Determination of the Planck constant with the METAS watt balance

To cite this article: Ali Eichenberger et al 2011 Metrologia 48 133

View the article online for updates and enhancements.

Related content
- Tracing Planck’s constant to the kilogram
  A Eichenberger, B Jeckelmann and P Richard

- Design of the new METAS watt balance experiment Mark II
  H Baumann, A Eichenberger, F Cosandier et al.

- The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass
  Ian A Robinson and Stephan Schlamminger

Recent citations
- Metrology in electricity and magnetism: EURAMET activities today and tomorrow
  F Piquemal et al

- Stability improvement for coil position locking of joule balance
  Yang Bai et al

- CODATA Recommended Values of the Fundamental Physical Constants: 2014
  Peter J. Mohr et al
Determination of the Planck constant with the METAS watt balance

Ali Eichenberger, Henri Baumann, Blaise Jeanneret, Beat Jeckelmann, Philippe Richard and Walter Beer

Federal Office of Metrology METAS, CH-3003 Bern-Wabern, Switzerland
E-mail: ali.eichenberger@metas.ch

Received 10 December 2010, in final form 6 February 2011
Published 30 March 2011
Online at stacks.iop.org/Met/48/133

Abstract
The METAS watt balance project was initiated slightly more than a decade ago. Over this time, the apparatus has been through an uninterrupted series of upgrades that have improved its reliability to a point where continuous series of measurements can be taken fully automatically over periods of several months. A comprehensive analysis of possible systematic errors has now been completed and a large set of data has been analysed to calculate a value for the Planck constant $h$. This paper describes the watt balance in detail, explains the data acquisition and analysis thoroughly and presents the uncertainty budget. The value of the Planck constant determined with our apparatus is $h = 6.6260691(20) \times 10^{-34}$ Js with a relative standard uncertainty of $0.29 \times 10^{-6}$. This value differs from the 2006 CODATA adjustment by $0.024 \mu$W W$^{-1}$.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Today, the kilogram is the last base unit of the International System of Units (SI) still based on an artefact: the international prototype of the kilogram $K$ which is kept at the Bureau International des Poids et Mesures (BIPM). The international comparisons between $K$ and national kilogram prototypes have clearly shown a long term relative drift [1]. Based on the BIPM official copies, this drift has been evaluated to be $0.5 \mu$g year$^{-1}$ whereas the absolute drift of $K$ is unknown and could even be larger. Since it is now possible to compare two mass standards of the same material with an uncertainty of about $1 \mu$g, the instability among the international mass prototypes—including $K$—is a major contribution to the final uncertainty. Clearly, such a situation is no longer satisfactory for the definition of an important unit of the SI.

In addition, a drifting mass unit also influences the electrical units through the definition of the ampere. In 1990, electrical reference standards based on the Josephson and the quantum Hall effects were introduced [2] by defining conventional values $K_{J,90}$ and $K_{K,0}$ for the Josephson constant and the von Klitzing constant. Since then, the electrical quantum standards have improved the worldwide consistency and uniformity of electrical measurements by two orders of magnitude. However, the results expressed in the 1990 conventional units may differ from their SI values. Moreover, $K_{J}$ and $K_{K}$ may fluctuate in time as a consequence of a possible drift of the kilogram.

There is now a general consensus in the metrology community that the time for a redefinition of the kilogram has come [3, 4]. The new definition should link the kilogram to fundamental constants with a relative uncertainty of a few parts in 10$^8$ [5]. Several experiments have been attempted to realize the new definition [6–8]. Up to now, the most successful electrical approach has been the watt balance which was proposed by Kibble in 1975 [9]. Its principle consists in linking the mass unit to the Planck constant using the equivalence between electrical and mechanical power.

Several watt balances are now in operation around the world (see [10] for the latest review). At the National Physical Laboratory (NPL, UK) and at the National Institute of Standards and Technology (NIST, USA) moving coil apparatus have been in operation for a long time and have already produced several results for the Planck constant [11, 12]. At the Laboratoire National de Métrologie et d’Essais (LNE, France), a balance is under development [13] and should soon provide a value for $h$. Other experiments are in progress at the BIPM [14, 15] and at the National Institute of Metrology (NIM, China) [16]. Recently, the NPL instrument was transferred...
Figure 1. (a) Static mode: the electromagnetic force acting on the current-carrying coil is balanced against the weight of the test mass \( m \). (b) Dynamic mode: the coil is moved in the vertical direction through the magnetic field and the induced voltage \( U \) is measured.

In the static phase, the force acting on a mass \( m \) placed in the local gravity field, \( \vec{g} \), is balanced by the vertical component of the electromagnetic force \( \vec{F} \) produced by a current \( I \) flowing in a coil immersed in a magnetic field \( \vec{B} \). The electromagnetic force can be expressed by

\[
\vec{F} = I \oint \vec{d}\ell \times \vec{B},
\]

where \( \ell \) is the conductor length of the coil. In the dynamic phase, the coil is moved vertically at a velocity \( \vec{v} \) in the magnetic field. This motion induces a voltage \( U \) across the coil that can be expressed by

\[
U = -\oint (\vec{d}\ell \times \vec{B}) \cdot \vec{v}.
\]

If the mechanical dimensions of the coil and the magnetic field are strictly identical in both modes, and under the hypothesis that the coil passes through its weighing position during the velocity mode with a strictly vertical motion, the combination of equations (1) and (2) leads to the expression

\[
U \cdot I = m \cdot g \cdot v.
\]

The watt balance thus allows a comparison between the electrical and the mechanical virtual powers by combining the static phase, where the velocities and voltages are not relevant, and the dynamic phase, where the forces and currents are not important. This means that real energy dissipation does not enter into the basic equation of the experiment. Using the expressions of the Josephson and quantum Hall effects, equation (3) can be rewritten as

\[
m = C \cdot f_J \cdot f'_J \cdot g \cdot v \cdot h,
\]

where \( C \) is a calibration constant, \( f_J \) and \( f'_J \) are the Josephson frequencies used during the static and the dynamic phases and \( h \) the Planck constant. The watt balance experiment relates therefore the unit of mass to the metre, the second and the Planck constant. A possible new definition for the mass unit could then be based on a fixed value of \( h \) [4].

3. The METAS watt balance

The original idea of the METAS design is to use a 100 g test mass (instead of the traditional 1 kg) with a commercial mass comparator to realize the static phase. The velocity mode is then performed with a separated mechanical system that translates the coil in the magnetic field generated by a permanent magnet. The small test mass enables an important reduction in the overall size of the apparatus whereas the use of two separate measurement systems, for the moving and the weighing modes, makes it possible to optimize each setup separately but forces the transfer of the measuring coil between the two systems during the measurement sequence.

Several improvements have been implemented to the initial project over the last few years, mainly related to alignment capabilities and control of the coil position during
Determination of the Planck constant with the METAS watt balance

Figure 2. A picture showing the METAS watt balance with the constant pressure chamber open. The mass comparator is placed on an aluminium table and the mechanical translation system (seesaw) is located under this table. On the right, the optical table for the velocity determination and, at the back, the absolute gravimeter are also visible. The base of the vacuum chamber is approximately 1 m × 1 m.

The measurement. A redesigned magnetic circuit was implemented to reduce hysteresis behaviour [20]. A new suspension coupled to a mass handler allowed the release of several degrees of freedom to facilitate the alignment procedure [21] and optical sensors were added to monitor the coil position in both modes [22]. A picture of the present system is shown in figure 2.

4. Results

Six sets of data representing a total of more than 3400 h of operation have been analysed to determine a value of the Planck constant. The different sets are composed of at least 500 data points. To minimize the influence of atmospheric pressure variations, the whole experiment is built in a hermetically sealed chamber whose pressure, temperature and relative humidity are monitored during the measurements. A single determination of $h$ can be achieved after a 60 min cycle. Each measurement cycle is composed of three sets of weighings and two sets of induced voltage measurements that are separated by a coil transfer; every operation lasts roughly one third of this total time. Different corrections of systematic effects have been taken into account. Residual misalignments of the coil position between the two modes have been corrected using an analytical model where the parameters are determined with a least square fit procedure. A detailed description of the measurement sequence and the data analysis is given in appendices A and B.

The value of $h = 6.626069(20) \times 10^{-34}$ J s extracted from this data set differs by 0.024 µW W$^{-1}$ from the CODATA 2006 value [23]. The results are presented in figure 3 where each point (open dot) is the mean value over 10 h and the mean value (black full dot) of each set is shown with its standard deviation.

Figure 3. Set of data used for the determination of the Planck constant. The open dots represent the mean value over ten individual determinations of $h$ (representing a period of 10 h) and the full dots are the mean values for each block, both with their associated standard deviation. Note that $h_{\text{ref}} = \frac{4}{R_{K-90}} K^2_{J-90}$.

Figure 4. Summary of the Planck constant determination with the three operational watt balances in the world compared with the CODATA 2006 value.

The time between data sets was used for consistency checks and secondary measurements (such as the verticality of the laser beam, transfer and horizontal motion of the coil). The standard deviation of the mean of these six values (0.07 µW W$^{-1}$) can therefore be considered as the reproducibility of the apparatus. A summary of the Planck constant determination with watt balances is presented in figure 4.

The uncertainty contributions are described in detail in appendix B and summarized in table 1. The global uncertainty associated with the Planck constant determination is 0.29 µW W$^{-1}$. Note that the dominant part of this budget is related to alignment issues and their combined contribution to the total uncertainty adds to 0.20 µW W$^{-1}$. Due to intrinsic limitations in the mechanical setup, it is not possible to significantly reduce this uncertainty contribution.

Based on the experience gathered over the last ten years, it was decided to start a new watt balance project at METAS. This new project is already in progress in close collaboration with external partners like the Ecole Polytechnique Fédérale de Lausanne EPFL (Lausanne, Switzerland), Mettler-Toledo Metrologia, 48 (2011) 133–141
(Greiffensee, Switzerland) and the European Organization for Nuclear Research (CERN, Geneva, Switzerland). This new experimental setup is meant to reach a relative uncertainty close to $10^{-8} \, \text{W}^{-1} \, \text{W}^{-1}$.

5. Conclusion

Over the last few years, several improvements have been implemented in the METAS watt balance. Recently, it has been possible to run the experiment for extended sets of measurements in a satisfactory stable mechanical situation. A value for the Planck constant has been extracted from these data to be $h = 6.6260691(20) \times 10^{-34} \, \text{Js}$ [0.29 x $10^{-6}$]. This value differs from the CODATA 2006 value by $0.024 \, \mu \text{W} \, \text{W}^{-1}$. With this result, the present apparatus has reached its limits. Additional improvements that should be implemented to reduce the uncertainty significantly become incompatible with the conception of the experimental setup.

Acknowledgments

The authors wish to acknowledge the help of all the former METAS collaborators J Butty, A Joyet, A Pourzand, H-U Schneiter, J Schwarz as well as the early collaboration with the optics group of Professor R Dändliker from the University of Neuchâtel.

The long-lasting technical support from our METAS colleagues from the electricity, mass and optics laboratories, the electronic and mechanic shops has without any doubt been significant to the success of the experiment and is gratefully acknowledged.

The steady collaboration with colleagues from NPL and NIST active in watt balance experiments, as well as the extensive exchange of ideas with our colleagues from LNE in the course of the e-MASS project were highly appreciated.

We gratefully thank S Benz and C Burroughs from NIST for their help in setting up the programmable Josephson voltage standard.

We thank the companies Mettler-Toledo and Metrotec for their support and advice in the understanding of the behaviour of the load cell.

Finally, the authors would like to express their gratitude to the former METAS direction board led by W Schwitz for its courage in undertaking such a challenging project as well as for its continuous support. The warm enthusiasm of the present METAS director Ch Bock in supporting the continuation of the project is also gratefully acknowledged and is certainly a major ingredient in the success of the second version of the METAS watt balance.

Part of this work was realized within the e-MASS EURAMET joint research project funded by the European Community’s Seventh Framework Programme, ERA-NET Plus, under Grant Agreement No 217257.

Appendix A. Details of the experimental setup

Appendix A.1. Mechanical setup

In contrast to the NPL and NIST watt balance projects, the METAS design is based on a physical separation of the two measurement phases (static and dynamic). This separation has been realized by means of a parallelogram structure that moves the coil through the magnetic field, and by two mechanical lifters which position and transfer this coil to the mechanical suspension hanging under the mass comparator used for the weighing mode. The coil position on the suspension can be kept within a range of the order of 1 µm in each direction during more than 500 transfers.

In the static phase (figure 5(a)), the weighings are performed with a customized commercial mass comparator from Mettler-Toledo. This phase is composed of two steps: the conventionally called positive step where the test mass is placed on the frame supporting the coil, and a stabilized current produces an electrical force to compensate half of the weight of the test mass, and a negative step where the test mass is lifted and the electrical current running in the coil is reversed to generate a force in the opposite direction. By combining these two steps the ratio $mg/I$, corresponding to the so-called ‘mechanical geometric factor $G_m$’ can be evaluated. In the dynamic phase (figure 5(b)), the coil is removed from the comparator frame and placed on the translation table. This table is clamped to the vertical side of the parallelogram structure and is rolling at the end of the two horizontal arms. The vertical movement is generated by a voice coil motor regulated at the desired velocity $v$ with the signal of a laser interferometer associated with a feedback loop. The induced voltage $U$ is measured and the ratio $U/v$ represents the ‘electrical geometric factor $G_e$’ that is compared with $G_m$.

In the laboratory, all electrical quantities are measured in terms of $K_{1-00}$ and $K_{9-99}$ and a relative difference $\delta h_{\text{meas}}$ between mechanical and electrical powers can be expressed by

$$\delta h_{\text{meas}} = \frac{G_m}{G_e} - 1. \tag{5}$$

Appendix A.2. The magnet coil assembly

The magnetic circuit of the METAS watt balance experiment is illustrated in figure 6. The circuit is composed of four permanent SmCo magnets, two trapezoidal spacers and two poles made of Armco® iron. The two spacers are used to adjust the parallelism as well as the width of the air gap. With a gap...

Table 1. Uncertainty budget.

<table>
<thead>
<tr>
<th>Contributions</th>
<th>Combined uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reproducibility</td>
<td>0.07</td>
</tr>
<tr>
<td>Operation at atm. pressure</td>
<td>0.09</td>
</tr>
<tr>
<td>Magnet temperature stability</td>
<td>0.05</td>
</tr>
<tr>
<td>Voltage measurements</td>
<td>0.10</td>
</tr>
<tr>
<td>$U/v$ ratio at weighing position</td>
<td>0.11</td>
</tr>
<tr>
<td>$F/I$ determination</td>
<td>0.12</td>
</tr>
<tr>
<td>Beam angle</td>
<td>0.12</td>
</tr>
<tr>
<td>Horizontal motions</td>
<td>0.10</td>
</tr>
<tr>
<td>Other contributions</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.29</td>
</tr>
</tbody>
</table>
Determination of the Planck constant with the METAS watt balance

Figure 5. (a) Static phase: the coil and the test mass are suspended under the comparator. A stabilized current is injected into the coil to produce the required force. (b) Dynamic phase: the coil is attached to the parallelogram and moved up and down in the magnetic field produced by a permanent magnet. The signal of a laser interferometer associated with a feedback loop is used to stabilize the coil velocity.

of 8 mm, a magnetic field of 0.56 T is obtained. In the centre region, the relative variation of the field is of the order of $10^{-4}$ over a region of 5 mm. The nominal temperature dependence of the magnetic material is $-360 \times 10^{-6}$ K$^{-1}$. Numerical simulations with finite element methods have been used to optimize the design in terms of temperature stability and homogeneity of the magnetic field. To reduce the temperature effects, the magnetic elements are placed as far away as possible from potential heating sources such as the coil. The large metallic pieces of the yoke act also as a temperature buffer and help us to reduce the temperature gradient of the magnets during the measurement cycles.

The main coil consists of a bi-filar winding of two times 2000 turns of 0.1 mm diameter copper wire wound in an 8-shape to minimize the sensitivity to external magnetic perturbations. A typical current of 3 mA injected in the coil generates the 0.5 N force required to balance half of the weight of the test mass in the static phase.

Appendix A.3. Velocity regulation

During the dynamic phase, the motion of the coil at a speed of $v = 3$ mm s$^{-1}$ produces an induced voltage $U = 0.5$ V. The coil displacement is measured with a laser interferometer [24] and maintained at a constant velocity. A partially reflecting flat reference mirror $M_{ref}$ fixed to the structure and a moving mirror $M_c$ attached to the coil form the Fabry–Perot cavity (figure 7).

The frequency of the interferometer signal is used in a software regulation loop to generate the feedback command for the voice coil motor maintaining the coil velocity at the target value. A time interval analyser (TIA) measures the time tags of the fringes, the voltage integration windows (DVM) and the shadow detector (PosDet), as shown in figure 8. This information is used to calculate the average velocity of the coil during the time interval while the induced voltage is integrated.
Figure 8. Schematic summary of the signals used to measure the coil velocity during the dynamic phase. The interferometer signal (fringes) is recorded as a function of time and combined with the voltage integration windows of the digital voltmeter (DVM) to compute the average velocity \( v_i = \frac{d_i}{\Delta t_i} \). The digital pulses of the position detector (PosDet) are used as an absolute reference to link the static and dynamic phases.

Figure 9. Schematic view of the alignment tool. The laser beam is split into two vertical beams to measure horizontal motions (CC) and tilts (M and L) of the balance arm (B) in the velocity mode.

The variation of the velocity at the scale of a fringe is small enough to be neglected during the interpolation within two time tags.

Appendix A.4. Alignment system

Appendix A.4.1. Verticality of the motion. In the velocity mode of the measurement sequence, only the voltage induced by a strictly vertical motion of the coil is relevant. It is therefore crucial to align and characterize this motion with respect to the vertical direction. To carry out this alignment, a tool has been developed. The system monitors the horizontal motions as well as the tilts of the coil along its path. As shown in figure 9, a set of two vertical laser beams is combined with a retroreflector corner cube (CC) and a mirror (M). These two optical elements are attached to the moving arm of the parallelogram structure holding the coil (B).

The verticality of the laser beam is adjusted using a liquid mirror as a horizontal reference and an autocollimation technique with an uncertainty smaller than 40 µrad. The positions of the reflected beams are read with position-sensitive detectors (PSD). The beam coming out of the retroreflector is sensitive to horizontal motions whereas the signal from the mirror side is proportional to the tilt of the coil. The lens (L) significantly reduces the influence of horizontal motions and parallax errors on the tilt measurement. These four components can be measured synchronously with the induced voltage. This helps us to evaluate the sensitivity of the system to tilts and horizontal motions of the coil.

Appendix A.4.2. Coil position and attitude. During the measurement, the coil attitude is monitored with two mirrors (M) and two corner cubes (CC) placed at 90° on the coil sides (figure 10).

Each optical device is linked to a laser diode and the reflected beam is analysed with a position-sensitive detector (PSD). With these four optical devices, the attitude of the coil can be measured with a resolution of 5 µm.

In addition, the mask of a shadow detector (PosDet) including three slits is attached to the coil and serves as an absolute position reference to lock the vertical scale in both modes. This position detector provides both analogue and digital signals to locate the position of the slits. The analogue signal from the centre slit is used to determine the altitude of the coil during the static phase with a resolution better than 200 nm. The digital pulses trigger the reading of the PSD when the coil goes through the weighing position in the velocity mode. The lower and upper slits trigger the induced voltage measurements in up and down directions, respectively (see figure 8).

Appendix A.5. Evaluation of the gravitational acceleration

During the operation of the METAS watt balance, the gravity acceleration \( g \) is measured synchronously with a commercial absolute gravimeter (Micro-g LaCoste FG5) [25]. From all the quantities that must be measured to satisfy the equality

\[ \text{Figure 10. Top view of the coil with four optical devices, two mirrors (M) and two corner cubes (CC) fixed for the determination of the coil attitude during the measurement. The position detector (PosDet) is used as a reference for the vertical position of the coil.} \]
described by equation (3), the value of the local gravity $g$ is the only one that cannot be accessed directly. For this reason, an appropriate evaluation technique has been developed to determine $g$ at the centre of mass of the test mass. This method uses a three-dimensional interpolation procedure based on an experimental absolute gravity network. Because any interpolation implies implicitly the continuity of the interpolated function, special care has been taken for the transfer of the gravity value, measured beside the experiment, through the vacuum chamber into the experiment. The method for the evaluation of the gravitational acceleration used in the METAS experiment has been discussed in detail in [26] and allows a transfer of the measured gravity to the reference position with a standard uncertainty of 2.5 nW W$^{-1}$.

Appendix A.6. Test mass and force measurement

In the static mode, the residual force is measured with a commercial flexure-strip mass comparator type AT106 from Mettler-Toledo, working on the substitution weighing principle, that has been specially adapted for the purpose of this experiment. The nominal charge of the comparator can be adjusted with the internal counter-mass between 1 kg and 2 kg and has a dynamic range of 1.2 g with a resolution of 0.1 µg. For the different measurements, a 100 g gold-plated copper test mass was used as a reference mass. Between 2004 and 2010 this reference was calibrated three times and exhibits a peak to peak scatter of 3 µg. Because the comparator is not used in a traditional way, its horizontality is of great importance. Indeed, in the usual scheme of measurement, a systematic error introduced by a misalignment of the horizontality is automatically compensated by the substitution procedure and a proper calibration step. In the present case, this systematic error is not cancelled and must therefore be minimized by adjusting the horizontality. The method used is to load the comparator with a test mass smaller than 1 g and to tune the horizontality until the indicated value of the comparator is maximized. With this approach, the residual systematic error has been evaluated to be lower than 10 nW W$^{-1}$.

Appendix B. Uncertainty evaluation

The different contributions to the uncertainty budget associated with the Planck constant determination (table 1) will now be reviewed. Only those with a contribution larger than 0.05 µW W$^{-1}$ are discussed in detail in the following paragraphs.

The measurement sequence adopted for the determination of the Planck constant is composed of a succession of weighings and induced voltage measurements. The complete sequence can be summarized as follows:

$$W_s W_w \rightleftharpoons U_{\text{ind}}^{+} \rightleftharpoons W_w W_s W_w \rightleftharpoons U_{\text{ind}}^{-} \rightleftharpoons W_s W_w, \quad (6)$$

where $W_w$ represents a weighing with a current in the coil producing a force down equivalent to half of the weight of the test mass ($\approx 0.5$ N) and $W_s$, a weighing with a reversed current (force up) when the test mass is placed on the suspension. Before and after each induced voltage measurement, a coil transfer ($\rightleftharpoons$) is necessary. The timing of the whole sequence is adjusted to ensure a symmetry and get rid of linear drifts. During the post-analysis, the mean magnet temperature during the four centre weighings is chosen as a reference.

Appendix B.1. Operation at atmospheric pressure

For technical and practical reasons (mainly linked to the stability of the optical alignment), the experiment was not run under vacuum conditions over long periods of time. However, special care was taken to do all experimental steps in a stable atmospheric environment (hermetically sealed chamber). The air buoyancy and the refractive index $n_{\text{air}}$ are the main corrections that need to be applied in the case of non-vacuum operation. For this purpose, the air pressure, temperature and CO$_2$ content must be determined. The air density $\rho_{\text{air}}$ and $n_{\text{air}}$ are evaluated using the usual formula [27]. The uncertainty associated with the operation at atmospheric pressure is estimated to be 0.09 µW W$^{-1}$.

Appendix B.2. Magnet temperature stability

As mentioned above, the magnetic field is produced by four SmCo permanent magnets. Because these magnets have a large temperature dependence, their temperature is permanently monitored with a set of four thermometers. The mean value of the magnet temperature during the four central weighings is chosen as a reference and the magnetic flux density is reduced to this temperature condition during the analysis. An indirect but efficient check of the temperature corrections is made every time the measurements are started after a period of time where the thermal load due to the current flowing in the coil has not followed the usual scheme. After a rest of one day or more, the magnet can take up to 12 h to reach a stable temperature with a typical excursion of 0.2 K. Larger excursions have also been studied and no residual effects are correlated with temperature variations, showing that the appropriate correction is applied. When the temperature of the magnet stabilizes, the overall variations do not exceed 2 mK. The residual contribution of the thermal corrections is estimated to be 0.05 µW W$^{-1}$.

Appendix B.3. Electrical measurements

Voltage measurements are made in the static and the dynamic mode. During the velocity phase, the induced voltage at the coil terminals is measured synchronously with the velocity. In the weighing mode, a calibrated 100 Ω resistor is connected in series with the coil and the current injected in this circuit is measured through the voltage drop across this resistor. The reference resistor is regularly calibrated against the quantum Hall resistance and a programmable Josephson voltage standard [28, 29] is used for the DVM calibration. An uncertainty of 50 nV contributes to the total uncertainty at the level of 0.10 µW W$^{-1}$.

Appendix B.4. $U/v$ ratio at weighing position

In the dynamic phase, the coil is translated vertically (up and down) through the magnetic field and the induced

Metrologia, 48 (2011) 133–141
voltage $U$ is integrated 50 times during 10 power line cycles (0.2 s). This integration time is long enough to achieve the required accuracy without straightening out the structure of the magnetic field. For each voltage value $U$, the average vertical velocity $v$ of the coil is evaluated with the interferometer signal registered as a function of time to calculate $G_e$ along the trajectory of the coil. The shadow detector serves as a position reference centred at the weighing position ($W_{pos}$) and allows $G_e$ to be interpolated at this position. A total of five ups and five downs are averaged and the uncertainty linked to this determination adds up to 0.11 µW W$^{-1}$. The interpolation at $W_{pos}$ is the dominant contribution.

Appendix B.5. Alignment

The different aspects of the alignment are illustrated in figure 11 where $F$ represents the electrical force, $g$ the local gravity field, $\dot{v}$ the velocity and $\lambda$ the laser beam direction.

In an ideal watt balance experiment, all the vector quantities, $\dot{F}$, $\dot{v}$ and $\lambda$, are aligned with the vertical given by $g$ and are acting at the centre of mass. Each misalignment in direction or position will potentially induce an error.

Appendix B.5.1. $F/I$ determination. In the static phase, $G_m$ is given by the ratio of the mechanical force (weight of the test mass) to the total current $I$ injected in the coil. As mentioned above, the measurement is made in two steps, with a direct and reversed current depending on the presence of the test mass. The coil is connected in series with a 100 Ω reference resistor to the source and the voltage drop across this resistor is used to stabilize and measure the currents. In both steps, the residual force is given by the Mettler-Toledo mass comparator AT-106H. After applying the corrections for non-vacuum operation and possible magnet temperature differences, these two measurements are combined to give the determination of $G_m$. The uncertainty associated with this determination is dominated by the alignment of the electrical forces. By measuring the position differences of the coil during the two weighing situations and the position at rest ($I = 0$), the contribution of this alignment to the uncertainty is evaluated to be 0.12 µW W$^{-1}$.

Appendix B.5.2. Beam angle. The orientation of the laser beam direction has a direct influence on the determination of the velocity. By construction, the moving mirror attached to the coil ($M_c$ in figure 7) has limited adjustment capabilities. The laser beam must therefore be aligned to be orthogonal to this flat mirror and cannot be directly adjusted to be vertical. The measurement of the residual beam angle is made off-line when the moving mirror is replaced with a two-dimensional PSD. The contribution associated with this measurement, mainly limited by the short travel of the coil, is 0.12 µW W$^{-1}$.

Appendix B.5.3. Horizontal motion of the coil. Only the voltage produced by a vertical motion of the coil is relevant when compared with the static phase. Depending on the geometry of the coil and the stray magnetic fields, a horizontal motion of the coil can also generate a voltage responsible for a systematic error. In the METAS design, parasitic motions are essentially due to the surface quality of the end of the horizontal arms to which the translation table is clamped. Even if this surface is regularly cleaned, horizontal movements of the coil during the dynamic phase cannot be completely avoided. The related uncertainty is evaluated by comparing the horizontal velocities and $G_e$ around $W_{pos}$ in different configurations. This contribution represents 0.10 µW W$^{-1}$.

Appendix B.5.4. Coil attitude. One essential condition for the experiment is that the evaluation of the mechanical and the electrical geometry factors, $G_m$ and $G_e$, is done when the coil is at the exact same position with the same orientation. A position or an attitude difference of the coil between the two modes will therefore lead to a parasitic contribution to $\delta h_{meas}$ from equation (5). These mechanical instabilities of the translation and transfer systems cannot be completely avoided. The error introduced by this type of misalignment can be modelled by

$$\delta h_{meas} = \delta h + \sum_{i=1}^{8} \beta_i \cdot \Delta p_i,$$

where

$$\delta h = h - h_{90} / h_{90},$$

is the deviation from $h_{90} = 4 \cdot (R_{K-90} \cdot K_{I-90}^2)^{-1}$ and $\beta_i$ the sensitivity factor for the position difference $\Delta p_i$ of the coil between the two modes measured by the PSDs. The index $i$ denotes the eight components measured by the four two-dimensional PSDs. The factors $\delta h$ and $\beta_i$ are determined by a least square adjustment and are summarized in table 2.

The high sensitivity factor of $M_2(y)$ can be explained by the fact that the M2 PSD is located much closer to the coil than the M1 PSD. The main result of this adjustment is nevertheless the deviation from $h_{90}$ given by $\delta h = 0.04 \mu W W^{-1}$ with an associated uncertainty of 0.06 µW W$^{-1}$. This uncertainty,
which takes into account every single value of $h$, is in excellent agreement with the standard deviation of the mean of the six data sets analysed above.

**Appendix B.6. Other contributions**

As mentioned at the beginning of this section, the different contributions smaller than 0.05 $\mu$W W$^{-1}$ will not be discussed. These include for example the reference resistance calibration, leakage resistance and dielectric losses of the coil, laser wavelength calibration, effect of laser beam diameter, eddy current effects and mass calibration. A last contribution of 0.10 $\mu$W W$^{-1}$ largely covers the neglected components and completes the uncertainty budget presented in table 1.

**References**


[7] Becker P 2003 Tracing the definition of the kilogram to the Avogadro constant using a silicon single crystal Metrologia 40 366–75


**Table 2. Summary of the parameters given by the correlation analysis of the residual misalignments.**

<table>
<thead>
<tr>
<th>Index $\beta_i$, 10$^{-6}$ µm$^{-1}$</th>
<th>$x$ axis</th>
<th>$y$ axis</th>
<th>$z$ axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSD</td>
<td>CC1</td>
<td>M1</td>
<td>CC2</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>0.043</td>
<td>0.063</td>
<td>0.063</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.006</td>
<td>0.014</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Determination of the Planck constant with the METAS watt balance

Metrologia, 48 (2011) 133–141

141