CALIBRATING THE UPDATED OVERSHOOT MIXING MODEL ON ECLIPSING BINARY STARS: HY Vir, YZ Cas, χ^2 Hya, and VV Crv

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ABSTRACT

Detached eclipsing binary stars with convective cores provide a good tool to investigate convective core overshoot. It has been performed on some binary stars to restrict the classical overshoot model which simply extends the boundary of the fully mixed region. However, the classical overshoot model is physically unreasonable and inconsistent with helioseismic investigations. An updated model of overshoot mixing was established recently. There is a key parameter in the model. In this paper, we use observations of four eclipsing binary stars, i.e., HY Vir, YZ Cas, χ^2 Hya, and VV Crv, to investigate a suitable value for the parameter. It is found that the value suggested by calibrations on eclipsing binary stars is the same as the value recommended by other methods. In addition, we have studied the effects of the updated overshoot model on the stellar structure. The diffusion coefficient of convective/overshoot mixing is very high in the convection zone, then quickly decreases near the convective boundary, and exponentially decreases in the overshoot region. The low value of the diffusion coefficient in the standard stellar models of low-mass stars with convective cores, is removed by partial overshoot mixing.

Key words: binaries: eclipsing - convection - stars: evolution

1. INTRODUCTION

The stellar parameters of eclipsing binary stars can be obtained by analyzing their light curves. If two components of an eclipsing binary star are detached and the period is long enough, then we can assume that each of them obeys the rules of stellar evolutionary theory for a single star. The observations of edipsing binary stars provide the possibility of testing stellar physics. Specifically, observations of detached eclipsing binary stars with the masses larger than 1.2 M_{\odot} can be used to restrict convective core overshoot mixing, which is an important factor affecting stellar evolution but is still not well studied. Observations and restricting convective core overshoot mixing have been performed on some detached eclipsing binary stars, e.g., CO And by Lacy et al. (2010), GX Gem by Lacy et al. (2008), and AQ Ser by Torres et al. (2014). Ribas et al. (2000) and Claret (2007) have studied the dependence of the size of the fully mixed overshoot region on stellar mass, and have suggested that a classical overshoot region with a size of $0.2 \leq \alpha_{\rm OV} \leq 0.25$ is best overall.

The investigations noted above are based on the classical overshoot model, which simply extends the convective boundary by a distance in order to become the boundary of the fully mixed region. This description of the overshoot is based on "ballistic" overshoot models (e.g., Shaviv & Salpeter 1973; Maeder 1975; Bressan et al. 1981). However, ballistic overshoot models are excluded by helioseismic investigations (Christensen-Dalsgaard et al. 2011). Recently, an updated overshoot mixing model has been established based on fluid dynamics equations (Zhang 2013). The updated model shows that overshoot mixing can be regarded as a diffusion process, and the efficiency of the mixing in the overshoot region is much lower than that in the convection zone. The key property of this model is the formula of the diffusion coefficient. The formula shows that physically D = $C_{\rm OV}L_{\rm Mix}^2/\tau$, where D is the coefficient, $L_{\rm Mix}$ is the characteristic length of the overshoot mixing, τ is the characteristic time, and

 $C_{\rm OV}$ is a dimensionless parameter which cannot be determined by the model itself. The key parameter $C_{\rm OV}$ is suggested to be 10^{-3} (Zhang 2013) based on the solar model and the restriction that the equivalent fully mixed overshoot region is less than 0.4 H_P. Since detached eclipsing binary stars provide a good probe for overshoot mixing, it is necessary to calibrate the key parameter $C_{\rm OV}$ by using observations of detached eclipsing binary stars.

In this paper, we calibrate the parameter C_{OV} for four detached eclipsing binary stars with a mass ratio that is not near unity, HY Vir, YZ Cas, χ^2 Hya, and VV Crv, and study the properties of the updated overshoot model. The method for modeling stars and the calibrations are introduced in Section 2. The numerical results of the calibrations and the properties of the updated overshoot model are described in Section 3. The conclusions are summarized in Section 4.

2. THE METHOD

2.1. The Stellar Evolutionary Code and Input Physics

The stellar evolutionary code YNEV (Zhang 2014) is adopted to calculate the stellar evolutionary models. The opacities are interpolated from OPAL opacity tables (Iglesias & Rogers 1996) and the F05 low temperature opacity tables (Ferguson et al. 2005). The equation of state (EOS) functions are interpolated from the OPAL-EOS tables (Rogers & Nayfonov 2002). A bicubic polynomial is used in the interpolations of opacity and in the EOS tables in order to obtain continuous derivatives. The rates of nuclear reactions are based on Angulo et al. (1999) and are enhanced by the weak screening model (Salpeter 1954). The T – τ relation of the Eddington gray model of stellar atmospheres is adopted in the atmosphere integral. In the YNEV code, two theories of stellar convection are optional: the mixing length theory (MLT) and the turbulent convection model (TCM) developed by Li & Yang (2007). The latter is a non-local turbulent convection theory which is based on hydrodynamic

equations and some modeling assumptions. In this paper, we focus on the effects of overshoot mixing, and thus the non-local TCM is adopted to deal with turbulent convection in the stellar interior. The implements of the TCM in the YNEV code are described by Zhang (2012c).

2.2. Overshoot Mixing Models

The traditional overshoot mixing model extends the convective boundary by a distance of $l_{\rm OV} = \alpha_{\rm OV} H_{\rm P}$ to become the boundary of the fully mixed region, where α_{OV} is a parameter and H_P is the pressure scale height. The temperature gradient in this extending region, i.e., the overshoot region, is adiabatic or radiative (radiative means that it ignores the convective flux in the overshoot region). The illustration is based on ballistic overshoot models (e.g., Shaviv & Salpeter 1973; Maeder 1975; Bressan et al. 1981), which trace the average fluid element overshooting from the convection zone into the radiative region. However, these ballistic overshoot models are physically unreasonable (Renzini 1987; Zhang 2013) and inconsistent with helioseismic investigations. Helioseismic investigations (Christensen-Dalsgaard et al. 2011) have shown that the temperature gradient smoothly changes from adiabatic to radiative near the convective boundary, so the ballistic overshoot models are excluded because they show an adiabatic overshoot region and a jump of the temperature gradient at the boundary of the overshoot region. It has been suggested that only the TCMs (e.g., Xiong 1981; Xiong et al. 1997; Canuto 1997, 2011; Canuto & Dubovikov 1998; Deng et al. 2006; Li & Yang 2007; Li 2012) can fit the restriction.

Another popular overshoot mixing model is the diffusion model with the diffusion coefficient D based on the characteristic turbulent velocity v and the characteristic length l, i.e., $D \propto vl$. However, in most diffusion overshoot mixing models (e.g., Freytag et al. 1996; Ventura et al. 1998; Lai & Li 2011; Zhang & Li 2012; Ding & Li 2014), the characteristic length l is assumed to be comparable with H_P. This is an analogy used to assume that the characteristic length in the overshoot region is similar with the characteristic length in the convection zone. Deng et al. (1996) have found that setting $l \sim H_{\rm P}$ in $D \sim vl/3$ leads to almost full mixing and have suggested a small characteristic length of $l \sim 10^{-5} l_0 \sim 10^{-5} H_P$ for the overshoot mixing in order to obtain a mixing timescale comparable with the evolutionary timescale. Zhang (2012a, 2012b, 2012c) have shown that when the characteristic length is assume to be comparable with H_P, the dimensionless parameter in $D = C_X v H_P$ should be small $C_X \sim 10^{-10}$ in order to fit some observations. This excessively small dimensionless parameter makes the assumption $l \sim H_P$ doubtful.

Recently, Zhang (2013) has developed an updated overshoot mixing model based on hydrodynamic equations and some modeling assumptions. This model focuses on the turbulent flux of the chemical component and calculates the diffusion coefficient for convective/overshoot mixing. It is found in the model that the diffusion coefficient in the overshoot region is different from that in the convection zone. In the convection zone,

$$D_{\rm CZ} = C_{\rm CZ} \frac{k^2}{\varepsilon},\tag{1}$$

and in the overshoot region,

where k is the turbulent kinetic energy, ε is the turbulent dissipation rate, N² describes the Brunt-Väisälä frequency, C_{CZ} is a parameter of the magnitude of the order of unity, and C_{OV} is another parameter which is recommended to be on the magnitude of the order of $C_{OV} \sim 10^{-3}$ based on the adopted TCM (and its parameters) and some observational restrictions (Zhang 2013). The physical meanings of Equations (1) and (2) have also been pointed out. Equation (1) is equivalent to the model $D \propto v H_P$. However, Equation (2) is physically different from $D \propto v H_P$. The diffusion coefficient in the overshoot region being Equation (2) is for the reason that fluid elements moving around their equilibrium location so the characteristic length is v/N (Zhang 2013).

In this paper, we use Zhang's (2013) overshoot mixing model (i.e., Equation (2)). The turbulent dissipation rate ε is calculated using TCM (Li & Yang 2007). Since the value of the dimensionless parameter $C_{\rm OV}$ for overshoot mixing is not determined in the theoretical model, the main aim of this paper is to use the observations of eclipsing binary stars to restrict the value of $C_{\rm OV}$.

2.3. On Calibrating Overshoot Mixing via Eclipsing Binary Stars

The masses and radii of the two components of an eclipsing binary star (by this we mean detached eclipsing binary star in this paper) can be observed via analyses of light curves. The masses, radii, and effective temperatures can be used to restrict the stellar evolution theory: the evolutionary track of a star with a given mass should pass the observed radius and effective temperature. For a main-sequence star with a convective core, the evolutionary track is sensitive to convective overshoot. Therefore, an eclipsing binary is a good tool to investigate convective overshoot. This has been performed on some eclipsing binary stars, e.g., CO And by Lacy et al. (2010) and AQ Ser by Torres et al. (2014). Ribas et al. (2000) and Claret (2007) have studied the dependence of the size of the fully mixed overshoot region on the stellar mass, and suggested that a classical overshoot region with a size of $0.2 \le \alpha_{OV} \le 0.25$ is the best overall.

In the investigations mentioned above, the overshoot region is assumed to be fully mixed. It is necessary to study the updated overshoot formula via eclipsing binary stars.

In the cases of standard stellar models with convective cores, the structure of a star is fixed when the mass, initial hydrogen abundance X, initial metallicity Z, age t, an overshoot parameter (i.e., α_{OV} for the classical overshoot or C_{OV} for the updated overshoot model), and a convection parameter α (i.e., α_{MLT} for the MLT theory or α_{TCM} for the TCM theory) are all fixed. Observations of a binary star give four restrictions for two stars with given masses, i.e., the radii and effective temperatures of the two components. We assume that the age and the chemical composition are the same for the two components. In this case, the initial hydrogen abundance X, initial metallicity Z, age t, and overshoot parameter of the binary star can be mathematically fixed when we adopt a fixed convection parameter because the number of variables is equal to the number of equations (Equations (A1) for two components). The standard errors can also be obtained based on the method described in the Appendix. Based on those properties, Zhang (2012c) tested the previous diffusion formula of the overshoot on the binary star HY Vir.

However, this method of calibration does not work in some cases. When the mass ratio $q \approx 1$, the masses, radii, and effective temperatures of the two components are very close

 Table 1

 Parameters of the Eclipsing Binaries: HY Vir, YZ Cas, χ^2 Hya, and VV Crv

	HY Vir	YZ Cas	χ^2 Hya	VV Crv
Observations				
M_A/M_{\odot}	1.838 ± 0.009	2.263 ± 0.012	3.613 ± 0.079	1.978 ± 0.010
M_B/M_{\odot}	1.404 ± 0.006	1.325 ± 0.007	2.638 ± 0.050	1.513 ± 0.008
R_A/R_{\odot}	2.806 ± 0.008	2.525 ± 0.011	4.384 ± 0.039	3.375 ± 0.010
R_B/R_{\odot}	1.519 ± 0.008	1.331 ± 0.006	2.165 ± 0.043	1.650 ± 0.008
$lg(T_A)$	3.836 ± 0.008	3.979 ± 0.006	4.066 ± 0.010	3.813 ± 0.013
$lg(T_B)$	3.816 ± 0.008	3.838 ± 0.015	4.041 ± 0.010	3.822 ± 0.013
Ref. Z	0.027	0.009 ± 0.003	0.025 ± 0.010	0.034 ± 0.013
Ref. <i>t</i> /Gyr	1.35 ± 0.10	0.52 ± 0.03	0.18 ± 0.02	1.2 ± 0.1
Ref.	(1)	(2)	(3)	(4)
Calibrations				
$\overline{C_{\rm OV} \times 10^3}$	0.9 ± 0.4	0.9 ± 0.4	0.7 ± 3.0	0.4 ± 1.5
X	0.64 ± 0.03	0.71 ± 0.02	0.72 ± 0.05	0.7 ± 0.1
Ζ	0.032 ± 0.009	0.012 ± 0.002	0.013 ± 0.003	0.03 ± 0.01
t/Gyr	1.3 ± 0.1	0.55 ± 0.07	0.21 ± 0.09	1.3 ± 0.4

References. (1) Lacy & Fekel (2011); (2) Pavlovski et al. (2014); (3) Clausen & Nordström (1978); (4) Fekel et al. (2013).

to each other. This leads to the problem that the Equations (A1) for the primary are almost identical to the Equations (A1) for the secondary, so there are only two independent equations. Therefore, we can only perform the calibration on eclipsing binary stars where the masses of the two components are obviously different. The effects of overshoot mixing on the stellar radius and effective temperature accumulate as the stellar age increases. Therefore, we cannot calibrate the overshoot parameter by using the stars near the zero-age main sequence (ZAMS) stage (e.g., UZ Dra Lacy et al. 1989 and V335 Ser Lacy et al. 2012).

Observation data for intermediate- and high-mass eclipsing binaries is scarce and not accurate enough. For high-mass stars, mass loss (Chiosi et al. 1978; Brunish & Truran 1982; Chiosi & Maeder 1986; Maeder& Meynet 1987; Meynet et al. 1994) and rotation (Meynet & Maeder 2000; Brott et al. 2011; Maeder& Meynet 2012) can also significantly affect stellar structure and evolution. Since rotation and mass loss are not well studied at the moment, we do not attempt to calibrate overshoot in high-mass stars. We focus on low-mass eclipsing binaries. In this paper, we use the methods above to find a suitable value of the overshoot parameter C_{OV} for the updated overshoot mixing model for four eclipsing binaries: HY Vir, YZ Cas, χ^2 Hya, and VV Crv.

3. NUMERICAL RESULTS

In this section, we show the results of the calibration of the eclipsing binaries and the properties of the updated overshoot mixing model in stellar interior. All of the stellar models evolve from pre-main sequences (PMS) with a center temperature of $T_{\rm C} = 10^5$ K. The metal composition is assumed to be the same as the solar metal composition AGSS09 (Asplund et al. 2009). The TCM (Li & Yang 2007) is adopted to calculate the turbulent variables (e.g., the turbulent dissipation rate required in the overshoot diffusion coefficient and the convective flux). The parameters of the TCM are the same as those in Zhang (2012c). The turbulent dissipation parameter $\alpha_{\rm TCM} = 0.8$ is based on solar calibration with the AGSS09 composition. The number of mesh points in the stellar models is typically 1000.



Figure 1. Evolutionary tracks on radius vs. effective temperature for HY Vir. The thick solid lines are for the model of $C_{OV} = 1 \times 10^{-3}$, the solid lines are for the calibrated stellar model, the dashed lines are for the stellar model with a 0.3H_P fully mixed classical overshoot region, and the dotted lines are for the standard stellar models without mixing outside the convection zones. The left plane is for the primary of HY Vir, and the right plane is for the secondary.

3.1. Calibration on Eclipsing Binary Models

We solve Equation (A1) for two components to calibrate the overshoot parameter $C_{\rm OV}$, composition (X and Z), and age t for four eclipsing binaries: HY Vir, YZ Cas, χ^2 Hya, and VV Crv. The mass range in the samples is about 1.3 < M/M_{\odot} < 3.6, which is comprised of seven low-mass stars and an intermediate-mass star (i.e., the primary of χ^2 Hya). The results of the calibrations are shown in Table 1. The radii and the effective temperatures of the calibrated stellar models match the observations in the accuracies of $\Delta R/R_{\odot} < 10^{-3}$ and $\Delta \lg T_{\rm eff} < 10^{-3}$.

The calibration results in Table 1 show that the best value of $C_{\rm OV}$ is about 1×10^{-3} . This is consistent with the suggested value via the test of the solar model and the classical restriction on convective core overshoot (Zhang 2013). Although the masses of the two components are different and there may possibly be a relationship between C_{OV} and the stellar mass, we fix $C_{\rm OV}$ in the calibration of each eclipsing binary. However, the results do not support an obvious dependency of C_{OV} on stellar mass. The calibration results show that the standard errors of $C_{\rm OV}$ are significant. It seems that the observational errors of M/M_{\odot} , R/R_{\odot} , and especially $\lg T_{\rm eff}$ should be less than 1%, otherwise the corresponding error of the overshoot parameter is too large. The metallicities and ages of those eclipsing binaries have been suggested in the references as shown in Table 1 (e.g., Ref. Z and t) by comparing the stellar model based on the fully mixed classical overshoot region with the observations. It has been found that our results for metallicities and ages based on the updated overshoot mixing model are similar to the results of the classical overshoot model. Only the suggested metallicity of χ^2 Hya is significantly higher than our calibration. This may result from the fact that Clausen & Nordström (1978) have used old opacity tables in modeling the stars. The calibrations of the chemical composition on HY Vir and YZ Cas support a helium enrichment law $\Delta Y / \Delta Z \approx 2$. The results of the chemical composition of χ^2 Hya and VV Crv are not accurate enough to validate the law.

The evolutionary tracks for effective temperature versus radius for the stellar models of the binaries are shown in Figures 1-4. The thick solid lines, solid lines, dashed lines,



Figure 2. Similar to Figure 1, but for YZ Cas.

and the dotted lines represent the evolutionary tracks with $C_{\rm OV} = 1 \times 10^{-3}$, the calibrated stellar models, the stellar models with a 0.3 H_P fully mixed classical overshoot region, and the standard stellar models without mixing outside the convection zones, respectively. The stellar models with a 0.3 H_P fully mixed overshoot region and the standard stellar models without mixing are calculated for comparison, and the MLT is adopted in those models to deal with the convective flux. The parameter $\alpha_{MLT} = 1.75$ in the MLT is based on solar calibration. The dashed lines are almost identical to the thick solid lines, indicating that the updated overshoot model with $C_{\rm OV} = 1 \times 10^{-3}$ leads to a similar mixing efficiency as for a 0.3 H_P fully mixed overshoot region in the stars with masses in the studied range. It is shown that the stellar effective temperature of the models in the PMS stage and near ZAMS is slightly affected by the adopted convection theory (MLT is used in the dashed lines and dotted lines, and the nonlocal TCM is used in the solid lines and thick solid lines). However, the differences are small because both of the turbulent dissipation parameters in the MLT and the TCM are based on solar calibrations and both convection theories show adiabatic convection in the convective core. It is interesting to note that for the stars with $M/M_{\odot} > 2$ (i.e., the two components of χ^2 Hya and the primary of YZ Cas), there are significant differences in the PMS stage, i.e., where $R/R_{\odot} \approx 2.5$ for the primary of χ^2 Hya, $R/R_{\odot} \approx 2.1$ for the secondary of χ^2 Hya, and $R/R_{\odot} \approx 1.9$ for the primary of YZ Cas. This can be explained as follows. At those locations, ¹²C is burned to be ¹⁴N in the center. Overshoot mixing could affect this process because of the refueling of ¹²C in the convective core. In the stellar models with a 0.3 H_P fully mixed overshoot region, overshoot mixing is assumed to be instantaneous. In the updated overshoot model, overshoot is a diffusion process, and the diffusion coefficient is not high enough to result in significant mixing in the short timescale of the PMS stage. Therefore, in Figure 3 and the left panel in Figure 2, the solid lines and the thick solid lines are located between the dotted lines and the dashed lines, and are very close to the dotted lines.

3.2. Properties of the Updated Overshoot Mixing Model in the Core Overshoot Region of Low-mass Stars

The hydrogen abundances in the stellar interior models are shown in Figure 5. The models for $1.5 M_{\odot}$ in the three cases, i.e., the standard model, the model with classical overshoot with $0.3H_P$, and the updated overshoot model with $C_{OV} = 10^{-3}$, and



Figure 3. Similar to Figure 1, but for χ^2 Hya.





Figure 5. Hydrogen abundance in the stellar interior. The arrows indicate the convective boundary of the models for the solid lines.

with different center hydrogen abundance ($X_C \approx 0.57, 0.40, 0.23$) are shown. It is found that the updated overshoot model results in a smooth profile of the hydrogen abundance. Near the convective boundary, the gradient is close to zero, indicating a large diffusion coefficient. Compared with the standard stellar models and the classical overshoot stellar models, the stellar models with the updated overshoot model show effects similar to the classical overshoot model with 0.3 H_p for refueling the



Figure 6. Diffusion coefficient of the convective overshoot mixing. *D* is the diffusion coefficient in units of $\text{cm}^2 \text{ s}^{-1}$. *P* is the pressure in the stellar interior in units of $\text{g cm}^{-1} \text{ s}^{-2}$.

core. This can be validated by estimating the difference of the areas below the solid lines and corresponding dotted lines. The distinction is that the hydrogen abundances of the updated overshoot model are always smooth, unlike the fact that there are no derivatives at the fully mixing boundaries in the classical overshoot model. The updated overshoot model showing effects similar to the classical overshoot model with 0.3 H_{P} explains that, as mentioned above, the evolutionary tracks of the two cases are identical and the calibrated metallicity and age are consistent with the suggested values based on stellar models with classical overshoot.

The formula for the diffusion coefficient of the updated overshoot mixing model (e.g., Equation (26) in Zhang 2013) has shown that the diffusion coefficient is very large in the convection zone and is low in the overshoot region, which intrinsically ensures a fully mixed convection zone and a partially mixed overshoot region (Zhang 2013). The approximation of the formula for the diffusion coefficient in the overshoot region is Equation (2), which is adopted in the numerical calculations. Figure 6 shows the profile of the diffusion coefficient for overshoot mixing. The stellar model is for the 1.5 M_{\odot} star with $X_C = 0.4$ and $C_{\rm OV} = 10^{-3}$. According to Equation (1), the diffusion coefficient in the convection zone is as large as $D \sim 10^{16}$. We set the upper limit of D to be 10^{10} in the calculations because an excessively large diffusion coefficient may lead to numerical instability and $D > 10^{10}$ ensures complete mixing. It is shown that D quickly decreases in a thin layer near the convective boundary and then exponentially decreases in the most of the overshoot region. After the quick decreasing, the geometric mean diffusion coefficient in the overshoot region is typically 10^2 and 10^0 for 0.5 H_P and 1 H_P, respectively. Accordingly, the timescale for a length L showing obvious mixing is $\tau \sim L^2/D$, i.e., about 10^{16} s for 0.5 H_P and about 10^{20} s for 1 H_P (H_P $\approx 10^{10}$ cm here). The former is of the same order of magnitude as the evolutionary timescale and the latter is much larger than the evolutionary timescale.

In the standard stellar model of a low-mass main sequence star with a convective core, a phenomenon called semi-convection occurs. There is a region near the convective boundary where $\nabla_R > \nabla_{ad}$ when this region is not mixed into the convective core, or $\nabla_{R} < \nabla_{ad}$ when this region is fully mixed into the convective core. This leads to the contradiction that the fully mixed region is not the convection boundary. In the framework of the classical ideal of local convection, this region should be partially mixed to reach the convective neutral condition $\nabla_{\rm R} = \nabla_{\rm ad}$, since the mixing process cannot continue when the neutral condition is satisfied (Schwarzschild & Härm 1958). The intrinsic reason for this phenomenon is that the mixing timescale is much less than the evolutionary timescale (the mixing timescale is zero in classical ideal local convection, e.g., instantaneous mixing), and thus we do not have sufficient time resolution to trace the variation of $\nabla_{\rm R}$ during the mixing process. However, semiconvection does not appear in our stellar models with the updated overshoot mixing model. Figure 7 shows the profiles of $\nabla_{\rm R}$ and $\nabla_{\rm ad}$ in stellar models with $M = 1.5 M_{\odot}$ and $X_C \approx 0.4$. The left panel shows the standard model and the right panel shows the model with updated overshoot mixing. In the region denoted as "SC", it is clearly shown that $\nabla_R > \nabla_{ad}$ when the region is not mixed and $\nabla_R < \nabla_{ad}$ when the region is fully mixed into the convective core. The radiative temperature gradient $\nabla_{\mathbf{R}}$ is discontinuous at the fully mixing boundary due to the jump of the chemical abundance. In the model with updated overshoot,



Figure 7. Convection status near the convective boundary for the standard stellar model and the stellar model with the updated overshoot model. ∇_R and ∇_{ad} are the dimensionless radiative temperature gradient and the adiabatic radiative temperature gradient, respectively.

there is no such phenomenon. The radiative temperature gradient is continuous because the updated overshoot model describes a weak mixing process and the profile of chemical abundance is continuous. Xiong (1981, 1986) has found that semi-convection in massive stars results from the local convection theory and can be removed by using the non-local turbulent convection theory. Our calculations show the similar result that the nonlocal effect of turbulent convection, i.e., overshoot mixing, can remove semi-convection in low-mass stars.

4. CONCLUSIONS

In this paper, we have used observations of four eclipsing binary stars, i.e., HY Vir, YZ Cas, χ^2 Hya, and VV Crv, to calibrate the updated overshoot mixing model recently developed by Zhang (2013). In addition, we have investigated the basic properties of stellar structures based on this model. The main results are as follows.

The dimensionless parameter in the updated overshoot mixing model is suggested to be $C_{\rm OV} = 10^{-3}$ for low-mass stars. Stellar models with this value can fit the observations of the concerned eclipsing binary stars in 1 δ . No obvious dependency of $C_{\rm OV}$ on the stellar mass is found in low-mass stars with $1.2 < M/M_{\odot} < 2.5$. The suggested value of $C_{\rm OV}$ in this paper is the same as the value in Zhang (2013), but is obtained using a different method.

The updated formula for overshoot mixing shows that the diffusion coefficient quickly decreases near the convective boundary and exponentially decreases in most of the overshoot region. This leads to a partial mixing region outside the convective core. The efficiency of overshoot mixing is high in the thin layer near the convective boundary due to the high diffusion coefficient, but is low in most of the overshoot region. Semi-convection, which appears in the standard stellar models of a low-mass star with a convective core, is removed by partial overshoot mixing.

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APPENDIX

THE CALCULATIONS OF THE STANDARD ERRORS

In this Appendix, we show the details for obtaining standard errors of (C_{OV}, X, Z, t) based on the standard errors of observed $(\lg T_A, \lg T_B, R_A, R_B, M_A, M_B)$.

The effective temperature and the radius of a star are determined by the mass M, overshoot parameter C_{OV} , hydrogen abundance X, metallicity Z, and age t, i.e.,

$$T = T(M, C_{OV}, X, Z, t), R = R(M, C_{OV}, X, Z, t).$$
(A1)

Accordingly, we determine the relation between variations of (C_{OV}, X, Z, t) and variations of $(\lg T_A, \lg T_B, R_A, R_B, M_A, M_B)$ as

follows:

$$\begin{bmatrix} dC_{\rm OV} \\ dX \\ dZ \\ dt \end{bmatrix} = \begin{bmatrix} \frac{\partial \lg T_A}{\partial C_{\rm OV}} & \frac{\partial \lg T_A}{\partial X} & \frac{\partial \lg T_A}{\partial Z} & \frac{\partial \lg T_A}{\partial t} \\ \frac{\partial \lg T_B}{\partial C_{\rm OV}} & \frac{\partial \lg T_B}{\partial X} & \frac{\partial \lg T_B}{\partial Z} & \frac{\partial \lg T_B}{\partial t} \\ \frac{\partial R_A}{\partial C_{\rm OV}} & \frac{\partial R_A}{\partial X} & \frac{\partial R_A}{\partial Z} & \frac{\partial R_A}{\partial t} \\ \frac{\partial R_B}{\partial C_{\rm OV}} & \frac{\partial R_B}{\partial X} & \frac{\partial R_B}{\partial Z} & \frac{\partial R_B}{\partial t} \end{bmatrix}^{-1} \\ \times \begin{bmatrix} d \lg T_A - \frac{\partial \lg T_A}{\partial M_A} dM_A \\ d \lg T_B - \frac{\partial \lg T_B}{\partial M_B} dM_B \\ dR_B - \frac{\partial R_B}{\partial M_B} dM_B \end{bmatrix}; \quad (A2)$$

where all derivatives are in independent variable sets (M, C_{OV}, X, Z, t) . All derivatives can be worked out numerically by alternately comparing the stellar model with corresponding (M, C_{OV}, X, Z, t) with the stellar models with small variations.

Equation (A2) shows the linear relations between (dC_{OV}, dX, dZ, dt) and $(dlgT_A, dlgT_B, dR_A, dR_B, dM_A, dM_B)$. The derivatives of (C_{OV}, X, Z, t) with respect to $(lgT_A, lgT_B, R_A, R_B, M_A, M_B)$ (also an independent variable set) can be calculated based on Equation (A2). For example, $\partial(C_{OV}, X, Z, t)/\partial R_A$ can be worked out by setting $(dlgT_A, dlgT_B, dR_A, dR_B, dM_A, dM_B) = (0, 0, 1, 0, 0, 0)$ in the r.h.s. and then the final result vector of the r.h.s. is $\partial(C_{OV}, X, Z, t)/\partial R_A$.

When the variables (x_i) are independent of each other, the standard errors of their functions $y_j = y_j(x_i)$ based on the assumption of a Gaussian distribution are as follow:

$$\sigma^{2}(y_{i}) = \sum_{j} \left(\frac{\partial y_{i}}{\partial x_{j}}\right)^{2} \sigma^{2}(x_{j}).$$
(A3)

However, in our case, the effective temperatures of the two components of an eclipsing binary star are highly dependent, and the ratio T_A/T_B is more accurate (Claret 2007). This leads to the restriction $dlgT_A \approx dlgT_B$. In this case, we define $dlgT = dlgT_A = dlgT_B$ in Equation (A2) and calculate the derivatives of (C_{OV}, X, Z, t) with respect to (lgT, R_A, R_B, M_A, M_B) based on that equation. (lgT, R_A, R_B, M_A, M_B) are assumed to be independent, and thus the standard errors of (C_{OV}, X, Z, t) can be worked out by using Equation (A3), where σlgT is calculated as $\sigma lgT = (\sigma lgT_A + \sigma lgT_B)/2$.

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