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Tracking changes in shell structure in neutron-rich nuclei as a function of spin

Robert V F Janssens

Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA
E-mail: janssens@anl.gov

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Abstract
Taking advantage of the resolving power of modern gamma-ray spectrometers in combination with different types of nuclear reactions, it has been possible to investigate to fairly high-spin neutron-rich nuclei in a number of regions of the nuclear chart. The primary motivations for such studies are to characterize changes in shell structure as a function of the neutron-to-proton ratio and to document the impact of a large neutron excess on global properties, such as the nuclear shape. Recent data on nuclei close to the ‘island of inversion’ near $^{32}$Mg are discussed first. They highlight challenges in describing the observations with modern effective interactions. Subsequently, the nature of excitations in neutron-rich fpg-shell nuclei between Ca and Ni is reviewed. A neutron sub-shell closure at $N = 32$ has been attributed to the monopole tensor force. Furthermore, the presence of collective excitations at moderate spin in neutron-rich Cr and Fe isotopes illustrates the role of the $g_{9/2}$ orbital in driving the nuclear shape. The observations suggest that mixing between ‘deformed’ and ‘shell-model’ states needs to be considered. Finally, recent results in the direct vicinity of $^{68}$Ni indicate that the impact of the $N = 40$ neutron shell closure is rather modest.

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(Some figures may appear in color only in the online journal)

1. Introduction

The structure of neutron-rich nuclei has recently become the focus of much theoretical and experimental effort. Central to the on-going investigations is the expectation that substantial modifications can occur to the intrinsic shell structure of nuclei with a sizable neutron excess [1]. Alterations to the energy spacings of the orbitals and/or to their ordering can have a considerable impact on global nuclear properties such as the nuclear shape or the type of excitations characterizing the level spectra.

As these proceedings amply demonstrate, a number of techniques have been developed to explore the properties of neutron-rich nuclei. Most take advantage of fragmentation of high-energy beams followed by selection of an isotope of interest through a separator. Measurements are then most often carried out either with the selected nucleus moving at high velocity [2] (Coulomb excitation, knockout reaction, etc) or after it has been brought to rest in the laboratory (mass measurements, $\beta$-decay studies, etc). This approach is particularly well suited for investigations of nuclear properties near the ground state. One of the purposes of the present contribution is to illustrate the power of a complementary approach. Using the resolving power of spectrometers such as Gammasphere [3] in combination with a variety of reactions ranging from fusion-evaporation with exotic beams or targets to multi-nucleon transfer reactions (so-called deep-inelastic reactions), it has been possible to investigate neutron-rich nuclei in a number of regions of the nuclear chart. In particular, over the last 10–15 years it has been shown that the yrast spectroscopy of hard-to-reach, neutron-rich nuclei, populated in deep-inelastic reactions, can be carried out successfully at energies 15–25% above the Coulomb barrier, when using $\gamma-\gamma$ coincidence measurements [4]. From a broader perspective, $\gamma$-ray intensity measurements from these reactions have provided a mapping of the A and Z distributions of product yields. In all cases, multi-nucleon transfers were shown to lead mostly to less neutron-rich
in the region close in mass to the heavy colliding partner. Conversely, species on the neutron-rich side of the light reaction partner were found to be favored as well. It is possible to understand these yield distribution patterns in terms of a general tendency toward N/Z equilibration of the dinuclear systems formed in deep-inelastic reactions. As stated above, this experimental approach is complementary to techniques with fast beams in that it provides a means to investigate neutron-rich nuclei to fairly high angular momentum, herewith providing the data required (i) to characterize further changes in shell structure while testing in detail large-scale shell-model calculations with the most modern interactions, and (ii) to characterize global properties, such as the evolution of the nuclear shape with spin.

This presentation will first discuss data gathered in nuclei close to the ‘island of inversion’ centered around $^{30}$Mg and highlight challenges in describing the observations with modern effective interactions. It will subsequently review the nature of excitations in neutron-rich fpg-shell nuclei between Ca and Ni, where interactions incorporating the monopole tensor force are proposed to be responsible for a neutron sub-shell closure at $N = 32$. Also of particular interest in this region is the role of the $g_{9/2}$ orbital in driving the nuclear shape as has been demonstrated by the presence of collective excitations at moderate spin in neutron-rich Cr and Fe isotopes. This suggests that mixing between the ‘deformed’ and the ‘shell-model’ states may have a strong influence on their properties. This presentation will conclude with a discussion of recent results obtained in the direct vicinity of $^{68}$Ni.

2. Structure of $^{30}$Mg and cross-shell excitations near the island of inversion

The ‘island of inversion’ in the neutron-rich sd shell is a region of the nuclear chart where the effects of shell evolution in nuclei far from stability have been studied for some time. Unexpectedly high binding energies, observed via mass measurements of $^{31,32}$Na [5], were later reproduced in Hartree–Fock calculations with neutron $f_{7/2}$ intruder configurations were included [6]. The same effects were subsequently seen in Mg and Ne isotopes around $N = 20$ [7]. Systematic trends in $E(2^+_1)$ and $B(E2; 0^+ \rightarrow 2^+)$ values have added weight to the arguments for large deformation in the ground- and low-lying states of $N = 20$ isotopes below $Z = 12$, while data for Si, S, Ar, and Ca nuclei are characteristic of a large $N = 20$ shell gap. The large ground-state deformation of nuclei within the island of inversion is understood as resulting from deformed neutron states of $f_{7/2}$ and $p_{3/2}$ spherical parentage falling below similar states arising from the $d_{3/2}$ orbital as the neutron number increases. Recent theoretical efforts to account for the experimental observations have focused on the monopole component of the nucleon–nucleon interaction [8], revealing that both the central and the tensor forces dominate shell evolution. The central force is always attractive, with similar strength between two d orbitals as for a d-and-f pair of states. The additional effect of the tensor force between protons and neutrons, however, is attractive between the $d_{3/2}$ and the $d_{5/2}$ orbitals and repulsive between the $d_{5/2}$ and $f_{7/2}$ states. As the central force is stronger than the tensor force, the net effect is that of an attraction between both the $d_{3/2}$ and $d_{5/2}$ and the $d_{5/2}$ and $f_{7/2}$ orbitals, but less so in the latter case.

The nucleus $^{30}$Mg is located at the boundary of the island of inversion. Recent neutron knockout experiments have confirmed this by contrasting the spectroscopic strength associated with the negative-parity orbitals in the ground states of $^{30}$Mg and $^{32}$Mg. The strength in $^{30}$Mg is a third of that found in $^{32}$Mg, highlighting a dramatic change when entering the region of inversion. $^{30}$Mg is, therefore, not expected to have significant ground-state deformation. However, the $N = 20$ gap should be smaller than for stable nuclei and, as such, configurations involving the $f_{7/2}$ and $p_{3/2}$ neutron orbitals should be observed at relatively low excitation energy. Furthermore, if intruder configurations involve $f_{7/2}$ excitations, enhanced proton–neutron interactions may lead to collectivity in these excited states. These considerations motivated the quest for high-spin states in $^{30}$Mg carried out by Deacon et al. [10]. $^{30}$Mg was populated with the $^{14}$C($^{18}$O,2p) reaction which provides access to states of higher energy and spin than had been studied previously. The cross section for the reaction channel of interest being rather small, the measurement required the resolving power of the Gammasphere array [3] in combination with the mass and isotope selection provided by the Argonne fragment mass analyzer [11]. The $^{30}$Mg level scheme can be found in figure 4 of [10]: of the 11 transitions identified, only two had been reported previously. Moreover, the coverage provided by Gammasphere enabled spin and parity assignments following an angular correlation analysis.

The experimental levels are compared to the results of the calculations in figure 1. An initial shell-model calculation was carried out with the universal sd (USD) effective interaction [12] within a full sd model space. The calculated states lie higher than those observed experimentally, and there also appear to be additional observed states compared to the calculation. In particular, states with negative parity are clearly outside the model space considered. Thus, calculations within the sd model space are insufficient to account properly for excitations in $^{30}$Mg, even at relatively low spin. Extensions to the model space are needed to produce counterparts for all experimental states, and increase the overall level of mixing, compressing the lowest-lying states in energy, to give a better match in energies. Such calculations were performed using the Monte Carlo shell-model method (MCSM) with the SDFP-M interaction [13], to judge the influence of excitations into the fp-shell orbitals. This interaction has previously been used in calculations for a number of neighboring nuclei, including data on $^{31}$Mg [14]. To illustrate the role of cross-shell excitations, the positive-parity states from the MCSM calculations are shown on the right-hand side of figure 1, where states are labeled by spin-parity, excitation energy, and $N_{fp}$, the average number of neutron excitations into the fp shell. A pure 0p–0h state would have $N_{fp} = 0$, whereas a pure 2p–2h state has $N_{fp} = 2$, if the contribution of 4p–4h excitations is considered to be negligible. As discussed in [10], the MCSM calculations provide a way to disentangle cross-shell excitations from those involving the sd orbitals only. Specifically, they provide a means to account for the deexcitation patterns observed experimentally. For
example, the first $2^+$ state is calculated to lie just 49 keV above the observed one, a closer level of agreement than for the USD result. Its structure is predominantly $0p$–$0h$ with $Np = 0.79$, as might be expected. The presence of a strong transition to this level from the observed $4^+$ state at 3379 keV suggests a similarity in structure. Therefore, the calculated $4^+$ state at 4450 keV would appear to be the theoretical partner, even though the energy is overestimated (see comment in what follows). In a similar fashion, the calculated states at 3000 and 3850 keV, both corresponding to cross-shell excitations with $Np \sim 1.5$, would appear to correspond to the observed $2^+$ and $4^+$ states at 2465 and 3455 keV, respectively. The calculated $0^+$ level at 2120 keV may correspond to the excited $0^+$ state reported in [15], observed via an $E0$ transition to the ground state, and with an excitation energy of 1789 keV. The calculations indicate a significant $2p$–$2h$ configuration for this state, consistent with the conclusions drawn in [15]. Despite these successes, the MCSM calculations appear to have several specific problems associated with the ‘intruder’ excitations ($Np \sim 2$) predicted at too high an excitation energy. This could be attributed to overprediction of the excitation energy of the $2p$–$2h$ configurations and/or overestimation of the mixing between $0p$–$0h$ and $2p$–$2h$ excitations. Comparison of the negative-parity ($3^-$) state with the $1p$–$1h$ calculations also hints at the need to include a reassessment of the energy of cross-shell configurations. Thus, studies of higher spin states in nuclei such as $^{30}$Mg suggest a path forward for future shell-model calculations, in terms of both improving single-particle energies and interactions and extending the model space available to cross-shell configurations. In this regard, it is worth pointing out that further information on $1p$–$1h$ excitations can be gathered from recent studies on less exotic nuclei such as $^{30}$Si or $^{30}$Al [16], for example. In these instances as well, comparisons between data and calculations suggest the need for improved single-particle energies.

### 3. Shell structure and the $N = 32$ sub-shell gap in neutron-rich nuclei

The neutron-rich Ca, Ti, and Cr isotopes have been the focus of many studies during recent years. This effort was prompted primarily by the observation of an $N = 32$ subshell closure in $^{52}$Ca [17], $^{54}$Ti [18], and $^{56}$Cr [19], which has been inferred from mounting experimental evidence such as systematic variations in $E(2^+_1)$ energies and $B(E2)$ values in the even–even Ca, Ti and Cr isotopes [20]. The two physical quantities are found to be anti-correlated; while the $E(2^+_1)$ energies increase significantly at $N = 32$, the $B(E2)$ strengths are lowest. The situation mirrors that seen for the well-known neutron shell gap at $N = 28$ in nuclei of this region. This new aspect of shell structure in neutron-rich nuclei is understood as resulting from a significant shift in energy of the neutron $v\pi f_{5/2}$ orbital caused by a strong proton–neutron monopole interaction [8]. When protons are removed from the $\pi f_{7/2}$ shell, the $v\pi f_{5/2}$ orbital shifts up in energy relative to the $v\pi p_{3/2}$ and $v\pi p_{1/2}$ levels due to the weakening of the attractive monopole interaction. Many calculations within the full pf shell-model space have been carried out with different Hamiltonians and most reproduce the features of the yrast level structures fairly well. However, the magnitude of the shift in energy of the $v\pi f_{5/2}$ orbital appears to be best reproduced by the GXFPI family of Hamiltonians developed by Homma et al [21]. By comparing the level sequences of nuclei near the $N = 32$ gap with shell-model
illustrates the ability of modern effective interactions to account properly for the monopole interaction, the observed behavior of the states with predominant $\nu p_{3/2}$ and $\nu f_{5/2}$ configurations suggests that the $p_{1/2} - f_{5/2}$ energy difference in Ca nuclei might be somewhat smaller than that predicted by the GXPF1A interaction.

The same conclusion was reached in [24] from a comparison between experiment and theory in the case of $^{53}$Ti ($N = 33$) [24]. Figure 3 provides the level scheme established by Zhu et al. [24] and compares the data with calculations involving excitations of $\nu p_{3/2}$ and $\nu f_{5/2}$ orbitals in the Ti isotopes, a result pointing to the preferred use of the GXPF1A interaction for the description of nuclei in the region. As discussed in [24], the coupling of a $p_{1/2}$ neutron to the $^{54}$Ti core. The GXPF1A calculations interpret the $5/2^+$ first excited level in a straightforward manner as resulting from the $^{54}$Ti $\otimes \nu f_{5/2}$ coupling. Thus, contrary to the prediction of the KG3B interaction, the $f_{5/2}$ state lies above the $p_{1/2}$ orbital in the Ti isotopes, a result pointing to the preferred use of the GXPF1A interaction for the description of nuclei in the region. As discussed in [24], the couplings of a $p_{1/2}$ neutron to the $^{54}$Ti $4^+$ and $6^+$ yrast excitations, accounts for the $9/2^-$ and $13/2^-$ states, and the calculations interpret the $17/2^-$ state as the $^{54}$Ti$(6^+) \otimes \nu f_{5/2}$ coupling. In order to generate a $19/2^-$ state, the neutron core has to be broken and a $p_{3/2}$ neutron hole created. This is reflected experimentally in the 1881 keV energy of the $19/2^- \rightarrow 17/2^-$ transition, and theoretically in the large probability (74%) of the $[\pi f_{5/2} \otimes \nu (f_{5/2})_2]$ configuration. While the agreement between experiment and theory for $^{53}$Ti level structure can be viewed as satisfactory, it is worth pointing out that the energy difference between the lowest levels with predominant $\nu p_{1/2}$ and $\nu f_{5/2}$ configurations (i.e. the $5/2^- - 1/2^-$ difference) is smaller experimentally than calculated by 307 keV. Thus, as in the case of the $N = 31$ isotones, it appears that the two single-particle orbitals may be somewhat closer in energy than the GXPF1A Hamiltonian would predict, although other

Figure 2. Comparison between experiment and shell-model calculations with the GXPF1A and KG3B Hamiltonians for states involving excitations of $\nu (p_{1/2})^2 f_{5/2}$ neutrons in the $N = 31$ isotones $^{51}$Ca, $^{52}$Sc, and $^{53}$Ti.

Figure 3. Level scheme for $^{55}$Ti adapted from [24] compared with GXPF1A and KG3B shell-model calculations.
effects may also contribute to this difference (such as the relative positions of the $v\pi_{1/2}$ and $p_{3/2}$ states, for example).

Over the last decade, many other tests of modern effective interactions for the description of neutron-rich nuclei in the region just above doubly-magic $^{56}$Ca have been carried out and the cases discussed above are only illustrative examples of the large experimental effort that has taken place. In this context, it is worth pointing to the work of [26], where the spectroscopy of $^{52}$Ti was expanded significantly by exploring and focusing on high-lying, non-yrast states. The comparisons between data and calculations focus not only on energies, spins and parities, but also on lifetimes as the latter provide additional opportunities to probe the many-body wavefunctions. The experimental approach combined data from a fusion-evaporation reaction producing $^{52}$Ti with information derived from $\beta$ decay into the same nucleus following a deep-inelastic reaction [26]. Thirty-two states were observed, 14 of which had not been reported previously. In addition, lifetimes of nine states were measured using the Doppler shift attenuation method (DSAM). Comparisons between the data and shell-model calculations favor, to a degree, the results obtained with the GXPF1A effective interaction over those derived with the FPD6 [27] and KB3G Hamiltonians.

4. Shape-driving effects of the $g_{9/2}$ neutron orbital in the Cr and Fe neutron-rich isotopes

As indicated by the systematics of the $2^+_1$ energies of the even–even Cr isotopes, the effect of the $N = 32$ shell gap is less pronounced in this isotopic chain. This is expected since two additional protons occupy the $f_{7/2}$ shell and the $\pi f_{7/2} - \nu f_{5/2}$ proton–neutron monopole interaction is correspondingly stronger. Nevertheless, results from a study of the $B(E2; 0^+ \rightarrow 2^+_1)$ transition probabilities indicate that the collectivity of the $2^+_1$ state in $^{56}$Cr (with $N = 32$) is significantly lower than that in the neighboring $N = 30$ and 34 isotopes, $^{54}$Cr and $^{58}$Cr, and is similar to that of $N = 28$, $^{52}$Cr [28].

As already alluded to above, another experimental observation is that for neutron numbers above $N = 32$, the $2^+_1$ energy starts to drop considerably: from 1007 keV in $^{56}$Cr to 880 keV in $^{58}$Cr, 645 keV in $^{60}$Cr, 446 keV in $^{62}$Cr and 420 keV in $^{64}$Cr. This trend of decreasing $2^+_1$ energies is associated with a small neutron gap at $N = 40$, which facilitates excitations to the $v\pi_{9/2}$ intruder orbital. Recent detailed studies of odd, neutron-rich Cr isotopes reveal that the yrast $9/2^+$ state, which is associated with this $v\pi_{9/2}$ intruder orbital, decreases in excitation energy with increasing neutron number: from 2087 keV in $^{55}$Cr to 1507 keV in $^{57}$Cr, and abruptly down to 503 keV in $^{59}$Cr. In $^{55,57}$Cr, decoupled rotational bands built on the $v\pi_{9/2}$ state have been reported [29, 30], and these bands provide strong experimental evidence for the shape-driving potential of this intruder level, in these specific cases toward substantial prolate deformation. On the other hand, the long-lived $9/2^+$ isomeric state observed in $^{59}$Cr appears consistent with an oblate core [31]. In any event, the evidence indicates an increasing involvement of the $v\pi_{9/2}$ orbital, when compared to the Ti isotopes. This point is illustrated further in figure 4, where the structure of $^{56}$Cr is presented. In this case, the level scheme is characterized by two distinct sets of states: positive parity states of single-particle character and a collective negative-parity sequence. Indeed, the positive-parity yrast sequence of $^{56}$Cr is reproduced satisfactorily by calculations with the GXPF1A Hamiltonian and the agreement between data and theory is comparable to that achieved in similar comparisons for the Ti isotopic chain. In fact, as discussed in [32], a similar agreement between low- and medium-spin levels and GXPF1A calculations is seen in $^{58}$Cr, but not in $^{60}$Cr where the experimental level structure is more compressed than that calculated. Thus, beyond $N = 34$, the fp model space appears to be too restrictive to account for the observations.

As for the negative-parity sequence, figure 5 illustrates the striking similarity between the negative-parity, spin $7–17$, sequence of $^{56}$Cr and the two decoupled bands in the odd neighbors. The aligned angular momentum $I$, experiences a similar increase with rotational frequency in all three bands, except perhaps at the highest spins where the beginning of a backbend is present in $^{57}$Cr. Total Routhian surface (TRS) calculations described in [30] were carried out in order to track both the ground-state configuration and a negative-parity, two quasi-neutron configuration similar to those considered in the odd neighbors, i.e. with one particle occupying the same prolate-driving $1/2^+[440]$ orbital of $g_{9/2}$. 

![Figure 4](image.png)
for neutron-rich nuclei of the region are badly needed to investigate the issues further. As a significant step in this direction, it is worth pointing to a recent development: the projected shell model (PSM) [33] has been applied to this mass region to investigate states in fp-shell systems at high spin. By exploiting the use of a deformed shell-model basis, the PSM can adopt a relatively large single-particle space that allows collective motion and cross-shell excitations to be taken into account. The PSM has successfully reproduced high-spin structural features in some fp-shell nuclei that involve $v_{g9/2}$ configurations (see, for example, [34]).

The role of the $g_{9/2}$ neutron orbital in driving the nuclear shape is illustrated further in the neutron-rich Fe nuclei which have also received much attention recently [35–38]. As seen in figure 5, the behavior of collective bands seen at moderate and high spin in $^{59,60}$Fe mirrors that discussed above for $^{55–57}$Cr in terms of the alignment as a function of frequency. Calculations again account for the observations by invoking deformed configurations associated with the occupation of at least one $v_{g9/2}$ neutron state. Similar bands are also present in the $N=32$ isotope $^{58}$Fe and they are reproduced satisfactorily by PSM calculations [35]. Perhaps, more noteworthy in this instance is the presence of two band structures consisting of low-energy $\Delta I=1\hbar$ transitions [35]. These are candidates for magnetic rotational bands. Unlike traditional deformed rotational bands, these structures are composed of strong magnetic dipole ($M_1$) and weak $E2$ transitions. The orientation of these rotors is not specified by the deformation of the overall density, but rather by the current distribution induced by specific nucleons moving in high-$j$ orbitals. In this interpretation, the angular momentum vectors of high-$j$ protons and neutrons form two blades of a pair of shears that are almost perpendicular to each other at the bandhead. Along the bands, energy and angular momentum are increased by closing the blades of the shears; i.e. by aligning the proton and neutron angular momenta. Consequently, rotational bands can be formed in spite of the fact that the shapes of these nuclei stay nearly spherical. The name ‘magnetic rotation’ then alludes to the fact that the magnetic moment is the order parameter inducing a violation of rotational symmetry and, therefore, causing the presence of rotational-like structures in the spectrum. As discussed in [35], calculations reproducing the observed sequences indicate that the respective proton and neutron spins forming the blades are likely associated with two aligned proton holes in the $\pi f_{7/2}$ orbital (i.e. the $\pi (f_{7/2})^{-2}$ configuration) and a $v_{f5/2}p_{3/2}\left(g_{9/2}\right)$ neutron excitation. To date, only one other case has been reported in this mass region where the $g_{9/2}$ orbital is involved in generating this collective mode: four candidate M1 bands have been identified in $^{60}$Ni [39].

5. Shape coexistence near $N=40$ in Cr and Fe?

A recent measurement at the NSCL on $^{64}$Cr [40], the Cr isotope with the most extreme proton–neutron asymmetry reachable today, has demonstrated that the shell gap at $N=40$ appears to vanish in neutron-rich Cr. The energy of the first $2^+$ and $4^+$ states, compared to lighter isotopes in figure 6, were found to be 420(7) and 1131(11) keV, respectively. This behavior contrasts that of the Fe isotopes

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**Figure 5.** Experimental aligned angular momenta, $I_+$, as a function of rotational frequency for the Cr and Fe isotopes indicated in the figure. Adopted from [35].
In fact, the similarity between the Cr–Fe and Hg–Pb regions is rather striking, as shown in figure 7. The influence of the mixing between coexisting configurations has recently also been considered in the Cr–Fe region [43]. Simple two-band mixing calculations were carried out assuming that the yrast bands of these neutron-rich Cr and Fe even–even isotopes result from mixing between spherical and deformed states [43]. The excitation energies of the shell-model states are calculated using the GXPF1A interaction, which does not include $g_{9/2}$ or $d_{5/2}$ orbitals in the model space. The deformed states are calculated by a rotational model, defined by the $2^+$ excitation energy relative to the $0^+$ deformed ground states. Once the level structures of the shell model and deformed states are defined, the yrast states can be calculated using a two-band mixing model by determining the excitation energy of the two $0^+$ states and the interaction strength between the two structures, which is assumed to be constant for all levels. It was found that the excitation energies of the known levels in $^{62,64,66,68}$Fe and $^{90,62,64}$Cr can be reasonably reproduced by assuming a similar interaction strength $V_{int} \sim 100$ keV for all isotopes. While all the states are mixed to some extent, the calculations indicate that the ground states of all the Fe isotopes are predominantly of spherical character, while the deformed (‘intruder’) $0^+$ state lies at 2.4 MeV in $^{60}$Fe and decreases in energy with increasing $N$ to $\sim 300$ keV in the heavier isotopes, $^{56,60}$Fe. In fact, these calculations suggest that the first excited state in $^{64,66,68}$Fe is the deformed $0^+$ state, making it challenging to detect. For $^{60,62,64}$Cr, the calculations result in a similar prediction that the excited $0^+$ state is the lowest excitation with an energy of $\sim 330$ keV in $^{60}$Cr and $\sim 200$ keV in $^{62,64}$Cr. However, unlike in the isotones $^{64,66}$Fe, the model predicts that the ground states of $^{62,64}$Cr are dominated by a deformed configuration. This result may well account for the difference in the relative behavior of the measured excitation cross sections in knockout between Cr and Fe isotopes discussed above [40]. Clearly, to test this hypothesis of shape coexistence further, additional measurements are required. Some of these are currently in the planning stage.

6. $^{67}$Ni and the $N = 40$ sub-shell gap

In discussions of magic numbers, neutron number $N = 40$ has historically been a subject of debate, especially in the case of the Ni isotopes. Proton number $Z = 28$ is magic, and, for neutrons, a sizable energy gap at $N = 40$ is thought to separate the pf shell from the intruder $g_{9/2}$ state, potentially making $Z = 28$, $N = 40$ $^{67}$Ni a doubly-magic nucleus. Experimentally, the occurrence of shell closures in $^{68}$Ni was first suggested based on the observation of a 1770 keV $0^+_2$ level as the lowest excited state, followed by a $2^+_1$ state of relatively high excitation energy (2034 keV) [44, 45]. The discovery of several isomeric states in $^{68}$Ni and in neighboring nuclei [46, 47] supported the case for its magic character further, as did the results from Coulomb excitation measurements indicating a $B(E2, 2^+_1 \rightarrow 0^+)$ reduced transition probability roughly three times smaller than the corresponding value for $^{56}$Ni [48, 49]. However, based on recent high-precision mass measurements in the neutron-rich Ni isotopes (up to $^{73}$Ni), the $N = 40$ shell closure appears to be more doubtful.
Figure 7. Comparison between data in the neutron-rich Fe and Hg nuclei illustrating the striking similarity in the behavior of the evolution of spin with rotational frequency, hereby suggesting the possibility of a common picture of shape coexistence.

Figure 8. Level scheme of \(^{67}\text{Ni}\) (reproduced from [56]. Copyright 2012 by the American Physical Society) and results of shell-model calculations with the JUN45 and jj44b effective interactions; The set of calculations to the left is identical to that on the right, except that the excitation energies were offset such that the \(9/2^+\) isomeric state matches the data. When inferred from changes in the two-neutron separation energies [50, 51]. It has been argued in the literature that the apparent contradiction between the \(B(E2)\) value and the separation energy is a consequence of the parity change across the \(N=40\) gap, with a sizable fraction of the low-lying \(B(E2)\) strength residing in excited states around 4 MeV, and the \(2^+_1\) level being associated predominantly with a neutron-pair excitation [48, 52]. The size of the \(N=40\) gap is then of the order of 2 MeV only and the corresponding discontinuity in the sequence of orbitals corresponds at most to a sub-shell closure.

From these considerations, it is clear that a satisfactory description of nuclear structure in this mass region is still lacking. This is reflected in on-going theoretical efforts to determine the most appropriate interactions for use in the calculations, as well as in further experimental work. In this area, the most recent efforts have focussed on studies of single-particle excitations in the even \(^{64,66,68}\text{Ni}\) isotopes [53], a search for collective excitations similar to those described above for Cr and Fe [54], a study of a possible new long-lived state in \(^{68}\text{Ni}\) [55], and a new investigation of the neutron-hole nucleus \(^{67}\text{Ni}\). The data on the latter nucleus are presented below, as they provide an opportunity to test the most modern interactions while investigating the nature of yrast excitations up to moderate spin [56]. A level scheme built on the previously known \(13\ \mu s\) isomer in \(^{67}\text{Ni}\) was established for the first time by exploring prompt and delayed coincidence relationships from deep-inelastic reaction products. The yrast and near-yrast sequences have been delineated up to an excitation energy of 5.3 MeV and a tentative spin and parity of \((21/2^-):\) see figure 8.

In order to gain further insight into the nature of the observed \(^{67}\text{Ni}\) states, large-scale calculations were carried out...
with the shell-model code ANTOINE [57] using both the jj44b [58] and the JUN45 [59] effective interactions. Both Hamiltonians were restricted to the neutron $f_{3/2}$, $p_{1/2}$, $p_{1/2}$, and $g_{9/2}$ valence space and assume a $^{56}$Ni core. However, the required two-body matrix elements and single-particle energies were obtained from fits to different sets of data. Specifically, the JUN45 interaction was developed by considering data in nuclei with $Z \sim 32$ and $N \sim 50$, and excludes explicitly the Ni and Cu isotopes as the $^{56}$Ni core is viewed as being rather ‘soft’ [59]. In contrast, experimental data from $Z = 28 - 30$ isotopes and $N \approx 48 - 50$ isotopes were incorporated in the fits in the case of the jj44b interaction [58].

The results of the calculations are compared with the experimental data in figure 8. With both interactions, the energy of the $9/2^+$ state is predicted lower than the measured value. This can be viewed as an indication that the adopted single-particle energy of the $g_{9/2}$ neutron orbital is too low in the two Hamiltonians. It is worth noting that the jj44b interaction calculates this state to lie within 192 keV of the data and indicates about a 25% admixture of the $1g_{9/2}$ configuration into the $9/2^+$ wavefunction. With the JUN45 interaction, the level is predicted to lie 498 keV lower than in the data with roughly 33% of the wavefunction involving three neutrons in the $g_{9/2}$ orbital. This is possibly the result of the location of the $g_{9/2}$ orbital at a lower energy in the JUN45 Hamiltonian, as compared to that used in the jj44b case, which leads to larger configuration mixing in the wavefunction of the $9/2^+$ state. Overall, the calculated spectrum with both interactions appears somewhat compressed when compared to the data, as illustrated on the right side of figure 8. The correspondence between data and calculations is rather satisfactory when the computed excitation energies are expressed relative to the $9/2^+$ isomer as is done on the left-hand side of figure 8.

As discussed in detail in [56], the levels above the $9/2^+$ isomeric state can be understood as neutron excitations, with contributions of protons across the $Z = 28$ gap playing a minor role at best. Calculations with both interactions are in fair agreement with the data. They attribute a significant role to the $g_{9/2}$ neutron orbital for every state observed in this measurement. In fact, in most cases, significant $1g_{9/2}$ and $1g_{9/2}$ configurations are part of the wavefunctions. Similar observations have been made for other nuclei close to $^{68}$Ni; see, for example, recent comparisons between calculations with the same jj44b and JUN45 interactions and data for $^{65,67}$Cu in [60]. From these findings, it is concluded that even in a nucleus only one neutron removed from $N = 40$, the impact of a neutron shell closure is rather modest. As the $g_{9/2}$ neutron orbital is shape driving, multi-particle-hole excitations involving this state may be expected to be associated with enhanced collectivity and it would be of interest to investigate the latter in future measurements.

7. Conclusions

Using the resolving power of spectrometers such as Gammasphere in combination with reactions ranging from fusion-evaporation with exotic beams or targets to deep-inelastic reactions, it has been possible to investigate to fairly high angular momentum neutron-rich nuclei in a number of regions of the nuclear chart. The approach provides tests of nuclear structure in these systems that are not available with other techniques.

The power of the techniques was illustrated through data from nuclei in the $A \sim 30$ and $\sim 60$ regions and challenges for current theoretical descriptions were discussed. Studies of high-spin states in nuclei near the $N = 20$ island of inversion suggest a path forward for future shell-model calculations in terms of both improving single-particle energies and interactions and extending the model space available to cross-shell configurations. The delineation to high angular momentum of yrast sequences in Ca, Ti and Cr nuclei has provided further evidence on the presence of a sub-shell gap at $N = 32$ as well as information on the interactions affecting the $p_{1/2}$, $p_{1/2}$ and $g_{9/2}$ orbitals. Collectivity manifests itself through the presence of rotational sequences at moderate spin in Cr and Fe isotopes near $N = 40$ and indications for mixing between spherical and collective excitations were presented. The data highlight the role of the $g_{9/2}$ orbital in driving the nuclear shape. Finally, recent results in the direct vicinity of $^{60}$Ni indicate that the impact of the $N = 40$ neutron shell closure is rather modest. Generally speaking, the results discussed in this contribution highlight the challenge for theory to adequately account for cross-shell excitations.

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