable for the reversible process and in highly unstable evaporating black hole (irreversible process) the information loss is possible.*

PACS numbers: 04.70.-s, 97.60.Lf
Key words: Kerr–Newman black hole, tunneling rate, Bekenstein–Hawking entropy

In 1974, Hawking proved theoretically that the black hole can radiate thermally, and the temperature of the black hole is true.[1] With the emission of thermal radiation, black holes could lose energy, shrink and eventually evaporate away completely. Thus the paradox of the information loss is created, which means the pure quantum state will be changed into the thermally mixture state and then the underlying unitary theory in quantum mechanics is not reliable. The created mechanism of the black hole's thermal radiation can be explained by either the tunneling effect in quantum mechanics or the vacuum fluctuation in quantum field theory. Namely, a pair of particles create just inside the horizon, the positive energy particle is tunneled out and the negative particle is absorbed by black hole. In other words, we can consider that the particles created just outside the horizon, the negative energy particle is tunneled into the horizon because the negative energy orbit only exists inside the horizon, thus the positive energy particle is left outside the horizon and moves towards the infinite distance and form Hawking thermal spectrum. Both the two narrative styles have a tunneling process, so the tunneling barrier should be found to truly describe the tunneling process and obtain the true radiate spectrum. But till now, the causes of the tunneling barrier are unclear for us. The related references do not use the language of quantum tunneling method to discuss Hawking radiation, so strictly speaking, it is not the quantum tunneling method. So, to derive the factually radial spectrum, the following two difficulties must be solved: Firstly, the formed mechanism of the potential hill; Secondly, the elimination of the coordinate singularity.

In fact, the background space-time is dynamical as a result of the loss of energy. But some existing methods, through which the Hawking radiation is derived, are mostly based on the fixed background and not taken the fluctuation of the space-time geometries into consideration.[2−7] Recently, Parikh and Wilczek have adopted the semi-classical tunneling picture to study the Hawking radiation via tunneling from the sphere-symmetrically static Schwarzchild and Reissner–Nordström black holes.[8−10] Under the consideration of the energy conservation and the unfixed background, the tunneling rates are obtained and meanwhile give out a correction to Hawking radiant spectrum, and prove that the factually radiant spectrum of black holes is not precisely thermal. In this model, Painlevé coordinate transformation is introduced to eliminate the coordinate singularity. After that, the semi-classical quantum tunneling method is extended to all kinds of sphere-symmetric black holes, and the results support the Parikh's opinion.[11−13] But, for axi-symmetric black holes, the research on the tunneling radiation is found to be fewness.[14−17]

Recently, Zhang investigated the Parikh–Wilzcek tunneling framework and argued that the Parikh–Wilzcek's work, the tunneling rate is consistent with an underlying unitary and satisfies the first law of the black hole thermodynamics, is only suitable for a reversible process.[18] The above mentioned is reversible process and the derived result is considered to be a support to the information conservation attributed ultimately to the underlying unitary theory. But, in fact, the factual emission process, because of an evaporating black hole highly unstable, is irreversible, thus the unitary theory will not be satisfied and the information loss is possible.

Being equipped with these insights, we are ready to investigate Hawking radiation via tunneling from new form of rotating Kerr–Newman solution by taking into account the energy conservation and the angular momentum conservation. The picture adopted in our discussion is as follows. A particle does tunnel out of a rotating black hole, and the tunneling barrier is created by the self-gravitation among the outgoing particle. If the total energy and total angular momentum are conserved, the outgoing particle must tunnel out a radial barrier to an observer resting

*The project supported by Fundamental Research Foundation of Sichuan Province of China under Grant No. 05JY029-092

in the dragging coordinate system. With the loss of energy and angular momentum, the black hole will shrink its size, correspondingly its dragging velocity will change also. That is to say, the geometry must be dynamic. Our results show that when self-gravitation is considered, the tunneling probability is related to the change of Bekenstein–Hawking entropy and the derived emission spectrum deviates from the pure thermal spectrum, but is consistent with the underlying unitary theory, and meanwhile the first law of the black hole thermodynamics for the reversible process is presented, and points out that the information conservation is only suitable to the reversible process and in highly unstable evaporating black hole (irreversible process) the information loss is possible.

In the case of a Schwarzschild black hole, it is convenient to recast the metric into the form of Painlevé–Schwarzschild coordinates,[19]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \Big(\mathrm{d}r + \sqrt{2M/r}\,\mathrm{d}t\Big)^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right) . \tag{1}$$

Here, in order to extend Parikh–Wilczek's work to stationary axi-symmetric black holes, we must adopt the general Painlevé coordinate transformation to eliminate the coordinate singularity. Recalling that the Kerr–Newman black hole solution can be expressed in the Boyer–Lindquist coordinate system as

$$\mathrm{d}s^2 = \mathrm{d}\bar{t}^2 - \frac{2Mr - Q^2}{\Sigma}\left(\mathrm{d}\bar{t} - a\sin^2\theta\,\mathrm{d}\bar{\phi}\right)^2 - \frac{\Sigma}{\Delta}\mathrm{d}r^2 - \Sigma\mathrm{d}\theta^2 - \left(r^2 + a^2\right)\sin^2\theta\,\mathrm{d}\bar{\phi}^2 , \tag{2}$$

where $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 + a^2 + Q^2 - 2Mr$. Carrying on a generalized Painlevé-type coordinate transformation,

$$\mathrm{d}\bar{t} = \mathrm{d}t - \frac{\sqrt{(2Mr - Q^2)\,(r^2 + a^2)}\,\mathrm{d}r}{\Delta}, \qquad \mathrm{d}\bar{\phi} = \mathrm{d}\phi - \frac{a}{\Delta}\sqrt{\frac{2Mr - Q^2}{r^2 + a^2}}\,\mathrm{d}r , \tag{3}$$

to Eq. (2), we have

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \Sigma\mathrm{d}\theta^2 - \Big[\sqrt{\frac{2Mr - Q^2}{\Sigma}}\left(\mathrm{d}t - a\sin^2\theta\,\mathrm{d}\phi\right) + \sqrt{\frac{\Sigma}{r^2 + a^2}}\,\mathrm{d}r\Big]^2 - \left(r^2 + a^2\right)\sin^2\theta\,\mathrm{d}\phi^2 , \tag{4}$$

which is a new form of the Kerr–Newman solution and is the obedient extension of Painlevé–Schwarzschild line element to the rotational case. Obviously, it also inherits a number of attractive features of the Painlevé–Schwarzschild line element: (i) The metric is well behaved at the event horizon; (ii) There exist Killing vectors ∂_t; (iii) The time coordinate represents the local proper time for radially free-falling observers; (iv) The hyper-surfaces of constant time slices are just flat Euclidean space in the oblate spheroidal coordinates; (v) it satisfies Landau's condition of the coordinate clock synchronization. These characters provide convenience for the research on the tunneling radiation of rotational black holes.

But, as the existence of the rotation, the Painlevé–Kerr–Newman metric (4) still brings us inconvenience. These reasons come from two aspects. On the one hand, the infinite red-shift surfaces $r_{\pm}^{\mathrm{TLS}} = M \pm \sqrt{M^2 - a^2\cos^2\theta - Q^2}$ is not coincident with the horizons $r_{\pm} = M \pm \sqrt{M^2 - a^2 - Q^2}$, so the geometrical optical limit is not reliable. On the other hand, the existence of a dragging effect in the stationary rotating spacetime results in the matter field in the ergosphere rotating spacetime near the horizon also dragged by the gravitational field. So, a rational and physical picture should be depicted in the dragging coordinate system.

Carrying out a dragging coordinate transformation,

$$\Omega = \frac{\mathrm{d}\phi}{\mathrm{d}t} = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{a\left(r^2 + a^2 - \Delta\right)}{\left(r^2 + a^2\right)^2 - \Delta a^2\sin^2\theta} , \tag{5}$$

to Eq. (4), we can derive the 3-dimensional dragged Painlevé–Kerr–Newman line element as

$$\mathrm{d}\hat{s}^2 = \frac{\Sigma\Delta}{\left(r^2 + a^2\right)^2 - \Delta a^2\sin^2\theta}\mathrm{d}t^2 - 2\frac{\Sigma\sqrt{(2Mr - Q^2)\,(r^2 + a^2)}}{\left(r^2 + a^2\right)^2 - \Delta a^2\sin^2\theta}\mathrm{d}t\mathrm{d}r - \frac{\Sigma}{r^2 + a^2}\mathrm{d}r^2 - \Sigma\mathrm{d}\theta^2 . \tag{6}$$

Now, we will investigate the tunneling behavior of massless particles across the horizon of the black hole. Since the tunneling processes take place near the event horizon, we may consider a particle tunneling from the event horizon as an ellipsoid shell and think that the particle should still be an ellipsoid shell during the tunneling process. Therefore, under these assumptions ($\mathrm{d}\hat{s}^2 = 0 = \mathrm{d}\theta$), the radial null geodesics followed by massless particle is

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\left(r^2 + a^2\right)^2}{\left(r^2 + a^2\right)^2 - \Delta a^2\sin^2\theta}\Big[-\sqrt{1 - \frac{\Delta}{r^2 + a^2}} \pm \sqrt{1 - \frac{\Delta^2 a^2\sin^2\theta}{\left(r^2 + a^2\right)^3}}\,\Big] , \tag{7}$$

where the $+(-)$ sign can be identified with outgoing (ingoing) radial motion.

As the tunneling process takes place near the event horizon, the outgoing particle can be treated as an ellipsoid shell. We adopt the picture of a pair of virtual particles spontaneously created just inside the event horizon. The positive energy particle can tunnel out of the event horizon and materializes as a real particle, and the negative particle is absorbed by the black hole, resulting in a decrease in the mass and angular momentum of the black hole and the shrink of the horizon. r_{in} and r_{out} are the locations of the event horizon before and after the horizon shrinks, and regarded as the two turning points of the tunneling barrier, the distance between them is dependent on the energy and angular momentum of the outgoing particle. Considering the self-gravitation among particles and under the condition of the energy conservation and angular momentum conservation, we fix the total energy of the spacetime and allow that of the black hole fluctuated. After the black hole radiates the particle of energy ω and angular momentum ωa, the mass and angular momentum of the black hole will be reduced to $M-\omega$ and $(M-\omega)\,a$ respectively. Then we should replace M with $M-\omega$ in Eqs. (5) $\sim$ (7) in order to describe the moving of the shell. For the moment, the radial null geodesics of the massless particle across the event horizon is

$$\dot{r}=\frac{\left(r^2+a^2\right)^2}{\left(r^2+a^2\right)^2-\tilde{\Delta}a^2\sin^2\theta}\Big[-\sqrt{1-\frac{\tilde{\Delta}}{r^2+a^2}}+\sqrt{1-\frac{\tilde{\Delta}^2a^2\sin^2\theta}{\left(r^2+a^2\right)^3}}\Big]\,, \tag{8}$$

where $\tilde{\Delta}=r^2+a^2+Q^2-2\,(M-\omega)\,r$ is the horizon equation after the emission of energy ω and angular momentum ωa. In the dragging coordinate system, the coordinate ϕ does not appear in the line element (6). That is, ϕ is an ignored coordinate in the Lagrangian function L. In order to eliminate the freedom completely, the imaginary part of the action can be written as

$$\mathrm{Im}S=\mathrm{Im}\int_{t_i}^{t_f}(L-P_\phi\dot{\phi})\mathrm{d}t=\mathrm{Im}\Big[\int_{r_{\rm in}}^{r_{\rm out}}P_r\,\mathrm{d}r-\int_{\phi_{\rm in}}^{\phi_{\rm out}}P_\phi\,\mathrm{d}\phi\Big]=\mathrm{Im}\Big[\int_{r_{\rm in}}^{r_{\rm out}}\int_0^{P_r}\mathrm{d}P_r'\mathrm{d}r-\int_{\phi_{\rm in}}^{\phi_{\rm out}}\int_0^{P_\phi}\mathrm{d}P_\phi'\mathrm{d}\phi\Big]\,, \tag{9}$$

where $r_{\rm in}$ and $r_{\rm out}$ represent the locations of the event horizon before and after the particle of energy ω and angular momentum ωa tunnels out, and (P_r,P_ϕ) are the canonical momentum conjugated to the coordinate (r,ϕ). Substituting Hamilton's canonical equation of motion,

$$\dot{r}=\frac{\mathrm{d}H}{\mathrm{d}P_r}\Big|_{(r;\phi,P_\phi)}\,,\qquad \mathrm{d}H_{(r;\phi,P_\phi)}=\mathrm{d}\,(M-\omega)\,,\qquad \dot{\phi}=\frac{\mathrm{d}H}{\mathrm{d}P_\phi}\Big|_{(\phi;r,P_r)}\,,\qquad \mathrm{d}H_{(\phi;r,P_r)}=\Omega\,\mathrm{d}J\,, \tag{10}$$

into Eq. (9), where $H=M-\omega$ and $P_\phi=J=(M-\omega)a$, and switching the order of integration yield the imaginary part of the action

$$\begin{aligned}\mathrm{Im}\,S&=\mathrm{Im}\int_{r_{\rm in}}^{r_{\rm out}}\int_M^{M-\omega}(\mathrm{d}H'-\Omega'\mathrm{d}J)\frac{\mathrm{d}r}{\dot{r}}=\mathrm{Im}\int_{r_{\rm in}}^{r_{\rm out}}\int_M^{M-\omega}(1-a\Omega')\frac{\mathrm{d}H'}{\dot{r}}\,\mathrm{d}r\\&=\mathrm{Im}\int_M^{M-\omega}\int_{r_{\rm in}}^{r_{\rm out}}\frac{\sqrt{(r^2+a^2)^4-\Delta'(r^2+a^2)^3}+\sqrt{(r^2+a^2)^4-\Delta'^2a^2\sin^2\theta(r^2+a^2)}}{\Delta'(r^2+a^2)}\\&\quad\times\Big[1-\frac{a^2\left(r^2+a^2-\Delta'\right)}{\left(r^2+a^2\right)^2-\Delta'a^2\sin^2\theta}\Big]\mathrm{d}r\,\mathrm{d}\,(M-\omega')\,,\end{aligned} \tag{11}$$

where

$$\begin{aligned}&\Delta'=r^2+a^2+Q^2-2\,(M-\omega')\,r=\left(r-r_+'\right)\left(r-r_-'\right),\\&r_+'=M-\omega'+\sqrt{(M-\omega')^2-a^2-Q^2},\qquad r_-'=M-\omega'-\sqrt{(M-\omega')^2-a^2-Q^2}\,,\\&r_{\rm in}=M+\sqrt{M^2-a^2-Q^2},\qquad r_{\rm out}=M-\omega+\sqrt{(M-\omega)^2-a^2-Q^2}\,.\end{aligned}$$

The above integral can be evaluated by deforming the contour around the single pole $r=r_+'$ at the event horizon. Doing the r integral first, we find

$$\begin{aligned}\mathrm{Im}\,S&=-\frac{1}{2}\int_M^{M-\omega}\frac{4\pi\big[(M-\omega')^2+(M-\omega')\sqrt{(M-\omega')^2-a^2-Q^2}-\frac{1}{2}Q^2\big]}{\sqrt{(M-\omega')^2-a^2-Q^2}}\mathrm{d}(M-\omega')\\&\quad-\frac{4\pi\big[(M-\omega')^2+(M-\omega')\sqrt{(M-\omega')^2-a^2-Q^2}-\frac{1}{2}Q^2\big]}{\sqrt{(M-\omega')^2-a^2-Q^2}}\Omega_+'\,\mathrm{d}J'\\&=-\frac{\pi}{2}\big[((M-\omega)+(M-\omega)\sqrt{(M-\omega)^2-a^2-Q^2})^2-(M+M\sqrt{M^2-a^2-Q^2})^2\big]\\&=-\frac{1}{2}(S_f-S_i)\,,\end{aligned} \tag{12}$$

where

$$\begin{aligned}&S_i=\tfrac{1}{4}A_i^2=\pi(r_{\rm in}^2+a^2)=\pi\big[(M+M\sqrt{M^2-a^2-Q^2})^2+a^2\big]\,,\\&S_f=\tfrac{1}{4}A_f^2=\pi(r_{\rm out}^2+a^2)=\pi\big[((M-\omega)+(M-\omega)\sqrt{(M-\omega)^2-a^2-Q^2})^2+a^2\big]\,,\end{aligned}$$

are the Bekenstein–Hawking entropies before and after the particle emission respectively. So the tunneling rate is[20]

$$\Gamma\sim\mathrm{e}^{-2\mathrm{Im}\,S}=^{(S_f-S_i)}=\mathrm{e}^{\Delta S}\,. \tag{13}$$

According to the definition of the surface gravity of the horizon, that is

$$\kappa_+'=-\frac{1}{2}\lim_{r\to r_+'}\sqrt{\frac{-g^{11}}{\hat{g}^{00}}}\frac{\partial}{\partial r}\ln\left(-\hat{g}^{00}\right)=\frac{r_+'-r_-'}{2\left(r_+'^2+a^2\right)}$$

$$= \frac{\sqrt{(M-\omega')^2 - a^2 - Q^2}}{2[(M-\omega')^2 + (M-\omega')\sqrt{(M-\omega')^2 - a^2 - Q^2} - \frac{1}{2}Q^2]} \,. \tag{14}$$

So equation (12) can be rewritten as

$$\mathrm{Im}\, S = -\frac{1}{2}\int_{M}^{M-\omega}\left[\frac{\mathrm{d}M'}{T'_+} - \frac{\Omega'_+ \mathrm{d}J'}{T'_+}\right] = -\frac{1}{2}\int_{S_i}^{S_f} \mathrm{d}S' = -\frac{1}{2}\left(S_f - S_i\right) \,, \tag{15}$$

where $M' = M - \omega'$ and

$$T'_+ = \frac{\kappa'_+}{2\pi} = \frac{\sqrt{(M-\omega')^2 - a^2 - Q^2}}{4\pi[(M-\omega')^2 + (M-\omega')\sqrt{(M-\omega')^2 - a^2 - Q^2} - \frac{1}{2}Q^2]} \,, \tag{16}$$

is the Hawking temperature of the event horizon after the particle emission. So from Eq. (15), we can easily find the differential form of the first law of the black hole thermodynamics

$$\frac{\mathrm{d}M'}{T'_+} - \frac{\Omega'_+ \mathrm{d}J'}{T'_+} = \mathrm{d}S' \,. \tag{17}$$

Obviously, Parikh–Wilczek's semi-classical tunneling formalism is so successfully that it satisfies the first law of the black hole thermodynamics, and meanwhile proves that the factual spectrum is not precisely thermal and the tunneling rate is related to Bekenstein–Hawking entropy, but is consistent with the underlying unitary theory.

Although Parikh and Wilczek treated Hawking radiation as a tunneling process and give a semi-classical but the first explicit calculation about the information conservation, the framework, which satisfies the first law of the black hole thermodynamics and consists with an underlying unitary theory, is only suitable for the reversible process and proves the information conservation. Equation (17) is the differential form of the first law of the black hole thermodynamics, which is combined with the energy conservation law $\mathrm{d}M - \Omega \mathrm{d}J = \mathrm{d}Q_h$ (where Q_h is the heat quantity) and the second law of the black hole thermodynamics $\mathrm{d}S = \mathrm{d}Q_h/T$ (where S is the entropy of the black hole). The equation of energy conservation is suitable for the reversible or the irreversible process, but the second law of the black hole thermodynamics $\mathrm{d}S = \mathrm{d}Q_h/T$ is only reliable for the reversible process. But in fact, the existence of the negative heat capacity, an evaporating black hole is a highly unstable system, and the thermal equilibrium between the black hole and the outside is unstable, there will be difference in temperature, so the process is irreversible, and for the moment the second law of the black hole thermodynamics should be $\mathrm{d}S > \mathrm{d}Q_h/T$. Thus the underlying unitary theory is not satisfied here, and therefore the information loss is possible during the evaporation, and the Parikh–Wilczek's tunneling framework cannot prove the information conservation.

For another, the preceding study is still a semi-classical analysis, which means that the radiation should be treated as point particles. Such an approximation can only be valid in the low energy regime. If we are to properly address the information loss problem, then a better understanding of physics at the Planck scale is a necessary prerequisite, especially that of the last stages of Hawking evaporation.

References

[1] S.W. Hawking, Commun. Math. Phys. **43** (1975) 199.

[2] J.B. Hartle and S.W. Hawking, Phys. Rev. D **13** (1976) 2188.

[3] T. Damour and R. Ruffini, Phys. Rev. D **14** (1976) 332.

[4] S.Q. Wu and X. Cai, IL Nuovo Cimento B **115** (2000) 143.

[5] B. Punsly, Phys. Rev. D **46** (1992) 1312.

[6] L. Liu and D.Y. Xu, Acta Phys Sin. **29** (1980) 1617 (in Chinese).

[7] Z. Zhao, Y.X. Gui, and L. Liu, Acta. Astrophy. Sin. **1** (1981) 141 (in Chinese).

[8] M.K. Parikh and F. Wilczek, Phys. Rev. Lett. **85** (2000) 5042.

[9] M.K. Parikh, Int. J. Mod. Phys. D **13** (2004) 2351.

[10] M.K. Parikh, 2004 hep-th/0402166.

[11] S. Hemming and E. Keski-Vakkuri, Phys. Rev. D **64** (2000) 044006.

[12] A.J.M. Medved, Phys. Rev. D **66** (2002) 124009.

[13] Q.Q. Jiang and S.Q. Wu, Phys. Lett. B **635** (2006) 151.

[14] J.Y. Zhang and Z. Zhao, Mod. Phys. Lett. A **22** (2005) 1673.

[15] J.Y. Zhang and Z. Zhao, Phys. Lett. B **618** (2005) 14.

[16] S.Z. Yang, Chin. Phys. Lett. **22** (2005) 2492.

[17] Q.Q. Jiang, S.Z. Yang, and H.L. Li, Commun. Theor. Phys. (Beijing, China) **45** (2006) 457.

[18] J.Y. Zhang, Y.P. Hu, and Z. Zhao, hep-th/0512121 (2005).

[19] C. Doran, Phys. Rev. D **61** (2000) 067503.

[20] P. Kraus and E. Keski-Vakkuri, Nucl. Phys. B **491** (1997) 249.