TECHNIQUES FOR HIGH-CONTRAST IMAGING IN MULTI-STAR SYSTEMS. I. SUPER-NYQUIST WAVEFRONT CONTROL

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ABSTRACT

Direct imaging of extra-solar planets is now a reality with the deployment and commissioning of the first generation of specialized ground-based instruments (GPI, SPHERE, P1640, and SCExAO). These systems allow of planets 10^7 times fainter than their host star. For space-based missions (EXCEDE, EXO-C, EXO-S, WFIRST), various teams have demonstrated laboratory contrasts reaching 10^{-10} within a few diffraction limits from the star. However, all of these current and future systems are designed to detect faint planets around a single host star, while most non-M-dwarf stars such as Alpha Centauri belong to multi-star systems. Direct imaging around binaries/ multiple systems at a level of contrast allowing detection of Earth-like planets is challenging because the region of interest is contaminated by the host star's companion in addition to the host itself. Generally, the light leakage is caused by both diffraction and aberrations in the system. Moreover, the region of interest usually falls outside the correcting zone of the deformable mirror (DM) with respect to the companion. Until now, it has been thought that removing the light of a companion star is too challenging, leading to the exclusion of many binary systems from target lists of direct imaging coronographic missions. In this paper, we will show new techniques for high-contrast imaging of planets around multi-star systems and detail the Super-Nyquist Wavefront Control (SNWC) method, which allows wavefront errors to be controlled beyond the nominal control region of the DM. Our simulations have demonstrated that, with SNWC, raw contrasts of at least 5×10^{-9} in a 10% bandwidth are possible.

Key words: binaries: visual – instrumentation: adaptive optics – planet–disk interactions – planetary systems – planets and satellites: detection

1. INTRODUCTION

The exoplanets field is rapidly expanding with the success of the *Kepler* mission (Burke et al. 2014 and references therein) and the emergence of direct imaging ground-based instruments such as GPI (Macintosh et al. 2014), SPHERE (Beuzit et al. 2008), SCExAO (Guyon et al. 2010), and P1640 (Hinkley et al. 2008). One of the most exciting prospects for future telescopes is finding other Earth analogues in our galaxy or solar neighborhood and ultimately detect life on them. The Kepler space telescope has already revealed that roughly 22% of stars have planets between 1 and 2 Earth radii in their habitable zone (Batalha 2014). However, Kepler does not perform spectral characterization of these targets. Direct imaging combined with spectroscopic characterization would allow us to determine the chemical composition of the planet's atmosphere, and constrain the presence of oxygen, water, and other elements necessary for life. Over the past decade, there have been more than a dozen direct imaging mission studies for space-based telescopes. Figure 1 shows a few representative missions into which a lot of these have evolved. An Earth-like planet orbiting the habitable zone of a Sun-like star would be 10 billion times dimmer than the star. Diffraction created by the telescope aperture as well as aberrations create a background several orders of magnitude brighter than the planet, making its detection very challenging. Several starlight suppression systems (Guyon et al. 2010; Kern et al. 2013) have demonstrated suppression of this background to 10^{-9} raw contrast or better in the laboratory.

These systems employ high-performance coronagraphs to suppress the star diffraction created by the telescope aperture and efficient wavefront control (WC) systems based on a deformable mirror (DM) to remove residual starlight leak (speckles) created by the aberrations from telescope optics. However, their designs have thus far been mostly limited to single-star systems and to planets or disks found within the DM Nyquist control zone, where speckles can be corrected by conventional means. Binary star systems are good candidates to search for planets since they are more common than single stars (at least if we exclude M-dwarfs). Good examples of such binary systems are Alpha Centauri (α Cen) and Sirius. Currently, these multi-star systems are typically excluded from mission target lists because there is no technical approach that can manage the technical challenges associated with double-star (or multi-star) high-contrast imaging (Bendek et al. 2014; Thomas et al. 2014). The three main challenges are as follow.

- 1. Multi-star separation is typically beyond the Nyquist frequency of the DM. In this paper, we propose a new method called Super-Nyquist Wavefront Control (SNWC) that uses a mild grating (or an existing pattern called print-through, commonly found on many DMs left over from their manufacturing process) to effectively alias low-spatial-frequency modes of the DM into higher frequencies, enabling the DM to suppress speckles well beyond the DM's Nyquist frequency. In effect, printthrough is used as a feature rather than a bug. This method already enables high-contrast imaging in binary stars if the on-axis star is independently suppressed (e.g., with a starshade).
- 2. The companion star creates a speckle background that is incoherent with respect to the host star. Therefore, conventional WC does not work because it relies on removing the speckle background through destructive interference. Our solution is called Multi-star Wavefront Control (MSWC), which uses independent DM modes to

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Figure 1. Mission concepts compatible with the methods proposed here. For these missions, no hardware changes are required as long as they have a deformable mirror with a typical print-through pattern.

remove the speckle backgrounds from each star independently (at the expense of reducing the discovery region). This method enables high-contrast imaging in multi-star systems if all stars are within the DM Nyquist separation angle (sub-Nyquist).

3. Combining SNWC and MSWC together. This enables independent suppression of starlight in multi-star systems in the general case, even when the separation angle is larger than the DM Nyquist angle (super-Nyquist). We call this combined technique Multi-Star Super-Nyquist Wavefront Control (MSSNWC).

Each of these techniques serves a specific science case that we will present in Section 2. In this paper, we focus on the Super-Nyquist (SNWC) algorithm. After discussing the challenges of observing multi-star systems in Section 3, we present the theoretical background of the method in Section 4. We also briefly explain the MSWC method (a detailed discussion of MSWC and MSSNWC is beyond the scope of this paper). Finally, we show simulation results in Section 5 in both monochromatic and polychromatic light.

2. SCIENCE MOTIVATION

2.1. Searching for Planets Around Multi-star Systems

The science cases addressed by several exoplanet detection missions such as the ones shown in Figure 1 may result in great leaps in our understanding of warm disks, exoplanet diversity, dynamics, and atmospheres. They also will deliver a census of exoplanets around nearby stars, and (for some missions) the detection and spectral characterization of Earth-like planets in the habitable zone. However, until recently, all of these missions generally excluded multi-star systems from their target lists, and most still do.

The method presented in this paper will greatly multiply the science yield of these missions since it enables direct imaging of planetary systems around multi-star systems, without additional hardware changes or costs as long as a DM with a typical print-through pattern is present on the mission. Enabling the study of multi-star systems is very important because the majority of all stars except for M-dwarfs are in multi-star systems. In particular, five out of the seven star systems within 4 pc containing K- or earlier type stars are multiples (α Cen, Sirius, Procyon, 61 Cyg, ϵ Ind), and only two are singles (ϵ Eri, τ Cet). While it is true that there are many more nearby M-dwarfs is arguably better done from the ground with Extremely Large Telescopes because: (a) they are dimmer and require larger apertures; (b) their planetary systems are



Figure 2. Simulation of a (hypothetical) Earth twin at maximum separation around every nearby star. On this graph of contrast vs. separation angle, the size of a circle corresponds to the planet's brightness and the its color represents the star type. We only marked α Cen A and B planets, all the other circles are all the other planets. α Cen is three times easier than any other star by almost any metric, except for the fact that it is a binary. The vertical line also shows the particular case of where $2\lambda/d$ would be placed for a 1.5 m aperture telescope at 550 nm.

closer to the star and require the angular resolution of larger apertures. Conversely, the study of K- and earlier type stars arguably favors space-based missions, for which the smaller apertures are sufficient, but the deeper contrasts their planets require may only be possible from space. Therefore, most of the stars best suited for space missions are in fact in multi-star systems. One case enabled by our methods deserves special attention: our nearest-neighbor star, α Cen. It is an extremely favorable and unusual outlier for direct imaging but is usually overlooked because of its multiplicity. With an inner working angle of $2\lambda/d$, a 30 cm telescope would be sufficient to image the habitable zone of α Cen A and B. To do this around any other FGKM star requires a telescope at least 1 m in diameter like Exo-S or Exo-C. Stars of comparable proximity to α Cen are all very dim, and stars of comparable brightness are about three times farther away. In particular, the next closest star earlier than M-type (ϵ Eri) is 2.4 times as far, and is known to have a thick disk that may interfere with detection of small planets. The next star of comparable proximity to α Cen is Barnard's star, which is 1.4 times farther, has a much dimmer

Figure 3. Simulations (courtesy of T. P. Greene 2015, private communication) showing that for the example of HR4796A, the disk is truncated with the current outer working angle of the AFTA instrument. The simulations were done using the Zodipic package, a general-purpose modeling tool for optically thin disks (Kuchner 2012).

magnitude (M = 10), and has a habitable zone only 30 mas wide, requiring at least a 4 m aperture just to resolve it (Figure 2).

Another reason why α Cen is an attractive target is because recent estimates of η_{\oplus} from *Kepler* have ranged from about 20% to 50%. Therefore α Cen has about a 40%–75% chance of harboring an exo-Earth around either the A or B star. A mission using the techniques proposed here may be the first to detect and spectrally characterize an Earth twin, if one exists around α Cen (Dumusque et al. 2012).

2.2. Exoplanetary Debris Systems

SNWC by itself also allows another specific science case. When using SNWC, the outer working angle (OWA) and thus the field of view (FOV) of most missions in Figure 1 is limited by the DM Nyquist frequency. As an example, for a 2.4 m telescope with a 48×48 DM such as WFIRST-AFTA the DMlimited OWA is about 1'' (depending on wavelength). Thus, the disk around ϵ Eri can only be seen in high contrast out to ~4 AU. For the case of α Cen, AFTA would not be able to observe beyond 1 AU even if the second star was not there. Figure 3 shows a simulation,¹ of how the disk around HR4796A is truncated with the current capabilities of the AFTA coronagraph instrument. The SNWC method relaxes this OWA limitation since it enables the extension of the dark zone for single-star systems past the DM Nyquist limit. SNWC does not increase the size of the dark zone, but it does allow it to be shifted to arbitrary locations. By stitching separately acquired sub- and super-Nyquist dark regions, imaging of arbitrarily wide planetary systems and disks is enabled in principle.

3. CHALLENGES OF MULTI-STAR AND EXTENDED DISK OBSERVING

Starlight suppression is challenging in the context of multiple-star systems because the unsuppressed, off-axis companion star leaks light into the region of interest around the suppressed on-axis target and is incoherent with respect to it. Moreover, the separation angle between the two stars is usually larger than the Nyquist-limited OWA set by the number

Figure 4. Simulation of a dark zone (high-contrast discovery region) created by conventional wavefront control around a star (which is itself mostly suppressed by a coronagraph). The dark zone size is limited by the number of actuators on the deformable mirror or equivalently the Nyquist frequency corresponding to the DM actuator sampling. We call this conventional control region "sub-Nyquist" and the region outside "super-Nyquist." SNWC enables suppression of speckles in the Super-Nyquist region. This shows the example of a DM with 32×32 actuators, for which the size of the sub-Nyquist region is $16\lambda/D \times 16\lambda/D$.

of DM actuators. In the context of extended disks, this OWA truncates wide disks even around single stars (see Figure 3).

Nyquist limit of the DM: The nominal region around the star over which we can create a dark zone is defined by the number of actuators on the DM. Figure 4 illustrates this. The square region around the central star is the controllable region and is limited by the Nyquist frequency f_N , which corresponds to an outer working sky angle of $N_{act} \times \lambda/2D$. It has been previously believed that the DM cannot control any speckles past this Nyquist-limited OWA, a limitation that is overcome by SNWC, as we will show later.

Light leakage from the companion star(s): When observing a target belonging to a binary system, the amount of light leaking from the off-axis companion (hereafter referred to as the companion) reduces the signal-to-noise ratio of the candidate planet around the on-axis target (hereafter referred to as the target) to a level such that the planet candidate might become undetectable. Two fundamentally different effects contribute to creating this light leak: diffraction and optical aberrations. These effects are complicated by the fact that the beams coming from the two stars are mutually incoherent.

To quantify the light leakage, we consider the case of searching for a planet or a disk $100\lambda/D$ away from a star. This could be a binary separated by 10'' observed through a 1.5 m telescope at a wavelength of 770 nm (Figure 5) or a single star observed with a large telescope at shorter wavelength. Figure 6 shows the light diffracted by the companion in the particular scenario where the on-axis star is totally suppressed (representative of a starshade mission for instance, for which the starshade practically removes all the light from the target star). The simulated contrast is the intensity at a given position in the

¹ Courtesy of Thomas P. Greene 2015, private communication.

Figure 5. Schematic showing the limitations of the multi-star coronagraphy. The image shows overlapping speckle fields in a binary star system, where A is the coronagraphically suppressed on-axis target and B is the off-axis companion. The black squares show the sub-Nyquist control region (for that star) and the rectangle called DZ is the dark zone region of interest, where we want to detect a planet. In this simulation, the mean and median raw contrast of this dark zone are, respectively, 5.4×10^{-5} and 3.49×10^{-5} before wavefront control.

image normalized by the peak intensity of the target. The simulation was done in the context of no aberrations (left image) and with 25 nm rms of aberrations with a power law of f^{-2} (right image). The figures also show the dark zone region (DZ) outlined by a red rectangle. We use these regions as a reference for the initial contrast before corrections.

The amount of light originating from the companion depends on the aberrations' amplitude and their spatial frequency distribution in the system. In monochromatic light and in the presence of no aberrations, the amount of light diffracted in the DZ (Airy rings) is of the order of 4×10^{-8} . With 25 nm rms of aberration, the contrast worsens to 4×10^{-7} . Figure 7 shows the polychromatic case, for a 10% bandwidth around the central wavelength of 550 nm. The median contrast intensity is of the order of 2×10^{-8} without aberrations and 2×10^{-7} with 10 nm rms of aberrations.

Until now the standard approach to control diffracted light from the second star in a binary system has been to design a coronagraph to block both stars (Cady et al. 2011). A coronagraph is only useful if the diffraction dominates over aberrations or otherwise cannot be removed by the wavefront control (WC) system. As a general rule, for typical mirror aberrations, diffraction dominates close to the star but aberrations dominate far from the star. All coronagraphs suppress diffraction, but they do not help with aberrations. A WC system (or equivalent) is required to suppress optical aberrations, and in addition to suppressing aberrations, is often capable of suppressing diffraction by at least an order of magnitude. Therefore, a coronagraph is neither sufficient nor necessary to suppress the leak of the off-axis star, while a WC system is both necessary and (as we will show in this paper) often sufficient to suppress both the aberrations and diffraction.

4. THEORY OF MSSNWC

Our full MSSNWC consists of two separate methods, SNWC and MSWC, each of which is useful in its own right. SNWC is the focus of this paper and in this section we present the theoretical foundation for SNWC and explore its theoretical capabilities and limitations. We also discuss how to design DM parameters to tune SNWC for a particular application. Finally, we outline the approach behind MSWC and the combination MSSNWC (which will be explored in detail in another paper).

4.1. Super-Nyquist Wavefront Control

SNWC enables a DM to overcome its conventional Nyquist limit. We start by introducing a simplified formalism for general coronagraphic WC that holds to some approximation for all coronagraphs.

4.1.1. Simplified Model of Coronagraphic Wavefront Control

Consider some arbitrary coronagraph with a DM and a science focal plane. Let $E_{DM}(x)$ and $E_f(\xi)$ be the electric fields in those two planes, where x and ξ are 2-vectors representing the normalized 2D coordinates in units of pupil size D and sky angle λ/D , respectively. Because a coronagraph is a passive linear system, the relationship between these two fields is given by some linear operator C:

$$E_{\rm f} = \mathcal{C} \{ E_{\rm DM} \}. \tag{1}$$

A change in the DM setting creates a change $\Delta E_{\rm DM}$ in the DM field and a corresponding change $\Delta E_{\rm f} = C\{\Delta E_{\rm DM}\}$ in the focal plane field. The purpose of any high-contrast WC algorithm is to solve for $E_{\rm DM}$, which gives some desired $E_{\rm f}$ that destructively interferes with stellar speckles to achieve high contrast.

For the purposes of our simplified treatment, we will assume that any coronagraph can be approximated as follows:

$$\mathcal{C}\left\{E_{\mathrm{DM}}(x)\right\}(\xi) = \mathcal{F}\left\{A(x)E_{\mathrm{DM}}(x)\right\}T(\xi)$$
(2)

where \mathcal{F} is the Fourier transform (with our normalization of *x* and ξ , the Fraunhofer integral reduces to the Fourier transform

Figure 6. Light diffracted at $100\lambda/D$ in monochromatic light, without aberrations (left) and with 25 nm rms aberrations (right). The size of the region of interest (red rectangle) is $4\lambda/D \times 8\lambda/D$. The top views are larger fields of view showing the companion star. The bottom is a zoom on the region of interest (close to the suppressed on-axis star). The median contrast intensity is of the order of 4×10^{-8} without aberrations and 4×10^{-7} with 25 nm rms of aberrations, before wavefront control. This simulation assumes the on-axis target has been totally suppressed and only the light from the companion remains.

to within a constant phase factor), A(x) represents the coronagraph aperture (or more generally, the cumulative effect of all apertures and pupil-plane masks projected or propagated onto the DM), and $T(\xi)$ represents the coronagraphic throughput as a function of sky angle, normalized to a maximum value of 1. Typically $T(\xi) = 1$ almost everywhere in the focal plane except the small blind spot with radius of a few λ/D corresponding to where the star is suppressed by the coronagraph. In what follows, we make the following simplifying assumptions: (a) we only consider the regions outside the coronagraphic blind spot, which allows us to set $T(\xi) = 1$; (b) A(x) = 1 across the aperture and 0 elsewhere, which is fairly accurate for high-performance coronagraphs and still holds to within a factor of order unity for most

other coronagraphs. With these assumptions, Equation (2) simplifies to

$$\Delta E_{\rm f} = \mathcal{F} \{ \Delta E_{\rm DM} \}. \tag{3}$$

We also normalize $E_{\rm DM}$ to unity amplitude, which with our other assumptions and normalizations implies that $\Delta E_{\rm f}$ will be in units of contrast (to within a factor of order unity) for most coronagraphs. In what follows we will treat it as such.

4.1.2. Basic Principle Behind SNWC

The basic principle behind SNWC is that modulations of E_{DM} can cause modulations of E_{f} even beyond the sub-Nyquist region because some of the light is often diffracted in super-Nyquist regions. Thus, control of those regions is in principle

Figure 7. Light diffracted in the vicinity of $100\lambda/D$ away from a star in polychromatic light (10% bandwidth), with 10 nm rms aberrations. The size of the region of interest (red square) is $4\lambda/D \times 4\lambda/D$. The median contrast intensity before wavefront control is of the order of 10^{-7} , too bright for most planets.

enabled essentially in the same way as in the sub-Nyquist region: solving (1) for $E_{\rm DM}$ given a desired $E_{\rm f}$. The main difference is that the modulation of $E_{\rm f}$ by $E_{\rm DM}$ will generally be weaker in super-Nyquist regions than in sub-Nyquist because the amount of light diffracted into those regions by the DM is generally small. However, the contrast of the speckles in super-Nyquist regions is often similarly small. We will quantify and show how to mitigate this effect in what follows.

Light can be diffracted into super-Nyquist regions due to either: (a) the periodic actuator nature of the DM for some specific influence functions or (b) the print-through pattern on the DM surface or an external grating (which may or may not share the actuator periodicity). We treat those separately in the two subsections below.

4.2. SNWC with Diffraction Caused by the DM Influence Function

The DM field is given by

$$E_{\rm DM}(x) = A(x)e^{i\phi_{\rm DM}(x)} = A(x) \Big[1 + i\phi_{\rm DM}(x) + o(\phi_{\rm DM}(x)^2) \Big]$$
(4)

where $\phi_{\text{DM}}(x)$ is the phase imparted to the electric field by the DM in radians. The first term of the above equation corresponds to the on-axis point-spread function (PSF), while the rest is the contribution is due to the DM, which we will call ΔE_{DM} . For small DM modulations, we can assume that only the leading term is significant $(i\phi)$ and use the influence function model:

$$\Delta E_{\rm DM}(x) = i\phi_{\rm DM}(x)$$
$$= i\sum_{n=1}^{N} a_n f(x - nd)$$
(5)

where f is the DM influence function, d is the spacing of actuators on the DM, and a_n (for n = 1...N) are the DM

actuator coefficients (note that we are using 1D notation for simplicity but the generalization to 2D is trivial if we treat *x*, *n*, and *d* as 2-vectors and *N* as representing the total number of DM actuators in 2D). We also dropped A(x) for simplicity and will simply assume the equivalent condition that $\phi_{DM}(x) = 0$ outside the aperture. We will adopt the convention of *f* being normalized to unity maximum value, which implies that a_n are in units of radians. We can re-express this equation as a convolution:

$$\Delta E_{\rm DM}(x) = i\alpha * f$$

where $\alpha = \sum_{n=1}^{N} a_n \delta(x - nd)$. This convolution can be seen on the left column of Figure 8. We now proceed to analyze the behavior in the focal plane by computing the electric field change created by the DM:

$$\Delta E_{\rm f} = \mathcal{F} \{ \Delta E_{\rm DM} \}$$

= $i \mathcal{F} \{ \alpha \} \mathcal{F} \{ f \}.$ (6)

Computing the intensities (which with our normalized units will be in units of contrast) gives

$$|\Delta E_{\rm f}|^2 = |\mathcal{F}\{\alpha\}|^2 |\mathcal{F}\{f\}|^2. \tag{7}$$

 $\mathcal{F}\{\alpha\}$ is a periodic function (see Figure 8, top row), because the periodic sampling of α by delta functions makes a_n the Fourier series of $\mathcal{F}\alpha$. Its different periods correspond to the sub-Nyquist region periodically copied, or aliased, into super-Nyquist regions (Figure 8, top right). In other words, if influence functions were delta functions (a non-physical hypothetical scenario), controlling or modulating speckles in the central sub-Nyquist region would be perfectly repeated in all super-Nyquist regions, enabling independent suppression of starlight in any super-Nyquist region in exactly the same fashion as in the sub-Nyquist region. In the case of realistic influence functions, we have a similar situation, except the super-Nyquist regions will get attenuated (as we show below), requiring larger DM strokes to suppress speckles of a given contrast in a super-Nyquist region as compared to the sub-Nyquist region. This imposes a coupling between the characteristics of the influence function, DM stroke, and maximum correctable contrast of errors that is not simple to characterize in the general case, but becomes very simple if we characterize it in a statistical sense. Specifically, consider the DM actuator coefficients a_n as independent random variables with standard deviation $\sqrt{\left\langle a_n^2 \right\rangle}$ radians (where without loss of generality we assume a 0 mean). This standard deviation is essentially a measure of DM stroke. For example, in a DM of 32×32 actuators, the peak stroke will typically be $3 \times$ this amount.

This "random" DM modulation will lead to a random modulation of the focal plane field $E_{\rm f}$. The average intensity of this modulation is given by

$$\left\langle \left| \Delta E_{f} \right|^{2} \right\rangle = \left\langle \left| \mathcal{F} \{ \alpha \} \right|^{2} \right\rangle \left| \mathcal{F} \{ f \} \right|^{2}$$
$$= \sum_{n=1}^{N} \left\langle a_{n}^{2} \right\rangle \left| \mathcal{F} \{ \delta (x - nd) \} \right|^{2} \left| \mathcal{F} \{ f \} \right|^{2}$$
$$= N \left\langle a_{n}^{2} \right\rangle \left| \mathcal{F} \{ f \} \right|^{2}. \tag{8}$$

Figure 8. Different influence functions cause different amounts of light to leak into super-Nyquist regions, enabling weaker or stronger super-Nyquist wavefront control. The left column shows the pupil plane and the right column the image plane (with color-coded examples). The top row shows the actuator coefficient α and its Fourier transform (Fourier series of a_n). The second row shows realistic influence functions and the bottom row shows the convolution or multiplication of the top two rows. The middle right plot is a measure of starting speckle contrast that can be corrected by SNWC.

This average intensity is essentially a measure of speckle contrast (as a function of position in the image plane) correctable by a DM with stroke $\sqrt{\langle a_n^2 \rangle}$. As expected, higher strokes lead to higher energy or contrast in the DM correction field and therefore the ability to correct for higher levels of error. Because the vast majority of the DMs are capable of strokes of several radians ($\sqrt{\langle a_n^2 \rangle} > 1$), and because our linearity assumption breaks down for $\sqrt{\langle a_n^2 \rangle} > 1$, we can assume that for the vast majority of cases, $\sqrt{\langle a_n^2 \rangle} = 1$, simplifying the above to

$$\left\langle \left| \Delta E_{\rm f} \right|^2 \right\rangle = N \left| \mathcal{F} \{f\} \right|^2$$
 (9)

which is the key result of this section and states that the Fourier transform of the influence function times the number of DM actuators N directly gives a measure of the contrast correctable by the DM, in sub- as well as super-Nyquist regions.

4.2.1. Application and Examples of Influence Function SNWC

Figure 8 shows practical applications and implications of this equation. The left column shows the pupil plane and the right column the image plane. The top row shows the actuator coefficient α and its Fourier Transform (Fourier series of a_n). This would be the situation if the influence function was a delta

function (unphysical). The second row shows realistic influence functions and the bottom row shows the convolution or multiplication of the top two rows. The middle right plot represents a measure of maximum speckle contrast that can be suppressed in super-Nyquist regions according to (9). We now consider four examples of influence functions.

Typical DM influence functions in continuous-sheet DMs (Figure 8, middle left, blue) are roughly the size of actuator spacing. The corresponding $\langle |\Delta E_f|^2 \rangle$ (Figure 8, middle right, blue) tells us that there is only significant correction in the sub-Nyquist region while super-Nyquist regions are attenuated too much to be controllable at any reasonable contrast levels. Roughly speaking, because such an influence function has an effective area comparable to a DM actuator (1/N) in our normalized units), then according to (9) the area of $\langle |\Delta E_f|^2 \rangle$ is N (in our normalized units of $(\lambda/D)^2$) and thus conservation of energy implies its mean value (a measure of maximum correctable speckle contrast) is 1/N. This simply states the well-known fact that conventional DMs are capable of suppressing starlight leak in the sub-Nyquist region of area N $(\lambda/D)^2$, and that by energy conservation, the mean contrast of that leak cannot start at higher than 1/N, averaged across the sub-Nyquist region. For example, in a 32×32 DM (N = 1024), the sub-Nyquist region is $1024(\lambda/D)^2$ in area $(32\lambda/D \times 32\lambda/D)$ and the controllable level of starlight leak is 10^{-3} contrast averaged across the sub-Nyquist region. For such

a DM, super-Nyquist control requires a grating or print-through pattern on the DM (covered in the next subsection).

Consider now a continuous-sheet DM with an unconventional influence function that is much narrower than the actuator spacings (green). Specifically, suppose it is a factor of *M* smaller than the blue influence function (in area for 2D). Such a DM would be unusual in the sense that it would not maintain a flat surface even if all actuators had the same bias applied, but it is possible to make (or to emulate by conventional DMs as shown at the end of this paragraph). Its Fourier transform intensity $|\mathcal{F}\{f\}|^2$ (Figure 8 middle right, green) shows its superior ability to correct in super-Nyquist regions as compared to a conventional DM with the same number of actuators. It will be a factor of M wider in area and a factor of *M* dimmer than the case of a conventional DM (blue), so as compared to a conventional DM, it gains the ability to control super-Nyquist regions at the expense of lowering the starting contrast of errors it can control in the sub-Nyquist region. Roughly speaking, such a DM is capable of correcting any one of M super-Nyquist regions, as well as the sub-Nyquist region, as long as the contrast of the errors is not higher than 1/MN, averaged across any one region. Note that we can create such an "unconventional" DM using a conventional DM. For example a conventional 64×64 DM with only every eighth row and column connected is effectively an 8×8 DM with undersized influence functions. In this case, $N = 8 \times 8$ and $M = 8 \times 8$, the DM can still control speckles anywhere out to $32\lambda/D$, but only one $8\lambda/D \times 8\lambda/D$ -sized region at a time. If the number of electrical lines is a cost or risk driver, and a mission emphasis is on planet characterization rather than search (i.e., planet location is known), such a DM may actually be preferred to the fully connected 64×64 DM.

Now consider the case of a conventional influence function (blue), but with a dip in the middle (blue dotted curve in Figure 8). This influence function was created simply by taking the difference between the blue and green influence functions. As a result, the corresponding $\langle |\Delta E_{\rm f}|^2 \rangle$ has essentially the same controllability of the sub-Nyquist region as the solid blue curve, but also can control super-Nyquist regions as well as the green case, and so combines the advantages of both cases. The relationship between the width of the narrow dip in the influence function, the contrast, and the number of super-Nyquist regions that can be controlled is the same as for the green case. This DM essentially has the middle of each actuator immobilized and is therefore somewhat unusual (but likely still manufacturable). However, as we will see in the next section, a print-through pattern typical for many already existing DMs achieves a very similar effect.

Finally, a segmented DM (red color) has both a full control of the sub-Nyquist region (to full contrast levels similar to the conventional continuous-sheet DM case in blue) and can correct super-Nyquist regions, as long as the error does not exceed the contrast shown by the red curve when spatially averaged across any super-Nyquist region. This ability comes from the side lobes of the red curve. Correction will be best at odd multiples of half-Nyquist frequency (peaks of side lobes) and there will be a blind spot at multiples of Nyquist frequency (zero-crossings of the red curve). These side lobes will be strongest along directions normal to the actuator edges (along "diffraction spikes" of the PSF).

4.3. SNWC with DM Print-through, Grating, or Beamsplitter

DM diffraction into super-Nyquist regions can be caused not only by a particular shape of the influence function, but also directly by a mild grating or beamsplitter. The case of a grating with periodicity matching the DM actuator spacing represents a print-through pattern that is almost ubiquitous on all DMs (and is usually considered an undesirable artifact).

In the previous subsection, the action of the diffractioncausing agent (influence function) was a convolution in the pupil plane and a multiplication in the image plane. In this subsection, the action of the diffraction-causing agent (grating) is the opposite: a multiplication in the pupil plane and a convolution in the image plane.

Assume the same scenario as in the previous subsection, except with a mild grating represented by a periodic function g(x) multiplying the DM field (10). Define the new DM field as

$$E_{\text{DM},g}(x) = E_{\text{DM}}(x)g(x)$$

= $A(x)(1 + \Delta E_{\text{DM}}(x))g(x)$
= $A(x)g(x) + \Delta E_{\text{DM}}(x)g(x)$ (10)

where $E_{\rm DM}$ is the field from the previous subsection (i.e., without the presence of the grating). In the focal plane, the first term will lead to the on-axis star PSF, together with fixed (DMindependent) attenuated copies of the star PSF diffracted by the grating. From the point of view of conventional sub-Nyquist WC, each of these diffracted PSF copies can be treated as the on-axis star around which conventional sub-Nyquist WC can be applied, thus enabling SNWC with respect to the original star. However, each diffracted PSF copy will create a small "blind spot" that the DM will not be able to remove, in exactly the same way as any star creates a blind spot in the center of the image that no WC can remove. (If the periodicity of the grating matches DM actuators, as would be the case for the DM printthrough pattern, then there will be a small blind spot in the exact middle of every super-Nyquist region.) For the purposes of this subsection, we will treat the Airy rings of these diffracted PSF copies as part of the star leak error to be suppressed along with actual aberrations. Because the Airy rings are independent of the DM, we do not book-keep them as part of the DM correction field and thus ignore the first term, focusing only on terms that are DM-dependent. The perturbation of the DM electric field by the DM is then

$$\Delta E_{\text{DM},g}(x) = \Delta E_{\text{DM}}(x)g(x). \tag{11}$$

Fourier transforming to the image plane leads to the perturbation by the DM of the image plane electric field:

$$\Delta E_{\mathrm{f},g}(\xi) = \Delta E_{\mathrm{f}}(\xi) G(\xi) \tag{12}$$

where ξ is the image plane position, ΔE_f is the image field perturbation from the previous subsection (i.e., without the grating), and $G = \mathcal{F}\{g\}$. Because *g* is a periodic function, its Fourier transform has the form $G = \sum g_n \delta(\xi - n\Delta\xi)$, where g_n are the Fourier series coefficients of g(x) and $\Delta\xi$ is the spacing between the diffraction orders in the focal plane (equal to the spacing between super-Nyquist regions if the grating periodicity is the same as the DM actuator periodicity).

Following the same statistical characterization method as in the previous subsection, we treat the perturbations of the image

Figure 9. SNWC enabled by a beamsplitter and a print-through pattern on an 8×8 DM. The left column shows the pupil plane and the right column the image plane. The top row shows the DM shape and its Fourier transform (Fourier series of a_n). The second row shows the print-through pattern (green) and a mild beamsplitter (blue). The bottom row shows the multiplication or convolution of the top two rows. The bottom right plot is a measure of contrast of speckles that can be corrected at different separations.

plane electric field by the DM as a random variable and compute its mean energy, which after some algebra becomes

$$\left\langle \left| \Delta E_{\mathrm{f},g} \right|^2 \right\rangle = N \left| \mathcal{F}\{f\} \right|^2 * |G|^2.$$
(13)

This is the main result of this subsection and as before represents a measure of correctable speckle contrast. This is almost identical to (9) except for a convolution with the grating term. This convolution implies that: (a) super-Nyquist control is enabled in a region around diffraction orders of the grating gand (b) as compared to the sub-Nyquist region, the maximum contrast of speckles controllable in the super-Nyquist region is lower by a factor equal to the contrast of the diffraction order. In effect, the DM redistributes a fraction of the energy contained within the diffracted PSF copy across the nearby region of interest, so the total energy of speckles to be corrected must be less than the total energy of the PSF copy. This is the key design parameter for the grating.

4.3.1. Examples of Grating and Beamsplitter-based SNWC

Two examples are shown in Figure 9. The blue case corresponds to the case where G only has one off-axis term, which is simply a mild beamsplitter bleeding off 1% of the total beam into $20\lambda/D$. This is the most efficient way to control a specific fixed super-Nyquist region. The green case in Figure 9 corresponds to a periodic amplitude grating such as a print-through on the DM. It acts in much the same way as a beamsplitter, except it creates many super-Nyquist control regions instead of just one.

Figure 10. Use of different DM modes to independently modulate light for two different stars. The first scheme on the left shows the different controllable regions for star A (black dot) and the middle the controllable regions for star B (gray dot). The figure on the right represents the superposition of the two and shows that there are two regions named I and II between star A and star B for which we can control A and B at the same time with the same DM. For region I we would use the modes from 0 to $8\lambda/D$ for B and $8-16\lambda/D$ for A and vice versa.

4.3.2. Summary of SNWC Grating or Print-through Design Principles

There are a few key principles relating the grating design characteristics, super-Nyquist control starting contrast, and region location: (a) when a grating creates a diffraction order (PSF copy) of contrast *C*, super-Nyquist control is enabled around that diffraction order and up to $(N/2)\lambda/D$ away, in exactly the same fashion as sub-Nyquist control around the original on-axis star; (b) the total energy of the error to be corrected cannot exceed the energy in that diffraction order. For example, if the diffraction order is 10^{-3} contrast and we have a 32×32 DM correcting in a $32\lambda/D \times 16\lambda/D$ half-region around that diffraction order, then the average contrast of the error corrected cannot exceed $10^{-3}/(32 \times 16) = 2 \times 10^{-6}$. (On the other hand, if the region of interest is only $3\lambda/D \times 3\lambda/D$, then speckles of up to 10^{-4} contrast can in principle be corrected.)

4.3.3. SNWC Implementation with an Astrometry Grating

Using dots on the pupil has been proposed previously to calibrate dynamic distortions on wide-field optical systems, enabling high-precision astrometric measurements (Guyon et al. 2012). For this technique the dots can be arranged in the pupil using a hexagonal geometry that allows higher azimuthal sampling. The diffractive pupil spacing can be adjusted to create PSF replicas to run the SNWC, and also to obtain high-precision astrometry on wide-field images. A description of the optimal hexagonal geometry for combining these two techniques has been published by Bendek et al. (2013).

4.4. Multi-star Wavefront Control

For the sake of completeness, we will outline here how to simultaneously suppress the speckle fields of both stars (MSWC), which will be covered in detail in a future paper. Consider the case of two stars, A and B, with (for now) a sub-Nyquist separation. The main challenge is that light from the two stars is mutually incoherent, and therefore light from each star can only be used to destructively interfere with its own speckle field but not the field from the other star. In order to suppress both stars, it is necessary to be able to independently modulate the speckle field of each star without affecting the speckle field of the other (in some region of interest). Figure 10 shows the special case of two stars separated by $16\lambda/D$. The left panel shows the (sub-Nyquist) control region of a 32×32 DM with respect to star A, and separates the region into four vertical sections, each controlled by a different and independent set of modes on the DM (the outer sections are controlled by modes on the DM corresponding to spatial frequencies of 8-16 cycles per aperture (cpa), and the inner regions are controlled by 0–8 cpa). The middle panel of Figure 10 shows the same thing, but with respect to star B. Finally, the right panel superimposes these control regions of the two stars. The two regions between the two stars are labeled as I and II. In region I, the 0-8 cpa modes modulate the speckle field of star B but not star A (to first order), and the 8-16 cpa modes modulate the speckle field of star A but not star B. In other words, in region I, the speckle field of star A can be suppressed by using the 8-16 cpa modes without affecting star B and the speckle field of star B can be independently suppressed by using the 0-8 cpa modes without affecting star A. In effect, we have reduced the MSWC problem to two separate conventional WC problems (each using different sets of DM modes), each of which is a solved problem. Solving these two conventional WC problems simultaneously results in simultaneous suppression of the speckle fields of both stars in region I. The same can be done in region II (but not simultaneously with region I). The final result is that a double-star dark zone can be created at the expense of reducing the size of the control region by a factor of two (one cannot overcome the limited number of degrees of freedom available on the DM), and there are two such regions. These two smaller regions can be suppressed separately and then stitched together to create the full FOV between the two stars.

This idea can be generalized to the case of arbitrary (sub-Nyquist) star separation. The result is that one can always partition the intersection of the sub-Nyquist regions of the two stars into two sections, each with half the area of the original control region, where speckle fields of both stars can be simultaneously suppressed. (These regions will have shapes different from Figure 10 and may consist of disconnected

Figure 11. Diagram of Super-Nyquist Multi-star Wavefront Control. The DM grating diffracts an attenuated replica of star B into a sub-Nyquist region of star A, just as in the case of single-star Super-Nyquist Wavefront Control. This reduces the problem to conventional (sub-Nyquist) Multi-star Wavefront Control, with the diffracted replica being treated as a sub-Nyquist star. In this image a coronagraph blocks the light originating from star A. This allows us to remove light from both stars in some region of interest (one example shown as "DZ") and search for planets around A.

parts.) A generalization to N stars implies N independent correction regions, each with 1/N of the area of the original control region. The shapes of these regions are related to the Voronoi partitions of a periodically extended star field, folded into the sub-Nyquist region. It should be noted that in practice it is impossible to completely decouple DM modes. Any DM mode will always affect all stars everywhere to some level. However, on the regions we constructed, one of the star's speckle fields is affected much more than the other, so the above algorithm works in a closed loop.

4.5. Super-Nyquist MSWC

In this section, we outline how to combine SNWC and MSWC and treat the case of two (or more) stars having a super-Nyquist separation. Figure 11 shows a diagram of an on-axis star A (suppressed by a coronagraph if present) and the companion off-axis star B. Both stars have sub-Nyquist control regions where conventional WC can suppress the speckle field of that star but not the other. Suppose that a mild DM grating diffracts the (attenuated) replica of star B inside the sub-Nyquist region of star A, just as in the case of SNWC. As discussed in Section 4.1, SNWC basically enables us to treat this diffracted replica as a sub-Nyquist star, effectively resulting in two stars with a sub-Nyquist separation. This reduces the problem to that of MSWC, which we outlined how to solve in Section 4.4. The main potential difficulty arises for the case of broadband light, where the diffracted replica of the off-axis star looks spectrally elongated. This will reduce the size of the correction region, but if SNWC works in broadband light then MSSNWC also works in principle with a reduced dark zone size.

5. SIMULATION OF SNWC

To demonstrate the SNWC, we chose to simulate the observation of a binary of equal brightness separated by $100\lambda/D$. This is similar to the separation of α Cen with a 1.5 m

telescope (such as Exo-S^2) in monochromatic as well as polychromatic light (10% bandwidth with a central wavelength of 770 nm). The expected separation of α Cen components A and B in the year 2025 is about 10 arcsec, which corresponds to about $100\lambda/d$ away at a wavelength of 770 nm. This is also applicable to imaging large disks. Since this paper demonstrates SNWC and not MSWC, we are assuming that the light from the target on-axis star has been completely removed (e.g., by a starshade) and only the off-axis companion remains. The goal is to demonstrate in simulation that it is indeed possible to create a dark zone beyond the Nyquist frequency of a DM in this configuration.

5.1. Simulation Description

In this paper, we have focused on demonstrating wavefront correction and not estimation, i.e., assuming we have perfect knowledge of the phase and amplitude of each star's speckle field in the region of interest.

The problem of correction is conceptually separate from the problem of estimation and is in some sense more fundamental, because the ability to correct implies estimation can also be implemented, but the converse does not hold.

Conventional WC estimation (e.g., in electric field conjugation (EFC) or stroke minimization, Give'on et al. 2007; Pueyo et al. 2010) uses the DM to create known probe fields to modulate the region of interest and then analyzes this modulation to reconstruct the phase and amplitude of the preexisting speckle field. Similarly to the correction problem, the estimation problem in all cases (SNWC, MSWC, MSSNWC) can also be reduced to conventional wavefront estimation as long as the DM is able to create known probe fields in the region of interest of sufficient energy, which will be the case as long as we demonstrate that correction is possible.

In Section 4.1, we described that SNWC requires the DM to diffract light beyond the Nyquist region with, for example, the

² http://exep.jpl.nasa.gov/stdt/exos/

Figure 12. Proof-of-concept simulations of Super-Nyquist wavefront control in the monochromatic case. The figures show a local region in the vicinity of $100\lambda/D$ away from the star (which is far to the left outside the bounds of the shown field of view). A faint diffracted copy of the star is in the center of the image, allowing SNWC to create dark zones in a region of interest (red rectangle, chosen to be a conservative $4\lambda/D \times 8\lambda/D$ in size for this example). Left: 0 nm rms aberrations (speckle field in the region of interest is dominated by Airy rings from the original PSF and fainter contributions by the PSF replica Airy rings). Right: 25 nm rms aberrations. The median contrast obtained is 2.5×10^{-10} without any aberrations and 1.7×10^{-9} with aberrations.

periodic print-through pattern left over from manufacturing. To create such a pattern, we used a grid of dots in the image plane, created from a mask of uniform intensity equal to 1 with values of 0 at locations of dots. The period of the grid is set such that there will be a diffracted (and attenuated) PSF copy within the sub-Nyquist separation of the target star. This will always be the case for a print-through pattern, but we arbitrarily chose a grating with a finer periodicity that still satisfies this condition for our particular target. Another parameter that can be adjusted in order to control the performance is the width of the dark zone region. Indeed the bigger the region, the more energy is required of the correction field and thus more DM stroke is needed according to (8) to achieve deeper contrast. We used a $4\lambda/d \times 8\lambda/d$ region for the demonstration, which we found to be a good compromise between the performance and the discovery region. The simulations were done both in monochromatic light (770 nm) and in polychromatic light with a 10% band. Finally, we also studied the case of a non-aberrated and an aberrated wavefront. For simplicity, we assumed the aberrations in our system are phase errors in the pupil plane, following a power law with a coefficient equal to -2. A more realistic case would need to include phase errors in planes not conjugated to the DM as well as amplitude errors on potential optics. The performance will then depend strongly on the geometry of the optical system, especially in broadband light. One potential solution would be to be very careful about where each optics is located in the system or to introduce several mirrors in different planes not conjugated to the DM. However, these considerations and challenges are not specific to SNWC. Broadband WC challenges at, say, $100\lambda/D$ are largely independent of whether we use SNWC or sub-Nyquist WC (with a higher number of DM actuators). Therefore a more general study of broadband control represents a topic separate from that of SNWC and is beyond the focus of this paper.

5.2. Results

Monochromatic Light: In monochromatic light, the introduced diffraction grid creates dots at $100\lambda/d$ with an intensity of 1.34×10^{-3} relative to the central star, which defines the maximum energy available for speckle suppression. Figure 12 shows the results of a (nonlinear) correction solution without aberrations (left) and with 25 nm rms of aberrations. The median contrast obtained without any aberrations is 2.5×10^{-10} and with aberration we reach a contrast of 1.7×10^{-9} . This corresponds to a factor 100 improvement from the no-grid simulation for both the aberrated and non-aberrated cases, demonstrating the ability of SNWC to create dark zones well outside the nominal Nyquist limit of the DM. These represent proof-of-concept results and we expect that deeper contrasts and larger zones are possible with further algorithm improvements and tuning.

Polychromatic Light: We now consider a more realistic scenario and study the effect of polychromatic light. We use the methods in Give'on et al. (2007) for polychromatic control within a 10% bandwidth, generalized to SNWC. In order to get an accurate image, we chose to sample the 10% bandwidth at three wavelengths. This allows a good compromise between computational speed and spectral resolution. For better contrasts or larger bandwidth, one can increase the sampling of the bandwidth. However, once we obtained the DM settings to be applied in order to create the dark zone, we used a better wavelength resolution (20 wavelengths over the 10% bandwidth) in order to simulate the real image. Figure 13 shows the results without and with 10 nm rms aberrations. The median contrast obtained without any aberrations is 4.9×10^{-9} , and very similarly for the case with aberrations we reach 5.3×10^{-9} . This suggests that we may be limited by chromaticity but nonetheless prove the principle that broadband SNWC is possible to a reasonable degree (at least in our case of pupil-plane phase errors). Note that we have not yet explored

Figure 13. Proof-of-concept simulations of Super-Nyquist wavefront control in the polychromatic case: 10% bandwidth around 550 nm and $100\lambda/D$ away from the star. The dark zone region has been conservatively set at $4\lambda/D \times 4\lambda/D$ in size. The image on the left shows the case of no aberrations and the image on the right with 10 nm rms of aberration. The median contrast obtained in this case is 4.9×10^{-9} without aberrations and 5.3×10^{-9} with aberrations. These images are the results of using 20 wavelengths sampled over the 10% bandwidth after using EFC with the sample of three wavelengths.

 Table 1

 Summary of Performance Achieved with the Simulations Before and After Wavefront Control (WFC) and for Monochromatic and Polychromatic Cases

	Aberration	Contrast
No grid, monochromatic, before WFC	0 nm	3.5×10^{-8}
No grid, polychromatic (10%), before WFC	0 nm	3.6×10^{-8}
Grid, monochromatic, after WFC	0 nm	2.6×10^{-10}
Grid, polychromatic (10%), after WFC	0 nm	4.9×10^{-9}
No grid, monochromatic, before WFC	25 nm	5.4×10^{-7}
No grid, polychromatic (10%), before WFC	10 nm	$8.7 imes 10^{-7}$
Grid, monochromatic, after WFC	25 nm	4.8×10^{-9}
Grid, polychromatic (10%), after WFC	10 nm	5.3×10^{-9}

the full power of this technique with better tuning of the algorithm parameters and that these results are raw contrasts. In practice, post-processing techniques may allow gains of a factor 10 or even 100 on the contrast, which is encouraging for detection and characterization of Earth-like planets.

Table 1 shows the summary of the different simulation results in the case of a equal brightness system.

6. CONCLUSION

In this paper, we have presented a method that will enable high-contrast imaging in multi-star systems as well as beyond the conventional OWA limited by the DM. This new concept is a collection of two independent methods as well as their combination, namely: (a) SNWC; (b) MSWC; (c) MSSNWC. We showed with simulations that it is possible to create a dark zone past the Nyquist frequency of a DM using a diffractive grid in the pupil or by using a DM with certain types of influence functions. The diffractive grid can be the printthrough pattern common to many DMs or an additional external mask. Proof-of-principle simulations were performed for the case of α Cen, an interesting target since it is the closest potential Earth-like planet host. SNWC by itself can be applicable to a perfect starshade coronagraph for which we totally block the light coming from the parent star but still need to remove the light coming from the companion in the dark zone of interest. MSSNWC enables the same thing for an internal coronagraph. This work potentially enables direct imaging of planetary systems and disks around multi-star systems as well as extends high-contrast capability to regions arbitrarily far from the star. This can be done at little additional hardware cost or changes to existing mission concepts, such as AFTA, Exo-C, Exo-S, and EXCEDE, and will greatly multiply the science yield of these missions.

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REFERENCES

Batalha, N. M. 2014, PNAS, 111, 12647

- Bendek, E., Belikov, R., Pluzhnik, E., & Guyon, O. 2013, PASP, 125, 204
- Bendek, E., Thomas, S., & Belikov, R. 2014, in Search for Life Beyond the Solar System Exoplanets, Biosignatures Instruments, ed. D. Apai & P. Gabor (Tucson, AZ: Univ. Arizona Press), 415
- Beuzit, J. L., Feldt, M., Dohlen, K., et al. 2008, Proc. SPIE, 7014, 701418
- Burke, C. J., Bryson, S. T., Mullally, F., et al. 2014, ApJS, 210, 19
- Cady, E., McElwain, M., Kasdin, N. J., & Thalmann, C. 2011, PASP, 123, 333
- Dumusque, X., Pepe, F., Lovis, C., et al. 2012, Natur, 491, 207

Give'on, A., Belikov, R., Shaklan, S., & Kasdin, J. 2007, OExpr, 15, 12338

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Guyon, O., Bendek, E. A., Eisner, J. A., et al. 2012, ApJS, 200, 11

- Guyon, O., Martinache, F., Garrel, V., et al. 2010, Proc. SPIE, 7736, 773624 Guyon, O., Pluzhnik, E., Martinache, F., et al. 2010, PASP, 122, 71
- Hinkley, S., Oppenheimer, B. R., Brenner, D., et al. 2008, Proc. SPIE, 7015, 701519

Kern, B., Guyon, O., Kuhnert, A., et al. 2013, Proc. SPIE, 8864, 88640R Kuchner, M. 2012, ascl soft, 1202.002

- Macintosh, B., Graham, J. R., Ingraham, P., et al. 2014, PNAS, 111, 12661
- Pueyo, L., Wallace, K., Troy, M., et al. 2010, Proc. SPIE, 7736, 77362A Thomas, S. J., Bendek, E., & Belikov, R. 2014, Proc. SPIE, 9143, 35