Journal of Physics C: Solid State Physics

Quasiparticle scattering time in P-wave-superconductors

To cite this article: L Coffey *et al* 1985 *J. Phys. C: Solid State Phys.* **18** L813

J. Phys. C: Solid State Phys. 18 (1985) L813–L816. Printed in Great Britain

LETTER TO THE EDITOR

Quasiparticle scattering time in P-wave-superconductors

L Coffey†, T M Rice† and K Ueda‡

† Theoretische Physik, ETH-Hönggerberg, 8093 Zürich, Switzerland

‡ Department of Applied Physics, The University of Tokyo, Tokyo 113, Japan

Received 7 June 1985

Abstract. We show that the quasiparticle scattering time, due to non-magnetic impurities diverges at zero energy in anisotropic superconductors with zeroes in the order parameter at points and on lines on the Fermi surface. We assume S-wave scattering from the impurities and carry out the calculations in the weak coupling limit. As a result of the energy dependence of the scattering time, the quasiparticle mean free path diverges at zero energy. This is in contrast to the BCS result where the mean free path is equal to its constant normal state value. The consequences of this for the ultrasonic attenuation are discussed.

Recent investigations of heavy-fermion superconductors have revealed results that deviate strongly from the BCS theory of singlet superconductivity [1]. These results include the low-temperature (T) dependences of the specific heat capacity, C_v, of UBe_{13} [2], the ultrasonic attenuation coefficient of UPt_3 [3] and the NMR spin–lattice relaxation rate in UBe_{13} [4] which have T^3, T^2 and T^3 behaviour, respectively.

A superconducting phase, with point zeroes of the order parameter on the Fermi surface, similar to the Anderson–Brinkman–Morel (ABM) phase of superfluid ^3He, has been used to analyse the specific heat data [2]. However, a phase similar to the polar phase of superfluid ^3He, with lines of zeroes in the order parameter, has been invoked to describe the latter two sets of results [3, 4].

Rodriguez [5] has pointed out that the ultrasonic attenuation experiment on UPt_3 was carried out in the hydrodynamic limit, $ql \ll 1$. He obtains evidence for an ABM-like state for UPt_3 by using the more correct picture of sound attenuation being due to the viscous damping of the electron gas, in this limit. However he assumes an energy-independent quasi-particle mean free path, $l_k = l_n$, in his analysis.

A number of authors [6–11] have investigated the nature of the superconducting phases of these materials more thoroughly by including the symmetry constraints on the order parameter that arise due to the crystal structure of these materials. The superconducting phases that result are more complicated than the simple ABM, Balian–Werthamer (BW) or polar phases of the isotropic liquid superfluid ^3He. One important point that emerges is that the stable states may have zeroes of the order parameter at points but never have them on lines.

The purpose of this Letter is to examine the quasiparticle scattering time due to impurities, τ_k. The calculation is carried out assuming a weak S-wave scattering due to the impurities and in the weak coupling limit of the superconductor. Strong coupling corrections that may arise due to a possible strong frequency dependence of the pairing interaction or that arise in higher order in the impurity potential are neglected.

The scattering time, τ_k, enters the calculation of the ultrasonic attenuation coefficient via the mean free path, l_k [5]. If l_k is dependent on the quasiparticle energy, E_k, the temperature dependence of the ultrasonic attenuation is affected quite markedly.

The calculation of the scattering time, τ_k, for the isotropic BCS singlet state results in a quasiparticle mean free path, l_k, that is independent of energy [12]. The present calculation is carried out using the anisotropic order parameters of the ABM and polar phases of superfluid ^{3}He [13]. These give scattering times, τ_k, which are generic for superconducting phases with point zeroes and lines of zeroes on the Fermi surface, respectively.

The scattering of electrons by impurities in the superconductor is described by the Hamiltonian

$$\mathcal{H} = \sum_{kq\delta} U(q) C^{\dagger}_{k+q,\delta} C_{k\delta}. \tag{1}$$

This is written in terms of the quasiparticle operators $\alpha_{k\delta}$ that diagonalise the superconducting Hamiltonian as

$$\mathcal{H} = \sum_{kq\delta} U(q) \boldsymbol{\alpha}^{\dagger}_{k+q} \begin{bmatrix} \hat{U}^{k+q\dagger} & - \hat{V}^{-k^*-q\dagger} \\ \hat{V}^{k+q\dagger} & - \hat{U}^{-k^*-q\dagger} \end{bmatrix} \begin{bmatrix} \hat{U}^{k} & \hat{V}^{-k} \\ \hat{V}^{-l^*} & \hat{U}^{-k^*} \end{bmatrix} \boldsymbol{\alpha}_k \tag{2}$$

where [13]

$$\begin{bmatrix} C_{k\uparrow} \\ C_{k\downarrow} \\ C^{\dagger}_{-k\uparrow} \\ C^{\dagger}_{-k\downarrow} \end{bmatrix} = \begin{bmatrix} \hat{U}^{k} & \hat{V}^{k} \\ \hat{V}^{-k^*} & \hat{U}^{-k^*} \end{bmatrix} \begin{bmatrix} \alpha_{k\uparrow} \\ \alpha_{k\downarrow} \\ \alpha^{\dagger}_{-k\uparrow} \\ \alpha^{\dagger}_{-k\downarrow} \end{bmatrix}. \tag{3}$$

The portion of this Hamiltonian describing scattering of quasiparticles is

$$\begin{aligned} \mathcal{H}^{\text{scatt}} = \sum_{\substack{k>0 \\ q\delta}} U(q)[& \alpha^{\dagger}_{k+q\uparrow} \alpha_{k\uparrow} (\hat{U}^{k+q^*\dagger} \hat{U}^{k} - \hat{V}^{-k-q\dagger} \hat{V}^{-k^*})_{11} \\ & + \alpha^{\dagger}_{k+q\downarrow} \alpha_{k\downarrow} (\hat{U}^{k+q^*\dagger} \hat{U}^{k} - \hat{V}^{-k-q\dagger} \hat{V}^{-k^*})_{22} \\ & + \alpha_{-k-q\uparrow} \alpha^{\dagger}_{-k\uparrow} (\hat{V}^{k+q^*\dagger} \hat{V}^{k} - \hat{U}^{-k-q\dagger} \hat{U}^{-k^*})_{11} \\ & + \alpha_{-k-q\downarrow} \alpha^{\dagger}_{-k\downarrow} (\hat{V}^{k+q^*\dagger} \hat{V}^{k} - \hat{U}^{-k-q\dagger} \hat{U}^{-k^*})_{22}]. \end{aligned} \tag{4}$$

Using the unitarity constraint on the matrix $\hat{U}^k$, $\hat{U}^k \hat{U}^{k\dagger} = \mathbb{1}$ and the diagonalisation formula relating the energy matrices $\hat{E}_k$ and $\hat{\xi}_k$ [13] it can be easily shown that, for ESP states where the off-diagonal components of the gap matrix are zero,

$$\hat{U}_k = \frac{1}{\sqrt{2}} \left(1 + \frac{\varepsilon_k}{E_k}\right)^{1/2} \mathbb{1} \tag{5}$$

and

$$\hat{V}_k = \frac{1}{\sqrt{2}} \left(1 - \frac{\varepsilon_k}{E_k}\right)^{1/2} \mathbb{1} \tag{6}$$

where $\Delta(\boldsymbol{k}) = \Delta_0(k_x + \mathrm{i}k_y)$ for an ABM phase and $\Delta(\boldsymbol{k}) = \Delta_0 k_z$ for a polar phase. In equations (5) and (6) $E_k = \sqrt{\varepsilon_k^2 + |\Delta(\boldsymbol{k})|^2}$ is the quasiparticle energy. The usual BCS results [12] apply also to the isotropic BW phase.

The scattering rate for quasiparticles of momentum $\boldsymbol{k}$ is then given by the Golden Rule formula

$$1/\tau_k = 2\pi/\hbar \sum_{k'} |U^k U^{k'} - V^{-k} V^{-k'}|^2 |U(k - k')|^2 \delta(E_k - E_{k'}). \tag{7}$$

For the singlet BCS and isotropic BW phases, equation (7) yields $1/\tau_k = \varepsilon_k/E_k$ $(1/\tau^{\text{normal}})$.

In the ABM-like phase, if $\boldsymbol{k}$ is initially along the z-axis where the gap vanishes, the evaluation of equation (7) is simple since V^{-k} vanishes. Such a situation is important at low temperatures since quasiparticles will exist with a higher probability at points on the Fermi surface where the gap is zero.

Therefore, for $\varepsilon_k < \Delta_0$,

$$1/\tau_k = (1/\tau^{\text{normal}})\tfrac{1}{2}\left[\frac{\varepsilon_k}{2\Delta_0}\ln\left|\frac{\varepsilon_k + \Delta_0}{\varepsilon_k - \Delta_0}\right| + \left(\frac{\varepsilon_k}{\Delta_0}\right)^2\right] \tag{8}$$

whereas for $\varepsilon_k > \Delta_0$

$$1/\tau_k = (1/\tau^{\text{normal}})\tfrac{1}{2}\left[\frac{\varepsilon_k}{2\Delta_0}\ln\left|\frac{\varepsilon_k + \Delta_0}{\varepsilon_k - \Delta_0}\right| + 1\right]. \tag{9}$$

The first term in square brackets in equations (8) and (9) is the energy-dependent density of states for the ABM phase. Thus the major difference in the case of an anisotropic triplet phase is the appearance of the density-of-states factor in place of the usual $(\varepsilon_k/E_k)N(0)$ factor that occurs in the singlet BCS and isotropic triplet BW phases.

At low temperatures, corresponding to ε_k vanishing, equation (8) yields

$$1/\tau_k \simeq 1/\tau^{\text{normal}}\left[\frac{\varepsilon_k}{\Delta_0}\right]^2. \tag{10}$$

When $\boldsymbol{k}$ is in a general direction on the Fermi surface, the same basic results as equation (8) and (9) hold. Defining $1/\tau(E) = \langle 1/\tau_k \rangle_E$ yields for example

$$1/\tau(E) = 1/\tau^{\text{normal}}\tfrac{1}{2}\left[\frac{E}{2\Delta_0}\left(\frac{3}{2} - \frac{\Delta_0^2}{2E^2}\right)\ln\left|\frac{E + \Delta_0}{E - \Delta_0}\right| + \tfrac{1}{2}\right] \tag{11}$$

where $E > \Delta_0$.

In the case of polar phase, the density of states factor for this phase, i.e. ε_k/Δ_0 for $\varepsilon_k < \Delta_0$ and $(\varepsilon_k/\Delta_0)\sin^{-1}(\Delta_0/\varepsilon_k)$ for $\varepsilon_k > \Delta_0$, enters the definition of $1/\tau_k$. Thus as ε_k goes to zero in the polar phase

$$1/\tau_k \simeq (1/\tau^{\text{normal}})(\varepsilon_k/\Delta_0). \tag{12}$$

The quasiparticle mean free path l_k, is defined by

$$l_k = v_k \tau_k \tag{13}$$

where v_k is the quasiparticle group velocity $(1/\hbar)(\partial E_k/\partial \boldsymbol{k})$ which reduces to $v_g = (1/\hbar)(\partial \varepsilon_k/\partial \boldsymbol{k})$ in regions where the gap vanishes. It can be seen that, from the preceding results, the mean free paths in the ABM and polar phases diverge as $l^{\text{normal}}[\Delta_0/\varepsilon_k]^2$ and $l^{\text{normal}}[\Delta_0/\varepsilon_k]$ respectively. This is in marked contrast to the usual BCS or BW phase result of $l_k = l^{\text{normal}}$.

The strong energy dependence of l_k should be treated carefully when attempting to draw conclusions about the nature of the superconducting state by comparing theoretical predictions of the ultrasonic attenuation coefficient with experimental results [5]. The use of the previous result for l_k for the ABM phase in the hydrodynamical formula for the ultrasonic attenuation yields a constant attenuation at zero temperature in contrast to the low temperature T^2 behaviour, obtained using a constant l_k. Therefore until the energy dependence of l_k including strong coupling effects is treated more carefully, the identification of the superconducting phase from the theoretical calculation of the ultrasonic attenuation is an open question.

Recent work by Ueda and Rice, and Gorkov and Kalajin [15] treating the effects of non-magnetic impurities on the triplet phases self-consistently, indicates similar effects in $\tau(E)$ to those that have been obtained here. The self-consistent treatment of impurity effects removes the singularity at $E_k = \Delta_0$ in the ABM density of states, for example. However it yields the same behaviour in the scattering rate, as ε_k vanishes, as found here.

Finally it is worth noting that a somewhat different lifetime arises in the calculation of the ultrasonic attenuation for an anisotropic superfluid when one uses a Landau kinetic equation approach to calculate the attenuation in the hydrodynamic limit [5]. The difference arises from the collision term on the right-hand side of the Landau kinetic equation and results in a factor of $(1 - \cos^2\theta)$, where θ is the scattering angle, in the expression for $1/\tau_k$. This is also different to the $(1 - \cos\theta)$ factor which occurs in a calculation of the transport scattering time in the conductivity of a normal electron gas.

However these factors do not significantly change the basic conclusions of this Letter since these are primarily dependent both on the form of the coherence factors and energy conservation.

More detailed calculations of the ultrasonic attenuation coefficient in impure anisotropic superconductors are presently being carried out.

We are grateful for useful discussions with R J Joynt and H Smith.

References

[1] Stewart G R 1984 *Rev. Mod. Phys.* **56** 755
[2] Ott H R, Rudigier H, Rice T M, Ueda K, Fisk Z and Smith J L 1984 *Phys. Rev. Lett.* **52** 1915
[3] Bishop D J, Varma C M, Batlogg B, Bucher E, Fisk Z and Smith J L 1984 *Phys. Rev. Lett.* **53** 1009
[4] MacLaughlin D E, Cheng Tien, Clark W G, Lan M D, Fisk Z, Smith J L and Ott H R 1984 *Phys. Rev. Lett.* **53** 1833
[5] Rodriguez J P 1984 *Phys. Rev. Lett.* **55** 250
[6] Anderson P W 1984 *Phys. Rev.* B **30** 4000
[7] Volovik G E and Gorkov L P 1984 *JETP Lett.* **39** 559
[8] Blount E I to be published
[9] Ueda K and Rice T M 1985 *Phys. Rev.* B **31** 7114
[10] Ueda K and Rice T M *Adv. Solid State Phys.* vol XXV
[11] Volovik G E and Gorkov L P preprint
[12] Rickayzen G 1969 *Superconductivity* vol I, ed. R D Parker (New York: Marcel Dekker)
[13] Anderson P W and Brinkman W F 1975 *Proc. 15th Scottish Universities Summer School in Physics, 1974* ed. J G M Armitage and I E Farquhar (New York: Academic) pp 315–416
[14] Ueda K and Rice T M *Proc. 8th Taniguchi Symposium of 'Theory of the Valence Fluctuating State', 10–13 April 1985* to be published
[15] Gorkov L P and Kalajin P A preprint