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Simplified fundamental force and mass measurements

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Abstract
The watt balance relates force or mass to the Planck constant \( h \), the metre and the second. It enables the forthcoming redefinition of the unit of mass within the SI by measuring the Planck constant in terms of mass, length and time with an uncertainty of better than 2 parts in \( 10^8 \). To achieve this, existing watt balances require complex and time-consuming alignment adjustments limiting their use to a few national metrology laboratories. This paper describes a simplified construction and operating principle for a watt balance which eliminates the need for the majority of these adjustments and is readily scalable using either electromagnetic or electrostatic actuators. It is hoped that this will encourage the more widespread use of the technique for a wide range of measurements of force or mass. For example: thrust measurements for space applications which would require only measurements of electrical quantities and velocity/displacement.

Keywords: watt balance, Planck constant, kilogram, mass measurement, force measurement, thrust balance

(Some figures may appear in colour only in the online journal)

1. Introduction

The watt balance relates force or mass to the Planck constant \( h \), the metre and the second. At present it is used to measure \( h \) in terms of the SI units of mass, length and time with an uncertainty of better than 2 parts in \( 10^8 \). In this way it will play a major role in the forthcoming redefinition of the kilogram, which will be based upon a fixed value of \( h \). After the redefinition, the technique will become a method of measuring a range of forces in SI units, using just SI measurements of voltage, current, length and time; with a knowledge of the local acceleration due to gravity \( g \), it can measure mass, but its complexity is a barrier to wider adoption. To achieve low uncertainties existing watt balances require complex and time consuming alignment adjustments to minimise unwanted forces and torques acting on parts of the balance. The need to make these adjustments makes the technique expensive, difficult to adopt for routine measurements and hinders efforts to scale it to measure small masses and forces, as the adjustment mechanisms become increasingly difficult to implement as the size of the instrument is reduced. This paper brings together a number of recent advances in the theory and practice of the watt balance and extends them to produce a technique which eliminates the need for these adjustments, requiring only accurate measurements of the principal quantities involved.

2. Background

The moving-coil watt balance was invented at the National Physical Laboratory (NPL) by Kibble in 1975 [1] and was originally conceived as a two part experiment linking electrical and mechanical virtual power. In conjunction with the SI ohm realised via the calculable capacitor [2] the watt balance was used to realise the SI ampere. The discovery of the quantum Hall effect [3] in 1980, in combination with the Josephson effect [4], enabled electrical power to be measured in terms of the Planck constant \( h \) and time. This allowed the watt balance to relate mass to \( h \), the metre and the second providing the possibility of redefining the unit of mass in the SI in terms of a fundamental constant. Many different forms of watt balance have been constructed by laboratories around the world [5], but this paper will concentrate on a particular class of watt balance, having a common mechanism for both...
weighing and moving. Such a balance, via reciprocity mechanisms, can exhibit immunity to a range of mechanical and electrical error mechanisms [6, 7]. The conditions required to achieve immunity to the mechanical error mechanisms in a watt balance were derived in [8] and were used to describe some novel watt balance designs. The descriptions given below refer to an electromagnetically actuated watt balance, but, with minor changes, can equally well be applied to an electrostatically actuated balance.

Figure 1 shows a possible, but somewhat unconventional, configuration of a watt balance. It consists of a coil, linked by a magnetic flux $\Phi$, which is attached to a sensitive balance and thereby is coupled to a mass carrier, on the opposite arm of the balance, which can support a mass $M$. In a conventional balance, designed to compare two masses, the inevitable small differences in the arm lengths of the balance would make an accurate direct comparison of masses on the two pans practically impossible. However in a watt balance of this type it is the stability of the ratio of the arm lengths of the balance that is important and the static value of the ratio is eliminated from the measurement. This allows the accurate measurement of the mass $M$ using forces generated by the coil on the opposite arm. The balance is constructed and operated so that the measured vertical velocity of the mass carrier $u_z'$ fully determines the motion and rotation of the coil. The vertical velocity of the point on the carrier where the mass is placed is measured, usually with a laser interferometer. The balance can be operated in one of two modes: in the weighing mode (switch position W), a current $I$ is passed through the coil and the resultant of the forces and torques produced by the coil oppose the weight $Mg$ of the mass $M$ on the opposing arm of the balance. In the measuring phase corresponding to this mode of operation the current $I$ is passed through a resistor of known value $R$ and the voltage drop across it is measured. The equilibrium condition for the balance is given by equation (1), this and equations (2)–(9) are derived from [8].

$$-Mg - I \left( \frac{\partial \Phi}{\partial x} \frac{\partial}{\partial x} + \frac{\partial \Phi}{\partial y} \frac{\partial}{\partial y} + \frac{\partial \Phi}{\partial z} \frac{\partial}{\partial z} \right) + \frac{\partial \Phi}{\partial \theta_x} \frac{\partial}{\partial \theta_x} + \frac{\partial \Phi}{\partial \theta_y} \frac{\partial}{\partial \theta_y} + \frac{\partial \Phi}{\partial \theta_z} \frac{\partial}{\partial \theta_z} \right) = 0. \tag{1}$$

The position of the mass carrier along a vertical axis is given by $z'$ and the coil has coordinates $(x, y, z)$ and its angles about these axes are $(\theta_x, \theta_y, \theta_z)$. To measure the relevant properties of the coil and magnet the mass is removed, the current switched off and the apparatus is placed in moving mode. The mass carrier is then moved which causes the coil to move and rotate with velocities $(u_x, u_y, u_z)$ and angular velocities $(\omega_x, \omega_y, \omega_z)$. The relationship between these and the vertical component of the mass carrier velocity $u_z'$ given by:

$$u_x = u_z' \frac{\partial x}{\partial z'} \tag{2}$$
$$u_y = u_z' \frac{\partial y}{\partial z'} \tag{3}$$
$$u_z = u_z' \frac{\partial z}{\partial z'} \tag{4}$$
$$\omega_x = u_z' \frac{\partial \theta_x}{\partial z'} \tag{5}$$
$$\omega_y = u_z' \frac{\partial \theta_y}{\partial z'} \tag{6}$$
$$\omega_z = u_z' \frac{\partial \theta_z}{\partial z'}. \tag{7}$$

In the measuring phase associated with this mode the velocity $u_z'$ and the voltage $V$ generated by the coil are measured (switch position M).

$$V = -u_z' \left( \frac{\partial \Phi}{\partial x} \frac{\partial}{\partial x} + \frac{\partial \Phi}{\partial y} \frac{\partial}{\partial y} + \frac{\partial \Phi}{\partial z} \frac{\partial}{\partial z} \right) + \frac{\partial \Phi}{\partial \theta_x} \frac{\partial}{\partial \theta_x} + \frac{\partial \Phi}{\partial \theta_y} \frac{\partial}{\partial \theta_y} + \frac{\partial \Phi}{\partial \theta_z} \frac{\partial}{\partial \theta_z}. \tag{8}$$
If the only significant forces or torques are produced by gravity \( g \) or the interaction of current and magnetic flux and the values of the partial derivatives in (2)–(7) and the space rates of change of \( \Phi \) do not change both during and between moving and weighing modes (the stability conditions) then it is possible to combine (1) and (8) to give the exact equation

\[ M_\omega \dot{u} = VI. \]  

(9)

In the case of the balance shown in figure 1 \( \partial z/\partial x' \) is equal to the ratio of the effective arm lengths and, providing the ratio is stable between the two modes, it cancels exactly when deriving equation (9). In principle this allows such a balance to be constructed with any practical ratio of arm lengths.

To relate the kilogram to fundamental constants the virtual electrical power \( VI \) is measured in terms of two quantum mechanical effects: the quantum Hall effect (QHE) [3] and the Josephson effect [4]. The Josephson effect allows voltage to be measured in terms of the ratio \( h/e^2 \) and frequency where \( e \) is the elementary charge. The quantum Hall effect allows the resistor \( R \) to be measured in terms of the ratio \( h/e^2 \) and, if the voltage across it is measured using the Josephson effect, the current through it can be measured in terms of \( e \) and frequency. If the voltage and current are measured in this way the elementary charge \( e \) cancels in their product and so mass can be related to the Planck constant \( h \), the metre and the second.

### 3. Existing watt balances

Some existing watt balances do not use a common weighing and moving mechanism and the cancellation of the relevant terms in equations (1) and (8) may not occur. Under these circumstances the contributions from the product terms in the equations, which contain \( x, y, \theta, \dot{\theta} \) and \( \theta \), represent error sources which must be adjusted to produce a negligible contribution to the desired overall uncertainty of the measurement. This requires careful, time consuming, alignment of the balance to minimise unwanted linear and angular motion of the coil and the associated forces and torques.

The adoption of a common weighing and moving mechanism is a necessary step on the way to removing the need for such adjustments but is not, of itself, sufficient. The stability conditions can be disturbed by systematic displacements and rotations of both the coil and mass carrier produced by force and torque changes generated during the normal operation of the conventional watt balance thereby increasing the uncertainty of the measurement. Thus the balance must still be carefully adjusted to minimise these effects, reducing the advantages of simple operation promised in [8]. However with a small number of changes to the way that a watt balance is constructed and operated it is possible to take full advantage of the theory in [8] without needing precise secondary adjustments.

### 4. The next generation watt balance

The first step in the process to take full advantage of the theory described in section 2 is to eliminate the force change in the transition between weighing and moving modes. This can be achieved by following a suggestion of the Bureau International des Poids et Mesures (BIPM) [9]. They combined the two measurement modes of the conventional two-mode two-phase (TMTP) watt balance, described above, into a single mode and made the moving and weighing measurements simultaneously in a single measurement phase, giving a single-mode one-phase (SMOP) watt balance. The original BIPM suggestion also involved the use of a superconducting coil to allow the weighing current to flow without affecting the voltage measurement. However, this adds significant error sources and complexity to the experiment and can be avoided by the use of the two strands of a bifilar wound coil to separate the weighing current and moving voltage as suggested in [10] allowing the BIPM technique to be used at room temperature with no loss in accuracy.

The single measurement phase of the BIPM technique has the significant practical advantage of eliminating the sensitivity of the apparatus to changes in the magnetic field of the magnet, but it is obtained at the cost of introducing the possibility of electromechanical coupling between the simultaneous weighing and moving measurements. For example: if the velocity of the coil is not perfectly constant the associated acceleration, acting on the whole moving part of the apparatus, will be seen as a variable force. At best this can introduce noise into the weighing measurement, at worst it can produce a systematic error. Also, if the weighing current is not constant, an additional voltage can be induced into the strand of the winding which is used to measure the induced voltage with similar results. It is very difficult to avoid these problems in a watt balance with a common moving and weighing mechanism, as, for example, the weighing current would need to be adjusted under servo control to keep the balance in equilibrium while it was moving. These problems can be eliminated by reverting to the two, temporally separated, measurement phases of the original TMTP watt balance to produce a single-mode two-phase (SMTTP) watt balance.

In the moving phase of such a balance the current in the main coil is maintained at a constant value and a separate drive coil and magnet makes up for any differences in force required to maintain a constant velocity over the range of movement of the balance. This arrangement minimises the voltage induced into the voltage measuring coil by changes in the weighing current. Noise in the current source will be seen as uncorrelated noise in the velocity measurement. A slow linear drift in the current from the source will be seen as a small steady voltage in the moving measurement which will be eliminated by the regular reversals of the movement of the coil. It is possible to measure the effects of changes in the current source on the voltage measured in moving mode by deliberately varying the current in a sinusoidal manner. This will vary the force on the balance and may affect its velocity. This effect can be eliminated using the usual correlation techniques. As long as the voltage samples are taken in a time much less than the period of the sinusoid, the required direct sensitivity of the moving voltage to changes in the current can be measured. This, plus a measurement of the noise and stability of the current source enables the measurement uncertainties arising
from this source to be estimated. In the weighing phase the coil now does not move and therefore the measurement works in a similar way to that of a conventional watt balance and the uncertainties can be analysed in the same manner.

The price to be paid for this change is that the balance is once again sensitive to temporal changes in the magnetic field. But, as the power dissipated in the coil is now constant, its heating effect on the magnet will also be constant, which makes it easier to either stabilise the magnet temperature, thereby stabilising its field, or correct for the effects of any changes when calculating the results of the measurement.

The final change eliminates the effects of alignment variations caused by the raising and lowering of the mass. In general, for a watt balance which has not been adjusted extremely precisely, the positions of both the coil and the mass carrier can change between the state where the mass is present on the mass carrier (the mass-down state) and the state where it has been lifted off the carrier (the mass-up state). These effects can violate the stability conditions increasing the uncertainty of the instrument. The mass carrier may move because of changes in its overall centre of gravity caused by the placement of the cylindrical mass. The coil may move as the current in it reverses between states and thereby reverses the forces and torques acting on it; this current reversal also changes the magnetic field of the magnet.

All these problems can be eliminated by considering the mass-up and mass-down states of the balance to represent two independent watt balances. Under these circumstances it is only necessary to achieve a stable, consistent, alignment and magnetic field in each state of the balance. As both the current, its direction and the associated forces and torques are constant in each state, the coil alignment and the magnetic field of the magnet are also stable. By replacing the usual cylindrical working mass with a spherical one, which locates reproducibly on three pins on the mass carrier, the mass carrier position can also be made extremely reproducible in each state. The moving measurements, made in each state, measure the magnetic flux associated with the weighing current in that state; this is an advantage of the BIPM single mode technique over the original watt balance technique which made moving measurements with no current in the coil and depended on the linearity of the magnet to relate moving and weighing measurements [11]. The principles of the watt balance, as introduced above, will apply to the two ‘virtual’ watt balances, so formed, and equation (9) will apply separately to each state allowing an accurate measurement of the mass in that state without needing precise alignment of the balance. Once the results from the two states of the balance have been calculated they can be combined to yield the value of the working mass.

The separation of the two states does have an effect on the measurement of the weighing current. In a conventional TMTPT watt balance the addition or removal of the mass causes a reversal of this current which is used to eliminate the effects of the unknown thermal EMFs in the voltage measurement circuit. In the case of a SMTP watt balance the two states of the balance must be considered to be independent and so the current reversal due to addition or removal of the mass can no longer be used for this purpose. This problem can easily be overcome by arranging to reverse the current through the measuring resistor independently of the mass state. The current circuit is not sensitive to thermal EMFs which allows a simple reversing relay to be employed which, in conjunction with a second relay to provide a bypass for the weighing current during the reversal, allows the reversal to take place with no effect on the equilibrium of the balance.

4.1. Measurement procedure

The following procedure describes the measurement of the working mass. For simplicity it is assumed that the acceleration due to gravity \( g \) is constant throughout the measurement and the thermal EMFs are time independent. For state \( i \) where the equilibrium current is \( I_i \), the thermal EMF in the voltage measurement circuit, associated with the resistor, is \( V_{ir} \), the measured voltages corresponding to forward and reverse current in the resistor are \( V_{fr} \) and \( V_{rr} \) respectively and the measured weight in each state is \( m_i g \). The moving measurements must be assigned to the state in which they are made and it is assumed that, as is usual, thermal EMFs in the coil measurement circuit have been eliminated between the upward and downward motion of the coil. The velocity of the coil is \( u_i \) and the corresponding induced voltage is \( V_i \). Assuming that in state \( i = 1 \) the mass \( M \) is raised and in state \( i = 2 \) it is lowered onto the mass carrier we can write:

\[
V_{fr} = I_1 R + V_{fr} \\
V_{rr} = -I_1 R + V_{rr} \\
I_1 = (V_{fr} - V_{rr})/(2R) \\
m_{1g} = V_1 I_1/u_1 \\
\]

Similarly for state 2 with the mass lowered \( I_2 = (V_{fr} - V_{rr})/(2R) \) \( m_{2g} = V_2 I_2/u_2 \)

The weighings can now be combined to give the value of the working mass \( M \):

\[
Mg = m_{2g} - m_{1g} \\
M = \frac{1}{2Rg} \left( \frac{V_2}{u_2} (V_{fr} - V_{rr}) - \frac{V_1}{u_1} (V_{fr} - V_{rr}) \right) 
\]

Periodically, the roles of the two strands of the bifilar winding would need to be exchanged and the measurement results combined, as described in [10], to eliminate differences between the windings.

The moving phase of the measurement is carried out with current in the coil this means that the motion of the coil may alter the magnetic state of the magnet in a way that alters with the direction of the motion. With a good magnet design this effect should be negligible but would need to be investigated as part of the design/commissioning process of the balance.
5. Seismometer-like watt balances

Although it is possible to build an SMTP watt balance in a similar way to that shown in figure 1 there are considerable advantages in size, scalability, simplicity and robustness to adopting the seismometer based design described in [8] and shown in figure 2. This initial design uses an electromagnetic offset system to provide the constant force required to counterbalance the weight of the frame less half the weight of the working mass, this eliminates the need for the tare mass used in conventional balance designs. It also uses flexures to guide the central balance frame. Flexures have been used for many years for the precise guiding of seismometer-like instruments such as the ‘Super Spring’ vibration isolators [12] used in gravimeters [13]. In the LNE watt balance flexures are used in the guidance mechanism [14] but, to obtain sufficient sensitivity for the weighing, a separate flexure-based balance is employed. In the design considered here specially designed flexures, suitable for both weighing and moving, are likely to be used initially but it is now possible to consider other approaches to this guidance problem.

5.1. Guidance techniques

The insensitivity of the technique described above to the alignment and motion of the watt balance coil may allow another issue of watt balance design to be addressed. Almost all balances use either flexures or balance knives to guide the coil and mass and to provide high sensitivity to departures from equilibrium during the weighing process. However, both have problems: knives exhibit hysteretic effects arising from anelasticity at the highly loaded interface between the knives and planes [15, 16]. Flexures are difficult to use in a watt balance of the type described here as the same flexures must be used for both weighing and moving and the long travel, which is required for the moving measurements, may give rise to problems associated with weighing sensitivity, anelasticity and fatigue. Hysteresis arising from anelasticity slows down the measuring procedures as it produces a systematic offset in the balance which must be erased before each weighing [17]. Using the techniques described it is now possible to consider a watt balance which uses other offset and guidance techniques such as combined magnetic levitation and guidance. With careful design the moving trajectory of the balance would be constrained to ensure that the moving measurements can be interpolated accurately to the weighing position but would avoid the problems associated with material guidance mechanisms. Wires would still be used to connect to the coils but these can be made sufficiently weak and elastic so as not to compromise the weighing sensitivity or introduce direction dependent offsets. If the considerable problems associated with this form of guidance and offset system can be solved the resulting instrument should gain commensurate advantages in sensitivity, elimination of the requirement for physical offset masses and lack of hysteretic effects.

6. Performance

Recently, conventional TMTP watt balances have achieved uncertainties of less than 20 parts in $10^{-9}$ [18] and are likely to achieve lower uncertainties in the next few years. The SMTP technique, introduced here, eliminates all of the secondary mechanical adjustments from the watt balance and the uncertainty of the instrument then depends upon the quality of the measurements of the principal quantities of the watt balance equation. When it is fully developed the uncertainties associated with the SMTP technique should be as good or better than those of the best existing watt balances but with the advantage that much less effort is required both to build and to operate the balance.

7. Measurement of smaller and larger masses and forces

While this technique is directly applicable to instruments operating at the kilogram level and above it has greater advantages at lower levels by using seismometer based forms of the balance [8]. The technique can also be applied to the measurement of forces. For example thrust balances for space
applications could be calibrated and operated with only measurements of current, voltage and velocity which can give considerable advantage when operating under vacuum. It is possible to substitute distance measurements for those of velocity providing that the coil voltage is integrated at the same time and over the same distance. This can be done either while the coil is moving, which is standard practice in most existing watt balance designs, or can be measured between differing positions where the coil is stationary, as described in [19].

7.1. Smaller masses

The methods required to achieve the secondary adjustments of a conventional watt balance [6] become increasingly difficult to apply as the scale of the balance reduces. The elimination of such adjustments, should allow the construction of extremely small, but accurate, instruments, which may use MEMS techniques. Good design and precise construction of the instrument can minimise effects that would otherwise have been nulled by adjustment and the need for further adjustment is then eliminated by the techniques described above. This is particularly relevant for MEMS based instruments as precise construction is much easier than the provision of multiple adjustment mechanisms.

7.2. Electrostatic balances

The technique is also applicable to watt balances which employ electrostatic actuators. The electrostatic, or voltage, watt balance technique was originally described in [20]. The original balance employed a fixed plate and a moving plate which was hung from a sensitive balance. A voltage $V_c$ applied between the plates generates a force $F_c = M g$ which depends on $V^2_c$ and space rates of change of capacitance in a similar way to that of the electromagnetic balances which depend on the current in the coil and space rates of change of flux $\Phi$. In a similar way to the electromagnetic case, the space rates of change of capacitance can be determined by measuring the current $I_c$ flowing between the plates when the plate attached to the balance is moved with a measured velocity $u$. Thus the equations (1) and (8) above can be converted into a form equivalent to those used for the electrostatic balance if $V_c/2$ replaces the current $I$ in equation (1), the plate current $I_c$ in moving mode replaces $V$ in equation (8) and the quantity $C V_c$ takes the place of the flux $\Phi$ in both equations. The fundamental equation then becomes $M g u = V_c / I_c$.

There are differences between the electrostatic and electromagnetic balances. The electrostatic field is generated by the voltage between the plates and so the vertical force is automatically present for both weighing and moving, making it a natural single mode watt balance. There is no advantage to be gained from the equivalent of the bifilar winding technique as there is no way to separate force generation and current generation in moving mode; however the low resistance of the plate allows the moving current to flow without a significant effect on the value and uniformity of the voltage on the plate. The force is dependent on $V^2_c$ and therefore cannot be reversed and so the working mass is measured as the difference in the forces generated by differing values of $V_c$. The separation of mass up and mass down states, as described above, yields the same advantages, eliminating the voltage coefficient of the capacitance and any changes to the stability conditions between the two states. Electrostatic actuation is often easier to fabricate in MEMS based systems and so this approach may have advantages over the electromagnetic form of the balance in some applications. The current generated is strongly dependent on the detailed design of the balance, but an order of magnitude calculation for a balance working at the $\mu$N level, indicates that the current would be some $10^7$ of pA, which could either be measured by conventional means, or by the quantised electron pumps which are being developed for direct realisation of the ampere in a redefined SI [21].

8. Conclusion

It is hoped that this paper will encourage the construction of watt balances to measure a range of masses from kilograms to milligrams and forces from many newtons down to the $\mu$N level. It should be understood that watt balance measurements require extreme care to ensure that the principal quantities are measured correctly to achieve the desired measurement uncertainties; the techniques described above do not relax that requirement but make it possible to consider achieving the desired uncertainty with less effort and in circumstances where precise alignment is difficult. For masses around 1 kg it is hoped that it will encourage more NMIs to build the most precise balances to achieve uncertainties of a few parts in $10^7$. After redefinition this will give direct access to the benefits of the redefined SI and enable increased numbers of NMIs to contribute to the maintenance of the world-wide mass scale. This scale, based upon a fixed value of the Planck constant, will then be realised directly in an extremely wide-ranging, robust and egalitarian manner.

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