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An orrery is often associated with a mechanical model of the solar system. The Human Orrery at Armagh [1] and The Peterborough Orrery [2] are based on a number of steel discs accurately placed to model the motion of the planets from Mercury to Saturn as well as the comets Halley and Encke to show the more pronounced elliptical orbits. The scale of both human orreries is 1 m to 1 AU. This scale allows the users to move around the inner orbits without colliding. It also allows a wide range of investigations into Kepler’s and Newton’s laws to be carried out without complex mathematical knowledge, although this is still available to more advanced users.

The model thus shows all the classical planets; that is, those bright enough to be visible with the naked eye and that have been known since antiquity.

One of the strengths of the Human Orrery is its ability to demonstrate the motion of the planets about the Sun at their correct relative speeds. As the distance between successive tiles on the ground corresponds to the same fixed time step for all of the planets, then the relative speeds are correct when people (one playing the part of each planet) move from one tile to the next in lockstep. This is known as ‘walking the orrery’. Choosing the length of the fixed time step roughly in inverse proportion to the scale of the model (for example, one should be able to move comfortably from one tile to the next for even the fastest moving objects) while ensuring that it is exactly divisible into the ~88-day orbital period of Mercury is at the heart of the human orrery design. At approximately 20 m in diameter the model can be used in a significant number of ways right across the curriculum [3]. Constructing such an orrery is a significant investment in time and money. However, it is not necessary to construct the whole orrery to investigate Kepler’s laws. Here we construct the orbit of a comet (Encke) and use this to show Kepler’s laws.

Download the A3 template ‘Encke plot’. Print this out onto paper, or better still cardboard, and fix it to a thin wooden board. If you drill a hole at each corner of the board you can fix the template while you construct your orbit.

Using a simple piece of inextensible string (length 4.2 m) tie a knot at one end so that a 4 inch nail or similar can be put through the hole. This will be the central point, or focus, of the orbit about the Sun. Now mark the following distances as accurately as you can on the string using the centre of the nail as your origin. Table 2 is in the order that you will use them. Each unit step is equivalent to a time step of 16 days. Because each disc step is five unit steps apart, then each labelled position is separated by 80 days of motion.

It is helpful to mark each of these points on the string with a small sticky label. This is not essential but helps to set out the positions quicker. Figure 1 shows the markers with different colours. In this case red is increasing distance towards aphelion. Purple is decreasing distance towards perihelion. Marking the odd-numbered positions in a different colour to the even-numbered positions can help to identify the distances to aphelion more easily.

Fix the template on the ground and move the string to the first line (marked in red). Continue by...
moving the string to the next line on the template in turn and marking the relevant distance in the same colour. Once you reach disc number 35, then the colour changes and the distances begin to reduce. At each point place a marker. This can be a nail, or a nail placed through a marker (e.g. small place mats with a hole through which to place the nail). Laminating these beforehand means that they are a little more weatherproof. It is possible to purchase plastic discs, but this can get expensive.

Once you have finished (see figure 2) you will have marked out an orrery plot of Encke, at a scale of 1 m to 1 AU (1:1.5 × 10^{11}). If you have a ball bearing of diameter 10 mm, this is approximately the diameter of the Sun to the same scale (actually 9.3 mm). It is instructive to place the ball bearing at the position of the Sun to show the size of the Sun compared with the orbit.

There are several ways to allow students to show Kepler’s laws.

**Kepler’s first law**  
All planets orbit in ellipses, with the Sun at one of the foci.

This is self-explanatory; the orbit is an ellipse. Allow students to look carefully at the distance between the markers as the value of \( r \) increases. Look at the motion of the comet as it approaches the Sun. As a crude comparison use a circle of radius 1 m to represent the orbit of the Earth (which is only very slightly elliptical). This can be achieved by using a 6.28 m length of string tied to form a circle. Look at where the comet crosses the path of the Earth on the inward leg of its orbit; this is where the dust from Encke’s comet can enter the Earth’s atmosphere, producing the meteors (or ‘shooting stars’) associated with the Taurid meteor shower, visible in early November every year.

**Kepler’s second law**  
*Each body sweeps out equal areas in a given time.*

Use the associated worksheet to calculate the area of the arc swept out by Encke. Either one of the methods will give good results if you allow for the fact that they are both approximate methods, designed to be within most students’ mathematical ability. A more complex solution gives more accurate results, but not by very much more than these methods.

**Kepler’s third law**  
The period of the orbit squared is directly proportional to the cube of its average distance from the Sun.

This can be done by using the Nine Planets website [4] to obtain values of \( a \) and \( T \) and plotting a graph of \( T^2 \) versus \( a^3 \) for a number of the main planets. Following the principle of students measuring
from the model, they can obtain the length of the major axis of Encke’s ellipse, setting $a$ equal to half this value. They can further estimate the time $T$ for the comet to complete one revolution, knowing that successive tiles are separated by five unit time steps, or 80 days. These values of $a$ and $T$ should fit perfectly on the graph of $T^2$ versus $a^3$.

Energy conservation

We can use the formula for the gravitational potential energy per unit mass $-GM/r$ where $M$ is the mass of the Sun and $G$ is the gravitational constant. The students will also know the formula for kinetic energy per unit mass, namely $\frac{1}{2}v^2$. So the total energy of an object orbiting the Sun is $E = -GM/r + \frac{1}{2}v^2$. If $E$ is constant, then $v$ has to be large when $r$ is small, and vice versa.

Students can verify this law of energy conservation using direct measurement from the Human Orrery. Measuring the distance between two adjacent discs and dividing by the time separating the two discs allows students to calculate $v$. The distance $r$ of Encke from the Sun varies, and we can use the average value of $r$ for the same two adjacent discs. This gives a pair of values $r$, $v$, which can be entered in a plot of $v^2$ against $1/r$. Repeating for more pairs of adjacent discs around Encke’s orbit will yield a straight-line graph with intercept corresponding to the value of $E$. (A further question for the student: is $E$ positive or negative, and why?)

In this way we allow our students to test a model (energy conservation) as simply as possible. For a much more detailed treatise, students can research the speed of an object in an elliptical orbit [5] and use the data from the discs accordingly.

Allowing students to take measurements from an accurate model takes them away from simply researching from the internet. The skills required to build the human orrery section and take measurements give them a real feel for the size of the orbit and its dynamics. Obviously, with more orbiting bodies the data can be tested even more and a class can compare data from planets and comets. Constructing an orrery model (as described in [1]), is a valuable learning exercise for students and teachers; it allows students to take measurements from the model and manipulate the data to test the ideas of Kepler and Newton.

**References**

[1] www.arm.ac.uk/orrery/


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