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Superconductivity in Disordered Thin Films

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Abstract

Superconductivity disappears in (mesoscale) disordered thin films at a sheet resistance $R_\square$ or order 10 k Ω per square. This and other observed features of systems with both atomic and meso-scale disorder are briefly reviewed, as also theoretical models mainly based on the Josephson junction lattice. These are known to be inadequate. A new approach is described, with the sole assumption that the pair amplitude $|\Delta|$ does not vary spatially. The energy cost of wavevector (q) and frequency (ω) dependent phase fluctuations $\theta_{q\omega}$ is expressed exactly in terms of disordered normal state properties. It is shown that because of perfect screening, the ω^2 dependent term has an extra factor $|q|$ in two dimensions relative to the Josephson junction lattice form. The q^2 or phase gradient term is calculated as a function of $R_\square$. A rapid and large drop in stiffness occurs for $R_\square \sim 10$ k Ω when the electron localization length and pair coherence length become comparable. At this level of disorder, the pair amplitude fluctuates spatially, so that the above approach becomes inadequate. Several indications are that a charged Bose system in a random potential would be a realistic model for disordered thin film superconductors at low temperature.

1. Introduction

The interplay between superconductivity and localization is a phenomenon of fundamental interest, for obvious reasons. Superconductivity is the most spectacular manifestation of coherence between electron (pair) states, whereas localization is the extreme consequence of disorder leading to disjointedness. One therefore expects that on increasing disorder so that states becomes localized, superconductivity in a given material will disappear. A number of attempts have been made to investigate this competition experimentally (Refs. [1–8] described work on films) and to discuss it theoretically [9–12]. We do not yet have a clear detailed picture of the phenomenon (or phenomena) or an understanding of its nature; I shall describe here briefly the situation for two dimensional systems, and present some new results which emphasize the role of electron localization.

The case of disordered thin superconducting films is especially intriguing, because the lower critical dimension for both superconductivity and localization is two. In two dimensions, global pair phase coherence is just possible, and states are localized no matter how weak the disorder. Consequently, the corresponding coherence lengths are both exponentially large, e.g., the localization length depends exponentially on the inverse disorder. This is in contrast to bulk or three dimensional systems [7], where a critical disorder is needed to localize states, and where the localization length diverges as a power of the fractional deviation from critical disorder. Thus, novel effects are expected (and found) in inhomogeneous thin films or metal insulator composites (also films) composed of superconducting metallic materials.

The experimental situation is summarized in the next section (Section 2). The existing theoretical models (e.g., the Josephson junction lattice model) are also reviewed (Section 3). I then argue that disorder is the essential

ingredient in any realistic model for thin films superconductors. One way it enters is the localization of underlying electronic states. The effects of this on the stiffness for pair phase fluctuations, and on universal random variations in this stiffness, are discussed here for the first time (Section 4). A possible scenario for explaining the observed phenomena is pointed out in the concluding section (Section 5).

2. Experimental Situation

The effect of disorder on superconductivity depends very much on its scale and size, the reason being the existence of several length scales in the systems. These are:- interatomic spacing a (or (Fermi momentum) $^{-1}$) ($\sim 1\text{--}2$ Å), electron mean free path l ($l > a$ unless disorder is on an atomic scale and is very strong, when $l \sim a$), film thickness d (10 to 1000 Å), zero temperature superconducting coherence length ξ_0 for the clean system ($\xi_0 \sim (\hbar v_F / k_B T_c) \sim 10\,000$ Å) and the same physical quantity ξ for the disordered system ($\xi \simeq \sqrt{\xi_0 l} \sim 100$ Å). These lengths are shown for comparison in Fig. 1. The film is considered two dimensional if $d < \xi$. We shall assume here that this is the case. Depending on how the film is made, the disorder can be on any length scale ranging from 2 Å to 200 Å.

Further, it is not easy to control or quantify the scale and size of disorder. This is one reason for the puzzling variety of experimental results.

An example of *atomic scale* disorder is a Mo–Ge film produced by codeposition, as described by Graybeal and Beasley [3, 4]. The material is amorphous. The Mo and Ge atoms are mixed together on a scale of 5–20 Å. For $x > 0.12$ in $\text{Mo}_x\text{Ge}_{1-x}$ the film is metallic and superconducting. Using various tricks, homogeneous films of constant thickness d can be prepared down to an astonishing 10–20 Å! In such systems, for a given composition (and for $d < \xi$) the superconducting transition temperature T_c is found to be a function *solely* of the resistance per square $R_\square$ of the film, decreasing rapidly and linearly with it (at least to begin with). One finds, approximately, that

$$\frac{\Delta T_c}{T_c^0} \cong R_\square$$

where $R_\square$ is in units of kilo ohms, and where, typically, $T_c^0 = 7$ K. T_c^0 drops to unmeasurably low values for $R_\square > 2.8$ k Ω .

Under most conditions, unless such special care is taken,

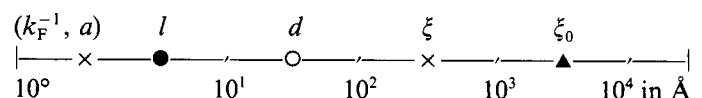


Fig. 1. Various length scales of interest in disordered thin film superconductors (see text for explanation of symbols).

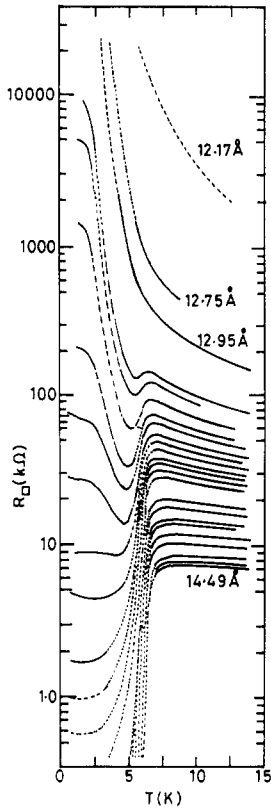


Fig. 2. Resistance per square $R_{\square}$ in $k\Omega$ versus temperature for different amorphous Ga thin films. (After Ref. [8]).

metal or metal-metal oxide films as deposited are inhomogeneous on a thickness or length scale of order $50 \text{ \AA}$ or so. They consist of blobs with this typical dimension, weakly connected via relatively narrow necks or via insulating oxide junctions. Such systems with *mesoscale disorder* have, experimentally, very different characteristics. For example, Goldman and coworkers [6, 8] have extensively studied thin metallic films whose coverage can be controlled very closely. Their data for Gallium films are shown in Fig. 2. The resistance of the films for different nominal thickness is plotted as a function of temperature. The superconducting transition temperature is relatively insensitive to disorder, and has nearly the bulk value, for $R_{\square}$ less than about 6 to $8 k\Omega$. For larger $R_{\square}$, there is a dip in resistance at the bulk transition temperature, but the resistance does not go to zero down to the lowest temperatures. It seems to settle to a finite value as $T \rightarrow 0$ (metallic behaviour). For stronger disorder, the film resistance rises more or less monotonically as temperature decreases (insulating regime).

It is fairly clear from these experiments that there is a critical sheet resistance beyond which global (pair) phase coherence does not exist. This critical value $R_{\square}^c$ seems to be in the range of 6 to $8 k\Omega$ per square. Figure 3 shows this dividing regime; the data points are for a number of different materials. Is the critical resistance relatively independent of the scale of disorder or does it depend upon it smoothly, i.e., as this scale is varied from atomic to mesoscale how does $R_{\square}^c$ change? This question has not been explored experimentally in any systematic way.

There is less agreement as to the number of possible phases. Are there three, namely superconductor, metal and insulator appearing with increasing disorder, as mentioned above, or are there basically two phases, superconductor

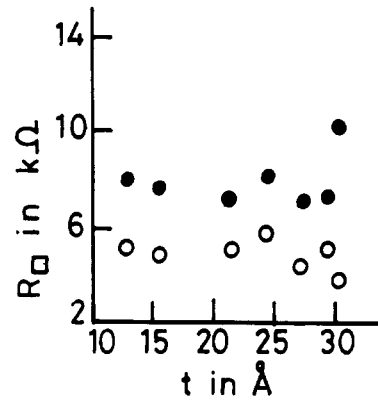


Fig. 3. Sheet resistance (in $k\Omega$) versus thickness in $\text{\AA}$ (nominal). The zero resistance films are marked $\circ$ while the ones which have a nonzero resistance at low temperature are marked $\bullet$. (After Ref. [6]).

(phase coherence) and insulator (localization?)? The apparently metallic state may go insulating at exponentially low temperatures if there is such a low energy or inverse length scale associated with the insulator, so the second alternative is not unlikely.

There have also been attempts [8] to correlate the low temperature $R_{\square}^l$ of the disordered thin film with its intrinsic disorder, i.e., the $R_{\square}^h$ at high temperature (obviously for $R_{\square}^h > R_{\square}^c$). Figure 4 shows the striking exponential connection.

For mesoscale disorder, it seems plausible that below the bulk transition temperature T_c^0 , Cooper pairs form, and well below T_c^0 , are bound nearly as strongly as in the bulk. A rough criterion for this to happen is that the spacing between electronic energy levels in a grain is smaller than the BCS gap Δ . This leads to a minimum grain size of order 10 to $20 \text{ \AA}$. For disorder on a significantly larger scale, the gap function is expected to be nearly the same as in bulk, and destruction of superconductivity is due to (pair) phase incoherence. Another direct confirmation of the former statement is provided by tunneling spectroscopy of quench condensed Sn thin films [7]. The quasiparticle density of states obtained from (dI/dV) is of the BCS form, with a gap Δ which *does not* depend on disorder (or $R_{\square}$) in the range $R_{\square} \leq 10 k\Omega$. T_c is not particularly sensitive to disorder either, for $R_{\square}$ in this range). The measured density of states, along with the BCS form,

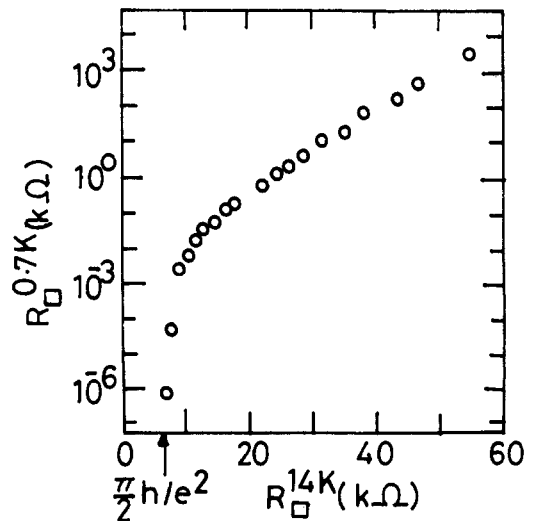


Fig. 4. Low temperature sheet resistance $R_{\square}^{0.7K}$ versus high temperature sheet resistance $R_{\square}^{14K}$ for amorphous Ga film (After Ref. [8]).

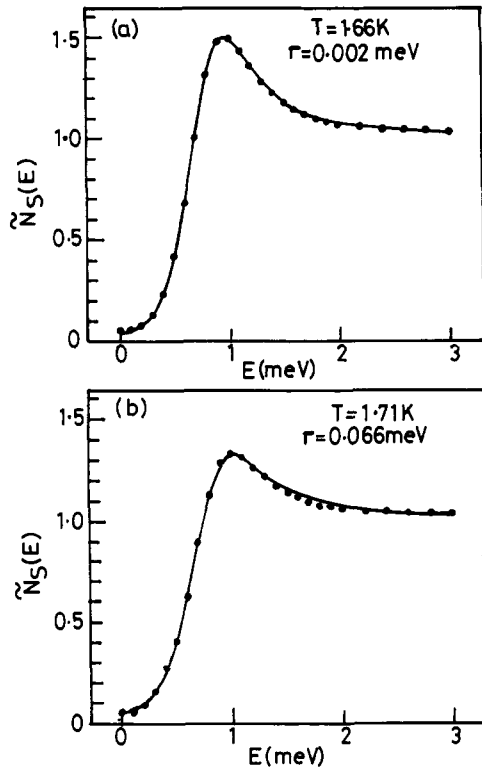


Fig. 5. The density of states deconvoluted from conductance data for:- (a) a 542 $\Omega/\square$ thin film and (b) a 9933 $\Omega/\square$ thin film. (After Ref. [7]).

broadened to include quasiparticle lifetime Γ^{-1} , is shown in Fig. 5. The agreement is excellent. (Even the size and $R_\square$ dependence of Γ are reasonable). In such systems then, the basic attraction between electrons, and pair formation, are unaffected (unlike in films with atomic scale disorder). Many more experiments are needed, however to establish in detail the features of these two presumed limiting cases, and to find out the nature of the crossover.

3. Theoretical models

Attempts to understand superconductivity in disordered thin films fall into two broad classes. One can think of the system as a (weakly) disordered metal, and try to find how Cooper pair formation is affected by disorder. This would be appropriate for systems with atomic scale disorder. Or, one can assume the system to consist of superconducting grains which are weakly and randomly coupled to each other. This Josephson junction or phase only model ought to be relevant for the low temperature behaviour of systems with mesoscale disorder. After a brief review of extant models in both classes, I describe some new results for the second or phase only kind of models.

3.1. Weakly disordered metal

In a disordered metal, localization and the dynamics of interacting electrons have characteristic effects on equilibrium and transport properties. The effects are singular (logarithmically) in two dimensions [9, 13]. The superconducting transition temperature is influenced in several ways, many of which have been discussed by Fukuyama and coworkers [14]. For example, the single particle density of states, the pair propagator, and the coulomb repulsion vertex are all changed, to first order in $(1/k_F l)$, for a film. This leads to a reduction in T_c of a film to first order in $(1/k_F l)$. In

bulk systems, the effect appears at order $(1/k_F l)^2$. Thus Anderson's theorem stating that T_c does not depend on non-magnetic static disorder, is valid to order $(1/k_F l)$ in three dimensions and is not valid in two (essentially because the disordered metallic film is anomalous). This $(1/k_F l)$ or $R_\square$ functional dependence is indeed observed for atomically disordered films [3, 4]. (Section 2). There is considerable uncertainty in the theoretically estimated coefficient; however the calculated values have the right size. The calculation cannot be carried through for strong disorder, and the disordered superconducting phase has not yet been considered in detail.

3.2. Josephson junction lattice

For mesoscale disorder, each grain i can be assumed to have a well defined pair amplitude, with a phase ϕ_i . The phases of neighbouring grains are coupled by pair (Josephson) tunneling. The size J_{ij} of this coupling depends on the integrain tunneling amplitude, i.e., inversely on the local electrical resistivity. The coupling tends to make phases the same, i.e., it is of the form $-J_{ij} \cos(\phi_i - \phi_j)$. This tendency towards global phase coherence is offset by charge fluctuations. A particular grain, if isolated, has a certain number of pairs or charge $2en_i$. This causes the phase to change at a steady rate. Thus if charge fluctuations are slow, phase incoherence is likely. The most important effects are due to single grain charging, and the Hamiltonian can then be written as:

$$H = \sum_i (2e)^2 \frac{n_i^2}{2C_i} - \sum_{i,j} J_{ij} \cos(\phi_i - \phi_j) \quad (1)$$

$$\int_0^\beta H(\tau) d\tau = \frac{1}{2} \int_0^\beta \left[\sum_i \frac{C_i}{(2e)^2} (\dot{\phi}_i(\tau))^2 - \sum_{i,j} J_{ij} \cos(\phi_i(\tau) - \phi_j(\tau)) \right] d\tau \quad (2)$$

where C_i is the capacitance of the grain i . Assuming that there is no randomness in the C_i and in the J_{ij} (which couple nearest neighbour grains) one has a Josephson junction lattice.

The above model is quite commonly used, and was first considered by Abeles [15], who pointed out the obvious analogy with lattice vibrations. The atomic mass is $(C/4e^2)$ and J_{ij} are the force constants. For small atomic mass (in relation to force constants) as in He^4 , zero point fluctuations are too large and lead to melting, i.e., loss of global phase coherence. Several authors, most recently Chakravarty, Ingold, Kivelson and Zimanyi [16] have used self consistent harmonic phonon theory to calculate phonon frequencies with the anharmonic potential of eq. (2), and find, like in high temperature calculations of vibrational instability [17] a sudden collapse of elastic stiffness. Chakravarty *et al.* include in addition a dissipative term in eq. (2) due to thermally excited quasiparticles causing supercurrent decay. In the phonon language, this amounts to assuming $\omega_q = qv_s - i\lambda q^2$ for small q , the imaginary part being due to dissipation. There is now more spectral density at low frequencies, so that "melting" (still discontinuous) occurs at smaller of the product CJ than before. For a sufficiently large value of λ however, the transition becomes continuous.

As applied to experimental systems, the model does predict a superconductor-insulator transition, generally discontinuous however since at low temperatures the dissipative

terms becomes exponentially small. Experimentally the transitions seems continuous. There is no universal critical $R_{\square}$ in the theory since the critical J_{ij} clearly depends on grain capacitance. There is no understanding yet in this picture of the relation between $R_{\square}^{\text{high}}$ and $R_{\square}^{\text{low}}$, or indeed whether there is an identifiable $R_{\square}^{\text{low}}$ region. In the lattice model disorder does not appear, though it could be crucial since in two dimensions *any* randomness localizes the phonon like excitations.

I present now an approach in which *no* assumption beyond the spatial constancy of the size of the Cooper pair amplitude $|\Delta|$ is made. The effect of disorder on phase fluctuation stiffness, and on its random variation, are related to measured normal state properties such as sheet resistance. The results point clearly to a new regime, namely one where $R_{\square}$ is such that the localization length $\xi_{\text{loc}} \lesssim \xi$ the coherence length.

4. Disordered film model

4.1. Preliminaries

For superconducting systems with mesoscale disorder, it is natural to assume that the magnitude of the superconducting order parameter has its bulk value and does not fluctuate very much spatially. The obvious question then is:- How does one describe the free energy of the system as a functional only of phase fluctuations? A method for answering such questions was proposed many years ago by deGennes [18]. He showed how one could work in the representation of the exact single particle states in the disordered medium, and express physically relevant interaction parameters or response functions in terms of measurable averages. The method has been since then used in the context of disordered superconductors by Kapitulnik and Kotliar [10], and by Ma and Lee [11]. I formulate it here a little differently, and apply it to two dimensional superconductivity.

Consider a collection of electrons in a random potential $V(\mathbf{x})$, interacting via a BCS zero range attractive term of strength g . The Hamiltonian of the system is

$$H = \int \sum_{\sigma} \psi_{\sigma}^{\dagger}(\mathbf{x}) \left[\frac{1}{2m} \left(\frac{\hbar}{i} \nabla - \frac{e\mathbf{A}(\mathbf{x})}{c} \right)^2 + V(\mathbf{x}) - \mu \right] \psi_{\sigma}(\mathbf{x}) d\mathbf{x} - g \int \psi_{\uparrow}^{\dagger}(\mathbf{x}) \psi_{\downarrow}^{\dagger}(\mathbf{x}) \psi_{\downarrow}(\mathbf{x}) \psi_{\uparrow}(\mathbf{x}) d\mathbf{x} \quad (3)$$

where $\mathbf{A}$ is the vector potential, and μ is the chemical potential. The partition function Z of this system can be written as a functional integral over the Grassmann fields $\psi(x) (= \psi(\mathbf{x}, \tau))$ and $\bar{\psi}(x)$:-

$$Z = \int D\{\psi_{\sigma}, \bar{\psi}_{\sigma}\} \exp \left[- \int_0^{\beta} d\tau \int d\mathbf{x} \left[\sum_{\sigma} \bar{\psi}_{\sigma}(x) \times \left(\frac{\delta}{\delta\tau} + \frac{1}{2m} \left(\frac{\hbar\nabla}{i} - \frac{e\mathbf{A}}{c} \right)^2 + V(\mathbf{x}) \right) \psi_{\sigma}(x) - g \bar{\psi}_{\uparrow}(x) \bar{\psi}_{\downarrow}(x) \psi_{\downarrow}(x) \psi_{\uparrow}(x) \right] \right] \quad (4)$$

The order parameter field $\Delta(x)$ is introduced by the usual Hubbard-Stratonovich transformation, i.e., by writing

$$\exp [g \bar{\psi}_{\uparrow}(x) \bar{\psi}_{\downarrow}(x) \psi_{\downarrow}(x) \psi_{\uparrow}(x)] = \left(\frac{1}{C} \right) \int d\Delta(x) \times d\Delta^{*}(x) \exp \left[\frac{-|\Delta(x)|^2}{g} - \Delta^{*} \psi_{\downarrow}(x) \psi_{\uparrow}(x) - \text{h.c.} \right] \quad (5)$$

where C is a normalization constant. We assume that $\Delta(x)$ can be written as a product of a spatially *constant* amplitude times a phase factor, i.e.,

$$\Delta(x) = \Delta e^{i\theta(x)} \quad (6)$$

and assume further that Δ has the free energy minimum or mean field value Δ_0 , i.e., that fluctuations with respect to the mean are not relevant for the physics. The mean field value Δ_0 satisfies the usual BCS Bogolubov equation, even for a disordered system, as shown by deGennes [18]. Ma and Lee [11] have argued that this is true so long as $\xi_{\text{loc}} > \xi$. One then has, on using eqs. (5) and (6) in eq. (4) for Z , and making a phase change $\psi_{\sigma}(x) \Rightarrow \psi_{\sigma} \exp i\theta(x)/2$ in the Fermi fields,

$$Z = \int D\{\psi, \bar{\psi}, \theta\} \exp [-d\tau \int d\mathbf{x} (H_0 + H_{\theta})] \quad (7a)$$

where

$$H_0 = \sum_{\sigma} \bar{\psi}_{\sigma}(x) \left[\frac{\delta}{\delta\tau} + \frac{p^2}{2m} + V(x) - \mu \right] \psi_{\sigma}(x) + \Delta_0 (\psi_{\downarrow}(x) \psi_{\uparrow}(x) + \text{h.c.}) - \frac{\Delta_0^2}{g} \quad (7b)$$

and

$$H_{\theta} = \sum_{\sigma} \bar{\psi}_{\sigma}(x) \left[\frac{i}{2} \frac{\delta\theta}{\delta\tau} + \frac{\mathbf{p} \cdot \mathbf{\Pi}_{\theta} + \mathbf{\Pi}_{\theta} \cdot \mathbf{p}}{4m} + \frac{\Pi_{\theta}^2}{8m} \right] \psi_{\sigma}(x)$$

with $\mathbf{\Pi}_{\theta} = \hbar\nabla\theta - (2e\mathbf{A}/c)$. If the fermion degrees of freedom are integrated out, the free energy is explicitly a functional of phase $\theta(x)$ alone. Further, the functional is clearly gauge invariant.

If θ fluctuates slowly in space and time, one need retain only the leading order (quadratic) terms of $\nabla\theta$ and in $\dot{\theta}$. Up to this order,

$$Z = \int D\{\theta\} \exp [-\beta H_{\text{eff}}\{\theta\}] \quad (8a)$$

where

$$H_{\text{eff}}\{\theta\} = -\frac{1}{8} \sum_{\mathbf{q}, \omega} \omega^2 \tilde{\chi}_{\mathbf{q}\omega} |\theta_{\mathbf{q}\omega}|^2 + \frac{1}{8} \sum_{\mathbf{q}, \mathbf{q}', \omega} D(\mathbf{q}, \mathbf{q}', \omega) i\mathbf{q}\theta_{\mathbf{q}, \omega} \cdot i\mathbf{q}'\theta_{\mathbf{q}', -\omega} \quad (8b)$$

In eq. (8b), ω is the Matsubara frequency ($2\pi i m k_B T$) with m an integer, and $\mathbf{q}$ is the wavevector. This phase Hamiltonian is very similar in form to that of the Josephson junction model (eq. (2)). The quantities $\tilde{\chi}_{\mathbf{q}\omega}$ and $D(\mathbf{q}, \mathbf{q}', \omega)$ describe the response of the disordered superconducting electron liquid to charge and current fluctuations, i.e.,

$$\tilde{\chi}_{\mathbf{q}\omega} = \int_0^{\beta} \langle T_{\tau} \{ \varrho_{\mathbf{q}}(\tau) \varrho_{-\mathbf{q}}(0) \} \rangle e^{-\omega\tau} d\tau \quad (9a)$$

and

$$D(\mathbf{q}, \mathbf{q}', \omega) = \frac{\hbar^2}{m} - \hbar^2 \int_0^{\beta} \langle T_{\tau} \{ v_x(\mathbf{q}, \tau) v_x(\mathbf{q}', 0) \} \rangle e^{-\omega\tau} d\tau \quad (9b)$$

$\tilde{\chi}$ is the inverse dielectric function, and D is the current correlation function (related to conductivity) for a particular impurity configuration, i.e., *before* configuration averaging. We discuss these two quantities now.

4.2. Inverse dielectric function

The inverse dielectric function for small wavevector q (and

zero frequency) is determined by the long-range coulomb interaction and its perfect screening in a Fermi liquid (normal or superconducting, clean or disordered). Quite generally, for small q , $\chi_{q\omega}$ can be written as

$$\tilde{\chi}_{q\omega} = [\chi_{q\omega}^0 / (1 + u_q \chi_{q\omega}^0)] \quad (10)$$

where u_q is the Fourier transform of the coulomb potential $e^2/|x|$, and goes as $[2e^2/|q|A]$ in two dimensions and as $[4\pi e^2/q^2 V]$ in three. In calculating $\chi_{q\omega}^0$ the coulomb interaction can be neglected (RPA). In the small q , $\omega = 0$ limit, $\chi_{q,\omega}^0$ the irreducible polarization, is equal to the normal metal density of states ρ_{EF} at the Fermi level. $\chi_{q,\omega=0}^0$ is found using the exact eigenstates method, to be

$$\chi_q^0 = \frac{\rho_{\text{EF}}}{4\pi} \int \int d\xi d\xi' \times \left\{ \frac{Dq^2}{(\xi - \xi')^2 + (Dq^2)^2} \right\} \left\{ \frac{\varepsilon\varepsilon' - \xi\xi' + \Delta^2}{\varepsilon\varepsilon'(\varepsilon + \varepsilon')} \right\} \quad (11)$$

where D is the electron diffusion constant in the disordered normal metal, and $\varepsilon = \sqrt{\xi^2 + \Delta^2}$. This has the clean limit, normal state value ρ_{EF} in the limit $q \rightarrow 0$, so that from Eq. (10) $\chi(q, 0) \simeq (+u_q)^{-1}$ (i.e., it goes to zero as $q \rightarrow 0$). This *incompressibility* of the electron gas due to the long range of coulomb interactions and the finite density of low energy excitations qualitatively alters the energy cost of temporal phase fluctuations, reducing it by factor q (in thin films or two dimensions) and by a factor q^2 (for three dimensional systems). Even in the quantum *insulator* limit (i.e., at $T = 0$ with states near Fermi level localized), for an overall neutral system, $\chi_{q,\omega=0}^0$ goes as q^2 for small q (namely for $q \ll \xi_{\text{loc}}^{-1}$). This is easily seen by expanding matrix elements involved in χ_q^0 as a power series in q . One thus has, in general, extra factors of q coming from the (in)compressibility χ_q of the electron liquid (or glass).

For the metallic regime, since $\tilde{\chi}_q \simeq (u_q)^{-1}$, (from eqs. (10) and (11)) the explicit effective phase Hamiltonian is

$$\tilde{H}_{\text{eff}}(\theta) = -\frac{1}{8} \sum_{q,\omega} \left(\frac{\pi}{e^2 k_F} \right) \left(\frac{q}{k_F} \right) \omega^2 |\theta_{q\omega}|^2 + \frac{1}{8} \sum_{q,q',\omega} D(q, q', \omega) i q \theta_{q,\omega} \cdot i q' \theta_{q',-\omega} \quad (12)$$

$D(q, q', \omega)$ has a nonzero limit for small values of its argument, and is diagonal in $(q, -q')$ on averaging over random potential configurations. (See Section 4.3). Comparing this equation with the phenomenological Josephson junction lattice Hamiltonian eqs. (2) and (1), the second term is seen to be similar in form, while the first is *not*. It has an extra factor of q whose origin has been discussed above. The Josephson junction lattice Hamiltonian is the result of a perturbation expansion in powers of intersite electron hopping. In such an expansion, D appears to second order, but perfect screening of coulomb interactions only in infinite order (RPA resummation of perturbation theory etc.). Also implied in the lattice model is the assumption that the Cooper pair binding energy Δ is the largest energy in the problem, i.e., larger than ε_F or ω_p . Since an extra q factor is present even for the disordered insulator, it seems that the Josephson junction lattice model qualitatively overestimates the energy cost of time dependent phase fluctuations. Therefore conclusions on the consequences of charging or zero point quantum phase fluctuations using this model cannot be taken as they stand; quantum fluctuation effects are much stronger.

4.3. Phase gradient stiffness

The phase gradient stiffness $D(q, q', \omega)$ (proportional to superfluid density!) is related to the current current correlation function (eq. (9b)). Using the exact eigenstates method, it can be related to the ac conductivity of the normal disordered metal (i.e., the disordered metal with everything the same, but *no* pairing term). The expression already appears in deGennes [18] and Ma and Lee [11]. The configuration averaged $D(q, q', \omega)$ is

$$D(q, q', \omega) = D(q, -q, \omega) = D_q(\omega) = \frac{\hbar}{2\pi e^2 \bar{n}} \times \left[\int_0^\infty d\xi \int_0^\infty d\xi' \left\{ \sigma_{xx}(q, \xi + \xi') \times \left(\frac{2}{\xi + \xi'} - \frac{\varepsilon\varepsilon' + \xi\xi' - \Delta^2}{\varepsilon\varepsilon'(\varepsilon + \varepsilon')} \right) - \sigma_{xx}(q, \xi - \xi') \left(\frac{\varepsilon\varepsilon' - \xi\xi' - \Delta^2}{\varepsilon\varepsilon'(\varepsilon + \varepsilon')} \right) \right\} \right] \quad (13)$$

where $\bar{n}$ is the electron density, and $\sigma(q, \omega)$ is the conductivity of the disordered metal in the absence of Cooper pairing (i.e., where the BCS attraction is absent, or g vanishes in eq. (3)). The effective phase gradient stiffness depends on normal state ac conductivity σ averaged over with a coherence factor which varies in the frequency range of Δ . Normally, $\sigma(\omega)$ varies on a scale τ^{-1} (Drude), so that since $\tau^{-1} \gg \Delta$, σ can be taken to be $(\bar{n}e^2\tau/m)$. In this limit,

$$D = D_0 = \tilde{D} \frac{\hbar^2}{m} \left(\frac{\Delta\tau}{\hbar} \right) = \frac{\tilde{D}\hbar^2}{m} \left(\frac{1}{\xi_0} \right) \quad (14)$$

the standard “dirty limit” value, where $\tilde{D}$ is a constant of order ten.

In a two dimensional disordered systems, however, all states are localized at $T = 0$ [9, 13], so that the conductivity goes to zero at low frequencies approximately as ω^2 . The crossover to the Drude form occurs for frequencies $\omega > \omega_0$, where $\omega_0 = \sqrt{2}\varepsilon_F \times \exp[-R_c/R_\square]$ with $R_c = (\pi^2\hbar/e^2) \simeq 40.5 \text{ k}\Omega$. Thus the effective stiffness can be very much smaller, because the ‘normal’ metal is very soft against current fluctuations. It is also obvious that when $\omega_0 \lesssim \Delta$, the stiffness D is likely to become very small. This means that the critical sheet resistance is

$$R_\square > R_c / \ln(\sqrt{2}\varepsilon_F/\Delta) \quad (15)$$

which comes to about $6 \text{ k}\Omega$ for $(\varepsilon_F/\Delta) \sim 10^3$. Another way of describing the condition $\omega_0 \lesssim \Delta$ is that the localization length $\xi_{\text{loc}} \lesssim \xi$ the superconducting coherence length (in the dirty limit).

A detailed calculation of D , which illustrates its rapid and large decrease can be made using an approximate form for $\sigma(q = 0, \omega)$ derived by Vollhardt and Wölfle in their self consistent theory of localization [20]. Their expression for $\sigma(\omega)$ is

$$\sigma(\omega) = (\bar{n}e^2\tau/m)\omega^2/(\omega^2 + \omega_0^4\tau^2) \quad (16)$$

The calculated value of the dimensionless stiffness coefficient $\tilde{D}$ for different values of the sheet resistance is shown in Fig. 6. A rather rapid and large drop in $\tilde{D}$ occurs in the range $R_\square \sim 8\text{--}20 \text{ k}\Omega$; $\tilde{D}$ decreases from nearly ten to about 0.4 in this range. Most of the stiffness for $R > 10 \text{ k}\Omega$ is due to localized states with localization length $\xi_{\text{loc}} < \xi$! In a normal

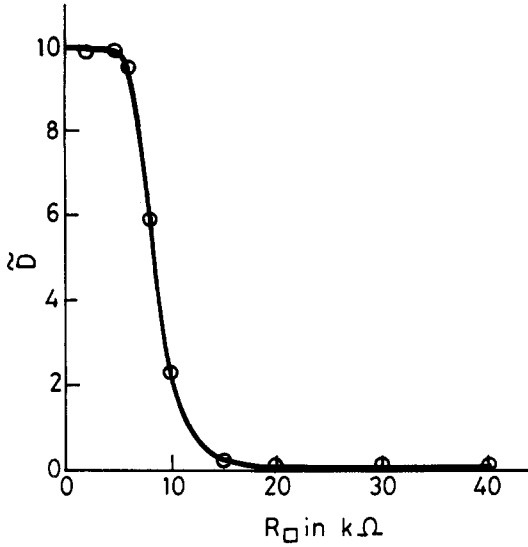


Fig. 6. The phase gradient stiffness D in dimensionless units versus sheet resistance $R_{\square}$ in $k\Omega$.

metal, inelastic processes mute localization effects; such processes are exponentially suppressed in the superconductor, so that the $T = 0$ limit and therefore strong localization effects are what one has in fact.

The precipitous softening of D for R beyond 8–10 $k\Omega$ does not by itself explain the superconductor insulator or superconductor metal transition; it makes such a transition easier, however, and points clearly to the relevance of a disorder scale such that $\xi_{loc} \lesssim \xi$ (the superconducting coherence length). An assumption is that even in systems with mesoscale disorder, the relation between localization length (or equivalently, ω_0) and the intrinsic disorder or $R_{\square}$ is given by the localization theory result previously mentioned. This is expected to be true so long as $\xi_{loc} > 1$, the mean free path, which in this case is the length scale of mesoscale disorder.

Another argument for the relevance of the criterion $\xi_{loc} \lesssim \xi$ is due to Ma and Lee [11], who point out that when this happens, i.e., when states near the Fermi energy have a $\xi_{loc} \lesssim \xi$, the number of states within a volume ξ^d fluctuates strongly. This leads to large local fluctuations in the pair amplitude $\Delta(x)$.

A third consequence of disorder is that the stiffness D fluctuates substantially from its mean value. This fluctuation is local in space, i.e., has a range ξ . It is related to conductance fluctuations in two dimensions which are universal.

4.4. Stiffness fluctuations

The phase stiffness $D(\mathbf{q}, \mathbf{q}', \omega)$ is not diagonal in $\mathbf{q}$ for a particular random potential configuration. The diagonal configuration averaged quantity $D(\mathbf{q}, -\mathbf{q}, \omega = \bar{D}(\mathbf{q}, \omega))$ has been discussed above (Section 4.3). In a disordered medium, D varies randomly from its mean value. In a Josephson junction lattice model, this means that the force constant J_{ij} fluctuates from bond to bond. An obvious measure of the spread in D is $(\bar{D}^2 - \bar{D}^2)$. Using the definition eq. (9b) for D , where the average is over Fermi fields for a given impurity configuration, and noticing that $(\bar{D}^2 - \bar{D}^2)$ involves a connected configuration average of four current operators, and relating this to conductance fluctuations, one finds that

$$\bar{D}^2 - \bar{D}^2 = \left(\frac{4 \ln 2}{\pi} \right) \frac{1}{N} \bar{D}^2 \left(\frac{\hbar}{\tau \Delta} \right) \quad (16)$$

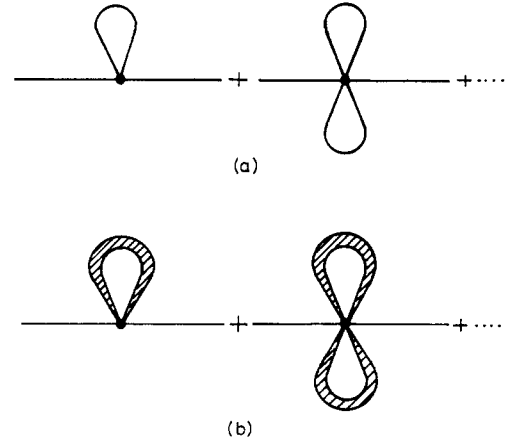


Fig. 7. Contributions to Sine-Gordon field (or vortex charge correlation function) self energy. (a) First two terms in infinite series of terms involving $\bar{D}$. (b) First two terms in infinite series involving $(\bar{D}^2 - \bar{D}^2)$, this being the cross-hatched ring or impurity configuration averaged $\langle \Phi \Phi \Phi \Phi \rangle$ correlation function.

where the q vectors of $D^2(q, q_2, q_3, q_4)$ are all taken to be zero. (This is the limit of interest for long wavelength stiffness fluctuations). Since the size of $\bar{D}$ is $(\Delta \tau / \hbar)(\hbar^2 / m)$, eq. (16) implies ‘universal’ fluctuations in the size of D . This has a number of consequences, one of which is the local reduction of vortex core energy. This is discussed now.

In a classical vortex unbinding or Kosterlitz Thouless model for two dimensional superconductivity, the static ($\omega = 0$) phase fluctuations, in the form of positive and negative vortices, are assumed the relevant degrees of freedom. This may not be a realistic model here, because phase fluctuations of different frequencies have nearly the same energies (Section 4.1), and so the $\omega = 0$ or static mode is not specially low in energy. Even in this situation, however if there are no terms for $H(\theta)$ higher than quadratic, phase fluctuations of different frequencies do not interact, and the result below is correct for the static fluctuations.

A convenient formulation of the vortex model is in terms of a Sine-Gordon field theory [21], for which the partition function for a given configuration is

$$Z = \int D\{\Phi\} \exp \left[- \frac{k_B T}{2} \int D^{-1}(\mathbf{x}, \mathbf{x}') \{ \nabla \Phi(\mathbf{x}) \cdot \nabla \Phi(\mathbf{x}') \} \right. \\ \left. \times d\mathbf{x} d\mathbf{x}' - \frac{2z}{z^2} \int \cos 2\pi \Phi(\mathbf{x}) d\mathbf{x} \right]. \quad (17)$$

Here Φ is a real field, and z is the vortex fugacity. The vortex charge correlation function is related to the propagator $\langle \Phi_q \Phi_{-q} \rangle$. For example if $z = 0$, $\langle \Phi_q \Phi_{-q} \rangle \simeq (\beta \bar{\delta} / q^2)$ where $\bar{\delta} = (\bar{D} / l^2)$ is of the order Δ . This corresponds to the logarithmic long range attraction between opposite sign vortices, while for $z > 0$, the interaction is screened at temperatures such that $\beta \bar{\delta} < 4$. This is the Kosterlitz–Thouless transition temperature. Consider the self energy $\Sigma(q = 0)$ of Φ propagator, to leading (first) order in z , and to all orders in Φ (perturbatively). The infinite series of terms, two of which are shown in Fig. 7(a), sum to a logarithmically (infrared) divergent term (see Ref. [21]). These terms involve $\bar{\delta}$. A second infinite series of terms arises from conductance fluctuations. Two of these are shown in Fig. 7(b). These can be summed, and lead to a renormalization of z , i.e.

$$z_{ren} = z \exp \{ \pi \beta^2 (\bar{\delta}^2 - \bar{\delta}^2) \}. \quad (18)$$

If the core energy of a vortex is E_c , then $z \simeq \exp(-\beta E_c)$ so that the above renormalization of z can be thought of as reducing the core energy, or as the first term in a high temperature expansion for the latter, the sum of which leads from $z \sim \exp(-\beta E_c)$ to $z' \sim \exp[-E_c/(k_B T + E_2)]$ where $E_2 \sim E_c$ for $R_\square \simeq 10 \text{ k}\Omega$. This decrease in core energy due to the possibility of soft spots in D , leads to a larger density of free vortices, and moves the bare parameters away from the Kosterlitz-Thouless line in the z - T plane. This could contribute to the destruction of the global phase coherence when $R_\square \sim 10 \text{ k}\Omega$ or so.

5. Conclusion

It is clear from what has been said in the previous two sections that there is as yet no credible theory of the phenomena observed with increasing (mesoscale) disorder. The Josephson junction lattice model focuses on the (generally discontinuous) phonon (phase excitation) instability as zero point quantum fluctuations increase due, say to an increase in sheet resistance. However, the actual energy spectrum of time dependent phase fluctuations is qualitatively different from that assumed in this model because of the incompressibility of the coulomb Fermi liquid (Section 4.2); this is expected to radically change conclusions of the lattice model. Secondly, disorder, neglected in the lattice model is known to cause localization of phonons no matter how small it is. This fact could be crucial!

The spatially constant order parameter or phase only model developed in this paper clarifies the microscopic meaning of coupling parameters i.e., temporal and spatial phase fluctuation stiffness. It shows how these depend on the measurable properties of the film, e.g., resistance per square or $R_\square$. It points to the significance of the criterion $\xi_{\text{loc}} \leq \xi$ for $R_\square \gtrsim 8\text{--}10 \text{ k}\Omega$ in at least two ways, viz. sharp reduction in phase gradient stiffness; and increase in its relative fluctuation. It is however not a complete dynamical theory, and does not describe the superconductor insulator phase change, ($R_\square^{\text{low}}$, $R_\square^{\text{high}}$) relations etc.

From the above, and from the fact that pairing *amplitude* fluctuations are significant for $\xi_{\text{loc}} \lesssim \xi$, it is clear that a model with *only* phase fluctuations is inadequate. More concretely, in a system with mesoscale disorder, some grains or islands may, purely by chance, be favourable for Cooper pairing, because of the way they are connected to other grains and consequent energy level structure. The reverse is also possible. Thus localization of $\Delta(x)$ due to randomness is an important effect. This Anderson localization will become important when the random fluctuation in $\Delta(x)$ is comparable to kinetic energy, i.e., to the size of the $|\nabla\Delta(x)|^2$ term. The former is crudely of order $\Delta \exp(-\xi_{\text{loc}}/\xi)$ (a function smoothly interpolating between the limits Δ and 0 for $\xi_{\text{loc}} \ll \xi$ and $\xi_{\text{loc}} \gg \xi$ respectively). The latter goes as $(\hbar^2/2m\xi^2)$. Equating the two, we again arrive at the criterion $\xi_{\text{loc}} \simeq \xi$. It thus seems plausible that localization of $\Delta(x)$ is a necessary ingredient of a complete theory.

A model which could be realistic on all counts is the charged Bose fluid in a random medium. We identify $\Delta(x)$ with the Bose field and the $|\nabla\Delta(x)|^2$ or Josephson term with boson kinetic energy. Thus the boson kinetic energy depends on disorder or $R_\square$. Interaction between bosons, with the uniform condensate present, leads to the phonon spectrum

characteristic of superfluidity. [In our case the bare interaction between pairs (or between bosons) is coulombic and the collective excitation spectrum plasmon like]. The randomness in pair amplitude due to localization of underlying electronic states is describable as a random one body potential acting on the bosons. The Bose glass problem has been discussed by several authors [22]; the general belief is that while bosons in a random potential condense into the lowest localized state the repulsive interaction between them counteracts this tendency, delocalizing them and restoring superfluidity. Thus for weak interaction (in relation to disorder) the system is a nonsuperfluid glass; for strong interaction it is a superfluid. There is a self consistent mode coupling calculation of this transition for both short and long range boson interactions [23], and very recently, a scaling analysis [24] of the critical exponents has been made. However, there is no theory yet for the case relevant here, namely a two dimensional random system where localization is endemic, with coulomb interactions between the charged bosons. Also, there is no mapping yet of the problem to the two dimensional superconductor nor a beginning of an attempt to connect observations with model predictions.

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