The Astrophysical Journal, 806:269 (6pp), 2015 June 20 doi:10.1088/0004-637X/806/2/269

ICECUBE NEUTRINOS AND LORENTZ INVARIANCE VIOLATION

Giovanni Amelino-Camelia[1], D. Guetta[2,3], and Tsvi Piran[4]
[1] Dipartimento di Fisica, Sapienza Università di Roma and INFN, Sez. Roma1, P.le A. Moro 2, I-00185 Roma, Italy
[2] Osservatorio astronomico di Roma, v. Frascati 33, I-00040 Monte Porzio Catone, Italy
[3] Department of Physics and Optical Engineering, ORT Braude, P.O. Box 78, Carmiel, Israel
[4] The Racah Institute for Physics, The Hebrew University of Jerusalem, Jerusalem 91904, Israel
Received 2013 June 19; accepted 2015 April 15; published 2015 June 23

ABSTRACT

The IceCube neutrino telescope has found so far no evidence of gamma-ray burst (GRB) neutrinos. We here notice that these results assume the same travel times from source to telescope for neutrinos and photons, an assumption that is challenged by some much-studied pictures of spacetime quantization. We briefly review previous results suggesting that limits on quantum-spacetime effects obtained for photons might not be applicable to neutrinos, and we then observe that the outcome of GRB-neutrino searches could depend strongly on whether one allows for neutrinos to be affected by the minute effects of Lorentz invariance violation (LIV) predicted by some relevant quantum-spacetime models. We discuss some relevant issues using as an illustrative example three neutrinos that were detected by IceCube in good spatial coincidence with GRBs, but hours before the corresponding gamma rays. In general, this could happen if the earlier arrival reflects quantum-spacetime-induced LIV, but, as we stress, some consistency criteria must be enforced in order to properly test such a hypothesis. Our analysis sets the stage for future GRB-neutrino searches that could systematically test the possibility of quantum-spacetime-induced LIV.

Key words: astroparticle physics – gamma-ray burst: general – neutrinos

Prominent on the agenda of the current generation of neutrino telescopes is the search for neutrinos emitted in the same gigantic explosion responsible for gamma-ray bursts (GRBs). The prediction of a neutrino emission associated with GRBs is generic within the most widely accepted phenomenological interpretation of these explosions, given in terms of the so-called fireball model (Piran 2000). But different variants of the model predict a different rate of neutrino production at the GRB source. According to the fireball picture, the energy carried by the hadrons in a relativistic expanding wind is dissipated through internal shocks between different parts of plasma. These shocks reconvert a substantial part of the kinetic energy to internal energy, which is then radiated as synchrotron and inverse-Compton radiation of shock-accelerated electrons. When the fireball has swept enough material, it collides with its surrounding medium, giving rise to reverse and forward shocks, and the latter would then be responsible for so-called afterglow emission (Mészáros 1999). Within this picture GRBs should produce neutrinos with energy of ~100 TeV through the interaction of high-energy protons with radiation, at the same region where GRB photons are produced (Guetta et al. 2001). Neutrinos may be produced also in other stages of fireball evolution and in particular within the afterglow or while a relativistic jet is still propagating within the stellar envelope (Mészáros & Waxman 2001).

Recently, IceCube reported (Abbasi et al. 2012) no detection of any GRB-associated neutrino in a data set taken from 2008 April to 2010 May. This is in conflict with earlier studies (Waxman & Bahcall 1997; Rachen & Mészáros 1998; Guetta et al. 2004; Ahlers et al. 2011) that predicted about 10 GRB neutrinos during this period. Those earlier estimates were largely calibrated assuming that ultra-high-energy cosmic rays (UHECRs) are produced by GRBs. The IceCube results then appear to rule out GRBs as the main sources of UHECRs or that the efficiency of neutrino production is much lower than had been estimated (Baerwald et al. 2011; Hummer et al. 2012; Zhang & Kumar 2012).

Since this issue ties in some of the most interesting and hotly debated aspects of high-energy astrophysics, it is interesting to explore alternatives to the conclusion suggested by this analysis (Abbasi et al. 2012). Of interest for the study we are here reporting is the fact that these assessments of the outcome of IceCube's GRB-neutrino searches are based on the expectation that such neutrinos should be detected in a temporal coincidence with the associated γ-rays or the early afterglow. We came to wonder how much of a difference it would make if the same data were analyzed from the perspective of a Lorentz invariance violation (LIV) scenario for propagation of GRB neutrinos first proposed by Jacob & Piran (2007) and more recently highlighted in an overall assessment of quantum-spacetime phenomenology by Amelino-Camelia & Smolin (Amelino-Camelia & Smolin 2009). This scenario is inspired by research on LIV seeded in quantum properties of spacetime and suggests that GRB neutrinos with energies of a few - TeV and above could be detected systematically much in advance of or much after the accompanying electromagnetic signal. Other authors have also considered the possibility of using neutrinos from GRBs to test LIV (Biesiada & Piorkowska 2007).

The main objective of the study we are here reporting is to show that the possibility of quantum-spacetime-induced LIV could play a significant role in GRB-neutrino searches and to discuss some aspects of the methodology that needs to be adopted in future data analyses in order to properly test the hypothesis of GRB neutrinos affected by LIV.

As a way to give more tangibility to our observations, we shall use as an illustrative case study the one of three weak candidates as GRB neutrinos reported by IceCube. Intriguingly, these three neutrinos were all detected by IceCube in good spatial coincidence with GRBs, but hours before the corresponding gamma rays. In the first IceCube data set, the IC40 data set, the two most significant candidate GRB neutrinos were both sizably in advance of the trigger of the accompanying electromagnetic signal: these were (Whitehorne 2010;

Table 1
The GRB and Neutrino Properties of the Considered GRBs

GRB	t_{90}^{i} (s)	Redshift	E_ν (TeV)	$t_{\rm obs}^{i}$
GRB 090417B	>260	0.35	1.3	−2249 s
GRB 090219	0.5	$0.1 < z < 1$[a]	3.3	−3594 s
GRB 091230A	120	$0.3 < z < 5$[b]	109	−14 hr

Note. We define $t_{\rm obs}^{i}$ as the time of observation of the ith neutrino event relative to the trigger time of the corresponding GRB$_i$, and t_{90}^{i} is the duration of this burst.
[a] This is a short burst; hence, we estimate its mostly likely range of redshifts between 0. 1 and 1.
[b] This is a long GRB; hence, we estimate its most likely range of redshifts between 0.5 and 5.

available at https://docushare.icecube.wisc.edu/dsweb/. See Figures 6.1 and 6.2 and their captions.) a 1.3 TeV neutrino 1°.95 off GRB 090417B, with localization uncertainty of 1°.61 and detection time 2249 s before the trigger of GRB 090417B, and a 3.3 TeV neutrino 6°.11 off GRB 090219, with a localization uncertainty of 6°.12 and detection time 3594 s before the GRB 090219 trigger. Neither of these two candidate GRB neutrinos could carry much significance, since they may well both be just a (not unlikely; Whitehorne 2010) chance fluctuation of the background noise constituted by atmospheric neutrinos, but they are well suited for the purpose of illustrating some aspects of the methodology we are proposing.

For the other IceCube data set on which GRB-neutrino searches have been conducted extensively (Whitehorne 2010; Abbasi et al. 2012), the IC59 data set, two other events were highlighted by the IceCube collaboration, a 35 TeV neutrino within 30 s of GRB 091026A, 4°.5 off-source, with a localization uncertainty of 10°.5, and a 109 TeV neutrino, within 0°.2 of GRB 091230A, with a localization uncertainty of 0°.2 and detected some 14 hr before the GRB 091230A trigger. While both these events were labeled as very likely cosmic-ray events rather than GRB neutrinos (Abbasi et al. 2012), in a more detailed account (Whitehorne 2010; also see CERN Courier 2012) it is observed that the 35 TeV event was very clearly a cosmic ray since it triggered the IceTop surface array, whereas for the 109 TeV event there was only one IceTop-tank trigger in time coincidence. This single IceTop-tank trigger may suggest that it was part of a cosmic-ray air shower, but (F. Halzen 2013, private communication) it could also be a background in the tank's photomultiplier. Following these remarks, we exclude the 35 TeV event, but we include the 109 TeV event in our triplet of events used for illustrating our proposed methodology. In Table 1 we summarize the properties of the three events on which our analysis focuses.

These three best GRB-neutrino candidates on which we focus should only be viewed as very weak candidates, considering the large number of trials involved in their search and the expected backgrounds (Whitehorne 2010; Abbasi et al. 2012). In particular, even the single IceTop-tank trigger in time coincidence with the 109 TeV event could be of concern. However, as shall become clear in the following, the triplet of events we consider here with these loose criteria makes up for a particularly interesting illustration of the methodology we are proposing rather for direct estimates of LIV. For a start, it is interesting for our purposes to observe that within the methodology adopted so far by IceCube data analyses, even if we were dealing with somewhat stronger candidate GRB neutrinos, the fact that all three of these candidates were detected sizably in advance of the triggers of the GRBs they could be associated with would actually obstruct any attempt to view them as GRB neutrinos: no current GRB model suggests that neutrinos could be emitted thousands of seconds before a GRB. But a collection of GRB-neutrino candidates all sizably in advance of (or all with a sizable delay with respect to) corresponding GRB triggers is just what one would expect on the basis of the quantum-spacetime-inspired LIV scenario (Jacob & Piran 2007; Amelino-Camelia & Smolin 2009), which is here of interest.

This scenario for the discovery of GRB neutrinos (Jacob & Piran 2007; Amelino-Camelia & Smolin 2009) was based on results for models of spacetime quantization suggesting that (see, e.g., Amelino-Camelia et al. 1998; Gambini & Pullin 1999; Alfaro et al. 2000; Amelino-Camelia & Majid 2000; Myers & Pospelov 2003) it is possible for the quantum properties of spacetime to introduce small violations of the special relativistic properties of classical spacetime. A key consequence of this picture would be that the time needed for an ultrarelativistic particle[5] to travel from a given source to a given detector is $t = t_0 + t_{\rm LIV}$. Here t_0 is the time that would be predicted in classical spacetime, while $t_{\rm LIV}$ is the contribution to the travel time due to quantum properties of spacetime. Notice that $t_{\rm LIV}$ might be negative (which is the case we focus on here), and in this case the particle's travel time is shorter than expected. For energies much smaller than $M_{\rm LIV}$, the scale of onset of these quantum-spacetime effects, one expects that at lowest order $t_{\rm LIV}$ is given by (Jacob & Piran 2008)

$$t_{\rm LIV} = s_{\pm} \frac{E}{M_{\rm LIV}} D(z), \tag{1}$$

where

$$D(z) = \int_0^z d\zeta \frac{(1+\zeta)}{H_0\sqrt{\Omega_\Lambda + (1+\zeta)^3 \Omega_m}}.$$

Here the information cosmology gives us on spacetime curvature is coded in the denominator for the integrand in $D(z)$, with z being the redshift and Ω_Λ, H_0, and Ω_0 denoting, as usual, the cosmological constant, the Hubble parameter, and the matter fraction, respectively. The "sign parameter" $s_\pm$, with allowed values of 1 or −1, and the scale $M_{\rm LIV}$ would have to be determined experimentally. The aspects of special relativity here at stake are indeed those connected to Lorentz invariance (Amelino-Camelia et al. 1998; Gambini & Pullin 1999; Alfaro et al. 2000; Amelino-Camelia & Majid 2000; Myers & Pospelov 2003; Jacob et al. 2010; and there is interest in this class of effects from the intrinsic Lorentz invariance test theory perspective (Mattingly 2005), with or without spacetime quantization). We must stress, however, that most theorists favor naturalness arguments suggesting that $M_{\rm LIV}$ should take a value that is rather close to the "Planck scale" $M_P = \sqrt{\hbar c^5 / G_N} \simeq 1.22 \cdot 10^{16}$ TeV.

[5] Of course, the only regime of particle propagation that is relevant for this manuscript is the ultrarelativistic regime, since photons have no mass and for the neutrinos we are contemplating (energy of a few TeV and above) the mass is completely negligible.

The picture of quantum-spacetime effects summarized in Equation (1) does not apply to all quantum-spacetime models. One can envisage quantum-spacetime pictures that do not violate Lorentz symmetry at all, and even among the most studied quantum-spacetime pictures that do violate Lorentz symmetry one also finds variants producing (see, e.g., Amelino-Camelia et al. 1998; Mattingly 2005; Amelino-Camelia & Smolin 2009) features analogous to Equation (1) but with the ratio $E/M_{\rm LIV}$ replaced by its square, $(E/M_{\rm LIV})^2$, in which case the effects would be much weaker and practically undetectable at present. We focus here on the most studied Lorentz-invariance-violating scenario, the one centered on Equation (1).

It is important that some quantum-spacetime models allow for laws roughly of the type shown in Equation (1) to apply differently to photons and neutrinos. An attractive hypothesis (Myers & Pospelov 2003; Mattingly 2005) is that the quantum-spacetime effects should still be accommodated within the formalism of effective quantum field theory, where effects of the type shown in Equation (1) would take the shape of dimension-5 operators added to the Lagrangian density and contributing to the particle's propagator. Within that effective field theory setup one can exactly formulate Equation (1) for neutrinos, but not for photons (though a variant of Equation (1) with an added polarization dependence is allowed for photons). And even among quantum-spacetime models that do not fully comply with the demands of a description within the effective field theory framework, neutrinos deserve dedicated interest. In particular, for the such quantum spacetime that is most studied, the so-called Moyal non-commutative spacetime, it is found (Szabo 2003), remarkably, that the implications of spacetime quantization for particle propagation end up depending on the standard-model charges carried by the particle and its associated coupling to other particles. Accurate studies of Equation (1) for neutrinos would be our first opportunity to tangibly constrain such possibilities for a particle carrying weak-interaction charge.

Testing the applicability of Equation (1) to GRB neutrinos is in principle simple. GRBs last anywhere between a few and $\sim$1000 s, and if $t_{\rm LIV} = 0$, the associated neutrinos are, of course, expected to be detected within approximately the same time window. If instead $t_{\rm LIV}$ is described by Equation (1), for sufficiently high energies and sufficiently high redshifts $t_{\rm LIV}$ would be large, and the neutrinos would be detected either significantly before or significantly after the time interval when the low-energy photons of the same GRB are observed. The same reasoning was applied to coincidences in arrival times of GeV photons and sub-MeV photons in some GRBs enabling the *Fermi* satellite to set limits for $M_{\rm LIV}$(photons) $\gtrsim M_P$ for photon propagation (Abdo et al. 2009; Vasileiou 2013).

Exploiting this strategy for GRB neutrinos imposes that we contend with the background of other neutrinos (in particular atmospheric neutrinos) that the telescopes detect. Assuming $t_{\rm LIV} = 0$, one can select candidate GRB neutrinos not only by requiring that they come from (roughly) the same direction of the GRB photons, but also requiring that they would be detected within a narrow time window around the time of arrival of the signal in photons. If, however, $t_{\rm LIV}$ is described by Equation (1), also considering that $M_{\rm LIV}$ has, as mentioned, a rather sizable "theoretical uncertainty" and E has a significant observational error, the temporal window should be made considerably larger and contending with background neutrinos may be a severe challenge.[6] Jacob & Piran (2007) have addressed this issue for GRB neutrinos of energies higher than those of interest here. In that case, they argue, the background noise is sufficiently low[7] that a detection of a neutrino from the direction of a GRB can be significant even when there is a sizable mismatch of detection times.

However, even at lower energies one can efficiently test Equation (1) upon adopting a change of approach such that the selection of GRB-neutrino candidates is based on rather tight directional criteria (the direction of the neutrinos should be determined to be rather accurately consistent within the point-spread function of the detector with the direction of the GRB potentially associated with it), while the time-window criteria for the selection of GRB neutrinos should be relaxed but in a systematic way allowing for Equation (1). If this strategy is adopted, we would gain the ability to test both the $t_{\rm LIV} = 0$ hypothesis and the hypothesis that $t_{\rm LIV}$ be described by Equation (1). If indeed GRBs are sources of TeV neutrinos and $t_{\rm LIV} = 0$, then at some point we will have quite a few such directionally selected GRB-neutrino candidates, and some of them will be established to be definitely GRB neutrinos because of a level of time coincidence with the associated GRBs that would allow us to exclude confidently the possibility of having caught a background neutrino. On the other hand, if $t_{\rm LIV}$ is described by Equation (1), one should expect that we might never have a specific neutrino that can be conclusively associated with a GRB and yet we could deduce that some of the neutrinos (without knowing which ones) did come from GRBs. The effect we are here contemplating could only be discovered statistically: considering the role of background neutrinos in these studies, according to Equation (1) one expects that the distribution of times of detection of directionally selected neutrinos would not be just random (as in the case of a sample of pure background neutrinos), but rather would manifest a higher probability of detecting neutrinos in a certain energy-dependent and redshift-dependent time window, governed by Equation (1), systematically advanced or delayed with respect to the gamma-ray trigger of the GRB.

One aspect of methodology in which we are here particularly interested concerns the organization of data on candidate GRB neutrinos, when analyzed from the perspective of Equation (1), particularly in light of the fact that the effect is redshift dependent. For most candidate GRB neutrinos there will not be a redshift measurement for the relevant GRB, and this will translate into an additional source of uncertainty that must be handled appropriately. We can use the triplet of IceCube neutrinos for illustrating how these challenges could be addressed. For this purpose, let us first notice that our

[6] A somewhat similar description of the challenges for testing Equation (1) at IceCube was given by Gonzalez-Garcia & Halzen (2007) a few years ago. Consistently with what we are here arguing, they concluded that these challenges would have to be reassessed once the first data from IceCube could be analyzed.

[7] Notice, however, that, as also observed by Jacob & Piran (2007), a detection of a single such neutrino is not enough on its own: only a consistent detection of several positionally coinciding and consistently time-shifted neutrinos from different GRBs would indicate an observation of $t_{\rm LIV}$ as described by Equation (1).

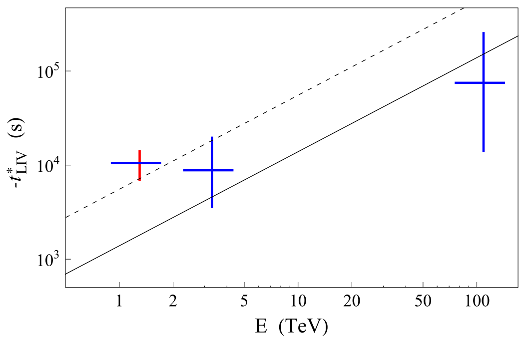

Figure 1. Conjectured quantum-spacetime effects of Equation (1) can be cast in the form of a prediction of a linear relationship between the energy of GRB neutrinos and the value of t^*_{LIV}, i.e., the value the measured t_{LIV} would have had if the source had been precisely at a redshift of 1. Two such linear relationships are shown in this log–log plot, one (solid) assuming $M_{LIV} = 0.04M_P$ and one (dashed) assuming $M_{LIV} = 0.01M_P$. We also provide here information on the three IceCube neutrino events, which (in the sense specified in our opening remarks) could be viewed as illustrative of possible properties of candidate GRB neutrinos. The horizontal segments reflect the energies of those three events, assuming (F. Halzen 2013, private communication) an energy uncertainty of 30%. The vertical segments reflect the uncertainty in the determination of t^*_{LIV} for each of the three candidate GRB neutrinos. This uncertainty Δt^*_{LIV} is affected not only by the accuracy with which one manages to measure t_{LIV} but also by the accuracy with which one manages to infer a value of redshift for the relevant GRB.

Equation (1) can be rewritten equivalently as

$$t_{LIV} = t^*_{LIV}\frac{D(z)}{D(1)} \tag{2}$$

with the definition

$$t^*_{LIV} = s_{\pm}\frac{E}{M_{LIV}}D(1), \tag{3}$$

This definition of t^*_{LIV} allows us to describe the relevant quantum-spacetime effects, which in general depend on both redshift and energy, as effects that depend exclusively on energy, through the simple expedient of focusing on the relationship between t_{LIV} and energy when the redshift has a certain chosen value, which in particular we chose to be $z = 1$. If one measures a certain t_{LIV} for a candidate GRB neutrino and the redshift z of the relevant GRB is well known, then one gets a firm determination of t^*_{LIV} by simply rescaling the measured t_{LIV} by the factor $D(1)/D(z)$. And, as we shall see, even when the redshift of the relevant GRB is not known accurately, one will be able to convert a measured t_{LIV} into a determined t^*_{LIV} with accuracy governed by how much one is still able to assume about the redshift of the relevant GRB. The net result is that, structuring the search of these quantum-spacetime effects from the viewpoint of 2D plots of t^*_{LIV} versus E, one ends up with a rather clear test of the hypothesis.

The triplet of weak GRB-neutrino candidates here discussed in the opening remarks allows us to illustrate some key aspects of this approach. Indeed, one of our key observations is that a high "quality" of GRB-neutrino candidates (for each of them, of course, being just opposite to the probability that it is just a background event) is not the only way to have conclusive results within our methodology: high statistics (a large number of weak GRB-neutrino candidates) can compensate for low quality of the candidates.

In Figure 1 we apply our strategy of analysis to our three candidate GRB neutrinos. The straight lines reflect the linear relationship between t^*_{LIV} and E, coded in Equation (3), for two different values of M_{LIV}:$M_{LIV} = 0.04M_P$ and $M_{LIV} = 0.01M_P$. All three of our GRB-neutrino-candidate data points are affected by uncertainties in both E and t^*_{LIV}, though the uncertainties in t^*_{LIV} have a different origin. We assigned to the three data points a 30% uncertainty (F. Halzen 2013, private communication) on the energy. As discussed in our opening remarks, for the highest-energy event one has an experimentally inferred value of t_{LIV} of 14 hr, but concerning the redshift of the relevant GRB, which is GRB 091230A, one should allow for a rather large uncertainty: the redshift of GRB 091230A was not determined experimentally, so one can only rely on the information that GRB 091230A was a long burst, which leads us to assume rather safely (Wanderman & Piran 2010) that it was at redshift $0.3 < z < 6$. Our corresponding data point in the E, t^*_{LIV} must then reflect this large redshift uncertainty in terms of a large uncertainty for t^*_{LIV}: t_{LIV} was determined rather accurately experimentally, but the redshift-dependent conversion of the measured value of t_{LIV} into a corresponding value of t^*_{LIV} is still sizably uncertain because of the little information we have on the redshift of GRB 091230A.

For the data point in Figure 1 concerning the event with energy of about 3.3 TeV, the same steps of analysis apply, the only difference being that the relevant GRB, which is GRB 090219, is a short burst. The relevant measured value of t_{LIV}, of −3594 s, must then be converted into an uncertain inferred value of t^*_{LIV} assuming for GRB 090219 a redshift of $0.2 < z < 1$, which is indeed appropriate for a short burst (Coward et al. 2012).

The analysis of the uncertainty on t^*_{LIV} shown in Figure 1 for the GRB-neutrino candidate with energy of about 1.3 TeV is instead rather different. For the relevant GRB, which is GRB 090417B, there is an observational determination (Holland et al. 2010) at $z \simeq 0.35$. So one would be tempted to convert the corresponding measured value of t_{LIV}, −2249 s, into a corresponding sharply determined value of t^*_{LIV}, $t^*_{LIV} = -(D(1)/D(0.35))2249$ s. However, it happens to be the case that GRB 090417B had an unusually long duration of some 2300 s. The values of t_{LIV} we are quoting are differences between the time of detection of the neutrino and the time of onset of the gamma-ray signal from the relevant GRB. However, as mentioned, our expectation is that GRB neutrinos are not necessarily produced at the very beginning of the GRB, but rather any time during the duration of the GRB emission. The two other GRBs mentioned above, GRB 091230A and GRB 090219, had duration shorter than 100 s (much shorter than that in the case of GRB 090219), and when considering values of t_{LIV} of thousands (or tens of thousands) of seconds and the uncertainty less than 100 s can be ignored, as we did. But for this GRB-neutrino candidate from GRB 090417B with t_{LIV} of −2249 s, the duration of 2300 s of GRB 090417B must inform us of a significant uncertainty in the simultaneity of emission between our candidate GRB neutrino and the onset of the gamma-ray signal. This uncertainty of 2300 s is reflected in the vertical segment farther to the left in Figure 1.

The method of analysis illustrated by our Figure 1 can be useful for a first assessment, from the LIV perspective, of a sample of candidate GRB neutrinos. If a given collection of candidate GRB neutrinos actually is just a pure collection of background events, one would expect completely random values for the $t_{\rm LIV}$ established within the (in that case, wrong) hypothesis that they be GRB neutrinos. A large enough sample of such pure-background events (which happened to only look like plausible GRB-neutrino candidates) should populate rather uniformly the region of the $t^*_{\rm LIV}$, E plane shown in our Figure 1 and actually should populate with equal density and uniformity also the region of the $t^*_{\rm LIV}$, E that we did not show, the one with negative $t^*_{\rm LIV}$. If a bunch of background neutrinos happen to just look like plausible candidate GRB neutrinos, the inferred values of $t^*_{\rm LIV}$ have an equal chance of being positive or negative.

Imagine instead a situation where the true value of $M_{\rm LIV}$ is, say, $0.04M_P$ and a large number of candidate GRB neutrinos have been accumulated. In such a scenario one might have at some point, say, 50 weak candidates of GRB neutrinos with, say, 30 of the corresponding values of $\{t^*_{\rm LIV}, E\}$ lining up roughly along the solid line in our Figure 1, and the remaining 20 values of $\{t^*_{\rm LIV}, E\}$ distributed rather uniformly in the $t^*_{\rm LIV}$, E plane, not only with positive but also with negative inferred values of $t^*_{\rm LIV}$. In such a hypothetical situation one would like to conclude that the 20 candidates providing data points off of the solid line actually are background, but at least a significant fraction of the 30 events compatible with the solid line would have to be true GRB neutrinos governed by the quantum-spacetime effects of Equation (1). The significance of such a conclusion should be then established on purely statistical grounds, in terms of the probability that a collection of 50 pure-background events happen to give by chance 30 of them compatible with the solid line in our Figure 1.

We use again our three IceCube neutrinos for a discussion of how such statistical analyses of significance could be performed. We propose that the core ingredient should be the likelihood $q_i(M_\star)$ that the ith event in the sample is a "GRB-LIV event with LIV scale $M_\star$" as

$$q_i(M_\star) = \int\int dz dE \quad \theta\left(t^i_{\rm obs}, t^i_{90}, E, z, M_*, \Delta t\right) p_E\left(E, E^i_{\rm obs}\right) p_z(z, \bar{z}_i) \qquad (4)$$

where

$$\theta = 1 \ \text{ if } \ t_{\rm LIV}(E, z, M_*) < t^i_{\rm obs} < t_{\rm LIV}(E, z, M_*) + t^i_{90} + \Delta t$$
$$\theta = 0 \quad \text{otherwise}$$

E is a value of energy for the neutrino, picked within a (weighted) interval derived from the observed energy $E^i_{\rm obs}$ and $p_E(E, E^i_{\rm obs})$, which is the probability of actually having energy E when $E^i_{\rm obs}$ is measured; z is the guessed redshift of GRB$_i$, taking values within a range fixed by the information on redshift available for the burst GRB$_i$, information also coded in the probability distribution[8] $p_z(z, \bar{z}_i)$; M_* is a trial value of $M_{\rm LIV}$; and Δt is a time window that we allow for the neutrino to be emitted at the source after $T^{\rm i}_{90}$.

We then define a binary vector n_i that takes values 0 or 1 corresponding to the question whether a neutrino is signal (1) or background (0). Our likelihood is then defined as follows:[9]

$$\mathcal{L}(M_*, \boldsymbol{n}) = \Pi^n_i\left[n_i q_i\left(1 - p_i\right) + \left(1 - n_i\right)^* p_i\right] \qquad (5)$$

where p_i is our estimate of the probability of having a background event with the properties of the$i$th event in our sample.

This likelihood is to be maximized over both M_* and the vector $\boldsymbol{n}$. That is, for each possible value of $\boldsymbol{n}$ (there are 2^N such binary vectors for a sample containing N events) one should find the optimal M_* and the value of $\mathcal{L}(M_*, \boldsymbol{n})$. Of course, one is interested in the highest values of likelihood from all the 2^N combinations, and particularly interesting is the comparison between the highest value of $\mathcal{L}$ among cases with at least one $n_i \neq 0$ and the value of $\mathcal{L}$ obtained assuming $\boldsymbol{n} = \vec{0}$.

Table 2 reports the results obtained testing this method on the three candidate GRB-neutrino events discussed above. This was done within some simplifying assumptions :

1. on the basis of information gathered from the IceCube literature (Whitehorne 2010; Abbasi et al. 2012) and from private communications with Halzen, we generously estimated $p_{1.3} = 0.4$ (probability of having a background event with the properties of the 1.3 TeV event in the relevant IceCube data set) and similarly $p_{3.3} = 0.4$ and $p_{109} = 0.2$; note that it would probably be more prudent to assign higher values to these p_1,p_1,p_3, but we shall see that even with such optimistic estimates the LIV interpretation is disfavored;
2. we assumed that the distribution $p_E(E, E^i_{\rm obs})$ is a Gaussian picked at the reported value of energy and with standard deviation estimated at 30% of the reported value of energy;
3. for the GRB tentatively associated with the 1.3 TeV event we assumed a redshift of 0.35; for the GRB tentatively associated with the 3.3 TeV event, which was a short burst, we assumed that the distribution p_z is $p_z = \text{const}$ for $0.1 < z < 1$ and 0 otherwise; for the GRB tentatively associated with the 109 TeV event, which was a long burst, we assumed that the distribution p_z is $p_z = \text{const}$ for $0.3 < z < 5$ and 0 otherwise.

Table 2
The Maximum Likelihood ($\mathcal{L}_{\rm max}$) Obtained at $M^{\rm est}_{\rm LIV}$ by Testing the Method Described in Equation (5) on the Three Candidate GRB-neutrino Events

109 TeV	3.3 TeV	1.3 TeV	$\mathcal{L}_{\rm max}$	$M^{\rm est}_{\rm LIV}/M_{\rm Plank}$
B	B	B	3.2×10^{-2}	N/A
B	S	S	5.5×10^{-5}	0.013
S	S	B	3.9×10^{-5}	0.023
S	B	S	1.7×10^{-5}	0.013
S	S	S	1.5×10^{-6}	0.014

Table 2 shows that, in spite of the possibly different impression one could get from the findings summarized in Figure 1, at a more careful level of analysis one finds that the interpretation of even just two of the neutrinos in our sample as LIV-affected GRB neutrinos is strongly disfavored. Within the set of hypotheses we tested, one should favor the one describing all three neutrinos as background events. This was

[8] If for one of the GRBs in the sample the redshift is well known, one can in principle replace $p_z(z, \bar{z}_i)$ with a Dirac δ function, but since there is always an uncertainty in any determination of redshift, one can always manipulate formulas assuming that z has allowed values within a certain range.

[9] Note that if all q_i vanish, then $\mathcal{L}(M_*, \boldsymbol{n})$ is just the binomial likelihood.

our primary goal here: we selected our three candidates with very loose criteria, somewhat biased toward the LIV hypothesis (enough so to produce Figure 1, encouraging for the LIV hypothesis), and yet our approach allowed us to expose that the LIV hypothesis was weak. This provides evidence of the strength of the approach we are advocating. If future data ever lead to a positive identification of some GRB neutrinos (so that the hypothesis "all background" would not appear in the analysis at all), then evidently one could use our likelihood estimator also to establish lower bounds on $M_{\rm LIV}$, using the profile of the likelihood function.

The analysis reported here sets the stage for other similar analyses that should be performed in the future, whose outcome could be particularly interesting when several (weak but) reasonably strong GRB-neutrino candidates are accumulated. Since we are arguing that the discovery of the validity of Equation (1) would have to be the outcome of a statistical analysis, evidently we favor an approach where the selection criteria for candidate GRB neutrinos are not too strict. For example, surely candidate GRB neutrinos for which one has a redshift measurement for the relevant GRB would give sharper information to such analyses (reduced size of the uncertainty on $t^*_{\rm LIV}$), but restricting the analyses to just GRBs whose redshift is measured would ultimately weaken the overall efficacy of the type of approach we are advocating, whose strength will ultimately reside on high statistics.

Eventually we might reach high enough statistics in this kind of study that it might be important to go even more in depth than done here. In particular, we here handled the uncertainty introduced by the fact that redshift is not precisely known using only very general properties of long and short GRBs. One could get an improved estimate (and therefore possibly a more accurate estimate of the significance of emerging evidence of the applicability of Equation (1)) by factoring in information on the fluence of the relevant GRBs, which is known to be at least weakly correlated with the redshift. One should also contemplate the possibility of allowing for the fact that the detected GRB redshift distribution is not necessarily the same as the intrinsic one.

If one wants to properly test the LIV hypothesis considered here, some changes in methodology would also be required for what concerns estimating for each single GRB-neutrino candidate the probability that it is a background event (estimates that, of course, will play a role in the statistical-significance approach we are advocating). Currently standard analyses, such as those in Whitehorne (2010), are strongly biased toward the hypothesis that GRB neutrinos should be detected in time coincident with the corresponding gamma-ray signal: the role of background events is minimized by looking for candidate GRB neutrinos first in a tiny time window around the GRB gamma-ray trigger, and then gradually opening the time window symmetrically around the time of the gamma-ray signal. This way events that are thousands of seconds before the observation of the corresponding gamma-ray signal are inevitably found only within time windows with sizable background-event probability. In order to properly test the hypothesis of, say, $M_{\rm LIV} = 0.04 M_P$, one should instead look for candidate GRB neutrinos in an energy-dependent but rather small time window for $t^*_{\rm LIV}$, with energy dependence governed by our Equation (3): the significance of each GRB-neutrino candidate found by such a procedure would be rather high (even for large values of $t^*_{\rm LIV}$) because of the smallness of the time window. The quality and reliability of the available data are sufficient for the illustrative purposes on which we focused in this work, but they are definitely insufficient for claiming any limits on the LIV scale, and therefore with this anlysis we are not able to put a limit on $M_{\rm LIV}$.

In summary, we find that in order to properly explore the quantum-spacetime hypothesis of our Equation (1) (or equivalently Equations (2) and (3)), several methodological adjustments are needed. Of course, the chances of success of this research program should be expected to be very small, but we feel that its significance should be gauged also considering the huge impact on fundamental physics that would be produced in the however unlikely event that this approach produces the first ever experimental discovery of a quantum property of spacetime.

We thank Francis Halzen, for several helpful discussions, and Fabrizio Fiore, for valuable comments on an early version of the manuscript. This research was partially supported by the John Templeton Foundation (GAC) and by an ERC advanced grant (T.P.). T.P. thanks the IUSS—Ferrara for hospitality while this research was done.

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