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THE *GAIA* INERTIAL REFERENCE FRAME AND THE TILTING OF THE MILKY WAY DISK

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ABSTRACT

While the precise relationship between the Milky Way disk and the symmetry planes of the dark matter halo remains somewhat uncertain, a time-varying disk orientation with respect to an inertial reference frame seems probable. Hierarchical structure formation models predict that the dark matter halo is triaxial and tumbles with a characteristic rate of ∼2 rad H_0^{-1} (∼30 μas yr^{-1}). These models also predict a time-dependent accretion of gas, such that the angular momentum vector of the disk should be misaligned with that of the halo. These effects, as well as tidal effects of the LMC, will result in the rotation of the angular momentum vector of the disk population with respect to the quasar reference frame. We assess the accuracy with which the positions and proper motions from *Gaia* can be referred to a kinematically non-rotating system, and show that the spin vector of the transformation from any rigid self-consistent catalog frame to the quasi-inertial system defined by quasars should be defined to better than 1 μas yr^{-1}. Determination of this inertial frame by *Gaia* will reveal any signature of the disk orientation varying with time, improve models of the potential and dynamics of the Milky Way, test theories of gravity, and provide new insights into the orbital evolution of the Sagittarius dwarf galaxy and the Magellanic Clouds.

Key words: astrometry – cosmology: observations – Galaxy: disk – Galaxy: formation – reference systems – space vehicles: instruments

1. INTRODUCTION

The *Gaia* space astrometry mission will make precision measurements of the positions and motions of both Galactic stars and distant quasars. Like *Hipparcos* before it, *Gaia* will utilize a small number of key measurement principles (observations above the atmosphere, two widely separated viewing directions, and a uniform "revolving scanning" of the celestial sphere) to create catalogs of star positions, proper motions, and parallaxes of state-of-the-art accuracies (Perryman et al. 2001; Lindegren et al. 2008). Crucially, both generate *absolute* trigonometric parallaxes, rather than the relative parallaxes accessible to narrow-field astrometric measurements from the ground. In both cases, the observations are effectively reduced to an internally consistent and extremely "rigid" catalog of positions and proper motions, but whose frame orientation and angular rate of change (spin) are essentially arbitrary, since the measured arc lengths between objects are invariant to frame rotation. Placing both positions and proper motions on an inertial system corresponds to determining these six degrees of freedom (three orientation and three spin components). They were derived after catalog completion for *Hipparcos*, and will be derived as a by-product of the observations/data reductions in the case of *Gaia*.

Will *Gaia* measure the same fundamental plane for quasars and for Galactic stars? Simulations of galaxy formation (Bailin & Steinmetz 2004; Bryan & Cress 2007) predict that most galaxy halos tumble with a characteristic rotation rate of ∼2 rad H_0^{-1}. Both analytical arguments (Nelson & Tremaine 1995) and numerical simulations (Dubinski & Kuijken 1995) suggest that dynamical friction in the inner regions of galaxies should tightly couple the inner disk to the halo, at least to $\sim R_{\mathrm{vir}}$ (Bailin & Steinmetz 2004), corresponding roughly to the region dominated by the baryons. Thus, if the angular momentum vectors of the inner disk and halo remain aligned (Libeskind et al. 2007), we would expect that the fundamental plane defined by the Galactic stars will rotate at a rate of ∼30 μas yr^{-1}. Even in the absence of a tumbling halo contribution, the disk orientation is expected to vary with time, due to a combination of the infall of misaligned gas (Shen & Sellwood 2006), the interaction of the infalling gas with the halo (Roškar et al. 2010), and the effect of the LMC (Weinberg & Blitz 2006). Simulations show the inner disk and the outer halo often decouple (Roškar et al. 2010), with average misalignments of 30°–40° (Croft et al. 2009; Bett et al. 2010; Hahn et al. 2010), such that the reference frame defined by the inner and outer disk stars may differ. Debattista et al. (2013) suggest that the observations of the Sagittarius stream may be better fit by models where the outer disk is not aligned with the principal plane of the dark matter halo. If any of these effects apply, *Gaia* may measure a different fundamental plane for quasars, inner disk stars, and outer disk stars.

For *Hipparcos*, typical positions and annual proper motions were of the order of 1 mas (milliarcsec), and the reference frame link was determined with an accuracy of about 0.6 mas in the three orientation components, and 0.25 mas yr^{-1} in the three spin components, determined by a variety of different link methodologies (described further below). *Gaia* will achieve accuracies of some 10 μas (microarcsec) in positions and annual proper motions for bright stars ($V \sim 10$), degrading to around 25 μas at $V = 15$, and to around 0.3 mas (300 μas) at $V = 20$ (Lindegren et al. 2008).

This paper addresses the accuracy with which *Gaia* can detect the rotation of the angular momentum vector defined by disk stars relative to the inertial frame defined by quasars. We will show that the reference frame link should be determined to better than 1 μas yr^{-1} in spin. Being significantly smaller than (for example) dynamical effects driven by a tumbling halo, we argue that accurately linking the *Gaia* catalog to an inertial reference system will, in addition to its expected impacts in many other fields of stellar kinematics, deepen our understanding of the larger scale dynamics and history of the Milky Way.

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2. REFERENCE SYSTEMS AND REFERENCE FRAMES

The IAU Working Group on Reference Frames and Reference Systems emphasizes the distinction between the theoretical construct of a celestial reference "system," and its practical materialization, referred to as a reference "frame," via a set of fiducial astronomical sources, whether at optical, radio, or other wavelengths.

Historically, celestial reference *systems* were referred to the position of the Earth's equator and equinox at some specified epoch. Thus the reference *system* B1950 specified positions with respect to (an outward extension of) the Earth's equator, and to the equinox location at epoch B1950.0. It was materialized by the FK4 reference *frame* comprising positions and proper motions of the 1535 stars of the fundamental catalog FK4. B1950/FK4 was later superseded by J2000/FK5, viz., the *reference system* J2000 (i.e., referred to the equator/equinox at epoch J2000), materialized by the FK5 reference *frame*, comprising improved positions/proper motions of the same 1535 primary reference stars, along with some 3000 others (the FK5 extension).

The International Celestial Reference System (ICRS) superseded the J2000 equator/equinox-based system, with the goal of placing positions and proper motions of celestial objects directly on an (extragalactic-based) inertial reference system. It was materialized by the International Celestial Reference Frame (ICRF), initially consisting of positions of 212 extragalactic radio sources, observed at 2.3 and 8.4 GHz by Mark III very long baseline interferometry (VLBI) through the middle of 1995, and with rms positional uncertainty between 100–500 μas (Ma et al. 1998). The IAU adopted the ICRF as the fundamental celestial reference frame, superseding the FK5 optical frame as of 1998 January 1. More recently, the ICRF2 has been extended to include positions of 3414 extragalactic radio sources observed by VLBI over 30 yr, with an improved noise floor of $\sim$40 μas, and an improved axis stability of $\sim$10 μas (Ma et al. 2009).

2.1. The Hipparcos Reference Frame

Finalizing the *Hipparcos* catalog included adjustments in both orientation and spin components such that the *Hipparcos* reference frame coincided with the ICRF, as already established in the radio. Following publication in 1997, the IAU adopted the *Hipparcos* catalog as the *optical* materialization of the ICRS. With a completeness limit of 7.3–9.0 mag, and a faint star limit of $V \sim 12$, *Hipparcos* included just one extragalactic object, the quasar 3C 273, and that with rather poor positional precision reflecting its faint magnitude. Accordingly, a number of different approaches were pursued, in parallel, to establish the six link parameters (Kovalevsky et al. 1997). These were (1) interferometric observations of radio stars by VLBI, MERLIN, and Very Large Array; (2) observations of quasars relative to *Hipparcos* stars via CCDs, photographic plates, and *Hubble Space Telescope*; (3) photographic programs to determine stellar proper motions with respect to extragalactic objects; and (4) comparison of Earth orientation parameters obtained by VLBI and by ground-based optical observations of *Hipparcos* stars. The various techniques generally agreed to within 10 mas in the orientation components, and to within 1 mas yr^{-1} in spin components. Weighted mean values were adopted for the definition of the system of positions and proper motions. As a result, the coordinate axes defined by the published catalog (at catalog mid-epoch, J1991.25) were considered aligned to the extragalactic radio frame with rms uncertainties estimated to be 0.6 mas in the three components of the orientation vector, $\boldsymbol{\varepsilon}$, and 0.25 mas yr^{-1} in the three components of the spin vector, $\boldsymbol{\omega}$ (we adopt Galactic coordinates, with ω_1 toward the Galactic center, ω_2 in the direction of Galactic rotation, and ω_3 toward the Galactic pole).

Numerous subsequent studies, including those with a longer temporal baseline, have largely confirmed these values. Some have hinted at slightly larger spin components in ω_3 (e.g., Bobylev 2004; Fedorov et al. 2011; Assafin et al. 2013), although this is the most sensitive to the effects of (differential) Galactic rotation. As an independent verification of the link, the kinematic bulk motion of Galactic disk stars within the adopted reference frame reveals no unexpected rotational component about axes in the plane of the Galaxy (ω_1, ω_2), although such bulk motions, even if present, would not in themselves invalidate the accuracy of the claimed link.

Subsequent kinematic investigations of the *Hipparcos* proper motions within $\sim$3 kpc have shown warp-like structures but of confusing and conflicting form (Smart et al. 1998; Drimmel et al. 2000). Warps are a common feature of a large fraction of spiral galaxies (Binney 1992; Sánchez-Salcedo 2006), and are thus either very long-lived or continuously regenerated, although both their origin and persistence remain topics of ongoing investigation. Current explanations invoke a tilt between the disk and triaxial dark matter halo, or a continuous infall of material with angular momentum misaligned with that of the disk (e.g., Weinberg & Blitz 2006; Shen & Sellwood 2006).

2.2. The Gaia Reference Frame

Gaia was launched on 2013 December 19. Over its 5 yr program, progressively more accurate catalogs will be released as the continuous sky scanning increases the number of individual measurements per star, and simultaneously extends the temporal baseline. Details of the astrometric data processing are given by Lindegren et al. (2012). As a result of on board detection thresholding, *Gaia* will observe *all* star-like objects down to a completeness limit of $V \sim 20$ mag (more strictly, the astrometry integrates over a broad-band response designated G). Out of its expected harvest of more than a billion objects, some 500,000 or more quasars are expected to be observed and identified, mostly in the range $z = 1.5$–2.0 (Claeskens et al. 2006; Mignard 2012). This will permit direct connection to an inertial reference system, with an accuracy estimated below.

Linking the *Gaia* catalog to the ICRS proceeds conceptually as follows: (1) the observations are reduced to an internally consistent catalog of positions and proper motions, with arbitrary system orientation and spin; (2) positions of the optical counterparts of radio sources in the ICRF will be compared with their radio positions, to give the orientation vector $\boldsymbol{\varepsilon}$ of the optical catalog with respect to ICRF; (3) the (apparent) proper motions of quasars will be analyzed to determine the quasi-inertial spin vector $\boldsymbol{\omega}$ of the catalog with respect to the extragalactic frame. The final *Gaia* catalog then results from applying a correction corresponding to $-\boldsymbol{\varepsilon}$ to all positions, and by applying a correction corresponding to $-\boldsymbol{\omega}$ to all proper motions. In practice, these steps will be incorporated within the iterative astrometric core processing (Lindegren et al. 2012; O'Mullane et al. 2011).

3. THE *GAIA* INERTIAL FRAME IN PRACTICE

3.1. Galactocentric Acceleration

In the practical realization of a non-rotating inertial reference frame at the μas level, the non-uniformity of the Galactic motion of the solar system barycenter is a manifestly non-negligible violation of inertiality. The principal observable effect is caused by the nearly constant (secular) acceleration of the barycenter with respect to the center of the Galaxy (Bastian 1995; Kovalevsky 2003; Kopeikin & Makarov 2006). This acceleration causes the aberration term to change slowly with time, and therefore results in a pattern of secular aberration observable as a systematic vector field of the apparent proper motions of distant quasars. The effect has been observed as residuals in the VLBI reference frame (Titov 2010; Titov et al. 2011; Xu et al. 2012), and has been identified as a contributing term in the orbital period change of the binary pulsar PSR 1913+16 (Damour & Taylor 1991).

The solar system's orbital velocity around the Galactic center, which we will adopt as $V_0 = 223$ km s^{-1} (see below), causes an aberration effect of $V_0/c \sim 2\rlap{.}'5$; its absolute velocity with respect to a cosmological reference frame similarly causes the dipole anisotropy of the cosmic microwave background (CMB). All measured star and quasar positions are therefore shifted toward Galactic coordinates $l = 90^\circ$, $b = 0^\circ$. For an arbitrary point on the sky the size of the effect is $2\rlap{.}'5$ ($\sin\eta$), where η is the angular distance to the point $l = 90^\circ$, $b = 0^\circ$. Adopting Oort constants $A = 14.82$, $B = -12.37$ (both in km s^{-1} kpc^{-1}) as derived from *Hipparcos* Cepheids (Feast & Whitelock 1997) giving $\Omega_0 = A - B = 27.19$ km s^{-1} kpc^{-1}, and a Galactocentric radius of the Sun of $R_0 = 8.2$ kpc (Ghez et al. 2008), results in a circular velocity at R_0 of $V_0 \equiv R_0\Omega_0 = 223$ km s^{-1}, and a Galactic orbital period for the Sun of $P_{\rm rot} = 2.26 \times 10^8$ yr. The resulting Galactocentric acceleration of the barycenter has the value

$$a_{\rm Gal} \equiv \frac{V_0^2}{R_0} = 2 \times 10^{-10}\,{\rm m\,s^{-2}} = 6 \times 10^{-3}\,{\rm m\,s^{-1}\,yr^{-1}}\,. \quad (1)$$

This causes a change in (first-order) aberration of $a_{\rm Gal}/c \sim 4\,\mu$as yr^{-1}, resulting in an apparent proper motion of a celestial object, toward the Galactic center, of $4\,\mu$as yr^{-1}($\sin\zeta$). This holds for all objects beyond about 200 Mpc, and in particular for quasars, for which their intrinsic proper motions, caused by real transverse motions, are assumed negligible. A proper motion of $4\,\mu$as yr^{-1} corresponds to a transverse velocity of $\sim$30,000 km s^{-1} at $z = 0.3$ for $H_0 = 70$ km s^{-1} Mpc^{-1}. Thus, all quasars will exhibit a distance-independent streaming motion toward the Galactic center. Within the Galaxy, on the other hand, the effect will be hidden in the local kinematics, e.g., corresponding to $\sim$0.2 km s^{-1} at 10 kpc.

3.2. Spin Vector

The spin vector, $\boldsymbol{\omega}$, will be determined from the $\sim$500,000 quasars, in the range $V = 12$–20 mag which will be observed by *Gaia* directly. Some of these, including large numbers from Two-Degree Field (2dF; Croom et al. 2004) and Sloan Digital Sky Survey (SDSS; Pâris et al. 2014) will be known a priori. In any case, all will be detected on board and therefore observed astrometrically and photometrically. Detailed studies (Claeskens et al. 2006) have shown that multi-parameter classification (based on color indices, photometric variability, and negligible parallax and proper motion) will be able to identify a large fraction of those quasars previously unknown, at the same time excluding stars at some expense of completeness (an essential process given that quasars will represent only some 0.05% of the observed objects).

For assessments of the accuracy of the link, the cumulative number density of quasars as function of magnitude was taken from Hartwick & Schade (1990), and restricted to redshifts $z < 2.2$. These authors already pointed out that the knowledge of the quasar luminosity function for $z < 2.2$ "appeared to be quite secure." This conclusion is broadly confirmed by the latest quasar compilation of Véron-Cetty & Véron (2010, with sky distributions given by Mignard 2012), complemented by a highly simplified full-sky extrapolation of the 2dF (Croom et al. 2004) and SDSS (Pâris et al. 2014) yields. At the same time, restriction to redshifts $z < 2.2$ probably gives some underestimate of the final numbers expected to be available for the link; larger surface densities were estimated by Mignard (2012) from an extrapolation of the highest densities found in Véron-Cetty & Véron (2010). Our adopted, and probably conservative, numbers are given in Tables 1 and 2.

3.3. Quasar Source Instabilities

The positional/proper motion stability of individual quasars will be affected by the following:

1. macrolensing by intervening galaxies: this may cause apparent proper motions of several μas yr^{-1}, but only if the impact parameter is close to the critical value (of the order of $1''$) where significant magnification occurs (Kochanek et al. 1996). The fraction of affected quasars is of the order of 1% (Kochanek 1996), and they usually have additional structure (multiple images and arcs) on scales that will be resolved by *Gaia*. For larger impact parameters, the proper motion of the single deflected image is smaller than the proper motion of the lensing galaxy, i.e., $\lesssim$0.2 μas yr^{-1} for a lens at $z \sim 0.1$;
2. gravitational lensing by stars in the Galaxy: some 1000 strong-lens quasars are expected to be discovered by *Gaia* (Claeskens et al. 2006), and excluded from the reference frame link. All quasars will be subject to weak lensing (Claeskens & Surdej 2002), leading to random, variable displacements of $\sim$1 μas (Sazhin et al. 1998). The typical effect on the mean proper motion over the *Gaia* lifetime will be $\lesssim$1 μas yr^{-1};
3. photocentric motion: most of the quasar optical emission comes from a region of $\lesssim$1 pc, corresponding to $\lesssim$200 μas at 1 Gpc. Assuming that the photocenter moves randomly within this region, a mean proper motion of $\lesssim$50 μas yr^{-1} may result over the 5 yr observation period. In a detailed study of the ultimate celestial reference for Gravity Probe B, the superluminal quasar 3C 454.3 has a 7 yr proper motion limit of $<$56 μas yr^{-1}. Photocentric motion is also induced by a variable nucleus combined with the much fainter, but much larger galaxy (e.g., Taris et al. 2011). This effect could reach some 100 μas yr^{-1}, but extreme cases might be recognized by the correlation between position and brightness; and
4. chromatic image displacement: although the *Gaia* telescopes are all reflective, they are nevertheless not strictly achromatic. Asymmetric wavefront errors, such as coma, introduce image centroids that depend on wavelength, and hence on the object's spectral energy distribution. For the typical wavefront errors expected in the astrometric field, of $\sim$50 nm rms, the centroid shift between early and late spectral types could reach several mas. This systematic "chromaticity" effect can therefore be many times larger than the photon statistical uncertainty of the estimated image location. It is thus essential to have a very good calibration of the spectral energy distribution of each observed source,

Table 1
Residual Spin of the *Gaia* Reference Frame, $\sigma(\omega_i)$, and Galactocentric Acceleration of the Solar System Barycenter, $\sigma(a_i/c)$, Estimated from a Simulation of Quasar Observations

V (mag)	P	$N_{\rm QSO}$	$\sigma_{\mu,\rm tot}$ (μas yr^{-1})	$\sigma(\omega_1)$	$\sigma(\omega_2)$ (μas yr^{-1})	$\sigma(\omega_3)$	$\sigma(a_1/c)$	$\sigma(a_2/c)$ (μas yr^{-1})	$\sigma(a_3/c)$
⩽15	1.0	40	14	2.5	2.5	3.0	2.5	2.5	3.0
15–16	1.0	230	21	1.5	1.5	1.8	1.5	1.5	1.8
16–17	0.9	1230	30	0.93	0.93	1.14	0.93	0.93	1.14
17–18	0.8	11500	45	0.46	0.46	0.57	0.46	0.46	0.57
18–19	0.6	60000	74	0.33	0.33	0.41	0.33	0.33	0.41
19–20	0.3	97000	130	0.46	0.46	0.56	0.46	0.46	0.56
⩽20		170000		0.22	0.22	0.27	0.22	0.22	0.27

Notes. This table assumes a contribution of $\sigma_0 = 10\,\mu$as yr^{-1} from source instability. Columns contain, for each range of magnitude: P, assumed probability that a quasar is unambiguously recognized from photometric indices; $N_{\rm QSO}$, expected number of recognized quasars with $z < 2.2$ and $|b| > 20°$; $\sigma_{\mu,\rm tot}$, mean standard errors in proper motion per object and coordinate, including a contribution of $\sigma_0 = 10\,\mu$as yr^{-1} from source instability; $\sigma(\omega_i)$, resulting precision of the spin components ($i = 1$ toward the Galactic center, $i = 2$ in the direction of Galactic rotation, $i = 3$ toward the Galactic pole); $\sigma(a_i/c)$, the resulting precision of the acceleration of the solar system barycenter along the Galactic axes.

Table 2
As Table 1, but with a Contribution of $\sigma_0 = 100\,\mu$as yr^{-1} from Source Instability

V (mag)	P	$N_{\rm QSO}$	$\sigma_{\mu,\rm tot}$ (μas yr^{-1})	$\sigma(\omega_1)$	$\sigma(\omega_2)$ (μas yr^{-1})	$\sigma(\omega_3)$	$\sigma(a_1/c)$	$\sigma(a_2/c)$ (μas yr^{-1})	$\sigma(a_3/c)$
⩽15	1.0	40	100	17.5	17.5	21.4	17.5	17.5	21.4
15–16	1.0	230	102	7.4	7.4	9.0	7.4	7.4	9.0
16–17	0.9	1230	104	3.3	3.3	4.0	3.3	3.3	4.0
17–18	0.8	11500	109	1.12	1.12	1.37	1.12	1.12	1.37
18–19	0.6	60000	124	0.56	0.56	0.68	0.56	0.56	0.68
19–20	0.3	97000	164	0.58	0.58	0.71	0.58	0.58	0.71
⩽20		170000		0.38	0.38	0.46	0.38	0.38	0.46

obtained on board from the blue and red photometers (Jordi et al. 2006). Since quasar spectra potentially show strong emission lines at any wavelength depending on redshift, their chromaticity correction will be more problematic than for stars, and could generate spurious proper motion of instrumental origin of ∼10 μas yr^{-1}.

In summary, the most important instabilities are expected to be due to variable source structure and residual telescope chromaticity. The likely range of the combined effects for typical quasars may lie between 10–100 μas yr^{-1} (a value of 30 μas yr^{-1} in each coordinate was inferred by Gwinn et al. 1997). These limits are used in the simulations described below, as summarized in Tables 1 and 2, respectively.

3.4. Condition Equations

The astrometric processing of the *Gaia* observations determines the positions, parallaxes, and proper motions of stars and quasars in an internally consistent, but provisional reference frame (Lindegren et al. 2012). In this frame, the quasars will have non-zero proper motions (μ_{l*}, μ_b) due to (1) the spin, $\boldsymbol{\omega}$, of the provisional frame with respect to the cosmological reference frame; (2) the apparent streaming motion caused by the acceleration, $\mathbf{a}$, of the solar system barycenter; and (3) observational errors and source instability. The first two effects are systematic while the third is assumed to be random and uncorrelated among the quasars. The spin vector should be determined simultaneously with the acceleration vector in a single least-squares solution, using the apparent proper motions of all the quasars. For a quasar at Galactic coordinates (l, b) the condition equations for the Galactic components of $\boldsymbol{\omega}$ and $\mathbf{a}$ are then

$$\mu_{l*} \equiv \mu_l \cos b = \mathbf{q}'\boldsymbol{\omega} + \mathbf{p}'\mathbf{a}\,c^{-1} + \text{noise} \qquad (2)$$

$$\mu_b = \mathbf{p}'\boldsymbol{\omega} + \mathbf{q}'\mathbf{a}\,c^{-1} + \text{noise}, \qquad (3)$$

where c is the speed of light, and $\mathbf{p} = (\sin l,\ \cos l,\ 0)'$ and $\mathbf{q} = (\sin b \cos l,\ \sin b \sin l,\ \cos b)'$ are unit vectors along $+l$, $+b$ tangent to the celestial sphere at the position of the quasar.

3.4.1. Simulations

Numerical simulations were made of the least-squares solution of $\boldsymbol{\omega}$ and $\mathbf{a}$, with the following assumptions (see Tables 1 and 2). The available quasar numbers were randomly distributed over the sky, except in the Galactic plane $|b| < 20°$, where zero density was assumed. Only a fraction $P(V)$ of all the quasars is used; this approximates to the use of various photometric and astrometric criteria to reject possible stars (Claeskens et al. 2006). Galactic coordinates were transformed to the ecliptic system, and the standard errors in $\mu_{\lambda*}$, μ_β were computed as a function of magnitude, and ecliptic latitude, β. A separate least-squares solution was made for each magnitude interval from 14–20, and one solution for the whole magnitude range. Only the covariance matrices are of interest; they were transformed back to the Galactic system, yielding the accuracy estimates in Tables 1 and 2.

To account for source instabilities, the quantity σ_0 (Section 3.3) was added in quadrature to the formal proper motion uncertainties. For Table 1, an optimistic value of σ_0 =

10 μas yr^{-1} was assumed, while for Table 2 the assumption was a rather pessimistic $\sigma_0 = 100\,\mu$as yr^{-1}. Sub-μas accuracy in the spin components is nevertheless reached in both cases due to the large number of sources. The accuracy is slightly lower about ω_3 (normal to the Galactic plane) than about the other two axes, due to the zone of avoidance. Comparable values have been considered by Mignard & Klioner (2012).

The solution for the acceleration **a** is practically orthogonal to that of $\boldsymbol{\omega}$, and of equal accuracy when expressed in comparable units ($\mathbf{a}/c$ has the dimension of proper motion, with $1\,\mu$as yr$^{-1} \equiv 4.606 \times 10^{-11}$ m s^{-2}). The Galactocentric acceleration of the solar system barycenter (Equation (1)) should be measurable at 5%–10% relative accuracy.

3.4.2. Analytical Solution

The simple structure of the condition equations also allows for an analytical accuracy estimate by making some plausible statistical assumptions. If quasars of apparent magnitude V are uniformly distributed on the celestial sphere, and the noise terms in the condition equations are uncorrelated with a standard deviation σ_μ that only depends on V, it is found that the determinations of the six unknowns ω_i, $a_i\,c^{-1}$ (where i = 1, 2, 3 for the Galactic axes) are approximately uncorrelated, each having a standard deviation given by

$$\sigma^2 \simeq \frac{3}{2}\left(\sum_V N(V)\,\sigma_\mu^{-2}(V)\right)^{-1}, \tag{4}$$

where $N(V)$ is the number of useful quasars per magnitude bin, and σ_μ combine the effects of observational errors and source stability. A comparison with the preceding simulations, which used a more detailed model of the quasar distribution (e.g., assuming no useful quasars for $|b| < 20°$) along with inhomogeneous observational noise, shows that Equation (4) is accurate within ±20% for the same total number, an adequate agreement given the likely uncertainties related to source instabilities and in the actual number of useful quasars.

3.5. Frame Orientation

Although unimportant for any kinematic interpretation, we can similarly estimate the accuracy of the *Gaia* reference frame orientation, $\boldsymbol{\varepsilon}$, with respect to the inertial frame. This will be established, consistent with the ICRF, by comparing radio sources positions in ICRF with those of their optical counterparts observed by *Gaia*. The number of radio sources in ICRF2 is currently 3414 (Ma et al. 2009), with positional uncertainties of $\gtrsim$40 μas. We assume that half can be observed optically by *Gaia*, and that most will be faint ($V \sim 19$) with positional accuracies of 100–200 μas. Equation (4) then suggests that the *Gaia* frame orientation will be defined with an uncertainty of ~5–10 μas in each component of $\boldsymbol{\varepsilon}$.

4. SIGNATURE OF THE TILTING DISK

The accuracy with which the final *Gaia* catalog represents an inertial frame is given by the uncertainties of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\omega}$. We have shown that the *Gaia* catalog will improve the accuracy of the optical materialization of ICRS by more than two orders of magnitude, allowing an examination of individual and bulk motions in the Galaxy's disk populations, with an enormous range of kinematic and dynamical applications. In the context of the Galaxy warp, for example, *Gaia* will extend detailed kinematic analyses to the probable disk edge, at $R \sim 15$ kpc, or some 7 kpc from the Sun, where the warp induces a mean offset out of the plane of ~1 kpc.

Specifically, *Gaia* will permit the identification of large-scale disk torques due to the progressive collapse of matter as guided by the ΛCDM structure formation paradigm. For example, a bulk rotation of the disk with a characteristic rate of 2 rad H_0^{-1} (30 μas yr^{-1}) about an axis in the plane of the Galaxy (Bailin & Steinmetz 2004; Bryan & Cress 2007) will significantly exceed the inertial reference frame residual rotation of some 0.2–0.5 μas yr^{-1}. If the disk and halo are misaligned (e.g., Debattista et al. 2013), then *Gaia* should detect a disk rotation rate that depends on Galactocentric radius.

The practical detection of such bulk motions may be viewed as follows. If the stars have, in addition to their component of Galactic rotation (of about 5000 μas yr^{-1}), an extra rotation of 30 μas yr^{-1} about an axis in the Galactic plane, then the net effect is a rotation about an axis that is offset by arctan(30/5000) = 0.°4 from the normal to the Galactic disk. Whether the plane of the disk can be determined that accurately from *Gaia* is not yet evident. However, there would also be a differential effect: assuming a flat rotation curve, the Galactic rotation varies from 10,000 to 2500 μas yr^{-1} between R = 4–12 kpc, so the offset would vary from 0.°2–0.°6, and there would be a differential effect of similar magnitude when comparing stars at different Galactic radii. This would create an additional warp-like structure in the kinematics, identifiable independently of the quasars. If the inner and outer disk are misaligned, then this will alter the radial dependance of this structure.

Might other effects mask these bulk motions? The small value of the CMB quadrupole seen by *COBE*, *Wilkinson Microwave Anisotropy Probe*, and *Planck* strongly constrains the net rotation of the universe (Barrow et al. 1985), while the low amplitude of the large-angle CMB modes also constrains any large-scale bulk quasar motions. Thus, scalar and vector perturbations terms are not likely to be significant. Gravitational waves may introduce additional structure in the apparent quasar proper motions over the sky, over a wide range of frequencies from the inverse of the observation period up to the Hubble time, but composed primarily of second-order transverse vector spherical harmonics (Gwinn et al. 1997; Jaffe 2004).

5. CONCLUSIONS

Our simulations using realistic quasar counts show that an accuracy of better than 1 μas yr^{-1} should be reached in all three inertial spin components of the *Gaia* reference frame, $\boldsymbol{\omega}$, even assuming somewhat conservative numbers of quasars used for the link, and rather pessimistic assumptions on the effects of variable source structure. At the same time, the *Gaia* reference frame orientation will be defined with respect to the ICRF with an uncertainty of ~5–10 μas in each component of $\boldsymbol{\varepsilon}$, while the Galactocentric acceleration of the solar system barycenter will be measured at 5%–10% relative accuracy.

These tight constraints on the inertial spin will allow the interpretation of individual and bulk motions in the Galaxy disk populations within the framework of an inertial reference frame defined by distant quasars.

Bulk stellar motions in the direction of Galactic rotation will reflect the many known complexities of the Galaxy's disk and halo structure, and its (differential) rotational motion (e.g., Makarov & Murphy 2007). To this extent, a variety of effects will likely mask any time-dependent influence of any external (e.g., halo-driven) effects on the spin component ω_3.

Bulk rotational motions about axes in the plane of the Galaxy (ω_1, ω_2) will reflect tilting of the Galaxy disk, regardless of origin, combined with any warp-like motions. Assuming that the latter are important only somewhat outside the solar circle, and assuming that the disk interior to the solar cycle responds as a solid body, then torque-induced motions of the order of 2 rad $H_0^{-1} \simeq 30\,\mu$as yr^{-1}, will formally be significantly above the accuracy with which the spin components are constrained from quasar observations.

Detection of a time-dependent rotation of the angular momentum of the of the Galactic disk population would contribute to an understanding of the dynamic effects of the dark halo on the disk (a basic test of Newtonian gravity) and will likely elucidate the dynamical history of the Milky Way. For example, the measurement of the halo rotation rate may change our interpretation of the Sagittarius stream, which appears to lie along the unstable intermediate axis orbit (Law & Majewski 2010; Debattista et al. 2013). In rapidly rotating halos, this intermediate axis orbit is stabilized (Heisler et al. 1982). Halo figure rotation would also alter models of the dynamics of the Magellanic Clouds (Besla et al. 2010).

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