# ACCRETION ONTO INTERMEDIATE-MASS BLACK HOLES REGULATED BY RADIATIVE FEEDBACK. I. PARAMETRIC STUDY FOR SPHERICALLY SYMMETRIC ACCRETION

KWANGHO PARK AND MASSIMO RICOTTI

Department of Astronomy, University of Maryland, College Park, MD 20740, USA; kpark@astro.umd.edu, ricotti@astro.umd.edu Received 2010 June 7; accepted 2011 June 17; published 2011 August 26

## ABSTRACT

We study the effect of radiative feedback on accretion onto intermediate-mass black holes (IMBHs) using the hydrodynamical code ZEUS-MP with a radiative transfer algorithm. In this paper, the first of a series, we assume accretion from a uniformly dense gas with zero angular momentum and extremely low metallicity. Our onedimensional (1D) and 2D simulations explore how X-ray and UV radiation emitted near the black hole regulates the gas supply from large scales. Both 1D and 2D simulations show similar accretion rates and periods between peaks in accretion, meaning that the hydro-instabilities that develop in 2D simulations do not affect the mean flow properties. We present a suite of simulations exploring accretion across a large parameter space, including different radiative efficiencies and radiation spectra, black hole masses, density, and temperature,  $T_{\infty}$ , of the neighboring gas. In agreement with previous studies, we find regular oscillatory behavior of the accretion rate, with duty cycle ~6%, mean accretion rate 3%  $(T_{\infty}/10^4 \text{ K})^{2.5}$  of the Bondi rate and peak accretion ~10 times the mean for  $T_{\infty}$ ranging between 3000 K and 15,000 K. We derive parametric formulae for the period between bursts, the mean accretion rate, and the peak luminosity of the bursts and thus provide a formulation of how feedback-regulated accretion operates. The temperature profile of the hot ionized gas is crucial in determining the accretion rate, while the period of the bursts is proportional to the mean size of the Strömgren sphere, and we find qualitatively different modes of accretion in the high versus low density regimes. We also find that a softer radiation spectrum produces a higher mean accretion rate. However, it is still unclear what the effect of a significant time delay is between the accretion rate at our inner boundary and the output luminosity. Such a delay is expected in realistic cases with non-zero angular momentum and may affect the time-dependent phenomenology presented here. This study is a first step to model the growth of seed black holes in the early universe and to make a prediction of the number and the luminosity of ultraluminous X-ray sources in galaxies produced by IMBHs accreting from the interstellar medium.

*Key words:* accretion, accretion disks – black hole physics – dark ages, reionization, first stars – hydrodynamics – methods: numerical – radiative transfer

Online-only material: color figures

## 1. INTRODUCTION

The occurrence of gas accretion onto compact gravitating sources is ubiquitous in the universe. The Bondi accretion formula (Bondi & Hoyle 1944; Bondi 1952), despite the simplifying assumption of spherical symmetry, provides a fundamental tool for understanding the basic physics of the accretion process. Angular momentum of accreted gas, in nearly all realistic cases, leads to the formation of an accretion disk on scales comparable to or possibly much greater than the gravitational radius of the black hole,  $r_g \sim GM/c^2$ , thus breaking the assumption of spherical symmetry in the Bondi solution. However, the fueling of the disk from scales larger than the circularization radius  $r_c \sim j^2/GM$ , where j is the gas specific angular momentum, can be approximated by a quasiradial inflow. Thus, assuming that numerical simulations resolve the sonic radius,  $r_s$ , the resolved gas flow is quasi-spherical if  $r_c \ll r_s$ . The Bondi formula, which links the accretion rate to the properties of the environment, such as the gas density and temperature, or the Eddington-limited rate is often used in cosmological simulations to model the supply of gas to the accretion disk from galactic scales (Volonteri & Rees 2005; Di Matteo et al. 2008; Pelupessy et al. 2007; Greif et al. 2008; Alvarez et al. 2009).

However, the Bondi formula is a crude estimation of the rate of gas supply to the accretion disk because it does not take into account the effect of accretion feedback loops on the surrounding environment. Radiation emitted by black holes originates from gravitational potential energy of inflowing gas (Shapiro 1973) and a substantial amount of work has been performed to understand the simplest case of spherical accretion onto compact X-ray sources or quasars. Several authors have used hydrodynamical simulations to explore how feedback loops operate and whether they produce time-dependent or steady accretion flows. A variety of feedback processes have been considered: X-ray preheating, gas cooling, photo-heating, and radiation pressure (Ostriker et al. 1976, 2010; Cowie et al. 1978; Bisnovatyi-Kogan & Blinnikov 1980; Krolik & London 1983; Vitello 1984; Wandel et al. 1984; Milosavljević et al. 2009a; Novak et al. 2010). Typically, the dominance of one process over the others depends on the black hole mass and the properties of the gas accreted by the black hole. The qualitative description of the problem is simple: gravitational potential energy is converted into other forms of energy such as UV and X-ray photons or jets, which act to reduce and reverse the gas inflow, either by heating the gas or by producing momentumdriven outflows (Ciotti & Ostriker 2007; Ciotti et al. 2009; Proga 2007; Proga et al. 2008). In general, these feedback processes reduce the accretion rate and thus the luminosity of the accreting black hole (Ostriker et al. 1976; Begelman 1985; Ricotti et al. 2008). Consequently, the time-averaged accretion rate differs from Bondi's solution. There have been works on self-regulated accretion of supermassive black holes (SMBHs) at the center of elliptical galaxies (Sazonov et al. 2005; Ciotti & Ostriker 2007; Ciotti et al. 2009) and radiation-driven axisymmetric outflow in active galactic nuclei (Proga 2007; Proga et al. 2008; Kurosawa et al. 2009; Kurosawa & Proga 2009a, 2009b). However, far less has been done in order to quantify the simplest case of spherical accretion onto IMBHs as a function of the properties of the environment. Recent theoretical (Milosavljević et al. 2009a, hereafter MBCO09) and numerical (Milosavljević et al. 2009b, hereafter MCB09) works explore accretion of protogalactic gas onto IMBHs in the first galaxies. MCB09 describe the accretion onto a  $100 M_{\odot}$  black hole from protogalactic gas of density  $n_{\rm H,\infty} = 10^7 \text{ cm}^{-3}$  and temperature  $T_{\infty} = 10^4 \text{ K}$ . Our study, which complements this recent numerical work, is a broader investigation of accretion onto IMBHs for a set of several simulations with a wide range of radiative efficiencies, black hole masses, densities, and sound speeds of the ambient gas. Our aim is to use simulations to provide a physically motivated description of how radiation modifies the Bondi solution and provide an analytical formulation of the problem (see MBCO09).

The results of the present study will help to better understand the accretion luminosities of IMBHs at high z and in the presentday universe (Ricotti 2009). Applications of this work include studies on the origin of ultraluminous X-ray sources (ULXs; Krolik et al. 1981; Krolik & Kallman 1984; Ricotti 2007; Mack et al. 2007; Ricotti et al. 2008), buildup of an early X-ray background (Venkatesan et al. 2001; Ricotti & Ostriker 2004; Madau et al. 2004; Ricotti et al. 2005), and growth of SMBHs (Volonteri et al. 2003; Volonteri & Rees 2005; Johnson & Bromm 2007; Pelupessy et al. 2007; Alvarez et al. 2009). For example, different scenarios have been proposed for the formation of quasars at  $z \sim 6$  (Fan et al. 2003): growth by mergers, accretion onto IMBHs, or direct formation of larger seed black holes from collapse of quasi-stars (Carr et al. 1984; Haehnelt et al. 1998; Fryer et al. 2001; Begelman et al. 2006; Volonteri et al. 2008; Omukai et al. 2008; Regan & Haehnelt 2009; Mayer et al. 2010) that may form from metal-free gas at the center of rare dark matter halos (Oh & Haiman 2002). Understanding the properties which determine the efficiency of self-regulated accretion onto IMBHs is important to estimate whether primordial black holes produced by Pop III stars can accrete fast enough to become SMBHs by redshift  $z \sim 6$ (Madau & Rees 2001; Volonteri et al. 2003; Yoo & Miralda-Escudé 2004; Volonteri & Rees 2005; Johnson & Bromm 2007; Pelupessy et al. 2007; Alvarez et al. 2009).

In this paper, we focus on simulating accretion onto IMBH regulated by photo-heating feedback in one-dimensional (1D) and 2D hydrodynamic simulations, assuming spherically symmetric initial conditions. We provide fitting formulae for the mean and peak accretion rates, and the period between accretion rate bursts as a function of the parameters we explore, including radiative efficiency, black hole mass, gas density, temperature, and spectrum of radiation. In Section 2, we introduce basic concepts and definitions in the problem. Numerical procedures and physical processes included in the simulations are discussed in Section 3. Our simulation results and the parameter study are shown in Section 4. In Section 5, we lay out a physically motivated model that describes the results of the

simulations. Finally, a summary and discussion are given in Section 6.

## 2. BASIC DEFINITIONS

#### 2.1. Bondi Accretion and Eddington Luminosity

The assumption of spherical symmetry allows us to treat accretion problems analytically. The solution (Bondi 1952) provides the typical length scale at which gravity affects gas dynamics and the typical accretion rate as a function of the black hole mass  $M_{\rm bh}$ , ambient gas density  $\rho_{\infty}$ , and sound speed  $c_{s,\infty}$ . The Bondi accretion rate for a black hole at rest is

$$\dot{M}_B = 4\pi \lambda_B r_b^2 \rho_\infty c_{s,\infty}$$
  
=  $4\pi \lambda_B \rho_\infty \frac{G^2 M_{bh}^2}{c_{s,\infty}^3},$  (1)

where  $r_b = GM_{bh}c_{s,\infty}^{-2}$  is the Bondi radius and  $\lambda_B$  is the dimensionless mass accretion rate, which depends on the polytropic index,  $\gamma$ , of the gas equation of state  $P = K\rho^{\gamma}$ :

$$\lambda_B = \frac{1}{4} \left[ \frac{2}{5 - 3\gamma} \right]^{\frac{5 - 3\gamma}{2(\gamma - 1)}}.$$
 (2)

The value of  $\lambda_B$  ranges from  $e^{3/2}/4 \simeq 1.12$  for an isothermal gas ( $\gamma = 1$ ) to 1/4 for an adiabatic gas ( $\gamma = 5/3$ ).

However, a fraction of the gravitational potential energy of the inflowing gas is necessarily converted into either radiation or mechanical energy when it approaches the black hole, significantly affecting the accretion process. Photons emitted near the black hole heat and ionize nearby gas, creating a hot bubble which exerts pressure on the inflowing gas. Radiation pressure may also be important in reducing the rate of gas inflow (see MBCO09). These processes may act as self-regulating mechanisms limiting gas supply to the disk from larger scales and, thus, controlling the luminosity of the black hole. We quantify the reduction of the accretion rate with respect to the case without radiative feedback by defining the dimensionless accretion rate

$$\lambda_{\rm rad} \equiv \frac{\dot{M}}{\dot{M}_B},\tag{3}$$

where  $\dot{M}_B$  is the Bondi accretion rate for isothermal gas ( $\dot{M}_B = e^{3/2}\pi G^2 M_{bh}^2 \rho_{\infty} c_{s,\infty}^{-3}$ ). This definition of  $\lambda_{rad}$  is consistent with the one adopted by MBCO09.

## 2.2. Luminosity and Radiative Efficiency

The Eddington luminosity sets an upper limit on the luminosity of a black hole. In this limit the inward gravitational force on the gas equals the radiation pressure from photons interacting with electrons via Compton scattering. Although this limit can be avoided in some special cases, observations suggest that black hole and SMBH luminosities are sub-Eddington. The Eddington luminosity is thus

$$L_{\rm Edd} = \frac{4\pi G M_{\rm bh} m_p c}{\sigma_T} \simeq 3.3 \times 10^6 L_{\odot} \left(\frac{M_{\rm bh}}{100 \, M_{\odot}}\right). \quad (4)$$

The luminosity of an accreting black hole is related to the accretion rate via the radiative efficiency  $\eta$ :  $L = \eta \dot{M} c^2$ . From

the Eddington luminosity, we define the Eddington gas accretion rate  $\dot{M}_{\rm Edd} \equiv L_{\rm Edd}c^{-2}$ , and the dimensionless accretion rate and luminosity as

$$\dot{m} \equiv \frac{\dot{M}}{\dot{M}_{\rm Edd}}$$
 and  $l \equiv \frac{L}{L_{\rm Edd}}$ . (5)

Hence, in dimensionless units, the bolometric luminosity of the black hole is  $l = \eta \dot{m}$ , where  $\dot{m}$  is the accretion rate onto the black hole. Note that our definition of  $M_{\rm Edd}$  is independent of the radiative efficiency  $\eta$ . Therefore, if we impose the sub-Eddington luminosity of the black hole, the dimensionless accretion rate ranges between  $0 < \dot{m} \leq \frac{1}{n}$ . The radiative efficiency,  $\eta$ , depends on the geometry of the accretion disk and on  $\dot{m}$ . For a thin disk,  $\eta \simeq 0.1$ , whereas  $\eta \propto \dot{m}$  for an advectiondominated thick disk or for spherical accretion (Shapiro 1973; Park & Ostriker 2001). In this study we consider two idealized cases for the radiative efficiency: the case of constant radiative efficiency  $\eta$  = constant, and the case in which the radiative efficiency has a dependence on the dimensionless accretion rate and luminosity,  $\eta = \text{constant}$  for  $l \ge 0.1$  and  $\eta \propto \dot{m}$  for l < 0.1. The second case we explored accounts for the lower radiative efficiency expected when the accreted gas does not settle into a thin disk. In both formulations, the radiative efficiency is one of the free parameters we allow to vary, and we do not find important differences between the two cases. Observations of Sgr A\*, the best studied case of low accretion rate onto an SMBH, suggest that the radiative efficiency is indeed low but not as low as implied by the scaling  $\eta \propto \dot{m}$ . Recent theoretical work by Sharma et al. (2007) demonstrates that there is indeed a floor on the radiative efficiency.

Because the Bondi rate,  $M_B$ , does not include radiation feedback effect, it provides an upper limit on the accretion rate from large scales to radii near the black hole. The Eddington rate provides the maximum accretion rate onto the black hole, limited by radiation feedback at small radii. Thus, numerical simulations are necessary to obtain realistic estimates of the accretion rates. If the accretion rate onto the black hole is lower than the gas accretion from large scales, the accreted material accumulates near the black hole, creating a disk whose mass grows with time. We cannot simulate such a scenario because it is too computationally challenging to resolve a range of scales from the Bondi radius to the accretion disk in the same simulation. Here we assume that accretion onto the black hole is not limited by physical processes taking place on radial distances much smaller than the sonic radius. For instance, even if the angular momentum of the accreted gas is small and the circularization radius  $r_c \ll r_s$ , further inflow will be slowed down with respect to the free-fall rate. The rate of inflow will be controlled by angular momentum loss (e.g., torques due to MHD turbulence) and there will be a delay between the accretion rate at the inner boundary of our simulation  $(r_{\min})$  and the accretion luminosity associated with it. The effect of the aforementioned time delay on the feedback loop is not considered in this paper but will be considered in future works. We also assume that the effect of self-gravity is negligible in our simulations since we have estimated that the mass within the HII region around the black hole is smaller than the black hole mass for  $M_{\rm bh} < 1000 M_{\odot}$ .

If the rate of gas supply to the disk is given by the Bondi rate, accretion onto the black hole is sub-Eddington for black hole masses

$$M_{\rm bh} < \frac{c_{s,\infty}^3}{Gn_{\rm H,\infty}\sigma_T c\eta} \sim 40 \, M_\odot \, T_{\infty,4}^{1.5} n_{\rm H,5}^{-1} \eta_{-1}^{-1}, \tag{6}$$

where we use the notations of  $T_{\infty,4} \equiv T_{\infty}/(10^4 \text{ K})$ ,  $n_{\text{H},5} \equiv n_{H,\infty}/(10^5 \text{ cm}^{-3})$ , and  $\eta_{-1} \equiv \eta/10^{-1}$ . Thus, in this regime we may assume that the accretion is quasi-steady in the sense that the mean accretion rate onto the black hole equals the gas supply from large scales when the accretion rate is averaged over a sufficiently long timescale.

#### **3. NUMERICAL SIMULATIONS**

#### 3.1. ZEUS-MP and Radiative Transfer Module

We perform a set of hydrodynamic simulations to understand accretion onto IMBHs regulated by radiation feedback. Numerical simulations of radiative feedback by black holes are challenging because they involve resolving a large dynamical range in length scales. In this study, we use ZEUS-MP (Hayes et al. 2006), a modified parallel version of the nonrelativistic hydrodynamics code ZEUS (Stone & Norman 1992). For the present work, we add a radiative transfer module (Ricotti et al. 2001) to ZEUS-MP to simulate the radiative transfer of UV and X-ray ionizing photons emitted near the black hole. A detailed description of the numerical methods used to solve radiative transfer and tests of the code are presented in Appendices A, B, and C.

As X-ray and UV photons ionize the surrounding medium, different reactions take place depending on the density and composition of the gas. Photo-ionization changes the ionization fraction of H and He. The detailed evolution of the Strömgren sphere depends on the cooling function  $\Lambda(T, Z)$  of the gas and thus on the metallicity, Z, and the fraction of gas in the molecular phase. For a gas of primordial composition, the cooling rate depends on the formation rates of  $H^-$  and  $H_2$ , which depend on both the redshift and the intensity of the local dissociating background in the  $H_2$  Lyman–Werner bands (e.g., Shapiro & Kang 1987; Abel et al. 1998; Ricotti et al. 2002a, 2002b). In addition, the cooling function may depend on redshift due to Compton cooling of the electrons by cosmic microwave background (CMB) photons. We adopt atomic hydrogen cooling for temperatures  $T > 10^4$  K, and use a simple parametric function to model complicated cooling physics of gas at T <10<sup>4</sup> K. Thus, the temperature structure inside the ionized bubble is appropriate only for a low-metallicity gas. For a subset of simulations we also include the effect of helium photo-heating and cooling. We assume that gas cooling at temperatures below  $T_{\infty}$  is negligible in order to achieve thermal equilibrium in the initial conditions far from the black hole. For the parameter space in which we can neglect the effect of radiation pressure, we find (see Section 5.1) that the accretion rate is a function of the temperature both outside and inside the HII region. The temperature outside the HII region depends on the cooling function of gas at  $T < 10^4$  K and on the heating sources. The temperature inside the H II region depends on the spectrum of radiation and cooling mechanism of gas at  $T > 10^4$  K. Thus, it depends on the gas metallicity and the redshift at which Compton cooling might become important. However, for the parameter space we have explored, we find that Compton cooling has a minor effect on the temperature inside and outside the Strömgren sphere.

The gas heating rate depends on the flux and spectral energy distribution (SED) of the radiation emitted near the black hole. We assume a luminosity of the black hole  $l = \eta \dot{m}$  (see Section 2.2), where  $\dot{m}$  is calculated at the inner boundary in our simulation (typically  $r_{\rm min} \sim 3 \times 10^{-5}$  pc). We adopt a single power law  $\nu^{-\alpha}$  for the SED, where the spectral

index  $\alpha$  is one of the parameters we vary in our set of simulations.

We use an operator-split method to calculate the hydrodynamic step and the radiative transfer and chemistry steps. The hydrodynamic calculation is done using ZEUS-MP, then for the radiative transfer calculation we use a ray-tracing module (Ricotti et al. 2001). The radiative transfer module calculates chemistry, cooling, and heating processes along rays outgoing from the central black hole, and thus is easily parallelized in the polar angle direction.

We perform 1D and 2D simulations in spherical coordinates. In both cases, we use a logarithmically spaced grid in the radial direction typically with 256–512 cells to achieve high resolution near the black hole. The size ratio between consecutive grids is chosen according to the free parameters of the simulation to resolve the ionization front and resolve the region where the gas is in free fall. In the 2D simulations we use evenly spaced grids in the polar angle direction and compute radiative transfer solutions in each direction. Flow-out inner boundary conditions and flow-in outer boundary conditions are used in the radial direction (r), whereas in polar angle directions ( $\theta$ ), reflective boundary conditions are used.

To determine the optimal box size of the simulations we make sure that we resolve important length scales in the problem: the inner Bondi radius,  $r_{b,in}$ , the outer Bondi radius,  $r_{b,\infty}$ , the sonic radius,  $r_s$ , and the ionization front,  $R_s$ . We select the value of the inner boundary (typically  $\sim 3 \times 10^{-5}$  pc for  $M_{\rm bh} = 100 M_{\odot}$ ) to be smaller than the sonic point or the inner Bondi radius (both still far larger than the Schwarzschild radius of the black holes). We find that once the sonic radius is resolved, reducing the inner boundary box size does not create significant differences in the results. In most cases the ionization front is located outside of the outer Bondi radius and the box size is selected to be large enough to cover both length scales. We select a box size that achieves the highest possible resolution with a given number of grids, making sure that the physical quantities around boundaries remain constant during the simulations. The box is sufficiently large to minimize the effect of spurious wave reflections at the outer boundary.

In this paper, the first of a series, we adopt idealized initial conditions of uniform density and temperature, zero velocity, and zero angular momentum of the gas relative to the black hole. In future work we will relax some of these assumptions by adding turbulence in the initial condition and considering the effect of black hole motion with respect to the ambient medium and considering the effect of a time delay between the accretion rate at the inner boundary of our simulations and the accretion luminosity. We assume monatomic, non-relativistic ideal gas with  $\gamma = 5/3$ , which is initially neutral (electron fraction  $x_e \sim 10^{-5}$ ). In this paper, we also neglect the effect of radiation pressure. Our goal is to add to the simulations one physical process at a time to understand which feedback loop is dominant in a given subset of the parameter space. We take this approach to attempt an interpretation of the simulation results in the context of a physically motivated analytical description of the accretion cycle. We will explore the effect of radiation pressure due to H I ionization and Ly $\alpha$  scattering in future works. However, a simple inspection of the relevant equations suggests that radiation pressure is increasingly important for large values of the ambient gas density  $(n_{\rm H,\infty} \sim 10^7 {\rm ~cm^{-3}}; {\rm see~MBCO09})$ and Section 6) since the accretion rate approaches the Eddington limit.

## 4. RESULTS

## 4.1. Qualitative Description of Accretion Regulated by Radiative Feedback

Our simulations show that UV and X-ray photons modify the thermal and dynamical structure of the gas in the vicinity of the Bondi radius. A hot bubble of gas is formed due to photo-heating by high-energy photons, and sharp changes of physical properties such as density, temperature, and ionization fraction occur at the ionization front. Figure 1 shows eight snapshots from one of our 2D simulations. The top half of each snapshot shows the gas density and the bottom half shows the hydrogen ionization fraction. We show the periodic oscillation of the density and the ionization fraction from a 2D simulation in Figure 1. The time evolution of the density, temperature, and ionization fraction profiles for the 1D simulation is shown in Figure 2. We can identify the following three evolutionary phases that repeat cyclically.

- 1. Once the Strömgren sphere is formed, it expands and the gas density inside that hot bubble decreases, roughly maintaining pressure equilibrium across the ionization front. At the front, gas inflow is stopped by the hot gas and the average gas density inside the bubble decreases due to the following two physical processes. First, the black hole continues accreting hot gas within an accretion radius,  $r_{\rm acc}$ , defined as the radius where the gravitational force of the black hole dominates the thermal energy of the hot gas. The accretion radius is similar to the Bondi radius defined by the temperature inside Strömgren sphere, but there exists a difference between them since the kinematic and thermal structures of gas is modified significantly by the photo-heating and cooling. Second, the gas between  $r_{\rm acc}$ and the ionization front moves toward the ionization front due to pressure gradients. The left panel of Figure 3 shows inflowing gas within  $r_{\rm acc}$  and outflowing gas outside  $r_{\rm acc}$ . A dense shell forms just outside the ionization front. Thus, the mass of the shell grows because gravity pulls distant gas into the system at the same time when gas within the hot bubble is pushed outward.
- 2. As the average density inside the hot bubble decreases, the accretion rate diminishes. During this process the radius of the Strömgren sphere remains approximately constant since the reduced number of ionizing UV and X-ray photons is still sufficient to ionize the rarefied hot bubble. Figure 5 illustrates this. Thus, the average gas temperature, ionization fraction, and the size of the HII region remain constant. As the accretion rate increases during the burst, it produces a rapid expansion of the Strömgren sphere radius. During one cycle of oscillation, there are small peaks in the Strömgren sphere radius which are associated with minor increases in the accretion rate. Rayleigh-Taylor (RT) instabilities develop quickly when the accretion rate increases. In these phases, the acceleration of the dense shell is directed toward the black hole, so the dense shell, supported by more rarefied gas, becomes RT unstable.
- 3. As gas depletion continues, the pressure inside the hot bubble decreases to the point where equilibrium at the ionization front breaks down. The outward pressure exerted by the hot bubble becomes too weak to support the gravitational force exerted on the dense shell. The dense shell of gas collapses toward the black hole, increasing dramatically the accretion rate and creating a burst of ionizing photons.

THE ASTROPHYSICAL JOURNAL, 739:2 (15pp), 2011 September 20

#### PARK & RICOTTI



**Figure 1.** Evolution of the gas density and ionization fraction in a simulation of an accreting black hole of mass  $M_{bh} = 100 M_{\odot}$ , gas density  $n_{H,\infty} = 10^5 \text{ cm}^{-3}$ , and temperature  $T_{\infty} = 10^4$  K. In each panel, the top halves show the density (number of hydrogen atoms per cm<sup>3</sup>) and the bottom halves show the ionization fraction,  $x_e = n_e/n_H$ , of the gas. The evolutionary sequences are shown in a clockwise direction. Top panels from left to right: a Strömgren sphere forms fueled by ionizing photons as the black hole accretes gas. The higher pressure inside the Strömgren sphere stops the gas inflow while the black hole at the center consumes the hot gas inside the ionization front. Inflowing gas accumulates in a dense shell outside the hot bubble while exponential decay of the accretion rate occurs due to decreasing density inside the hot bubble as gas depletion continues. Although the number of emitted ionizing photons decreases, the ionized sphere maintains its size because of the decrease in density inside the hot bubble. Bottom panels from right to left: the density of hot gas inside the Strömgren sphere keeps decreasing until pressure equilibrium across the front can no longer be maintained. Middle left: the dense shell in front of the Strömgren sphere collapses onto the black hole and this leads to a burst of accretion luminosity. Top left: the Strömgren sphere reaches its maximum size and the simulation cycle repeats.

The ionization front propagates outward in a spherically symmetric manner, creating a large Strömgren sphere and returning to the state where the high pressure inside the Strömgren sphere suppresses gas inflow from outside.

## 4.2. Comparison of 1D and 2D Simulations

In agreement with previous studies, our simulations show that radiation feedback induces regular oscillations of the accretion rate onto the IMBH. This result is in good agreement with numerical work by MCB09 for accretion onto a 100  $M_{\odot}$  black hole from a high density ( $n_{\rm H,\infty} = 10^7 {\rm cm}^{-3}$ ) and high temperature ( $T_{\infty} = 10^4 {\rm K}$ ) gas. Periodic oscillatory behavior is found in all our simulations for different combinations of parameters, when assuming spherically symmetric initial conditions and a stationary black hole. This oscillation pattern is quite regular and no sign of damping is observed for at least ~10 cycles.

For the same parameters, our 1D and 2D simulations are nearly identical in terms of oscillatory behavior in accretion



**Figure 2.** Top to bottom: radial profiles of density, temperature, and neutral/ ionization fractions in 1D simulation for  $\eta = 0.1$ ,  $M_{\rm bh} = 100 M_{\odot}$ ,  $n_{\rm H,\infty} = 10^5 \, {\rm cm}^{-3}$ , and  $T_{\infty} = 10^4 \, {\rm K}$ . Different lines indicate profiles at different times: t = 0.0 (dotted),  $t = 1.13 \times 10^4$  (solid),  $t = 1.28 \times 10^4$  (short dashed),  $t = 1.43 \times 10^4$  (long dashed),  $t = 1.58 \times 10^4$  (dot-short dashed), and  $t = 1.71 \times 10^4$  yr (dot-long dashed). Solid lines: at the maximum expansion of the Strömgren sphere. Dot-long dashed lines: at the collapsing phase of the dense shell. Physical properties inside the Strömgren sphere change as a function of time. The number density and temperature of hydrogen decrease with time after the burst. The neutral fraction increases as a function of time from the burst.



**Figure 4.** Accretion rates as a function of time in 1D and 2D simulations with  $\eta = 0.1$ ,  $M_{\rm bh} = 100 M_{\odot}$ ,  $n_{\rm H,\infty} = 10^5 {\rm cm}^{-3}$ , and  $T_{\infty} = 10^4 {\rm K}$ . Both results show similar oscillation patterns with the same period and average accretion rate.

rate and Strömgren sphere size. Figure 4 shows accretion rate in 1D and 2D simulations for  $M_{\rm bh} = 100 \ M_{\odot}$ ,  $T_{\infty} = 10^4 \ K$ , and  $n_{\rm H,\infty} = 10^5 \ {\rm cm}^{-3}$ . Note the similar pattern in accretion rate and period between bursts. This indicates that the 1D result adequately represents 2D cases when the accretion flow does not have significant angular momentum.

Moreover, this result demonstrates that RT instabilities which we observe in the 2D simulations do not affect the mean accretion rate or the period of oscillations. The RT instability develops during the phase when the dense shell in front of the ionization front is supported against gravitational accretion by the low-density medium inside the hot bubble (Whalen & Norman 2008a, 2008b). The top panels in Figure 1 show small instabilities when ionization fronts move outward, which largely



**Figure 3.** Gas density and velocity field for the simulation with  $\eta = 0.1$ ,  $M_{bh} = 100 M_{\odot}$ ,  $n_{H,\infty} = 10^5 \text{ cm}^{-3}$ , and  $T_{\infty} = 10^4 \text{ K}$ . Left: when a Strömgren sphere is formed, gas inside the hot bubble is depleted by accretion onto the black hole and the outflow toward the dense shell due to pressure gradient. Right: gas depletion inside the Strömgren sphere leads to the collapse of the dense shell, creating a burst of accretion. (A color version of this figure is available in the online journal.)



**Figure 5.** Evolution of the Strömgren radius with time for 2D simulation with  $\eta = 0.1$ ,  $M_{\rm bh} = 100 M_{\odot}$ ,  $n_{\rm H,\infty} = 10^5 \, {\rm cm}^{-3}$ , and  $T_{\infty} = 10^4 \, {\rm K}$ . The solid line shows the mean size of the Strömgren radius and dotted lines show the minimum and maximum Strömgren radii. It shows the same period of oscillation seen in the accretion rate as a function of time. In general, the Strömgren radius is proportional to the accretion rate which determines the number of ionizing photons. When the accretion rate is maximum, the size of the Strömgren sphere also has its maximum size.

decay over time. The pressure gradient inside the Strömgren sphere creates an outward force which helps suppress the development of the instability.

In summary, we believe that 1D simulations can be used in place of higher dimension simulations to determine the cycle and magnitude of the periodic burst of gas accretion onto the IMBH. This allows us to reduce the computational time required to explore a large range of parameter space.

#### 4.3. Parameter Space Exploration

In this section we present the results of a set of 1D simulations aimed at exploring the dependence of the accretion rate and the period of oscillations of the black hole luminosity as a function of the black hole mass  $M_{\rm bh}$ , the ambient gas density  $n_{\rm H,\infty}$ , temperature  $T_{\infty}$ , and the radiative efficiency  $\eta$ . In Section 5 we present results in which we allow the spectrum of ionizing radiation to vary as well. The accretion can be described by three main parameters:  $\tau_{\rm cycle}$ , the mean period between bursts,  $\lambda_{\rm rad,max}$ , the maximum value of the dimensionless accretion rate (at the peak of the burst), and  $\langle \lambda_{\rm rad} \rangle$ , the time-averaged dimensionless accretion rate. These parameters are typically calculated as the mean over ~5 oscillation cycles and the error bars represent the standard deviation of the measurements.

After reaching the peak, the luminosity decreases nearly exponentially on a timescale  $\tau_{on}$  that we identify as the duration of the burst. Both  $\tau_{on}$  and the duty cycle,  $f_{duty}$ , of the black hole activity (i.e., the fraction of time the black hole is active), can be expressed as a function of  $\tau_{cycle}$ ,  $\lambda_{rad,max}$ , and  $\langle \lambda_{rad} \rangle$ :

$$\tau_{\rm on} \equiv \frac{\langle \lambda_{\rm rad} \rangle}{\lambda_{\rm rad,max}} \tau_{\rm cycle},\tag{7}$$

$$f_{\rm duty} \equiv \frac{\tau_{on}}{\tau_{\rm cycle}} = \frac{\langle \lambda_{\rm rad} \rangle}{\lambda_{\rm rad,max}}.$$
 (8)



**Figure 6.** Dependence of accretion rate and period of oscillations on the radiative efficiency  $\eta$ . From top to bottom, the evolution of accretion rate is shown for  $\eta = 0.1, 0.03, 0.01$ , and 0.003. The peak accretion rate does not change much with  $\eta$ , but intervals between oscillations decrease with decreasing  $\eta$ .

The values of  $\lambda_{rad,max}$  and  $f_{duty}$  as a function of the black hole mass, the density, and the temperature of the ambient medium are important for estimating the possibility of detecting IMBHs in the local universe because these values provide an estimate of the maximum luminosity and the number of active sources in the local universe at any time. On the other hand, the mean accretion rate  $\langle \lambda_{rad} \rangle$  is of critical importance for estimating the IMBH growth rate in the early universe.

The four panels in Figure 7 summarize the results of a set of simulations in which we vary the free parameters one at a time. We find that, in most of the parameter space that we have explored, the period of the oscillations and the accretion rates are described by a single or a split power law with slope  $\beta$ . In the following paragraphs, we report the values of  $\beta$  derived from weighted least-squares fitting of the simulation results. The weight is  $1/\sigma$ , where  $\sigma$  is the standard deviation of  $\langle \lambda_{rad} \rangle$ or  $\lambda_{rad,max}$  over several oscillations.

(1) Dependence on the radiative efficiency. First, we explore how the accretion depends on the radiative efficiency  $\eta$ . This parameter describes the fraction,  $\eta$ , of the accreting rest-mass energy converted into radiation while the remaining fraction,  $1 - \eta$ , is added to the black hole mass. We have explored both constant values of the radiative efficiency and the case  $\eta \propto \dot{m}$ for l < 0.1 (see Section 2.2). The simulation results shown in this section are obtained assuming  $\eta$  is constant. We find similar results for  $\langle \lambda_{rad} \rangle$ ,  $\lambda_{rad,max}$ , and  $\tau_{cycle}$  when we assume  $\eta \propto \dot{m}$ . The radiative efficiency for a thin disk is about 10%. Here, we vary  $\eta$  in the range 0.2%–10%. The other free parameters are kept constant with values  $n_{\rm H,\infty} = 10^5 \text{ cm}^{-3}$ ,  $M_{\rm bh} = 100 M_{\odot}$ , and  $T_{\infty} = 10^4 \text{ K}$ . Figure 6 shows the accretion rate as a function of time for different values of the radiative efficiency:  $\eta = 0.1, 0.03, 0.01, \text{ and } 0.003.$  Panel (a) in Figure 7 shows the dependence on  $\eta$  of the three parameters that characterize the accretion cycle. The maximum accretion rate increases mildly with increasing  $\eta$  (log slope  $\beta = 0.13 \pm 0.06$ ). The average accretion rate is  $\langle \lambda_{rad} \rangle \sim 2.9\% \pm 0.2\%$ , which is



Figure 7. For each panel, peak accretion rate, average accretion rate, and period between bursts are shown from top to bottom as a function of a given parameter. Error bars represent the standard deviation around the mean values over ~5 accretion cycles. (a) Dependence on  $\eta$ .  $\langle \lambda_{rad} \rangle \sim \text{constant while } \tau_{cycle} \propto \eta^{1/3}$ . Open symbols indicate the simulations including helium photo-heating and cooling, which show ~41% lower accretion rate and ~42% shorter period. (b) Same plots as a function of  $M_{bh}$ .  $\langle \lambda_{rad} \rangle \sim \text{constant while } \tau_{cycle} \propto M_{bh}^{2/3}$ . (c) Same plots as a function of  $n_{H,\infty}$  of gas. At low densities,  $\tau_{cycle} \propto n_{H,\infty}^{-1/6}$ , whereas at higher density,  $\tau_{cycle} \propto n_{H,\infty}^{-1/3}$ . (d) Same plots as a function rate  $\langle \lambda_{rad} \rangle \propto T_{\infty}^{2.5}$ . With an exception at lowest temperature  $\tau_{cycle} \propto T_{\infty}^{-0.5}$ . (A color version of this figure is available in the online journal.)

nearly independent of  $\eta$  ( $\beta = -0.04 \pm 0.01$ ). The period of the oscillations increases with  $\eta$  as  $\tau_{cycle} \propto \eta^{1/3}$ . We also show the simulation results including helium photo-heating and cooling, shown as open symbols in the same panel of Figure 7. We find that including helium does not change the qualitative description of the results, but does offset the mean accretion rate, which is ~41% lower, and the period of the accretion bursts, which is ~42% shorter. This offset of the accretion rate and period with respect to the case without helium is due to the higher

temperature of the gas inside the HII region surrounding the black hole.

(2) Dependence on black hole mass. We explore a range in black hole mass from  $100 M_{\odot}$  to  $800 M_{\odot}$ , while keeping the other parameters constant ( $\eta = 0.1$ ,  $n_{\rm H,\infty} = 10^5$  cm<sup>-3</sup>, and  $T_{\infty} = 10^4$  K). The results are shown in panel (b) of Figure 7. The mean accretion rate is  $\langle \lambda_{\rm rad} \rangle \sim 2.7\% \pm 0.4\%$  and the maximum accretion rate is  $\lambda_{\rm rad,max} \sim 42\% \pm 12\%$  ( $\beta = -0.26 \pm 0.20$ ). They

are both independent of  $M_{\rm bh}$  within the error of the fit. The period of the bursts is well described by a power-law relation  $\tau_{\rm cycle} \propto M_{\rm bh}^{2/3}$ .

(3) Dependence on gas density of the ambient medium. Panel (c) in Figure 7 shows the dependence of accretion rate and burst period on the ambient gas density,  $n_{\rm H,\infty}$ . We explore a range of  $n_{\rm H,\infty}$  from  $5 \times 10^3$  cm<sup>-3</sup> to  $10^7$  cm<sup>-3</sup>, while keeping the other parameters constant at  $\eta = 0.1$ ,  $M_{\rm bh} = 100 M_{\odot}$ , and  $T_{\infty} = 10^4$  K. For densities  $n_{\rm H,\infty} \ge 10^5$  cm<sup>-3</sup>,  $\langle \lambda_{\rm rad} \rangle$  and  $\lambda_{\rm rad,max}$  are insensitive to  $n_{\rm H,\infty}$  ( $\beta = -0.04 \pm 0.08$  and  $\beta = -0.18 \pm 0.13$ , respectively). However, for  $n_{\rm H,\infty} \le 10^5$  cm<sup>-3</sup>,  $\langle \lambda_{\rm rad} \rangle$  and  $\lambda_{\rm rad,max}$ are proportional to  $n_{\rm H,\infty}^{1/2}$  ( $\beta = 0.44 \pm 0.02$  and  $\beta = 0.37 \pm 0.09$ , respectively).

The bottom of Figure 7(c) shows the effect of density in determining the oscillation period. For densities  $n_{\rm H,\infty} \ge 1/3$  $10^5$  cm<sup>-3</sup>,  $\tau_{\rm cycle}$  is fitted well by a power law with  $\tau_{\rm cycle} \propto n_{\rm H,\infty}^{-1/3}$ , and for the densities  $n_{\rm H,\infty} \leq 10^5 \,{\rm cm}^{-3}$  it is fitted well by a power law  $\tau_{\rm cycle} \propto n_{\rm H,\infty}^{-1/6}$ . However,  $\tau_{\rm cycle}$  at  $n_{\rm H,\infty} = 10^7 \,{\rm cm}^{-3}$  is lower than predicted by the power-law fit for  $n_{\rm H,\infty} \ge 10^5 {\rm cm}^{-3}$ . Although Figures 4 and 6 do not clearly show the magnitude of accretion rate during the inactive phase, it is evident in a log-log plot that accretion rate at minima is four orders of magnitude lower than during the peak of the burst. This is the case for all simulations but the ones with  $n_{\rm H,\infty} = 10^7 {\rm ~cm^{-3}}$  in which the accretion rate at minima is two orders of magnitude higher than in all other simulations. The simulations show that the ambient gas density is an important parameter in determining the accretion luminosity and period between bursts of the IMBH. One of the reasons is that the gas temperature inside the hot ionized bubble and the thickness and density of the dense shell in front of it depend on the density via the cooling function. The drop in the accretion rate we observe at low densities can be linked to an increase of the temperature within the sonic radius with respect to simulations with higher ambient density. This results in an increase in the pressure gradient within the ionized bubble that reduces the accretion rate significantly.

(4) Dependence on the temperature of the ambient medium. Panel (d) in Figure 7 shows the dependence of accretion rate and period of the bursts on the temperature of the ambient medium,  $T_{\infty}$ . We vary  $T_{\infty}$  from 3000 K to 15,000 K while keeping the other parameters constant at  $\eta = 0.1$ ,  $M_{\rm bh} = 100 M_{\odot}$ , and  $n_{\rm H,\infty} = 10^5 \text{ cm}^{-3}$ . We find that  $\langle \lambda_{\rm rad} \rangle$  and  $\lambda_{\rm rad,max}$  depend steeply on  $T_{\infty}$  as  $T_{\infty}^{5/2}$  ( $\beta = 2.44 \pm 0.06$ ). Except for the simulation with  $T_{\infty} = 3000$  K, the period of the accretion cycle is fitted well by a single power law  $\tau_{\rm cycle} \propto T_{\infty}^{-1/2}$ .

## 5. ANALYTICAL FORMULATION OF BONDI ACCRETION WITH RADIATIVE FEEDBACK

In this section we use the fitting formulae for  $\langle \lambda_{\rm rad} \rangle$ ,  $\lambda_{\rm rad,max}$ , and  $\tau_{\rm cycle}$  obtained from the simulations to formulate an analytic description of the accretion process. For ambient densities  $n_{\rm H,\infty} \ge 10^5$  cm<sup>-3</sup>, we have found that the dimensionless mean accretion rate  $\langle \lambda_{\rm rad} \rangle$  depends only on the temperature of the ambient medium. It is insensitive to  $\eta$ ,  $M_{\rm bh}$ , and  $n_{\rm H,\infty}$ . Thus, for  $n_{\rm H,\infty} \ge 10^5$  cm<sup>-3</sup> we find

$$\langle \lambda_{\rm rad} \rangle \sim 3.3\% \, T_{\infty,4}^{5/2} \, n_{\rm H,5}^{-0.04} \sim 3.3\% \, T_{\infty,4}^{5/2},$$
 (9)

while for  $n_{\rm H,\infty} \leq 10^5 {\rm ~cm^{-3}}$  we find

$$\langle \lambda_{\rm rad} \rangle \sim 3.3\% \, T_{\infty,4}^{5/2} \, n_{\rm H,5}^{1/2}.$$
 (10)

As mentioned above, the dependence of  $\langle \lambda_{rad} \rangle$  on the density is due to the increasing temperature inside the ionized bubble at low densities. The period of the accretion cycle depends on all the parameters we have investigated in our simulation. In the range of densities  $n_{\rm H,\infty} \ge 10^5$  cm<sup>-3</sup>, we find

$$\tau_{\text{cycle}} = (6 \times 10^3 \,\text{yr}) \,\eta_{-1}^{\frac{1}{3}} M_{\text{bh},2}^{\frac{2}{3}} \,n_{\text{H},5}^{-\frac{1}{3}} \,T_{\infty,4}^{-\frac{1}{2}}, \tag{11}$$

where we use the notation  $M_{bh,2} \equiv M_{bh}/(10^2 M_{\odot})$ . However, at lower densities  $n_{\rm H,\infty} \leq 10^5 \text{ cm}^{-3}$ , we find

$$\tau_{\text{cycle}} = (6 \times 10^3 \,\text{yr}) \, \eta_{-1}^{\frac{1}{3}} \, M_{\text{bh},2}^{\frac{2}{3}} \, n_{\text{H},5}^{-\frac{1}{6}} \, T_{\infty,4}^{-\frac{1}{2}}, \tag{12}$$

in which only the dependence on  $n_{\rm H,5}$  changes. The different dependence of  $\tau_{\rm cycle}$  on  $n_{\rm H,\infty}$  is associated with a change of the mean accretion rate  $\langle \lambda_{\rm rad} \rangle$  for each density regime. The deviation of  $\tau_{\rm cycle}$  from the power-law fit at  $n_{\rm H,\infty} = 10^7 \text{ cm}^{-3}$  is not associated with any variation of the mean accretion rate. Our value of  $\tau_{\rm cycle}$  for  $n_{\rm H,\infty} = 10^7 \text{ cm}^{-3}$  is in good agreement with the value found by MCB09.

### 5.1. Dimensionless Accretion Rate: $\langle \lambda_{rad} \rangle$

In this section we seek a physical explanation for the relationship between the mean accretion rate  $\langle \lambda_{rad} \rangle$  and the temperature of the ambient medium found in the simulations. The model is valid in all parameter spaces we have explored with a caveat in the low density regime ( $n_{\rm H,\infty} < 3 \times 10^5$  cm<sup>-3</sup>) and at low ambient temperatures ( $T_{\infty} < 3000$  K).

Figure 8 shows the time-averaged temperature profiles for simulations in which we vary  $\eta$ ,  $M_{bh}$ ,  $n_{H,\infty}$ , and  $T_{\infty}$ . In the case of different  $M_{bh}$  the radii are rescaled so that direct comparisons can be made with the case of  $100 M_{\odot}$ . Vertical lines indicate the accretion radius  $r_{acc}$ , inside of which gas is accreted and outside of which gas is pushed out to the ionization front. We find that the value of  $r_{acc}$  is generally insensitive to the parameters of the simulation as is the gas temperature at  $r_{acc}$ .

Accretion onto the black hole of gas inside the hot ionized sphere is limited by the thermal pressure of the hot gas and by the outflow velocity of the gas that is produced by the pressure gradient inside the Strömgren sphere. Thus, the accretion radius,  $r_{acc}$ , is analogous to the inner Bondi radius,  $r_{b,in}$ , modified to take into account the temperature and pressure gradients inside the hot bubble.

Let us assume that the average accretion rate onto the black hole is

$$\langle M \rangle = 4\pi \lambda_B r_{\rm acc}^2 \rho_{\rm in} c_{s,\rm in}, \qquad (13)$$

where  $\rho_{in}$  and  $c_{s,in}$  (and the corresponding temperature  $T_{in}$ ) are the density and the sound speed at  $r_{acc}$ . Based on the results illustrated in Figure 8, we expect the accretion rate to depend only on  $\rho_{in}$ , since  $r_{acc}$  and  $c_{s,in}$  can be taken to be constants.

When a Strömgren sphere is formed, the gas inside the hot bubble expands and its density decreases. Inside the ionization front, the temperature is about  $10^4-10^5$  K. Thus, assuming pressure equilibrium across the ionization front, we find the dependence of  $\rho_{in}$  on  $T_{\infty}$ :

$$\rho_{\rm in} \approx \rho_{\infty} \frac{T_{\infty}}{T_{\rm in}} = \rho_{\infty} \left(\frac{c_{s,\infty}}{c_{s,\rm in}}\right)^2.$$
(14)

We find  $f = r_{\rm acc}/r_{b,\rm in} \sim 1.8$  and the temperature at  $r_{\rm acc}$  is  $T_{\rm in} \sim 4 \times 10^4$  K independent of  $\eta$ ,  $M_{\rm bh}$ ,  $n_{\rm H,\infty}$ , and  $T_{\infty}$  for a fixed spectral index of radiation  $\alpha = 1.5$ . The dimensionless



Figure 8. Time-averaged temperature profiles as a function of simulation parameters. Different vertical lines indicate accretion radii,  $r_{acc}$ , for each parameter. Top left:  $\eta$  ranges from 0.1 (solid line) to 0.003 (long dashed line). Top right:  $M_{bh}$  ranges from 100  $M_{\odot}$  (solid) to 800  $M_{\odot}$  (long dashed). Bottom left: density ranges from 10<sup>4</sup> cm<sup>-3</sup> (solid) to 10<sup>7</sup> cm<sup>-3</sup> (long dashed). Bottom right:  $T_{\infty}$  ranges from 5000 K (solid) to 15,000 K (long dashed).

accretion rate inside of the Strömgren sphere normalized by the Bondi accretion rate in the ambient medium is then

$$\begin{aligned} \langle \lambda_{\rm rad} \rangle &\simeq \lambda_B \frac{r_{\rm acc}^2 \rho_{\rm in} c_{s,\rm in}}{r_{b,\infty}^2 \rho_{\infty} c_{s,\infty}} \simeq \frac{1}{4} (1.8)^2 \left(\frac{\rho_{\rm in}}{\rho_{\infty}}\right) \left(\frac{c_{s,\rm in}}{c_{s,\infty}}\right)^{-3} \\ &\simeq 3\% T_{\infty,4}^{2.5}, \end{aligned} \tag{15}$$

where we have used  $\lambda_B = 1/4$ , appropriate for an adiabatic gas. Thus,  $\langle \lambda_{\rm rad} \rangle \propto T_{\infty}^{5/2}$ , which is in agreement with the simulation result, given that  $r_{\rm acc}$  and  $T_{\rm in}$  remain constant when we change the simulation parameters. However,  $r_{\rm acc}$  and  $T_{\rm in}$  may not stay constant if we modify the cooling or heating function, for instance by increasing the gas metallicity or by changing the spectrum of radiation; this result suggests that the accretion rate is very sensitive to the details of the temperature structure inside the Strömgren sphere, which shows a dependence on  $n_{\rm H,\infty}$ . The temperature profile changes significantly for  $n_{\rm H,\infty} < 3 \times 10^4$  cm<sup>-3</sup> and this is probably the reason why our model does not perfectly fit  $\langle \lambda_{\rm rad} \rangle$  from the simulations in the lower density regime. In the next section we test whether Equation (13) is still a good description of our results when we change the thermal structure inside the H II region.

#### 5.1.1. Dependence on Temperature at Accretion Radius

In this section we study the dependence of the accretion rate on the time-averaged temperature  $T_{\rm in}$  at  $r_{\rm acc}$ . We change the temperature  $T_{\rm in}$  by varying the spectral index  $\alpha$  of the radiation spectrum and by including Compton cooling of the ionized gas by CMB photons. Here we explore the spectral index of the radiation spectrum in the range  $\alpha = 0.5, 1.0, 1.5, 2.0, \text{ and } 2.5,$ with the energy of photons from 10 keV to 100 keV. The other parameters are kept constant at  $\eta = 0.1, M_{\rm bh} = 100 M_{\odot},$  $n_{\rm H,\infty} = 10^5 \text{ cm}^{-3}$ , and  $T_{\infty} = 10^4 \text{ K}.$ 

Figure 9 shows the different time-averaged temperature profiles for different values of  $\alpha$ . Spectra with lower values of the spectral index  $\alpha$  produce more energetic photons for a given bolometric luminosity, increasing the temperature inside the ionized bubble. Simulations show that the averaged accretion rate  $\langle \lambda_{rad} \rangle$  increases for a softer radiation spectrum. Different slopes (0.5  $\leq \alpha \leq 2.5$ ) of the power-law spectrum lead to different  $T_{in}$  (59,000 K to 36,000 K) and  $\langle \lambda_{rad} \rangle$  (0.0076 to 0.0509). Adopting a harder spectrum (with  $\alpha = 0.5$ ) instead of the softer one ( $\alpha = 2.5$ ) increases  $T_{in}$  by a factor of 1.6 and  $\langle \lambda_{rad} \rangle$  decreases by a factor 6.7. The fit to the simulation results in Figure 9 show that  $\langle \lambda_{rad} \rangle$  depends on temperature at  $r_{acc}$  as

$$\langle \lambda_{\rm rad} \rangle \propto T_{\rm in}^{-4} \propto c_{s,\rm in}^{-8}.$$
 (16)



Figure 9. Left: average temperature profiles of the H II region as a function of spectral index  $\alpha$ . Smaller  $\alpha$  results in smaller  $r_{acc}$  and higher  $T_{in}$ . Right: relation between temperature at  $r_{acc}$  and average accretion rate  $\langle \lambda_{rad} \rangle$ . We find  $\langle \lambda_{rad} \rangle \propto T_{in}^{-4}$ .

The dependence on  $c_{s,in}$  differs from Equation (15). However, this is not surprising because in these simulations the values of  $r_{acc}$  and  $c_{s,in}$  do not remain constant while we vary the value of the spectral index  $\alpha$ . This is due to a change of the temperature and pressure gradients within the H II region. The accretion radius,  $r_{acc}$ , can be expressed as a function of the Bondi radius inside the hot bubble,  $r_{b,in} = GM_{bh}c_{s,in}^{-2}$ . From the simulations, we obtain the following relationship between these two radii:

$$f = \frac{r_{\rm acc}}{r_{b,\rm in}} \simeq 1.8 \left(\frac{T_{\rm in}}{4 \times 10^4 \,\rm K}\right)^{-0.7 \pm 0.2}.$$
 (17)

Thus, if our model for the accretion rate summarized by Equation (13) is valid, we should have

$$\begin{aligned} \langle \lambda_{\rm rad} \rangle &\simeq \frac{1}{4} \frac{r_{\rm acc}^2 \rho_{\rm in} c_{s,\rm in}}{r_{b,\infty}^2 \rho_{\infty} c_{s,\infty}} \simeq \frac{(1.8)^2}{4} \left(\frac{\rho_{\rm in}}{\rho_{\infty}}\right) c_{s,\rm in}^{-5.9} c_{s,\infty}^3 \\ &\simeq 3\% T_{\infty,4}^{2.5} \left(\frac{T_{\rm in}}{4 \times 10^4 \,\rm K}\right)^{-4}, \end{aligned} \tag{18}$$

in agreement with the simulation results  $\langle \lambda_{\rm rad} \rangle \propto T_{\infty}^{2.5} T_{\rm in}^{-4}$ , where the dependence on  $T_{\rm in}$  was not explored initially. Thus, Bondi-like accretion on the scale of  $r_{\rm acc}$  is indeed a good explanation of our results. Given the steep dependence of the value of accretion rate  $\langle \lambda_{\rm rad} \rangle$  on  $T_{\rm in}$ , it is clear that it is very sensitive to the details of the thermal structure inside the H II region. This means that  $\langle \lambda_{\rm rad} \rangle$  depends on the spectrum of radiation and gas metallicity.

#### 5.2. Accretion Rate at Peaks and Duty Cycle: $\lambda_{rad,max}$ , $f_{duty}$

We estimate  $f_{duty}$  by comparing  $\lambda_{rad,max}$  and  $\langle \lambda_{rad} \rangle$  using Equation (8). This quantity gives an estimate of what fraction of black holes are accreting gas at the rate close to the maximum. Within the fitting errors, the log slopes of  $\lambda_{rad,max}$  and  $\langle \lambda_{rad} \rangle$  as a function of the parameters  $M_{bh}$  and  $T_{\infty}$  are zero. Thus, we assume that the dimensionless accretion rates are independent of these parameters.

For  $n_{\rm H,\infty} \ge 10^5 \,{\rm cm}^{-3}$ ,  $\lambda_{\rm rad,max}$  can be expressed as  $\lambda_{\rm rad,max} \sim 0.55 \eta_{-1}^{0.13} n_{\rm H,5}^{-0.18} T_{\infty,4}^{2.0}$  and the dependence of  $f_{\rm duty}$  on these parameters can be expressed using Equation (9) as

$$f_{\text{duty}} \sim 6\% \,\eta_{-1}^{-0.13} \,n_{\text{H},5}^{0.14} \,T_{\infty,4}^{0.5},\tag{19}$$

where we include the mild dependence of  $\langle \lambda_{rad} \rangle$  on the density. For  $n_{\rm H,\infty} \leq 10^5 {\rm cm}^{-3}$ ,  $\lambda_{\rm rad,max} \sim 0.55 \eta_{-1}^{0.13} n_{\rm H,5}^{0.37} T_{\infty,4}^{2.0}$  has a different power-law dependence on the density and we get  $f_{\rm duty}$  as

$$f_{\rm duty} \sim 6\% \eta_{-1}^{-0.13} n_{\rm H,5}^{0.07} T_{\infty,4}^{0.5},$$
 (20)

where  $f_{duty}$  shows a milder dependence on the gas density. Thus, we expect about 6% of IMBHs to be accreting near the maximum rate at any given time. This value depends weakly on  $\eta$ ,  $n_{H,\infty}$ , and  $T_{\infty}$ .

### 5.3. Average Period between Bursts: $\tau_{cycle}$

In this section we derive an analytical expression for the period of the luminosity bursts as a function of all the parameters we tested. Although  $\tau_{cycle}$  shows a seemingly complicated power-law dependence on the free parameters, we find that  $\tau_{cycle}$  is proportional to the time-averaged size of the Strömgren sphere. This is shown in Figure 10. The linear relation between  $\tau_{cycle}$  and the average Strömgren radius  $\langle R_s \rangle$  explains the dependence of  $\tau_{cycle}$  on every parameter considered in this work.

The number of ionizing photons created by accretion onto a black hole is determined by the average accretion rate and the radiative efficiency  $\eta$ . The average accretion rate itself can be expressed as a fraction of the Bondi accretion rate  $\langle \lambda_{rad} \rangle$ . Therefore, the average number of ionizing photons emitted near the black hole can be expressed as

$$N_{\rm ion} \propto \eta \langle \lambda_{\rm rad} \rangle \dot{M}_B \propto \eta \langle \lambda_{\rm rad} \rangle \frac{G^2 M_{\rm bh}^2}{c_{s,\infty}^3} \rho_{\infty}.$$
 (21)

It follows that

$$\begin{aligned} \pi_{\text{cycle}} &= t_{\text{out}} \approx \frac{\langle R_s \rangle}{v_{\text{out}}} \propto \left(\frac{3N_{\text{ion}}}{4\pi\alpha_{\text{rec}}n_{\text{H}}^2}\right)^{\frac{1}{3}} \\ &\propto \left(\frac{1}{n_{\text{H}}^2}\right)^{\frac{1}{3}} \left(\eta \langle \lambda_{\text{rad}} \rangle \frac{G^2 M_{\text{bh}}^2}{c_{s,\infty}^3} \rho_{\infty}\right)^{\frac{1}{3}}, \end{aligned} \tag{22}$$

where we find  $v_{\text{out}} \sim \frac{1}{3}c_{s,\text{in}}$ . Ignoring constant coefficients and using Equation (9) for  $n_{\text{H},\infty} \ge 10^5 \text{ cm}^{-3}$ , we find

$$au_{\text{cycle}} \propto \eta^{\frac{1}{3}} M_{\text{bh}}^{\frac{2}{3}} n_{\text{H},\infty}^{-\frac{1}{3}} T_{\infty}^{-\frac{1}{2}},$$
 (23)



**Figure 10.** Period of accretion bursts as a function of the average Strömgren radius. All simulation results from all the parameters are plotted together. The average size of the Strömgren sphere shows a linear relation with the period  $\tau_{cycle}$ . The only exception happens at the highest density ( $n_{H,\infty} = 10^7 \text{ cm}^{-3}$ ), but this result is in agreement with the work of MCB09 (symbol with error bar). (A color version of this figure is available in the online journal.)

or using Equation (10) for  $n_{\rm H,\infty} \leq 10^5$  cm<sup>-3</sup>, we find

$$\tau_{\rm cycle} \propto \eta^{\frac{1}{3}} M_{\rm bh}^{\frac{2}{3}} n_{\rm H,\infty}^{-\frac{1}{6}} T_{\infty}^{-\frac{1}{2}},$$
(24)

which are exactly as in the empirical fitting formulae in both density regimes and also in good agreement with the analytical work by MBCO09. This explains the dependence of  $\tau_{cycle}$  on any tested parameter  $\eta$ ,  $M_{bh}$ ,  $n_{H,\infty}$ , and  $T_{\infty}$ . In Figure 10 we also show simulation results assuming  $\eta \propto \dot{m}$ . All simulations show the same relationship between  $\tau_{cycle}$  and  $\langle R_s \rangle$ . However, the simulation with the highest ambient density ( $n_{H,\infty} = 10^7 \text{ cm}^{-3}$ ) deviates from the linear relationship, but is in agreement with the numerical simulation of MCB09. It appears that in the high density regime,  $\tau_{cycle}$  decreases steeply with decreasing  $\langle R_s \rangle$ .

We can interpret  $\tau_{cycle}$  as the timescale at which the gas inside H<sub>II</sub> region gets depleted. If the gas depletion inside the Strömgren sphere is dominated by the outward gas flow, then  $\tau_{cycle} \propto \langle R_s \rangle / c_{s,in}$ , in agreement with the empirical linear relation in Figure 10. However, the depletion timescale may be different if the accretion by the black hole dominates gas consumption inside the Strömgren sphere. We can derive this timescale as

$$t_{\rm in} = \frac{M_{\rm H\,II}}{\dot{M}} = \left(\frac{\langle R_s \rangle}{r_{\rm acc}}\right)^2 \frac{\langle R_s \rangle}{3c_{s,\rm in}} \sim \left(\frac{\langle R_s \rangle}{r_{\rm acc}}\right)^2 \frac{t_{\rm out}}{9}.$$

Roughly, we expect  $\tau_{\text{cycle}} = \min(t_{\text{out}}, t_{\text{in}})$ . So, for  $\langle R_s \rangle / r_{\text{acc}} \leq 3$ , the period of the cycle scales as  $\langle R_s \rangle^3$ . This may explain the deviation of the period for  $n_{\text{H},\infty} = 10^7 \text{ cm}^{-3}$  from the linear relation. We see in Figure 8 that the ratio  $\langle R_s \rangle / r_{\text{acc}} \sim 5$  for  $n_{\text{H},\infty} = 10^7 \text{ cm}^{-3}$ , which is much smaller than the ratio found for other densities.

#### 5.3.1. Rayleigh-Taylor Instability

In 2D simulations we find that RT instability develops across the Strömgren radius, but it decays on short timescales. This can be explained by the pressure gradient inside the Strömgren sphere which does not allow the RT to grow. In the linear regime, the growth timescale of the RT instability of wavelength  $\lambda$  is

$$au_{
m RT} \simeq \sqrt{rac{
ho_{
m sh} + 
ho_{
m in}}{
ho_{
m sh} - 
ho_{
m in}}} rac{2\pi\lambda}{g} \simeq \sqrt{rac{2\pi\lambda}{g}}$$

where  $\rho_{\rm sh}$  is the density of the shell and  $g \simeq GM_{\rm th} \langle R_s \rangle^{-2}$  is the gravitational acceleration at the shell radius. Thus, the RT timescale can be expressed as

$$\pi_{\rm RT} \simeq \frac{\langle R_s \rangle}{c_{s,\rm in}} \sqrt{\frac{2\pi\,\lambda}{r_{b,\rm in}}}.$$
(25)

So, during one cycle perturbations grow on scales:

$$\lambda_{\mathrm{RT}} < \left(\frac{\tau_{\mathrm{RT}}}{\tau_{\mathrm{cycle}}}\right)^2 \frac{r_{b,\mathrm{in}}}{2\pi} < \frac{r_{b,\mathrm{in}}}{2\pi},$$

where  $r_{b,\text{in}}$  is the inner Bondi radius. Thus, only instability on angular scales  $\theta \sim \lambda_{\text{RT}}/2\pi \langle R_s \rangle \leq r_{b,\text{in}}/(2\pi)^2 \langle R_s \rangle$  grows in our simulation.

#### 6. SUMMARY AND DISCUSSION

In this paper we simulate accretion onto IMBHs regulated by radiative feedback assuming spherically symmetric initial conditions. We study accretion rates and feedback loop periods while varying radiative efficiency, mass of black hole, density and temperature of the medium, and spectrum of radiation. The aim of this work is to simulate feedback-regulated accretion in a wide range of the parameter space to formulate an analytical description of processes that dominate the self-regulation mechanism. Thus, in this first paper we keep the physics as simple as possible, neglecting the effect of angular momentum of the gas and radiation pressure, and assuming a gas of primordial composition (i.e., metal and dust free). We will relax some of these assumptions in future works. However, the parametric formulae for the accretion presented in this paper should provide a realistic description of quasi-spherical accretion onto IMBH for ambient gas densities  $n_{\rm H,\infty} \lesssim 10^5 - 10^6 {\rm cm}^{-3}$ , as radiation pressure should be minor for these densities.

We find an oscillatory behavior of the accretion rate that can be explained by the effect of UV and X-ray photo-heating. The ionizing photons produced by the black hole near the gravitational radius increase gas pressure around the black hole. This pressure prevents the surrounding gas from being accreted. An overdense shell starts to form just outside the Strömgren sphere. Due to the decreased accretion rate, the number of emitted ionizing photons decreases and the density inside the Strömgren sphere also decreases with time. Gas accretion onto the black hole is dominant in decreasing the density inside the H II region only for ambient gas density  $n_{\rm H,\infty} \gtrsim 10^7 {\rm ~cm^{-3}}$ ; for lower values of the ambient gas density, the gas inside the HII region is pushed outward toward the dense shell by a pressure gradient that develops behind the ionization front. Eventually, the pressure gradient inside the Strömgren sphere is not able to support the weight of the overdense shell that starts to fall toward the black hole. The accretion rate rapidly increases and the Strömgren sphere starts to expand again.

However, the introduction of a small, non-zero, angular momentum in the flow could change the time-dependent behavior of the accretion and feedback loop. The inflow rate in the accretion disk that will necessarily develop, and which is not resolved in our simulations, is typically much slower than the free-fall rate since the viscous timescale in units of the free-fall time is  $t_{\rm visc}/t_{\rm ff} \sim \alpha^{-1} \mathcal{M}^2$ , where  $\alpha$  is the dimensionless parameter for a thin disk (Shakura & Sunyaev 1973) and  $\mathcal{M}$  is the gas Mach number. Therefore, angular momentum may produce a long delay between changes in the accretion rate at the inner boundary of our simulation and their mirror in terms of output luminosity. Hence,  $\alpha \simeq 0.01$ –0.1 and  $\mathcal{M}$  at the inner boundary of our simulations is of order of unity; time delays of 10-100 free-fall times are shorter if the disk is smaller than the inner boundary of the simulation. We have started investigating the effect of such a delay on the periodic oscillations and preliminary results show that the period of the oscillations can be modified by the time delay but the oscillatory behavior is still present (at least for delays of 10-100 free-fall times calculated at the inner boundary). As long as the time delay is shorter than the oscillation period, which depends mainly on the gas density, it does not seem to affect the results. We are carefully investigating this in the low and high density regimes where the oscillation pattern and the periods are different. At low densities, a time delay of a few hundred free-fall times is much smaller compared to the oscillation period, whereas at the high densities the maximum time delay that we have tested is comparable to the oscillation period. We will publish more extensive results on this effect in our next paper in this series.

We find that the average accretion rate is sensitive to the temperature of the ambient medium and to the temperature profile inside the ionized bubble, and so it depends on the gas cooling function and SED of the radiation. The period of the accretion bursts is insensitive to the temperature structure of the H II region, but is proportional to its radius.

Our simulations show that 1D results adequately reproduce 2D results in which instabilities often develop. We find that the accretion rate is expressed as  $\lambda_{rad} \simeq 3\% T_{\infty,4}^{2.5} (T_{in}/4 \times 10^4 \text{ K})^{-4}$ . We also derive  $\tau_{cycle}$  as a function of  $\eta$ ,  $M_{bh}$ ,  $n_{H,\infty}$ , and  $T_{\infty}$ . The dependencies of  $\langle \lambda_{rad} \rangle$  and  $\tau_{cycle}$  on our free parameters can be explained analytically. Assuming pressure equilibrium across the Strömgren sphere is a key ingredient to derive the dependence of  $\langle \lambda_{rad} \rangle$  on  $T_{\infty}$ , whereas the linear relation between the average size of the Strömgren sphere and  $\tau_{cycle}$  is used to derive the dependence of  $\tau_{cycle}$  on all the parameters we varied.

The qualitative picture of the feedback loop agrees with the description of X-ray bursters in Cowie et al. (1978). After extrapolating our analytical formulae to black holes of a few solar masses studied by Cowie et al. (1978), we find that the average accretion rate is in good agreement (L  $\sim$  $2 \times 10^{35}$  erg s<sup>-1</sup>). However, the details of the accretion rate as a function of time, burst period, and peak accretion rates show qualitative differences. The Cowie et al. (1978) simulations do not show periodic oscillation while our simulations have a welldefined fast rise and exponential decay of accretion followed by quiescent phases of the accretion rate. This regular pattern of accretion bursts is possible only when spherical symmetry is maintained on relatively large scales during oscillations. An axisymmetric radiation source (Proga 2007; Proga et al. 2008; Kurosawa et al. 2009; Kurosawa & Proga 2009a, 2009b) or inhomogeneous initial condition on the scale of the Bondi radius can break the symmetry.

Our simulations are also in excellent qualitative agreement with simulations by MCB09 who studied accretion onto a  $100 M_{\odot}$  black hole for the case  $n_{\rm H,\infty} = 10^7 \, {\rm cm}^{-3}$ . However, we find a dimensionless accretion rate  $\langle\lambda_{rad}\rangle\sim 3\%$  (  $\langle\lambda_{rad}\rangle\sim 2\%$ including helium heating/cooling) that is about one order of magnitude larger than in MCB09. The cycle period,  $\tau_{cycle}$ , is in better agreement since  $au_{
m cycle} \propto \langle R_s \rangle \propto \langle \lambda_{
m rad} 
angle^{1/3}$ . The discrepancy in the mean accretion is likely produced by the effect of radiation pressure on HI, which becomes important for large  $n_{\rm H,\infty}$  and which we have neglected. In addition, our results indicate that the qualitative description of the feedback loop starts to change at ambient densities  $>10^6-10^7$  cm<sup>-3</sup>: the oscillation period decreases much more rapidly with increasing ambient density as gas depletion inside the ionized bubble becomes dominated by accretion onto the IMBH. The accretion luminosity during the quiescent phase of the accretion also increases and the two phases of growth and collapse of the dense shell become blended into a smoother modulation of the accretion rate. Hence, further numerical studies are required to characterize accretion onto IMBH in the high density regime.

As mentioned above, in this study we have neglected three important physical processes that may further reduce the accretion rate: (1) Compton heating, (2) radiation pressure, and (3) Ly $\alpha$  scattering processes. The importance of these processes is thoroughly discussed in MBCO09. Ricotti et al. (2008) and MBCO09 find that Compton heating is not an important feedback mechanism in regulating the accretion rate onto IMBHs, in which the gas density inside the Strömgren sphere is roughly independent of the radius. However, MBCO09 suggest that both radiation pressure on HI and Ly $\alpha$  radiation pressure can contribute to reducing the accretion rate onto IMBH. At higher densities, accretion becomes Eddington limited ( $\langle \lambda_{\rm rad} \rangle \dot{M}_B \lesssim \dot{M}_{\rm Edd}$ ). By assuming  $\langle \lambda_{\rm rad} \rangle \sim 1\% T_{\infty,4}^{2.5}$ , which is suggested by simulations including helium, and manipulating Equation (6) we obtain  $M_{\rm bh,2} T_{\infty,4} n_{\rm H,5} \eta_{-1} \gtrsim 40$  as a criterion for the Eddington-limited condition. This expression predicts that radiation pressure becomes important at gas density  $n_{\rm H.5} \sim 10$ –100, with other parameters fixed to unity. It also implies that this critical density depends on the black hole mass, gas temperature, and radiative efficiency. We expect radiation pressure to play a minor role at low densities also because the accretion luminosity is negligible during the quiescent phases of accretion (between accretion bursts). However, at densities  $n_{\rm H,\infty} \ge 10^7 \, {\rm cm}^{-3}$  the accretion rate during the quiescent phases is not negligible, thus radiation pressure can be important in this regime. The main effect of the Thomson radiation pressure is to prevent the accretion luminosity from exceeding the Eddington limit. The continuum radiation pressure due to HI ionization can instead be important for sub-Eddington luminosities, but will be strong only at the location of the ionization front during the peak of the accretion burst. We will present more extensive results in the next paper of this series.

The results of this study provide a first step in estimating the maximum X-ray luminosity and period of oscillations of an accreting IMBH from a medium with given physical conditions. Hence, they may be useful for modeling detection probability of ULX originating from accreting IMBHs in the local universe. From the average growth rate of IMBHs accreting in this manner, it is also possible to estimate the maximum masses of quasars at a given redshift starting from seed primordial black holes. One of the main motivations of this study is to derive simple analytical prescriptions to incorporate the growth of seed black holes from Population III stars into a large-scale cosmological



Figure 11. Left: simulated Bondi accretion rate ( $\lambda_B$ ) as a function of minimum radius with given adiabatic index  $\gamma$  without radiative feedback. Dashed lines are analytically estimated values for each  $\gamma = 1.2$ , 1.4, and 1.6. In order not to overestimate the accretion rate, the sonic point should be resolved where the velocity of the inflowing gas becomes supersonic. Right: test of the Strömgren radius with a given number of ionizing photons. Solid line is the prediction for the given number of ionizing photons from  $10^{40}$  to  $10^{50}$  s<sup>-1</sup>. Triangle symbols represent the location where the ionization fraction of  $n_{\rm H}$  ( $x_{\rm H_I}$ ) is 0.50. Squares are for  $x_{\rm H_I} = 0.90$  and circles are for  $x_{\rm H_I} = 0.99$ .

(A color version of this figure is available in the online journal.)

simulation. However, before we are able to use these results in a cosmological simulation, we need to understand the effects of relaxing our assumption of spherically symmetric accretion: we need to simulate accretion onto moving black holes (Hoyle & Lyttleton 1939; Shima et al. 1985; Ruffert & Arnett 1994; Ruffert 1996) and use more realistic initial conditions, including gas with non-zero angular momentum or a multi-phase turbulent interstellar medium (Krumholz et al. 2005, 2006).

The simulations presented in this paper were carried out using high performance computing clusters administered by the Center for Theory and Computation of the Department of Astronomy at the University of Maryland ("yorp"), and the Office of Information Technology at the University of Maryland ("deepthought"). This research was supported by NASA grants NNX07AH10G and NNX10AH10G. The authors thank the anonymous referee for constructive comments and feedback.

## APPENDIX A

## BASIC TESTS OF THE CODE

We test the Bondi accretion formula using ZEUS-MP for the adiabatic indexes  $\gamma = 1.2$ , 1.4, and 1.6. For a given equation of state, the sonic point where the gas inflow becomes supersonic must be resolved so as not to overestimate the accretion rate  $\lambda_B$ . The left panel of Figure 11 shows the steady accretion rate as a function of radius at the inner boundary normalized by the Bondi radius. Different lines show results for  $\gamma = 1.2$ , 1.4, and 1.6.

We also test whether our radiative transfer module produces radii of the Strömgren spheres in agreement with the analytical prediction:  $(4\pi/3)R_s^3n_en_{\rm H}\alpha_{\rm rec} = N_{\rm ion}$ , where  $R_s$  is the Strömgren radius and  $N_{\rm ion}$  is the number of ionizing photons emitted per unit time. The right panel of Figure 11 shows the test of the 1D radiative transfer module without hydrodynamics. Different symbols indicate the radii for the different ionization fractions:  $x_e = 0.99$  (circle), 0.90 (square), and 0.50 (triangle).

#### APPENDIX B

### RADIATIVE TRANSFER MODULE AND TIME STEPPING

Our hydrodynamic calculation is performed using ZEUS-MP, returning the density and gas energy at each time step to the radiative transfer module. The operator-splitting method is applied to mediate between hydrodynamics and radiative transfer with a photon-conserving method (Whalen & Norman 2006). For each line of sight, radiative transfer equations are solved in the following order.

- 1. At the inner boundary, the average inflow mass flux  $\dot{M}$  is calculated.
- 2. The mass flux is converted into accretion luminosity *L*, and thus into the number of ionizing photons for a given radiative efficiency  $\eta$ .
- 3. The photon spectrum is determined using a power-law SED with spectral index  $\alpha$ . We use up to 300 logarithmically spaced frequency bins for photons between 10 eV and 100 keV.
- 4. The ordinary differential equation for time-dependent radiative transfer cooling/heating and chemistry of the gas are solved using a Runge–Kutta or semi-implicit solver for each line of sight with a maximum of 10% error. Photoheating, cooling for a given cooling function, and Compton cooling are calculated.
- 5. The energy density and the abundances of neutral and ionized hydrogen are updated.

Parallelization is easily implemented in the polar angle direction because radiative transfer calculations along each ray are independent of one another.



Figure 12. Comparisons between simulations of  $\eta = 0.1$ ,  $M_{\rm bh} = 100 M_{\odot}$ ,  $n_{\rm H,\infty} = 10^5 \text{ cm}^{-3}$ , and  $T_{\infty} = 10^4 \text{ K}$  with various resolutions. Solid: 384 grid run. Dotted: 512 grid run. Long dashed: 768 grid run. Short dashed: 512 grid with a Courant number of 0.05.

(A color version of this figure is available in the online journal.)

## APPENDIX C

# **RESOLUTION STUDIES**

We perform a resolution study to confirm that the number of grid zones does not affect the results. The number of zones from 384 to 768 are tested and they all show similar outputs in terms of accretion rate at peaks, average accretion rate, decaying shape, and the period between peaks. Figure 12 shows that the details of the accretion rate history from simulations are not identical but the physical quantities which we are interested in (average accretion rate, peak accretion rate, and period of the bursts) do not show significant deviation from each other. In general, a Courant number of 0.5 is used for most simulations, but we try a Courant number which is one order of magnitude smaller to investigate how the results are affected by reducing the hydro-time step by an order of magnitude. The chemical/ cooling time steps are calculated independently by the radiation transfer module.

#### REFERENCES

- Abel, T., Anninos, P., Norman, M. L., & Zhang, Y. 1998, ApJ, 508, 518 Alvarez, M. A., Wise, J. H., & Abel, T. 2009, ApJ, 701, L133
- Begelman, M. C. 1985, ApJ, 297, 492
- Begelman, M. C., Volonteri, M., & Rees, M. J. 2006, MNRAS, 370, 289
- Bisnovatyi-Kogan, G. S., & Blinnikov, S. I. 1980, MNRAS, 191, 711
- Bondi, H. 1952, MNRAS, 112, 195
- Bondi, H., & Hoyle, F. 1944, MNRAS, 104, 273
- Carr, B. J., Bond, J. R., & Arnett, W. D. 1984, ApJ, 277, 445
- Ciotti, L., & Ostriker, J. P. 2007, ApJ, 665, 1038

- Ciotti, L., Ostriker, J. P., & Proga, D. 2009, ApJ, 699, 89
- Cowie, L. L., Ostriker, J. P., & Stark, A. A. 1978, ApJ, 226, 1041
- Di Matteo, T., Colberg, J., Springel, V., Hernquist, L., & Sijacki, D. 2008, ApJ, 676.33
- Fan, X., Strauss, M. A., Schneider, D. P., et al. 2003, AJ, 125, 1649
- Fryer, C. L., Woosley, S. E., & Heger, A. 2001, ApJ, 550, 372
- Greif, T. H., Johnson, J. L., Klessen, R. S., & Bromm, V. 2008, MNRAS, 387,
- Haehnelt, M. G., Natarajan, P., & Rees, M. J. 1998, MNRAS, 300, 817
- Hayes, J. C., Norman, M. L., Fiedler, R. A., et al. 2006, ApJS, 165, 188
- Hoyle, F., & Lyttleton, R. A. 1939, Proc. Camb. Phil. Soc., 35, 405
- Johnson, J. L., & Bromm, V. 2007, MNRAS, 374, 1557
- Krolik, J. H., & Kallman, T. R. 1984, ApJ, 286, 366
- Krolik, J. H., & London, R. A. 1983, ApJ, 267, 18
- Krolik, J. H., McKee, C. F., & Tarter, C. B. 1981, ApJ, 249, 422
- Krumholz, M. R., McKee, C. F., & Klein, R. I. 2005, ApJ, 618, 757
- Krumholz, M. R., McKee, C. F., & Klein, R. I. 2006, ApJ, 638, 369
- Kurosawa, R., & Proga, D. 2009a, MNRAS, 397, 1791
- Kurosawa, R., & Proga, D. 2009b, ApJ, 693, 1929
- Kurosawa, R., Proga, D., & Nagamine, K. 2009, ApJ, 707, 823
- Mack, K. J., Ostriker, J. P., & Ricotti, M. 2007, ApJ, 665, 1277
- Madau, P., & Rees, M. J. 2001, ApJ, 551, L27
- Madau, P., Rees, M. J., Volonteri, M., Haardt, F., & Oh, S. P. 2004, ApJ, 604, 484
- Mayer, L., Kazantzidis, S., Escala, A., & Callegari, S. 2010, Nature, 466, 1082
- Milosavljević, M., Bromm, V., Couch, S. M., & Oh, S. P. 2009a, ApJ, 698, 766
- Milosavljević, M., Couch, S. M., & Bromm, V. 2009b, ApJ, 696, L146
- Novak, G. S., Ostriker, J. P., & Ciotti, L. 2010, arXiv:1007.3505
- Oh, S. P., & Haiman, Z. 2002, ApJ, 569, 558
- Omukai, K., Schneider, R., & Haiman, Z. 2008, ApJ, 686, 801
- Ostriker, J. P., Choi, E., Ciotti, L., Novak, G. S., & Proga, D. 2010, ApJ, 722, 642
- Ostriker, J. P., Weaver, R., Yahil, A., & McCray, R. 1976, ApJ, 208, L61
- Park, M., & Ostriker, J. P. 2001, ApJ, 549, 100
- Pelupessy, F. I., Di Matteo, T., & Ciardi, B. 2007, ApJ, 665, 107 Proga, D. 2007, ApJ, 661, 693
- Proga, D., Ostriker, J. P., & Kurosawa, R. 2008, ApJ, 676, 101
- Regan, J. A., & Haehnelt, M. G. 2009, MNRAS, 396, 343
- Ricotti, M. 2007, ApJ, 662, 53
- Ricotti, M. 2009, MNRAS, 392, L45
- Ricotti, M., Gnedin, N. Y., & Shull, J. M. 2001, ApJ, 560, 580
- Ricotti, M., Gnedin, N. Y., & Shull, J. M. 2002a, ApJ, 575, 33
- Ricotti, M., Gnedin, N. Y., & Shull, J. M. 2002b, ApJ, 575, 49
- Ricotti, M., & Ostriker, J. P. 2004, MNRAS, 350, 539
- Ricotti, M., Ostriker, J. P., & Gnedin, N. Y. 2005, MNRAS, 357, 207
- Ricotti, M., Ostriker, J. P., & Mack, K. J. 2008, ApJ, 680, 829
- Ruffert, M. 1996, A&A, 311, 817
- Ruffert, M., & Arnett, D. 1994, ApJ, 427, 351
- Sazonov, S. Y., Ostriker, J. P., Ciotti, L., & Sunyaev, R. A. 2005, MNRAS, 358, 168
- Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
- Shapiro, P. R., & Kang, H. 1987, ApJ, 318, 32
- Shapiro, S. L. 1973, ApJ, 180, 531
- Sharma, P., Quataert, E., Hammett, G. W., & Stone, J. M. 2007, ApJ, 667, 714
- Shima, E., Matsuda, T., Takeda, H., & Sawada, K. 1985, MNRAS, 217, 367
- Stone, J. M., & Norman, M. L. 1992, ApJS, 80, 753
- Venkatesan, A., Giroux, M. L., & Shull, J. M. 2001, ApJ, 563, 1
- Vitello, P. 1984, ApJ, 284, 394
- Volonteri, M., Haardt, F., & Madau, P. 2003, ApJ, 582, 559
- Volonteri, M., Lodato, G., & Natarajan, P. 2008, MNRAS, 383, 1079
- Volonteri, M., & Rees, M. J. 2005, ApJ, 633, 624
- Wandel, A., Yahil, A., & Milgrom, M. 1984, ApJ, 282, 53
- Whalen, D., & Norman, M. L. 2006, ApJS, 162, 281 Whalen, D., & Norman, M. L. 2008a, ApJ, 673, 664
- Whalen, D. J., & Norman, M. L. 2008b, ApJ, 672, 287
- Yoo, J., & Miralda-Escudé, J. 2004, ApJ, 614, L25