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Decoherence and the paradox of the observed observer in quantum mechanics

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Abstract

If an atomic system is being observed, and its observer is also observed, would the first observer be in a superposition of states and evolve deterministically before being observed? In this paper, a simple model of non-demolition measurement is analyzed in order to elucidate the so-called 'Wigner's friend' paradox. The model illustrates the decoherence of an atomic system and its observer (the 'friend') as the latter is being observed (by Wigner).

Keywords: quantum mechanics, decoherence, measurement

1. Introduction

According to the usual interpretation of quantum mechanics, an atomic system, if not observed, is in a superposition of states described by a wave function evolving deterministically. It is the process of measurement that introduces an element of uncertainty by a reduction of the wave function. This interpretation, being rather counterintuitive, has led to many discussions and criticisms, such as the well known Schrödinger cat paradox, or a closely related situation discussed by Wigner in 1961 [1] (see also [2]). Wigner proposed to replace the cat by a conscious being, a friend, who would observe the decay of an atom (obviously without a killing mechanism). Clearly, according to the friend's perception, his joint state with the atom will be either $|+\rangle|f_+\rangle$ or $|-\rangle|f_-\rangle$, where $|\pm\rangle$ are the two possible states of the atom and $|f_\pm\rangle$ the corresponding states of the friend. However, for an outside observer, friend and atom must be in the joint state $c_+|+\rangle|f_+\rangle + c_-|-\rangle|f_-\rangle$, with probabilities $|c_+|^2$ and $|c_-|^2$, as long as they are not observed.

As pointed out by Wigner, the simplest way out of the difficulty is to assume that the joint system, observer plus atomic system, cannot be described by a wave function but by a mixture [1].

Everett pointed out a similar paradox in his thesis on the 'many-worlds interpretation' [3]. Suppose an observer in a room performs an experiment believing that the outcome of his measurement is undetermined according to the rules of orthodox quantum mechanics. However, for a second observer outside the room, as long as he does not open the door to the room, the evolution of the first observer should be completely deterministic, including the process of measurement.

It is usually accepted nowadays that the solution to this kind of paradoxes is the key concept of decoherence: a continuous transition from a pure state described by a wave function, to a mixture described by a density matrix [4–6].

A simple model of measurement and decoherence was worked out some time ago by the present author [7]. The model illustrated how the process of measurement, which brings a meter to a macroscopic state, is accompanied by a decoherence taking place in a time scale inversely proportional to the energy of the observed system, the decoherence being inevitable even at zero temperature due to vacuum fluctuations.

The aim of the present paper is to extend this previous analysis to an equally simple model of a (two states) atomic system S and two observers, modelled by harmonic oscillators, the first playing the role of Wigner's friend and performing a non-demolition measurement on S, and the second interacting strongly with the first one after a certain time. The complete Hamiltonian of the system is taken quite generally as non stationary [8]. The observers' meters are coherent states that, under suitable conditions, become macroscopic. In this way, the decoherence of the whole system and of each of its parts, together with the evolution of the meters, are made explicit.

The plan of the paper is as follows. In section 2, the analysis of [7] (for zero temperature) is briefly reviewed for the sake of completeness. Section 3 illustrates the way in which decoherence takes place, and a particular example is worked out. The results are discussed and interpreted in section 4.

2. Atomic system and observer

Let H_0 be the Hamiltonian of an atom with two levels of energy, say $\pm\epsilon$, and assume that H_0 commutes with all the other operators (it can be treated as a c-number for practical purposes, except when applied to an atomic-state).

Let the measuring apparatus, the 'observer's meter', be described by a harmonic oscillator with frequency ω and creation and annihilation operators a and $a^\dagger$ satisfying the usual commutation relation $[a, a^\dagger] = 1$. Then the total atom-observer hamiltonian can be modelled as

$$H_{01} = H_0 + \hbar\omega a^\dagger a + \mathrm{i}H_0\Big(fa^\dagger - f^* a\Big), \tag{2.1}$$

where $f = f(t)$ is a certain function describing the interaction between the two systems. This Hamiltonian represents a 'non-demolition' measurement since H_0 commutes with the full Hamiltonian [9, 10].

The corresponding evolution operator U_{01}, satisfying the equation

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}U_{01} = H_{01}\, U_{01}, \tag{2.2}$$

is

$$U_{01} = \exp\Big\{(\mathrm{i}\hbar)^{-1}H_0\, t - \mathrm{i}a^\dagger a\omega t\Big\}V, \tag{2.3}$$

where V is the evolution operator in the interaction picture. It satisfies the equation $\mathrm{i}\hbar\ \mathrm{d}V/\mathrm{d}t = H'V$, where

$$H' = \mathrm{i}H_0\Big(f\mathrm{e}^{\mathrm{i}\omega t}a^\dagger - f^*\mathrm{e}^{-\mathrm{i}\omega t}a\Big). \tag{2.4}$$

The solution is [7]

$$\begin{aligned} V &= \exp\Big\{\mathrm{i}\hbar^{-2}\nu H_0^2 + \hbar^{-1}H_0\Big(Fa^\dagger - F^* a\Big)\Big\} \\ &= \mathrm{e}^{\hbar^{-2}(\mathrm{i}\nu - |F|^2/2)H_0^2}\,\mathrm{e}^{\hbar^{-1}H_0 F a^\dagger}\,\mathrm{e}^{-\hbar^{-1}H_0F^* a} \\ &= \mathrm{e}^{\mathrm{i}\hbar^{-2}\nu H_0^2}D\Big(\hbar^{-1}H_0 F\Big), \end{aligned} \tag{2.5}$$

where the functions $F(t)$ and $\nu(t)$ are solutions of

$$\frac{\mathrm{d}}{\mathrm{d}t}F = f(t)\mathrm{e}^{\mathrm{i}\omega t}, \tag{2.6}$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} = \frac{\mathrm{i}}{2}\left(F\frac{\mathrm{d}F^*}{\mathrm{d}t} - F^*\frac{\mathrm{d}F}{\mathrm{d}t}\right), \tag{2.7}$$

and

$$D(\alpha) \equiv \mathrm{e}^{\alpha a^\dagger - \alpha^* a} \tag{2.8}$$

is a displacement operator, satisfying the condition

$$D^{-1}(\alpha)aD(\alpha) = a + \alpha. \tag{2.9}$$

The two (orthogonal) states of the atom, $|\pm\rangle$ are such that

$$H_0|\pm\rangle = \pm\epsilon|\pm\rangle.$$

If the meter is initially in the vacuum state $|0\rangle$, then after a time t

$$U_{01}(t)|\pm\rangle\,|0\rangle = \exp\Big\{\pm(\mathrm{i}\hbar)^{-1}\epsilon t + \mathrm{i}\nu\epsilon^2\Big\}|\pm\rangle\,\Big|\pm\alpha\Big\rangle, \tag{2.10}$$

and the meter is in the coherent state $|\pm\alpha\rangle = D(\pm\alpha)|0\rangle$, with

$$\alpha = \hbar^{-1}\epsilon F(t)\mathrm{e}^{-\mathrm{i}\omega t}. \tag{2.11}$$

If the coupling function $f(t)$ is at resonance with the oscillator, i.e. $f(t) = f_0\,\mathrm{e}^{-\mathrm{i}\omega t}$ (with f_0 constant), then $F(t) = f_0\, t$. The important point in this case is that the amplitude of the oscillations increases linearly in time and can eventually reach a macroscopic value [7].

If the general atomic state is a superposition

$$c_+|+\rangle + c_-|-\rangle$$

of two states $|+\rangle$ and $|-\rangle$, then

$$\begin{aligned} U_{01}(t)\big(c_+|+\rangle + c_-|-\rangle\big)|0\rangle = \mathrm{e}^{\mathrm{i}\nu\epsilon^2}\Big(&c_+\mathrm{e}^{(\mathrm{i}\hbar)^{-1}\epsilon t}|+\rangle|\alpha\rangle \\ &+ c_-\mathrm{e}^{-(\mathrm{i}\hbar)^{-1}\epsilon t}|-\rangle|-\alpha\rangle\Big), \end{aligned} \tag{2.12}$$

which is an entangled pure state.

The reduced density operator of the meter is simply

$$\rho_1 = \big|c_+\big|^2|\alpha\rangle\langle\alpha| + \big|c_-\big|^2|-\alpha\rangle\langle-\alpha| \tag{2.13}$$

and the reduced density operator of the atomic state is

$$\begin{aligned} \rho_0 = &\big|c_+\big|^2|+\rangle\langle+| + \big|c_-\big|^2|-\rangle\langle-| \\ &+ \mathrm{e}^{-2(\epsilon F/\hbar)^2}\Big(c_+c_-^*\mathrm{e}^{2(\mathrm{i}\hbar)^{-1}\epsilon t}|+\rangle\langle-| + \mathrm{h.\,c.}\Big) \end{aligned} \tag{2.14}$$

(since $\mathrm{Tr}\,|-\alpha\rangle\langle\alpha| = \langle\alpha|-\alpha\rangle = \exp\{-2|\alpha|^2\}$). Equation (2.14) exhibits the phenomenon of decoherence: for a macroscopic measurement, $f_0 \propto 1$ and $F \propto t$, the non-diagonal terms of the density matrix decay exponentially in time as $(t/t_{\mathrm{dec}})^2$, the decay time being $t_{\mathrm{dec}} \sim \hbar/\epsilon$, where ϵ is the characteristic energy of the measured atomic system. Then

$$|\alpha| \sim \frac{t}{t_{\mathrm{dec}}},$$

which implies that a realistic measurement (i.e. one with a macroscopic reading) requires a time $t \gg t_{\mathrm{dec}}$.

3. Atom and observed–observer

Let us now suppose that there is a second observer with an identical measuring apparatus modelled as the harmonic oscillator of the first observer. This second apparatus is described by creation and annihilation operators b and $b^\dagger$, and is strongly coupled to the first one. Accordingly, the Hamiltonian of the atom and the two observers can be taken as

$$H_{012} = H_{01} + \hbar\omega b^\dagger b + \hbar\kappa\left(ab^\dagger + a^\dagger b\right), \tag{3.1}$$

where κ is a measure of the interaction between the two oscillators. The evolution operator of the complete system is

$$U = \exp\left\{ (\mathrm{i}\hbar)^{-1}H_0\, t - \mathrm{i}\omega t\left(a^\dagger a + b^\dagger b\right)\right\} U_I, \tag{3.2}$$

where, in the interaction picture, $\mathrm{i}\hbar\ \mathrm{d}U_I/\mathrm{d}t = H_I U_I$ and

$$H_I = \mathrm{i}H_0\left(f\mathrm{e}^{\mathrm{i}\omega t}a^\dagger - f^*\mathrm{e}^{-\mathrm{i}\omega t}a\right) + \hbar\kappa\left(ab^\dagger + a^\dagger b\right). \tag{3.3}$$

The same procedure used in the previous section to obtain the evolution operator (2.5) can be generalized to the present case. Using the formulas

$$\mathrm{e}^{\mathrm{i}k(ab^\dagger + a^\dagger b)}a\ \mathrm{e}^{-\mathrm{i}k(ab^\dagger + a^\dagger b)} = a\cos k - \mathrm{i}b\sin k,$$

$$\mathrm{e}^{\mathrm{i}k(ab^\dagger + a^\dagger b)}a^\dagger\ \mathrm{e}^{-\mathrm{i}k(ab^\dagger + a^\dagger b)} = a^\dagger\cos k + \mathrm{i}b^\dagger\sin k, \tag{3.4}$$

(with the corresponding expressions with $a \leftrightarrow b$), it can be seen by direct substitution that the unitary evolution operator for the Hamiltonian (3.3) is

$$U_I = \mathrm{e}^{-\mathrm{i}k(ab^\dagger + a^\dagger b) + \mathrm{i}\hbar^{-2}H_0^2 w}\ \mathrm{e}^{\hbar^{-1}H_0(Ga^\dagger - G^* a)}\ \mathrm{e}^{\hbar^{-1}H_0(Jb^\dagger - J^* b)}, \tag{3.5}$$

where $k(t)$ and $w(t)$ are real functions, and $G(t)$ and $J(t)$ are complex functions satisfying the equations

$$\frac{\mathrm{d}k}{\mathrm{d}t} = \kappa$$

$$\frac{\mathrm{d}G}{\mathrm{d}t} = \frac{\mathrm{d}F}{\mathrm{d}t}\cos k, \quad \frac{\mathrm{d}J}{\mathrm{d}t} = \mathrm{i}\frac{\mathrm{d}F}{\mathrm{d}t}\sin k, \tag{3.6}$$

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{1}{2\mathrm{i}}\left(\frac{\mathrm{d}G}{\mathrm{d}t}G^* - G\frac{\mathrm{d}G^*}{\mathrm{d}t} + \frac{\mathrm{d}J}{\mathrm{d}t}J^* - J\frac{\mathrm{d}J^*}{\mathrm{d}t}\right). \tag{3.7}$$

Suppose now that the complete system is initially in the state $|\Psi_\pm(0)\rangle = |\pm\rangle|0\rangle_1|0\rangle_2$ (subscripts 1 and 2 refer to the first and second oscillators). Since, in general

$$\mathrm{e}^{-\mathrm{i}k(ab^\dagger + a^\dagger b)}|\alpha\rangle_1|\beta\rangle_2 = |\alpha\cos k - \mathrm{i}\beta\sin k\rangle_1| - \mathrm{i}\alpha\sin k + \beta\cos k\rangle_2,$$

it follows from equation (3.5) that

$$\left|\Psi_\pm(t)\right\rangle = \mathrm{e}^{\pm(\mathrm{i}\hbar)^{-1}\epsilon t + \mathrm{i}\epsilon^2 w(t)}|\pm\rangle|\pm A\rangle_1|\pm B\rangle_2, \tag{3.8}$$

where $|\pm A\rangle_1$ and $|\pm B\rangle_2$ are coherent states of the first and second oscillators respectively, with

$$A = \hbar^{-1}\epsilon[G\cos k - \mathrm{i}J\sin k]\mathrm{e}^{-\mathrm{i}\omega t}$$

$$B = \hbar^{-1}\epsilon[-\mathrm{i}G\sin k + J\cos k]\mathrm{e}^{-\mathrm{i}\omega t}. \tag{3.9}$$

If the interaction between the two oscillators does not take place before a certain time t_2, then, for $t < t_2$, we have $k = 0$ and therefore $G = F$ and $J = 0$, or equivalently $A = \hbar^{-1}\epsilon F$ and $B = 0$. Accordingly, the state of the second oscillator remains as $|0\rangle_2$, independently of the state of the atomic system, and the wave function of the complete system is just the wave function of the atom and first observer multiplied by $|0\rangle_2$; as such, it is a pure state. As expected, it is only after switching on the interaction that the decoherence takes place.

Summing over the states of the second oscillator, the reduced density operator of the atom and first oscillator turns out to be

$$\rho_{01} = \left|c_+\right|^2|+\rangle\langle+||A\rangle\langle A| + \left|c_-\right|^2|-\rangle\langle-||-A\rangle\langle-A|$$

$$+\left(c_+c_-^*|+\rangle\langle-||A\rangle\langle-A| + \mathrm{h.\,c.}\right)\mathrm{e}^{-2|B|^2}. \tag{3.10}$$

Notice that for $t < t_2$, $B = 0$ and thus ρ_{01} corresponds to a pure state.

As for the atomic system, its reduced density operator is

$$\rho_0 = \left|c_+\right|^2|+\rangle\langle+| + \left|c_-\right|^2|-\rangle\langle-|$$

$$+\left(c_+c_-^*|+\rangle\langle-| + \mathrm{h.\,c.}\right)\mathrm{e}^{-2(|A|^2+|B|^2)}, \tag{3.11}$$

just as in equation (2.14) for $t < t_2$. The intervention of the second observer accelerates the process of decoherence.

3.1. A particular example

Consider the resonant case described in section 1, with $F = f_0 t$. For $t < t_2$, we have $G = f_0 t$ and $J = 0$ as explained above. For $t > t_2$ we set $k(t) = \kappa(t - t_2)$ and then the coupling term in equation (3.1) is the constant κ. The solutions of equation (3.6) are

$$G = f_0\left(\kappa^{-1}\sin k(t) + t_2\right)$$

$$J = \mathrm{i}\frac{f_0}{\kappa}(1 - \cos k(t)), \tag{3.12}$$

and thus

$$A = \frac{\epsilon f_0}{\hbar\kappa}\left(\kappa t_2\cos k + \sin k\right)\mathrm{e}^{-\mathrm{i}\omega t}$$

$$B = -\mathrm{i}\frac{\epsilon f_0}{\hbar\kappa}\left(1 - \cos k - \kappa t_2\sin k(t)\right)\mathrm{e}^{-\mathrm{i}\omega t}. \tag{3.13}$$

These functions are oscillatory; their amplitudes is constant on the average and of magnitude $f_0\epsilon t_2/\hbar$.

4. Discussion of results

From the previous results it can be inferred that, as long as the first oscillator does not interact with the second one, its amplitude increases linearly in time until eventually becoming macroscopic. As soon as the second oscillator is coupled, say at time t_2, this growth stops and both meters oscillate with amplitudes of order $f_0\epsilon t_2/\hbar$.

If we identify the oscillators with observers, a pictorial description of the above results can be as the follows. The first observer looks at his meter and deduces that the atom is in one of the states $|+\rangle$ or $|-\rangle$ according to the direction of oscillation (with probabilities $|c_+|^2$ or $|c_-|^2$); this is what equation (2.13) for the density operator implies. During such a measurement, the state of the atom undergoes decoherence and is no longer a pure state; its density operator, equation (2.14), without non-diagonal terms is in exact correspondence with that of the observer. At a certain time, the second observer intervenes and what he sees is obviously the first observer in one of two possible states. For this second observer, however, the joint state of atom and first observer was a pure state at $t < t_2$, as it follows from their density operator (3.10); accordingly, this joint system evolved in a deterministic way. After the second intervention, the decoherence of the complete system continues with the typical time scale t_{dec}.

In conclusion, Wigner's original statement remains valid [1]: 'the joint system of friend plus object cannot be described by a wave function after the interaction—the proper description of their state is a mixture.'

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