LETTER TO THE EDITOR

Some problems concerning the use of the CODATA adjusted values of fundamental constants in the definition of measurement units

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Letter to the Editor

Some problems concerning the use of the CODATA adjusted values of fundamental constants in the definition of measurement units

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Abstract
This note discusses some problems that should be taken into account in the implementation of the proposed use of fundamental constants in the definition of measurement units: use of CODATA adjusted values of the constants for this specific purpose; how the CODATA Task Group could continue performing meaningful least squares adjustments of the fundamental constants taking into account future data.

Keywords: fundamental constants, metrology, measurement units, SI, CODATA, stipulation

Online supplementary data available from stacks.iop.org/Met/51/L1/mmedia

1. Introduction

In recent times, a proposal has been brought forward to use some fundamental constants ($c_0$, $h$, $e$, $kB$, $N_A$), by stipulating their numerical values, as the bases for the redefinition of some of the present base units of the International System of Units, the SI [1]. This proposal was submitted at its 2011 meeting to the CGPM, which decided in its Resolution 1 to “take note of the intention of the CIPM” for “the possible future revision of the International System of Units” [2].

The merit of the proposal is an issue outside the scope of this Letter, dealing only with some problems that, in the author’s opinion, should be more discussed to be taken into account should the proposal be implemented. These problems, possibly discussed in regulatory bodies or panels, or presented on websites (e.g. the FAQ on [3]), but not specifically treated in the open scientific literature, are the following.

1. The use of CODATA adjusted values of the constants correct for the above specific purpose.
2. Possible limits to the CODATA task arising from the use of their values in the definitions.

This Letter does not aim at discussing the CODATA authority with respect to performing their analyses, nor the general validity of the least squares adjustment for specific purposes. It aims at stimulating the debate by illustrating problems and suggesting solutions. The online supplementary information contains some more supporting material (stacks.iop.org/Met/51/L1/mmedia). For formal problems concerning the stipulation of the values or algebraic expressions of them see [4].

A convention is implied in the following: only the numerical values assigned to the ‘constant value’ of the relevant fundamental constants are considered; the physical nature of the constants is never addressed. Two principles to be followed in performing any adjustment of a system of units are also implied.

– The numerical values assigned to any quantity used in the definitions are entirely determined by the current set of measurement units, and their stipulation is derived from the uncertain ‘best values’—in a statistical sense—obtained from critically selected experimental determinations available at the time of stipulation.
– The above-stipulated numerical values are critical in that one wants to avoid significant discontinuities with the historical assignments.


2. CODATA adjusted values: meaning and limits

Since 1973, a CODATA Task Group has performed a unique check of consistency of a large set of “fundamental constants” using the so-called least squares adjustment (LSA) algorithm. At regular intervals in time, these analyses produce a set of ‘adjusted values’ and of the associated uncertainties (see the full list of relevant references in the online supplementary information (stacks.iop.org/Met/51/L1/mmedia)). The aim of the CODATA Task Group is reported by BIPM [5] as being “to periodically provide the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry based on all of the relevant data available at a given point in time”.

The use of the LSA technique in connection with the studies on fundamental constants was initiated as early as 1955 [6]. More recently, the LSA was indicated in [7] as “one of the few ways in which the over-all consistency of physical theory can be systematically investigated” (emphasis added). In [8] obtaining a recommended set of values using the LSA was indicated as “only of secondary importance” because the main importance was attached to the “information gained during the course of the critical review which necessarily accompanies the adjustments”. Clear warnings were also given in [7,8] that the LSA-adjusted values should be used with caution, in particular because “of the intimate relationship which exist [is set by the LSA] among least-squares adjusted values of the fundamental constants”, so that “a significant shift in the numerical value of one will generally cause significant shifts in others. Consequently, for any critical application of these numbers, the user is urged to refer to the original article[s] as well as to the current literature . . .” (emphasis added) [8].

These early warnings raise questions about the possible use of LSA-adjusted values computed by CODATA in the definition of measurement units, clearly a particularly critical application, where they should be considered seriously.

It is a fact that consistency is a most important requirement on a set of constants. However, its check is performed with tools suitable for that purpose, such as the LSA, which are different from the tools used to obtain the numerical values of the chosen summary statistics directly evaluated from the—critically selected—experimental data available for each constant. In fact,

(a) in the LSA, the value of (at least) one member of the set—chosen arbitrarily—is to be set fixed, so that all the adjusted numerical values are relative to this choice\(^1\.\) They are not absolute evaluations, but only relatively adjusted values;\(^2\)

(b) the values of the adjustable members of the set depend on the chosen inter-subjective criteria, and their initial values are altered according to the LSA optimization algorithm. They are not equivalent to the statistical ‘best values’. In doing so, the LSA operation sets an “intimate relationship” [8] between all members of the set, which may have no physical meaning for some of them in the specific case of the fundamental constants. The issue of interest here is not a possible resulting ‘discrepancy’ between data values, but the fact that the obtained relatively adjusted values optimize the consistency, i.e. the value of the statistical parameter used for evaluating the overall uncertainty of the set. However, for the values of the set there is a cost: divorcing by a certain amount from the original physical world. This cost is generally irrelevant in other applications of the CODATA analysis. However, in the specific case of the measurement units, the CIPM may want (or need) to take advantage of the best accuracy allowed by the current experimental data, and this is a critical condition making unsuitable the use of the CODATA relatively adjusted values, which may even result in missing the aimed goal\(^3\)—see at the end of this section;

(c) the above difficulties are generally also involving the adjusted uncertainties associated with the relatively adjusted values, typically lower than the experimental ones: it may happen again that the aimed uncertainty level (see footnote 3) is satisfied by some adjusted constants, but not yet achieved in actual experimental determinations.

Thus, confounding the results of the LSA with the choice of a statistical summary (e.g., mean, weighted mean, median, etc), and associated uncertainty, should be considered incorrect. See for example [9, 10] for a comparison of several of such summaries for \(h\): only involving ‘random effects’ models [9], also ‘fixed effects’ models [10].

It is relevant to recall here what happened at the time preceding the stipulation of \(c_0\) for the purpose of the definition of the unit of length. See more details in the online supplementary information (available from stacks.iop.org/Met/51/L1/mmedia).

The only experimental value used for the 1973 CODATA adjustment was \(c_0 = 299 792 456.2(1.1) \text{ m s}^{-1}\)—all others being more than ten times less accurate. However, the CCDM preferred a different value for \(\lambda(\text{CH})\), bringing to the different value \(c_0 = \lambda \cdot v = 299 792 458.33(1.2) \text{ m s}^{-1}\) [8]. The CODATA 1973 adjusted value came out on request of the CCDM as \(c_0 = 299 792 458(1.2) \text{ m s}^{-1}\) [11]. In contrast, the 1973 LSA, performed prior to the CCDM decision, used the value \(c_0 = 299 792 456.2(1.1) \text{ m s}^{-1}\), barely compatible with the other choice—whose stipulation would have given instead \(c_0 = 299 792 456 \text{ m s}^{-1}\).

Two issues should be noted: (i) the CCDM preference for the experimental value and (ii) the external input to the CODATA Task Group, asking to avoid any adjustment affecting the \(c_0\) value; the latter accepted this constraint. Thus, the outcomes of the CODATA task can be ‘flexible’ also in this respect. A metrological need can lead to a preference different from the ‘general science’ need for the best-consistent value.

\(^1\) Only pair differences are invariant to the choice of the fixed member(s).

\(^2\) So, best consistency of a set of constants is not a property also ensuring fulfilment of the second principle in the introduction.

\(^3\) CGPM (2007) stipulated that revision of the SI shall take place “if the results of experiments are found to be satisfactory and the needs of users met” [12].

\(^4\) Similarly to the LSA, all methods with fixed effects are generally not suitable for the purpose—see reason (a).
In all instances, the CODATA provide only the uncertain value: its stipulation is outside the CODATA task. The stipulation in view of a measurement unit creates non-trivial problems [4].

3. The CODATA least squares adjustment after stipulation of several fundamental constants in measurement units

If the CGPM eventually adopts the CIPM proposal and uses the CODATA adjusted values, more constants will have their numerical values stipulated, even more would also get fixed numerical values because of the interrelations between constants. According to the present CODATA method, this fact would considerably reduce the number of adjustable constants. The remaining number might be so small that the whole CODATA task could become irrelevant or even terminated.

A question arises: is it useful (or correct, depending on the viewpoint) that the CODATA analysis and outcomes always incorporate constraints arising from a basically regulatory field, like metrology, for decisions concerning the measurement units? It is true that the numerical values of the constants completely depend on the size of the units on which the measurements are based. However, LSA evaluations of consistency are actually biased by the metrological issue.

A single example is available today. After 1983, when the above value of \(c_0\) was stipulated in the definition of the SI unit length, CODATA included \(c_0\) among the non-adjustable constants, and then also \(\varepsilon_0 = \frac{1}{\mu_0 \cdot c_0^2}\). In fact, if they had remained adjustable in the CODATA analyses, the values of \(c_0\) and \(\varepsilon_0\) would obviously have changed also after 1983, due to their “intimate relationship” [8] with other adjusted constants.

The impossibility for further CODATA analyses would also place delicate question marks on the handling of future experimental determinations of the constants, namely of the stipulated ones. Instead, ‘general science’ will certainly remain more interested in the CODATA consistency checks of the physical theories than being strictly tied by the needs of metrology.

Metrologists will need, in all instances, to demonstrate the degree of traceability of their \textit{mises en pratique}, by providing facts supporting evidence up to the level of the new definitions. Therefore, the new proposed units will still cause the metrological need for future direct measurements of those constants.

In this respect, national traceability is only demanding, within each NMI, to (i) realize apparatus for the measurement of the constants, (ii) assign the outcome of the measurements the exact stipulated value of each constant\(^6\), and (iii) establish a relationship between these results and the ‘\textit{mises en pratique}’ in that NMI\(^7\).

However, in order to establish the \textit{International} traceability at the level of the new SI definitions, the need will remain to detect systematic differences between NMIs—here meaning between determined values of the realized constants. The metrologists will resort, as usual, to between-laboratories information, i.e. to \textit{inter-comparisons} [13], where the differences will show up for the \textit{whole set of constants}.

In addition, as scientists, they might intend to take advantage of the same apparatus to also perform new independent determinations of the constants, and to assign independent numerical values to them\(^8\). By assuming that significant discontinuities between the new and the previous units are avoided, the experimenters can then treat the local realizations of the constants as they were with the former units, and compute the new \textit{actual} values as they have done so far.

How will these new determinations be taken into account in future? Possibly, it will not be in the interest of metrology to adjourn the stipulated value of a specific constant, while it would certainly be advantageous for ‘general science’ to have the new information duly taken into account, namely by the CODATA Task Group.

4. Conclusions

Concerning the use of the \textit{relatively} adjusted numerical values of the CODATA Task Group, for the purpose of stipulating the numerical value of the constant in the definition of measurement units, the LSA should not be considered as a proper tool for obtaining an enhanced evaluation of the ‘best value’ for each constant. These values should be determined, for each individual constant at the time of the stipulation, only by strict statistical criteria from the set of critically selected experimental data. Statistical methods such as, e.g., those in [9, 10] look appropriate—except, as already said, fixed-effect methods.

The above also applies to the \textit{adjusted uncertainties}, which should not be conceptually confused with the uncertainties actually achieved experimentally, and should not replace the latter.

Concerning the effect of the new definitions on the future CODATA task and on handling future experimental determinations of the constants stipulated in metrology, a serious concern is expressed for a possible divorcing of the metrological regulatory procedures and outcomes from the ‘general science’ interests—and needs. To ensure the future possibility to take into account the fact that new experimental determinations might possibly lead to different ‘best’ values according to normal scientific practice, a possibility would be that the CODATA slightly modify its procedure. In this perspective, beyond the critical review of data, the LSA analysis on the set of constants should concern exclusively the check for their best consistency, limiting the number of non-adjusted constants to one, and not using the values stipulated for metrological use. E.g., for \(\mu_0\) being the fixed constant, adjustments should be recalculated back from 1973 with \(c_0\).

\(^5\) The author is aware of a current opinion that CODATA will no longer adjust any constant whose numerical value has been fixed, and that, once in the new SI, there will not be any other measurement of \(h, \ k, \ N, \) only \textit{mises en pratique} of the respective units, as happened for \(\mu_0\) and \(\varepsilon_0\).

\(^6\) For example, when realizing the triple point of water, all realizations are assigned the stipulated value 273.16 K.

\(^7\) In the case of constant-based \textit{interrelated} unit definitions, these checks in NMIs are likely to be rather more indirect and complex—and costly—than the present ones.

\(^8\) As happens, e.g., when an interpolating gas thermometer, built for realizing the ITS-90, is also used for new determinations of the thermodynamic temperature scale.
and $\varepsilon_0$ adjustable. This procedure should replace the present one, or it should become a separate additional analysis if the present one remains feasible in future.

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