A method for MREIT-based source imaging: simulation studies

To cite this article: Yizhuang Song et al 2016 Phys. Med. Biol. 61 5706

View the article online for updates and enhancements.

Related content
- Topical Review
  Eung Je Woo and Jin Keun Seo
- Hybrid algorithm in MREIT
  Kiwan Jeon, Hyung Joong Kim, Chang-Ock Lee et al.
- Note
  Byung Il Lee, Chunjae Park, Hyun Chan Pyo et al.

Recent citations
- Direct detection of neural activity in vitro using magnetic resonance electrical impedance tomography (MREIT)
  Rosalind J. Sadleir et al.
A method for MREIT-based source imaging: simulation studies

Yizhuang Song\textsuperscript{1}, Woo Chul Jeong\textsuperscript{2}, Eung Je Woo\textsuperscript{2} and Jin Keun Seo\textsuperscript{3,4}

\textsuperscript{1} School of Mathematical Sciences, Shandong Normal University, Jinan, People’s Republic of China
\textsuperscript{2} College of Electronics and Information, Kyung Hee University, Gyeonggi-do, Korea
\textsuperscript{3} Department of Computational Science and Engineering, Yonsei University, Seoul, Korea

E-mail: seoj@yonsei.ac.kr

Received 12 April 2015, revised 20 March 2016
Accepted for publication 7 June 2016
Published 12 July 2016

Abstract
This paper aims to provide a method for using magnetic resonance electrical impedance tomography (MREIT) to visualize local conductivity changes associated with evoked neuronal activities in the brain. MREIT is an MRI-based technique for conductivity mapping by probing the magnetic flux density induced by an externally injected current through surface electrodes. Since local conductivity changes resulting from evoked neural activities are very small (less than a few %), a major challenge is to acquire exogenous magnetic flux density data exceeding a certain noise level. Noting that the signal-to-noise ratio is proportional to the square root of the number of averages, it is important to reduce the data acquisition time to get more averages within a given total data collection time. The proposed method uses a sub-sampled $k$-space data set in the phase-encoding direction to significantly reduce the data acquisition time. Since the sub-sampled data violates the Nyquist criteria, we only get a nonlinearly wrapped version of the exogenous magnetic flux density data, which is insufficient for conductivity imaging. Taking advantage of the sparseness of the conductivity change, the proposed method detects local conductivity changes by estimating the time-change of the Laplacian of the nonlinearly wrapped data.

Keywords: MREIT, source imaging, neuroimaging

(Some figures may appear in colour only in the online journal)
1. Introduction

Functional MRI, the most commonly used neuroimaging technique, is an indirect method of visualizing changes in cerebral blood flow associated with neuronal activities (Smith 2012). There have been numerous studies to detect endogenous magnetic field induced by evoked neuronal currents from MR magnitude and phase images (Bodurka and Bandettini 2002, Luo et al 2011). However, the currently available MRI scanners seem to lack the sensitivity needed to produce any reproducible results (Huang 2014). Electroencephalogram (EEG) and magnetoencephalogram (MEG) source imaging methods entail the measurements of electric potential on the head and magnetic field outside the head, respectively, associated with neuronal current sources inside the head. The solutions of the corresponding inverse problems are not unique, and these neuroimaging methods have their own limitations (Wendel et al 2009).

The effective tissue conductivity inside the brain depends on molecular composition, amounts of intra- and extra-cellular fluids and cellular structure. Cell membranes are known to be highly resistive, hindering the DC current from passing through the cells. The opening of ion channels in cell membranes leads to an increase in the effective tissue conductivity since ion channels allow DC current to pass through the intracellular space. Animal studies have shown that neuronal depolarization leads to a net increase in the tissue conductance (Klivington and Galambos 1967, Velluti et al 1968, Oh et al 2011, Santos et al 2016). On the other hand, cell swelling accompanying epileptic seizures tend to decrease the effective conductivity. Santos et al (2016) characterized and visualized cortical conductivity changes during interictal and ictal activities in the anaesthetized rat. The decreased cerebral tissue conductivity during seizure activities can be attributed to the shrinkage of extracellular space by cell swelling that follows intensive neural activities (Van Harreveld and Schade 1962, Elazar et al 1966, Dietzel et al 1980, Olsson et al 2006).

There have been research efforts to detect conductivity changes associated with neuronal depolarization. If successful, this will provide a new direct neuroimaging method. Recent experimental studies show the feasibility of detecting fast neural evoked activities on rat cerebral cortex using an impedance imaging method employing a planar epicortical electrode array (Oh et al 2011, Aristovich et al 2014). This method may find numerous clinical applications where the invasive procedure to place epicortical electrodes is acceptable.

Magnetic resonance electrical impedance tomography (MREIT) produces cross-sectional images of a conductivity distribution inside the human body by injecting currents and measuring induced magnetic flux densities using an MRI scanner. We may apply the technique to visualize local conductivity changes associated with evoked neuronal activities in the brain (Sadleir et al 2010, Woo 2011). The major technical challenge in this MREIT-based source imaging or so called functional MREIT is how to acquire the magnetic flux density data with an sufficient signal-to-noise ratio (SNR) to detect neural activities within an acceptable imaging time.

In MREIT, an almost constant current is injected into an imaging subject to produce a magnetic flux density \( \mathbf{B} = (B_x, B_y, B_z) \) inside the body. The component \( B_z \) is then found from the measured phase changes in the MR images. MREIT exploits the nonlinear relationship between the conductivity \( \sigma \) and \( B_z \) via the Biot–Savart law, making a key observation that the Laplacian of the \( B_z \) data probes changes in the logarithm of the conductivity distribution along any equipotential curve in each imaging slice (Seo et al 2003, Woo and Seo 2008, Seo and Woo 2011, Seo et al 2013b). Given that the conductivity distribution changes with time due to neuronal activity, we consider \( \sigma \) and \( B_z \) as functions of position \( \mathbf{r} = (x, y, z) \) and time \( t \). The relation between \( \frac{\partial}{\partial t} \sigma \) and \( \frac{\partial}{\partial t} B_z \) can then be expressed by the Biot–Savart law: for \( \mathbf{r} \in \Omega \),

\[
\frac{\partial}{\partial t} \sigma \quad \text{and} \quad \frac{\partial}{\partial t} B_z.
\]
\[
\frac{\partial}{\partial t} B_z(r, t) = \frac{\mu_0}{4\pi} \int_{\Omega} \left( \frac{\partial}{\partial t} \ln \sigma \right) \mathbf{J}(r', t) + \sigma \frac{\partial \mathbf{E}}{\partial t}(r', t) \times \mathbf{e}_z \right] \frac{\text{dr}'}{|r - r'|^3},
\]
\[\text{(1)}\]

where \( \mathbf{e}_z = (0, 0, 1) \), \( \Omega \) represents the imaging domain, and \( \mathbf{J} \) and \( \mathbf{E} \) are the current density and electric field, respectively, induced by the injection current. The inverse problem for MREIT source localization is to estimate location of the support of \( \frac{\partial}{\partial t} \sigma \) and its magnitude from knowledge of the measured data \( \frac{\partial}{\partial t} B_z \) and the relation \( (1) \).

Noting that the magnitude of \( \frac{\partial}{\partial t} \sigma \) associated with neural activity is very small (less than a few %), the major challenge facing MREIT source localization is to obtain a sufficiently high SNR of \( \frac{\partial}{\partial t} B_z \) subject to a given amount of a given injection current and a fixed acquisition time. The SNR of \( \frac{\partial}{\partial t} B_z \) is proportional to that of the \( k \)-space data. In this paper, we use a sub-sampled \( k \)-space data set in the phase-encoding direction to increase SNR significantly for a fixed acquisition time. Unfortunately, it is very difficult to extract \( \frac{\partial}{\partial t} B_z \) with this sub-sampled \( k \)-space data set, because the \( B_z \) data in the inverse Fourier transform of the sub-sampled \( k \)-space data are wrapped not only circularly but also nonlinearly. To overcome this difficulty, we take advantage of the sparsity of its Laplacian \( \nabla^2 \frac{\partial}{\partial t} B_z \), which is closely related to \( \frac{\partial}{\partial t} \sigma \) being assumed to be sparse.

Numerical simulations demonstrate the performance of the proposed method and support its feasibility in future experimental studies. We will also address some technical issues for MREIT source imaging to advance it toward in vivo animal and human experiments.

2. Method

Assume that an imaging object occupies a three-dimensional domain \( \Omega \) and that its conductivity distribution is \( \sigma \). In MREIT, we inject a pulsed DC current of \( I \) mA into \( \Omega \) using a pair of surface electrodes \( \mathbf{E}^+ \) and \( \mathbf{E}^- \) to produce the time-independent current density \( \mathbf{J} \), electric field \( \mathbf{E} \) and magnetic flux density \( \mathbf{B} \) inside \( \Omega \) as shown in figure 1. Since the conductivity \( \sigma(r, t) \) depends on position \( r = (x, y, z) \) and time \( t \) due to the conductivity change associated with neuronal activity, the static fields \( \mathbf{J}, \mathbf{E}, \mathbf{B} \) also depend on time \( t \) due to the conductivity change \( \frac{\partial}{\partial t} \sigma \). Throughout this section, we fix \( z = z_0 \) and consider all these quantities only on a slice \( \Omega_{z_0} = \Omega \cap \{z = z_0\} \) of \( \Omega \).

Assuming that the \( z \)-axis is the axial magnetization direction of the scanner, the MR signal contains only the information of the \( z \)-component of \( \mathbf{B} \) (Joy et al. 1989, Scott et al. 1991, 1992). To be precise, the DC current of \( I \) mA is injected in pulses with the pulse width of \( T_c \) between the 90° and 180° RF pulses and also between the 180° RF pulse and the read gradient, as shown in figure 1. The corresponding \( k \)-space MR signal due to the injection current can be described as

\[
S(k_x, k_y, t) = \int_{\Omega_{z_0}} M(x, y)e^{j(k_x x + k_y y + \gamma \delta z) + jT_c(k_x x + k_y y)} \text{d}x \text{d}y,
\]
\[\text{(2)}\]

where \( M \) is the MR magnitude image, \( \gamma = 26.75 \times 10^7 \text{ rad T}^{-1} \cdot \text{s} \) the gyromagnetic ratio of hydrogen, and \( \delta \) any systematic phase artifact. If \( k \)-space sampling is designed to meet the Nyquist criteria (Seo and Woo 2013a), the inverse discrete Fourier transformation (DFT) of the \( k \)-space MR signal \( S(k_x, k_y, t) \) provides the complex MR image...
Hence, with the full sampling in $k$-space data, $B_z(x, y, t)$ can be extracted from
\[ M(x, y) \exp[i\delta(x, y) e^{i\gamma T_C B_z(x, y, t)} e^{i(x k_x + y k_y)}] \] (through unwrapping procedure), provided the time-independent quantities $M(x, y)$ and $\delta(x, y)$ are known a priori.

On the other hand, if we skip the $k$-space violating the Nyquist criteria, the inverse DFT does not provide the image of $M(x, y)$ and $\delta(x, y)$ is known a priori.

Let $\mathcal{M}(x, y, t) := M(x, y) e^{i[k \cdot B_z(x, y, t) \cdot \omega(x, y)]}$.

\[ \mathcal{M}(x, y, t) := \sum_{k=0}^{N-1} M(x, y + k) \frac{e^{i(x k_x + y k_y)}}{N} \]

where $\mathcal{M}(x, y) = M(x, y) e^{i\delta(x, y)}$. Figures 2(a) and (b) illustrate the real and imaginary parts of $\mathcal{M}$ with $N = 4$ respectively.

In this paper, we use the sub-sampled $k$-space data set $S^N$ to increase its SNR significantly with a fixed acquisition time. The data acquisition time of $k$-space data $S^N$ is inversely proportional to the factor $N$ due to the time consuming phase encoding, so that $S^N$ has higher averaging cycle with a fixed scan time.

Taking the time derivative on both sides of (4), we obtain
We need to extract \( \frac{\partial B_z}{\partial t} \) from the above equation (5) with knowledge of \( \frac{\partial M^N}{\partial t} \), but this is difficult due to the nonlinear relationship between \( \frac{\partial B_z}{\partial t} \) and \( \frac{\partial M^N}{\partial t} \). To overcome this difficulty, we take advantage of the sparsity assumption of \( \sigma \). Assume that the support of \( \frac{\partial M^N}{\partial t} \) occupies a small source region \( D \) as shown in figure 3.

Various numerical simulations show that \( \nabla \cdot \frac{\partial B_z}{\partial t} \) is very small away from the small source region \( D \), the support of \( \frac{\partial M^N}{\partial t} \). To see this, we note that the distribution of \( \nabla \cdot \frac{\partial B_z}{\partial t} \) is related to \( \sigma \) through the following identity, which can be obtained by taking the Laplacian to the z-component of Biot–Savart law (1):

\[
-\frac{1}{\mu_0} \nabla \cdot \frac{\partial B_z}{\partial t} = \mathbf{J}^\perp \cdot \nabla \ln \sigma + \frac{\partial \mathbf{E}}{\partial t} \cdot \nabla \sigma,
\]

where \( \mathbf{J}^\perp = (J_y, -J_x, 0) \) and \( \mathbf{E}^\perp = (E_y, -E_x, 0) \). Clearly, the term \( \mathbf{J}^\perp \cdot \nabla \ln \sigma \) in (6) is supported in \( D \). Next, consider the second term \( \frac{\partial \mathbf{E}}{\partial t} \cdot \nabla \sigma \) in (6). Since \( \nabla u = -\mathbf{E} \) satisfies \( \nabla \cdot (\sigma \nabla u) = 0 \), we have the Poisson equation for \( u \):

\[
\nabla \left( \sigma \nabla \frac{\partial u}{\partial t} \right) = -\nabla \cdot \left( \frac{\partial \sigma}{\partial t} \nabla u \right).
\]

Since the support of \(-\nabla \cdot \left( \frac{\partial B_z}{\partial t} \right) \nabla u \) in (7) is contained in \( D \), \( u \) can be viewed as a solution of the Poisson equation (7) with its source term supported in \( D \). Hence, according to the standard PDE theory, \( \mathbf{E} = -\nabla u \) decays rapidly with the distance from \( D \). Hence, we may assume that \( \nabla \cdot \frac{\partial B_z}{\partial t} \) is very small away from \( D \). We also assume \( \tilde{M}(t, x, y) = \tilde{M}(x, y) e^{i \mathcal{J}(t, x, y) / 2} \) (at time \( t = 0 \)) is known. This background image can be computed from the standard MREIT method.
In this paper, we compute \( \psi = \frac{\partial}{\partial B} t \) by minimizing the following energy functional

\[
\Psi[\psi] = \int_{\Omega_0} \left[ \sum_{k=0}^{N-1} \hat{M} \left( x, y + \frac{k}{N}, t \right) \psi \left( x, y + \frac{k}{N}, t \right) - \frac{\partial M^N}{\partial t} \left( x, y, t \right) \right]^2 dx dy + \lambda \int_{\Omega_0} |\nabla \psi(x, y, t)|^2 dx dy.
\]  

The first integral in (8), called the fidelity term, forces the residual \( \sum_{k=0}^{N-1} \hat{M} \left( x, y + \frac{k}{N}, t \right) \psi \left( x, y + \frac{k}{N}, t \right) - \frac{\partial M^N}{\partial t} \left( x, y, t \right) \), which is an approximation of (5), to be small. The second integral of (8), called the regularization term, enforces the sparsity of \( \nabla \psi \), since it is small away from the small source region \( D \). Here, \( \lambda \) is the regularization parameter which controls the tradeoff between the residual norm and the sparsity.

For technical simplicity, instead of solving the misfit functional (8), we consider the following modified version:

\[
\Psi[\psi] = \int_{\Omega_0} \left[ \sum_{k=0}^{N-1} \hat{M} \left( x, y + \frac{k}{N}, t \right) \psi \left( x, y + \frac{k}{N}, t \right) - \frac{\partial M^N}{\partial t} \left( x, y, t \right) \right]^2 dx dy + \lambda \int_{\Omega_0} |\nabla \psi(x, y, t)|^2 dx dy.
\]  

A simple calculation yields that a minimizer \( \psi \) satisfies the following partial differential equation approximately:

\[
-\lambda \nabla^2 \psi(x, y, t) + \sum_{k=0}^{N-1} \hat{M} \left( x, y + \frac{k}{N}, t \right) \psi \left( x, y + \frac{k}{N}, t \right) - \frac{\partial M^N}{\partial t} \left( x, y, t \right) = 0.
\]  

The equation (10) can be viewed as an approximation of the Euler–Lagrange equation corresponding to (9). To solve the equation (10), we set the homogeneous Dirichlet boundary condition; because \( \frac{\partial \psi}{\partial n} \) decays very fast away from the small source region \( D \). One may solve the equation (10) iteratively with the initial guess \( \psi^{(0)} = 0 \), using the standard finite element method (FEM).

We should note that solving the equation (10) provides only a folded image of \( \nabla^2 \frac{\partial \psi}{\partial t} \), as shown in figure 4. Using a simple thresholding method, we can segment the following folded area of the source region \( D \) from the Laplacian of the computed solution \( \psi \) of (10):
Figure 4. Schematic diagram of the proposed MREIT-based source imaging. A head is placed in an MRI scanner. We use a sub-sampled $k$-space data set in the phase-encoding direction, in which a nonlinearly wrapped version of the exogenous magnetic signal is available.

$$D^N = \Omega_{\sigma_0} \cap \{(x, y) : (x, y + \frac{k}{N}) \in D \text{ for some integer } k\}. \quad (11)$$

Hence, it remains to select $D$ out of the $N$-folded area $D^N$. For the selection, we need to use the fact that the change $\frac{\partial B_t}{\partial t}$ induced by the injection current reflects the interrelation between the source $\frac{\partial B_t}{\partial t}$ and the positions of electrodes, where current is injected. To be precise, assume that $D_0, D_1, \ldots, D_{N-1}$ are $y$-directional translations of $D$ so that $D^N = \bigcup_{k=0}^{N-1} D_k$. For simplicity, let $(x_0, y_0 + \frac{k}{N})$ be the center of $D_k$ and $r_k = (x_0, y_0 + \frac{k}{N}, z_0)$. Let $\chi_{r_k}$ be the characteristic function of the unit voxel containing $r_k$. In the special case when $\frac{\partial \sigma}{\partial t} = \chi_{r_k}$, the corresponding $\frac{\partial B_t}{\partial t}$ in (1) can be approximated roughly by the following function $\zeta_k$:

$$\zeta_k(r) : = \frac{\mu_0}{4\pi} \frac{r_k - r}{|r - r_k|^3} \cdot \left[ (\nabla u_0(r_k) + \sigma_0(r_k) \nabla v_k(r_k)) \times e_z \right], \quad (12)$$

where $u_0$ is the potential corresponding to a reference conductivity $\sigma_0$:

$$\begin{cases} \nabla \cdot (\sigma_0 \nabla u_0) = 0 & \text{in } \Omega \\ I = \int_{E^-} \sigma_0 \frac{\partial u_0}{\partial n} \, ds = - \int_{E^+} \sigma_0 \frac{\partial u_0}{\partial n} \, ds \\ \nabla u \times n = 0 & \text{on } E^- \cup E^+ \\ \sigma_0 \frac{\partial u_0}{\partial n} = 0 & \text{on } \partial \Omega \setminus \overline{E^+ \cup E^-} \end{cases} \quad (13)$$

and $v_k$ is a solution of Poisson’s equation:
Here, the reference conductivity $\sigma_0$ can be obtained by the standard MREIT algorithm, under the assumption that $\frac{\partial \sigma}{\partial t}$ is small, and $u_0$ is computed by solving the forward problem (13). From (12)–(13), we observe that $\zeta_k$ depends on the positions of electrodes $E^+ , E^-$. Now, we use the fact that, if $D = D_k$, the spatial distribution of $\zeta_k$ is similar to that of $\frac{\partial B_t}{\partial t}$. We select $k$ from $\{ \}$ which minimizes the difference:

$$\min_{\eta} \int_{\Omega_0} \| \mathcal{M}^N(x,y,1) - \Upsilon_k(x,y,\eta) \|^2 dx dy$$

where $\Upsilon_k$ is defined by

$$\Upsilon_k(x,y,\eta) := \eta \left[ \sum_{k=0}^{N-1} M \left( x, y + \frac{k}{N} \right) e^{i \theta_k} \zeta_k \left( x, y + \frac{k}{N}, z_0 \right) \right].$$

Here, $\eta$ is a scaling factor related to the percentage increase in conductivity. Figure 4 shows a schematic diagram of the proposed MREIT-based source imaging method.

### 3. Results

#### 3.1. Numerical simulation results

We carried out numerical simulations to test the feasibility of the proposed MREIT-based source localization method, which is described in the previous section. The procedure of the proposed method is summarized roughly as follows:

(i) Using MRI scanner and the standard MREIT method, we get the MR magnitude image $M$ and the reference information of $B_z(x, y, 0)$ and $\sigma(x, y, 0)$ on $\Omega_0$ at time $t = 0$.

(ii) Get the sub-sampled $k$-space data $\mathcal{S}^N$ and the $N$-folded image of $\mathcal{M}^N$ via inverse FFT for each time $t \geq 0$.

(iii) Solve the equation (17) iteratively with the initial guess $\psi^0 = 0$: For $n = 1, 2, \cdots$ and each $t$, solve

$$-\lambda \nabla^2 \psi^n(x,y,t) = - \sum_{k=0}^{N-1} M \left( x, y + \frac{k}{N} \right) \psi^{n-1} \left( x, y + \frac{k}{N}, t \right) + \frac{\partial M^N}{\partial t}(x,y,t)$$

with the homogeneous boundary condition $\psi^n|_{\partial \Omega_0} = 0$.

(iv) Segment the $N$-folded region $D^N := \{(x,y) \in \Omega_0 : |\nabla^2 \psi(x,y,t)| > \text{threshold}\}$ and select $D$ using the forward model with positions of electrodes, through which current is injected.

We construct a realistic three dimensional head model as shown in figure 1 (a). We place a spherical source domain $D$ inside the subject, whose conductivity changes with time as $\sigma = 0.09 + 0.0009 t$ for $t \in 3, 5, 8$. The conductivity distribution when $t = 5$ of the slice $\Omega_{z0}$ is shown in figure 1(c). The domain of head was meshed into finite elements as in figure 1(b). Two pairs of surface electrodes are attached to reconstruct the background conductivity. We generate the simulated data $B_t$ using the forward solver of MREIT (Lee et al 2003). We use the standard MREIT algorithm, called the harmonic $B_t$ algorithm (Seo et al 2003), to compute the...
reference conductivity distribution using two different reference $B_z$ data at $t = 0$, as shown in figure 6. Next, we consider the head model in figure 7, which is a bit different in terms of the size of electrodes. Figure 8 illustrates the temporal distribution of the conductivity. Through this pair of electrodes we inject a current with amplitude of 1 mA. Temporarily, we do not add any noise in the obtained data. Figure 9 illustrates the simulated data $M(x, y, t)$ with $N = 4$.

Using $M^N$, we get the solution $\psi \approx \frac{\partial \psi}{\partial z}$ by solving the equation (17) iteratively with the initial guess $\psi^0 = 0$ and the parameter $\lambda = \gamma T_z$. The first row of figure 10 shows the image of its Laplacian $\nabla^2 \psi \approx \nabla^2 \frac{\partial \psi}{\partial z}$. The second row of figure 10 shows the segmentations of the image of $\nabla^2 \psi$ at three different $t = 3, 5, 8$ by setting the threshold away from the value $\nabla^2 \psi$ of the source and the background. This image shows that $\nabla^2 \frac{\partial \psi}{\partial z}$ is closely related with the source region $D$, the support of $\frac{\partial \psi}{\partial z}$. Figure 10 shows that the images of $\nabla^2 \psi \approx \nabla^2 \frac{\partial \psi}{\partial z}$ for $t = 3, 5, 8$ clearly identify the folded area.
To test the noise tolerance of the algorithm, we add Gaussian random noise to the simulated data $\mathcal{M}^N(x, y, t)$. From the noise analysis in Sadleir et al (2005), Scott et al (1992), the noise standard deviation $s_B$ of $B_z$ is given by $s_B = 1/(2\gamma T_c \text{SNR})$, where SNR is the signal-to-noise ratio of the MR magnitude image. Using the noisy $B_z(x, y, t)$, we can produce the sub-sampled $k$-space data and validate our algorithm. In this experiment, SNR = 2000 and $T_c = 50$ ms. Figure 11 shows the image of the Laplacian $\nabla^2 \psi \approx \nabla^2 \partial_\tau$ at three different $t = 3, 5, 8$ and their segmentations by choosing the threshold away from the value $\nabla^2 \psi$ of the background. It remains to choose one out of three. Let $D_0, D_1, D_2$ represent each separated domain that contained in $\Omega_{cr}$. For each fixed time $t = 3, 5, 8$, using the forward solver of MREIT (Lee et al 2003), we can generate three different folded images of $\mathcal{M}^j, j = 0, 1, 2$ corresponding to three different supports $(D_0, D_1, D_2)$ of $\partial \tau$. Comparing them with the simulated data $\mathcal{M}^N$, we can localize $D$ successfully. Table 1 illustrates the process of selecting $D$ from $D^3$ at $t = 5$ by evaluating the difference $\text{Err}_k$ defined by

$$D^N = \{ (x, y) \in \Omega_{cr} : (x, y + \frac{kN}{N}) \in D \text{ for some integer } k \}. \quad (18)$$

Figure 7. (a) Head model with a pair of small electrodes. (b) Mesh of the corresponding forward model. (c) Spatial distribution of the conductivity in the slice $\Omega_{cr}$. Here, we multiply the true $\partial \tau$ by 20 to enhance the contrast.

Figure 8. Time-dependent conductivity distribution. (Left) $\sigma(x, y, 5)$, the conductivity at time $t = 5$; (middle) $\sigma(x, y, 0)$, the conductivity at $t = 0$; (right) the conductivity change $\sigma(x, y, 5) - \sigma(x, y, 0) \approx \frac{\partial \tau}{\partial t}$, supported in a source region in $\Omega_{cr}$. To enhance the image contrast, we multiply $\frac{\partial \tau}{\partial t}$ by 20.
Figure 9. Images of $\mathcal{M}^N(x, y, t)$ for $t = 3, 5, 8$. (Top row) Real parts of $\mathcal{M}^N(x, y, t)$ for $t = 3, 5, 8$. (Bottom row) Imaginary parts of $\mathcal{M}^N(x, y, t)$ for $t = 3, 5, 8$.

Figure 10. Laplacian of $\psi$ without adding any noise. (Top row) Images of $\nabla^2 \psi \approx \nabla^2 \frac{\partial \psi}{\partial t}$ for $t = 3, 5, 8$. (Bottom row) Segmented regions of the images in the top row using a simple thresholding. We get the folded area of the source region $D$. 
\[ \nu = \frac{1}{\nu} \left( \int_{\Omega_{xy}}^{\Omega_{xy}} M^N(x, y, 1) - \nabla^2 \eta \right)^{1/2}, \]  
(19)

where \( \nu \) is a scaling constant given by \( \nu = \frac{1}{\int_{\Omega_{xy}}^{\Omega_{xy}} (\int_{\Omega_{xy}} M^2 \, dx \, dy)^{1/2}} \). In this experiment, we choose \( \eta \) as 5% increase in conductivity:

\[ \eta = \frac{0.05}{\int_{\Omega_{xy}} \sigma(x, y, 0) \, dx \, dy}, \]  
(20)

where \( \sigma(x, y, 0) \) is the reference conductivity reconstructed from the standard MREIT algorithm (Seo et al 2003). According to table 1, we have \( Err_1 < Err_2 < Err_0 \) in both cases of SNR = \( \infty \) and 2000, and hence \( D = D_1 \) is selected.

Next, we performed numerical experiments for an F-shaped source region as shown in figure 12. Figures 13 and 14 show that \( \nabla^2 \psi \approx \Delta^2 \frac{\partial \psi}{\partial t} \) can localize the support of \( \frac{\partial \psi}{\partial t} \) when SNR = \( \infty \) and 2000 respectively. Table 2 shows that \( Err_1 < Err_2 < Err_0 \) in both cases of SNR = \( \infty \) and 2000. Hence, \( D = D_1 \) is selected.

We can see from numerical experiments in this section, the proposed algorithm is capable of visualizing the time changes of the conductivity less than 5% from the sub-sampled k-space data. The proposed method of detecting the support of \( D^N \) with the undersampled k-space data works well with different simulation models. The performance does not change with a more.
realistic model with heterogeneous background. The performance of selecting $D$ depends on the distance between $D$ and electrode locations. Generally speaking, the nearer $D$ is to the electrodes, the better the performance.

### 3.2. Phantom experiment

To experimentally test the proposed method, we built a saline-filled cylindrical phantom where an insulated carbon wire is immersed. Figure 15 shows the phantom with its diameter of 250 mm and height of 500 mm. The diameter of the carbon wire was 1 mm including insulation and 0.2 mm without insulation. We injected 1 mA imaging current through a pair of carbon electrodes attached on the boundary of the imaging plane $\Omega \cup \{z = z_0\}$. The size of the carbon electrode was $10 \times 10$ mm. We additionally passed 30 $\mu$A current through the insulated carbon wire to generate an equivalent current dipole, mimicking neural activity. The wire was partially exposed to saline in the imaging plane so that the dipole current could spread out inside the saline water. A 9.4T MRI scanner was used with a gradient echo-based MREIT pulse sequence (Seo et al 2013b). Imaging parameters were $\text{TR/TE} = 300/3$ ms, $T_c = 27$ ms,

<table>
<thead>
<tr>
<th>$D^4$</th>
<th>$D_0$</th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>

| $\text{Err}_k$ (SNR = $\infty$) | 0.3325 | 0 | 0.1035 |
| $\text{Err}_k$ (SNR = 2000) | 3.7624 | 2.6323 | 3.7050 |

*Note:* The first row illustrates that $D^4 = D_0 \cup D_1 \cup D_2$; in the third and fourth rows, we give the values of $\text{Err}_k$ when SNR = $\infty$ and 2000, respectively.
Figure 13. Images of $\nabla^2 \psi \approx \nabla^2 \frac{\partial B}{\partial t}$ for an F-shaped source model when $t = 3, 5, 8$ and $\text{SNR} = \infty$. (Top row) Images of $\nabla^2 \psi \approx \nabla^2 \frac{\partial B}{\partial t}$ for $t = 3, 5, 8$. (Bottom row) Segmented regions of these images using a simple thresholding.

Figure 14. Images of $\nabla^2 \psi \approx \nabla^2 \frac{\partial B}{\partial t}$ for an F-shaped source model when $t = 3, 5, 8$ and $\text{SNR} = 2000$. Each figure has similar meaning to that in figure 13.
The total scan time was 50 min. For the reader’s convenience, we briefly explain the reason why the dipole current corresponds to an equivalent conductivity change. The 30 μA current spreads out in the imaging slice over a neighborhood of the uninsulated carbon wire segment. Now let us imagine that there occurred a conductivity change in the region which is the neighborhood of the uninsulated wire segment. Denoting by the corresponding potential change, the time-difference satisfies approximately \( \nabla \cdot (\sigma \nabla \frac{\partial u}{\partial t}) \approx - \nabla \cdot \left( \frac{\partial \sigma}{\partial t} \nabla u \right) \chi_D \) in \( \Omega \) with the homogenous Neumann boundary data, provided that \( D \) and \( \frac{\partial \sigma}{\partial t} \) are small. This \( - \frac{\partial \sigma}{\partial t} \nabla u \) can be viewed as a dipole current over \( D \).

Figures 16(a) and (b) show the real and imaginary parts of \( \mathcal{M}^4 \). Figure 16(c) shows the image of \( \nabla^2 \psi \) by solving the equation (10) iteratively. From table 3, we chose \( D_1 \) using the same selection technique as before with (19).

### Table 2. For an ‘F’ shape source domain, select \( D \) from the folded \( D^4 \).

<table>
<thead>
<tr>
<th>( D^i )</th>
<th>( D_0 )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( Err_k ) (SNR = ( \infty ))</th>
<th>0.3354</th>
<th>0</th>
<th>0.1077</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Err_k ) (SNR = 2000)</td>
<td>4.5756</td>
<td>3.1712</td>
<td>4.6255</td>
</tr>
</tbody>
</table>

Note: Meaning of each row is the same as that in table 1.
Neuronal activities increase tissue conductivity values in a local region regardless of the directions of associated neuronal currents. Since this spontaneous relation is direct and instantaneous, neural activity can be in theory directly mapped by visualizing changes in the conductivity distribution. MREIT is capable of providing high-resolution conductivity images with a pixel size of about 1 mm, provided that the SNR of $B_z$ data is sufficiently high.

For MREIT-based neuroimaging, the most challenging issue is to achieve a sufficiently high SNR of the acquired MR data to allow the detection of conductivity changes of less than 5%. Such an improvement of the SNR must occur under the constraint of a fixed total scan time. Assuming a certain MRI scanner with a given performance level, we may improve the SNR by increasing the number of averages. In order not to increase the total scan time, the proposed method uses sub-sampled $k$-space data. This is accomplished through expediting the data acquisition by skipping the time-consuming phase encoding lines. Using 20% $k$-space data and one injection current, for example, we can reduce the data acquisition time by about 90%, and hence the MR data can be averaged 10 more times.

Although the sub-sampled $k$-space data may be insufficient to obtain the complete $B_z$ data, it is capable of computing a folded version of $\nabla^2 \psi$, from which we can visualize local...
perturbations of conductivity values. The results of numerical experiments show that 20% k-space data can successfully detect local conductivity changes associated with neural activities. The proposed method can either significantly reduce the scan time without deteriorating the quality of source images or improve the image quality without increasing the scan time.

Acknowledgments

Song was supported by Mathematical Tianyuan Foundation (Grant No. 11426147), the Research Fund for Excellent Young and Middle aged Scientists of Shandong Province (No. BS2014SF020) and National Science Foundation of Shandong Province (ZR2015AM001). Woo was supported by the National Research Foundation of Korea (NRF) grant (NRF-2014R1A2A1A09006320). Seo was supported by NRF grant 2015R1A5A1009350.

References


Dietzel I, Heinemann U, Hofmeier G and Lux H D 1980 Transient changes in the size of the extracellular space in the sensorimotor cortex of cats in relation to stimulus-induced changes in potassium concentration Exp. Brain Res. 40 432–39

Elazar Z, Kado R T and Adey W R 1966 Impedance changes during epileptic seizures Epilepsia 7 291–307


Joy M, Scott G and Henkelman M 1989 In-vivo detection of applied electric currents by magnetic resonance imaging Magn. Reson. Imaging 7 89–94


Sadleir R et al 2005 Noise analysis in MREIT at 3 and 11 Tesla field strength Physiol. Meas. 26 875–84

Sadleir R J, Grant S C, Woo E J 2010 Can high-field MREIT be used to directly detect neural activity? Theoretical considerations NeuroImage 52 205–16


Seo J K and Woo E J 2011 Magnetic resonance electrical impedance tomography (MREIT) SIAM Rev. 53 40–68
Seo J K and Woo E J 2013a Nonlinear Inverse Problems in Imaging (Chichester: Wiley)
Smith K 2012 Brain imaging: fMRI 2.0 Nature 484 24–6
Van Harreveld A and Schade J 1962 Changes in the electrical conductivity of cerebral cortex during seizure activity Exp. Neurol. 5 383–400
Woo E J and Seo J K 2008 Magnetic resonance electrical impedance tomography (MREIT) for high-resolution conductivity imaging Physiol. Meas. 29 R1–26
Woo E J 2011 Functional brain imaging using MREIT and EIT: requirements and feasibility NFSI & ICBEM, Banff, Canada 13–5