COSMOLOGICAL EVOLUTION OF MASSIVE BLACK HOLES: EFFECTS OF EDDINGTON RATIO DISTRIBUTION AND QUASAR LIFETIME

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ABSTRACT

A power-law time-dependent light curve for active galactic nuclei (AGNs) is expected by the self-regulated black hole growth scenario, in which the feedback of AGNs expels gas and shut down accretion. This is also supported by the observed power-law Eddington ratio distribution of AGNs. At high redshifts, the AGN life timescale is comparable with (or even shorter than) the age of the universe, which sets a constraint on the minimal Eddington ratio for AGNs on the assumption of a power-law AGN light curve. The black hole mass function (BHMF) of AGN relics is calculated by integrating the continuity equation of massive black hole number density on the assumption of the growth of massive black holes being dominated by mass accretion with a power-law Eddington ratio distribution for AGNs. The derived BHMF of AGN relics at z = 0 can fit the measured local mass function of the massive black holes in galaxies quite well, provided the radiative efficiency ~0.1 and a suitable power-law index for the Eddington ratio distribution are adopted. In our calculations of the black hole evolution, the duty cycle of AGN should be less than unity, which requires the quasar life timescale $\tau_Q \gtrsim 5 \times 10^8$ years.

Key words: accretion, accretion disks - black hole physics - galaxies: active - quasars: general

1. INTRODUCTION

There is evidence that most nearby galaxies contain massive black holes at their centers, and the central massive black hole mass is found to be tightly correlated with the velocity dispersion of the galaxy (Ferrarese & Merritt 2000; Gebhardt et al. 2000), or the luminosity of the spheroid component of its host galaxy (e.g., Magorrian et al. 1998; Marconi & Hunt 2003). These correlations of the black hole mass with velocity dispersion/host galaxy luminosity were widely used to estimate black hole masses and to derive the mass functions of the central massive black holes in galaxies (e.g., Yu & Tremaine 2002; Marconi et al. 2004; Shankar et al. 2004; Tamura et al. 2006; Graham et al. 2007; Shankar 2009). On the other hand, quasars are powered by accretion onto massive black holes, and the growth of massive black holes could be dominantly governed by mass accretion in quasars. The massive black holes are therefore the active galactic nucleus (AGN) relics (Soltan 1982), and the luminosity functions (LFs) of AGNs provide important clues on the growth of massive black holes. The black hole mass function (BHMF) of AGN relics can be calculated by integrating the continuity equation of massive black hole number density on the assumption of the growth of massive black holes being dominated by mass accretion, in which the activity of massive black holes is described by the AGN LF (e.g., Cavaliere et al. 1971; Soltan 1982; Chokshi & Turner 1992; Small & Blandford 1992; Shen 2009; Li et al. 2009). There are two free parameters: the radiative efficiency η_{rad} and the mean Eddington ratio $\lambda = L_{bol}/L_{Edd}$ for AGNs, adopted in most such calculations on the cosmological evolution of massive black holes (e.g., Yu & Tremaine 2002; Marconi et al. 2004; Shankar et al. 2004). The derived BHMF of AGN relics in this way is required to match the measured local BHMF at redshift z = 0by tuning the values of two parameters η_{rad} and λ , which usually requires almost all AGNs to be accreting close to the Eddington limit (e.g., Yu & Tremaine 2002; Marconi et al. 2004; Shankar et al. 2004).

In principle, the mean Eddington ratio for AGNs λ is not a free parameter, which can be estimated from a sample of AGNs with measured black hole masses. One of the most effective approaches for measuring masses of black holes in AGNs is the reverberation mapping method (Peterson 1993; Kaspi et al. 2000). Using the tight correlation between the size of the broad-line region and the optical luminosity established with the reverberation mapping method for a sample of AGNs, the black hole masses of AGNs can easily be estimated from their optical luminosity and width of broad emission line. The mean Eddington ratio $\lambda \simeq 0.1$ at $z \sim 0.2$ to $\simeq 0.4$ at $z \sim 2$ was derived from a large sample of AGNs with the analyses of the Sloan Digital Sky Survey (SDSS) by McLure & Dunlop (2004) (see also Warner et al. 2004; Kelly et al. 2010). Kollmeier et al. (2006) pointed that the samples selected from the SDSS are heavily weighted toward high-luminosity objects due to the limited sensitivity of the SDSS. Kollmeier et al. (2006) estimated the Eddington ratios of AGNs discovered in the AGN and Galaxy Evolution Survey (AGES), which is more sensitive than the SDSS. The derived Eddington ratio distribution at fixed luminosity is well described by a single log-normal distribution peaked at ~ 0.25 (also see Shen et al. 2008; Trump et al. 2009). However, some other investigations showed that the Eddington ratios of local AGNs spread over several orders of magnitude (e.g., Ho 2002; Hopkins et al. 2006). The Eddington ratio distribution for AGNs exhibits a power-law distribution with an exponential cutoff at a high Eddington ratio (Merloni & Heinz 2008; Hopkins & Hernquist 2009), or a power-law distribution with an additional log-normal component (Kauffmann & Heckman 2009). Such a power-law Eddington ratio distribution is qualitatively consistent with the self-regulated black hole growth scenario, in which the feedback of AGN expels gas and shut down accretion (e.g., Hopkins et al. 2005a, 2005b; Hopkins & Hernquist 2009). This means that an AGN with bolometric luminosity L_{bol} may contain a relatively small black hole accreting at a high rate or a more massive black hole accreting at a lower rate. In this work, we adopt a



Figure 1. Minimal Eddington ratios as functions of redshift z for different AGN life timescales: $\tau_Q = 10^8$ (red), 2.5×10^8 (green), 5×10^8 (blue), and 7.5×10^8 years (black), where $\lambda_{min,0} = 10^{-4}$ is adopted.

power-law Eddington ratio distribution with an exponential cutoff at a high ratio to derive the BHMF of AGN with an LF, with which the continuity equation for black hole number density is integrated to calculate the cosmological evolution of the BHMF of AGN relics. The resultant BHMF of AGN relics is constrained by the measured local BHMF. The conventional cosmological parameters $\Omega_{\rm M} = 0.3$, $\Omega_{\Lambda} = 0.7$, and $H_0 = 70 \,\rm km \, s^{-1} \, Mpc^{-1}$ have been adopted in this work.

2. THE EDDINGTON RATIO DISTRIBUTION FOR ACTIVE GALACTIC NUCLEI

Hopkins & Hernquist (2009) suggested that the quasar light curve can be described by

$$\frac{dt}{d\log\lambda} = \tau_{\rm Q} \left(\frac{\lambda}{\lambda_{\rm peak}}\right)^{-\beta_l} \exp\left(-\frac{\lambda}{\lambda_{\rm peak}}\right),\tag{1}$$

where $\lambda = L_{\text{bol}}/L_{\text{Edd}}$ ($L_{\text{Edd}} = 1.3 \times 10^{38} M_{\text{bh}}/M_{\odot} \text{ erg s}^{-1}$), τ_{Q} is the quasar life timescale, and the parameter λ_{peak} describes the peak luminosity of quasars. This power-law light curve is consistent with the self-regulated black hole growth model, in which feedback produces a self-regulating decay or blowout phase after the AGN reaches some peak luminosity and begins to expel gas and shut down accretion (e.g., Hopkins et al. 2005a). This light curve can be translated to an observed Eddington ratio distribution $\zeta(\lambda)$,

$$\zeta(\lambda) = \frac{d\mathcal{N}}{\mathcal{N}d\log\lambda} = C_l \left(\frac{\lambda}{\lambda_{\text{peak}}}\right)^{-\beta_l} \exp\left(-\frac{\lambda}{\lambda_{\text{peak}}}\right), \quad (2)$$

where C_l is the normalization, if the switch-on of AGN activity is balanced with the switch-off of AGN activity, and τ_Q is significantly shorter than the age of the universe at redshift z. The power-law Eddington ratio distribution is consistent with those derived with samples of nearby AGNs (e.g., Heckman et al. 2004; Yu et al. 2005; Hopkins & Hernquist 2009). Such an Eddington ratio distribution has a lower cutoff at $\lambda = \lambda_{\min,0}$, below which the sources are no longer regarded as AGNs. In this work, we adopt $\lambda = \lambda_{\min,0} = 10^{-4}$ in all the calculations.

At high redshifts, the quasar life timescale τ_Q is comparable with (or even shorter than) the age of the universe at redshift z. In this case, the time after the birth of the first quasars is so short that most of them are still very luminous (i.e., accreting at high rates), and the lower limit on the Eddington ratios for AGNs should be significantly higher than $\lambda_{\min,0}$. The first quasars are predicted to have formed at $z_{\rm fq} \sim 10$ (e.g., Haiman & Loeb 2001; Bromm & Loeb 2003), with which we can estimate the minimal Eddington ratio $\lambda'_{\rm min}$ for AGNs at redshift *z* as

$$\int_{\lambda'_{\min}(z)} \tau_{\rm Q}\left(\frac{\lambda}{\lambda_{\rm peak}}\right)^{-\beta_l} \exp\left(-\frac{\lambda}{\lambda_{\rm peak}}\right) d\log\lambda = t(z), \quad (3)$$

where t(z) is the age of the universe at z measured from $z_{\rm fq} = 10$ when the first quasars formed. For simplicity, we adopt $\lambda_{\rm min} = \max[\lambda_{\rm min,0}, \lambda'_{\rm min}(z)]$ in all our calculations on the black hole evolution. In Figure 1, we plot the minimal Eddington ratios as functions of redshift z for different quasar life timescales $\tau_{\rm Q}$.

For a given BHMF $N_{AGN}(M_{bh}, z)$ and Eddington ratio distribution, the AGN LF $\Phi(z, L_{bol})$ can be calculated with

$$\Phi(z, L_{\text{bol}}) = \int_{\lambda} N_{\text{AGN}}(M_{\text{bh}}, z) \frac{d \log M_{\text{bh}}}{d \log L_{\text{bol}}} \zeta(\lambda) d$$
$$\times \log \lambda \text{Mpc}^{-3} (\log L_{\text{bol}})^{-1}, \qquad (4)$$

where $M_{\rm bh}/M_{\odot} = L_{\rm bol}/(1.3 \times 10^{38} \lambda)$, $N_{\rm AGN}(M_{\rm bh}, z)$ is the AGN BHMF at z, and the Eddington ratio distribution $\zeta(\lambda)$ is given by Equation (2). In this work, we assume that the AGN BHMF has the same form as the AGN LF,

$$N_{\rm AGN}(M_{\rm bh}, z) = \frac{N_{\rm AGN,0}(z)}{(M_{\rm bh}/M_{\rm bh}^*)^{\beta_{\rm m1}} + (M_{\rm bh}/M_{\rm bh}^*)^{\beta_{\rm m2}}} {\rm Mpc}^{-3} (\log M_{\rm bh})^{-1}, \quad (5)$$

where the parameters $N_{\text{AGN},0}$, M_{bh}^* , β_{m1} , and β_{m2} are to be determined. Substituting Equation (5) into Equation (4), we can calculate the AGN LF with a given Eddington ratio distribution $\zeta(\lambda)$ provided the values of the four parameters in the AGN BHMF are specified. In this work, we tune the values of these parameters until the observed LF can be well reproduced by that calculated with Equation (4). We adopt the LF given by Hopkins et al. (2007b), which is calculated by using a large set of observed quasar LFs in various wave bands, from the IR through optical, soft, and hard X-rays (see Hopkins et al. 2007b, for the details),

$$\Phi(L_{\rm bol}, z) = \frac{\phi_*}{(L_{\rm bol}/L_*)^{\gamma_1} + (L_{\rm bol}/L_*)^{\gamma_2}} \mathrm{Mpc}^{-3} (\log L_{\rm bol})^{-1},$$
(6)

with normalization ϕ_* , break luminosity L_* , faint-end slope γ_1 , and bright-end slope γ_2 . The break luminosity L_* evolves with redshift and is given by

$$\log L_* = (\log L_*)_0 + k_{L,1}\xi + k_{L,2}\xi^2 + k_{L,3}\xi^3, \qquad (7)$$

and the two slopes γ_1 and γ_2 evolve with redshift as

$$\gamma_1 = (\gamma_1)_0 \left(\frac{1+z}{1+z_{\rm ref}}\right)^{k_{\gamma_1}},$$
(8)

and

$$\gamma_2 = \frac{2(\gamma_2)_0}{\left(\frac{1+z}{1+z_{\rm ref}}\right)^{k_{\gamma_2,1}} + \left(\frac{1+z}{1+z_{\rm ref}}\right)^{k_{\gamma_2,2}}}.$$
(9)



Figure 2. Top: the comparison between the best-fitted LFs (lines) calculated form the AGN BHMF with a given Eddington ratio distribution and the LFs (dots) given by Hopkins et al. (2007b) at different redshifts: z = 0 (red), 3 (green), and 5 (blue). The parameters $\beta_l = 0.3$, $\lambda_{peak} = 2.5$, and $\tau_Q = 5 \times 10^8$ years are adopted. Bottom: the derived AGN BHMFs at different redshifts: z = 0 (red), 3 (green), and 5 (blue).

The parameter ξ is

$$\xi = \log\left(\frac{1+z}{1+z_{\rm ref}}\right),\tag{10}$$

and $z_{\text{ref}} = 2$ is fixed. The other parameters of this LF are as follows: $\log \phi_*(\text{Mpc}^{-3}) = -4.825 \pm 0.060$, $[\log L_*(3.9 \times 10^{33} \text{ erg s}^{-1})]_0 = 13.036 \pm 0.043$, $k_{L,1} = 0.632 \pm 0.077$, $k_{L,2} = -11.76 \pm 0.38$, $k_{L,3} = -14.25 \pm 0.80$, $(\gamma_1)_0 = 0.417 \pm 0.055$, $k_{\gamma_1} = -0.623 \pm 0.132$, $(\gamma_2)_0 = 2.174 \pm 0.055$, $k_{\gamma_2,1} = 1.460 \pm 0.096$, and $k_{\gamma_2,2} = -0.793 \pm 0.057$ (see Hopkins et al. 2007b, for the details). This LF includes both the Comptonthin and thick sources.

We give some examples of the AGN BHMFs derived from an AGN LF with a given Eddington ratio distribution at different redshifts z in Figure 2. It is found that the AGN LF can be well reproduced by the calculations from the AGN BHMF with the form given in Equation (5).

3. THE COSMOLOGICAL EVOLUTION OF MASSIVE BLACK HOLES

The evolution of massive black hole number density is described by (Small & Blandford 1992)

$$\frac{\partial n(M_{\rm bh}, t)}{\partial t} + \frac{\partial}{\partial M_{\rm bh}} [n(M_{\rm bh}, t) < \dot{M}(M_{\rm bh}, t)] = 0, \qquad (11)$$

where the BHMF $n(M_{bh}, t)$ is in units of Mpc⁻³ M_{bh}^{-1} , $\langle \dot{M}(M_{bh}, t) \rangle$ is the mean mass accretion rate for the black holes with M_{bh} , and the efforts of mergers of black holes are neglected (see Shankar et al. 2009; Cao & Li 2008, for detailed discussion). The BHMF $N(M_{bh}, t)$ used in this work is in units of Mpc⁻³(log M_{bh})⁻¹. Equation (11) can be rewritten as

$$\frac{1}{M_{bh}} \frac{\partial N(M_{bh}, t)}{\partial t} + \frac{\partial}{\partial M_{bh}} \left[\frac{N(M_{bh}, t) \langle \dot{M}(M_{bh}, t) \rangle}{M_{bh}} \right] = 0, \quad (12)$$

since $N(M_{bh}, t) \equiv \ln 10 M_{bh} n(M_{bh}, t).$

It was suggested that the standard thin accretion disk will transit to an advection dominated accretion flow (ADAF) when the dimensionless mass accretion rate \dot{m} is lower than a critical value $\dot{m}_{\rm crit}$ ($\dot{m} = \dot{M}/\dot{M}_{\rm Edd}$; $\dot{M}_{\rm Edd} = 1.3 \times 10^{38} M_{\rm bh}/0.1 M_{\odot} c^2$; Narayan & Yi 1995). The radiative efficiency for ADAFs is significantly lower than that for standard thin disks, and it decreases with decreasing mass accretion rate \dot{m} . It was suggested that the radiative efficiency $\eta_{\rm rad}$ can be described with

$$\eta_{\rm rad} = \begin{cases} \eta_{\rm rad,0}, & \text{if } \dot{m} \ge \dot{m}_{\rm crit}; \\ \eta_{\rm rad,0} \left(\frac{\dot{m}}{\dot{m}_{\rm crit}}\right)^s, & \text{if } \dot{m} < \dot{m}_{\rm crit}, \end{cases}$$
(13)

where $\dot{m}_{\rm crit} = 0.01$ is adopted in this work (see Narayan 2002, for the detailed discussion and the references therein), and the parameter s = 1 is suggested in Narayan & Yi (1995). The calculations of the ADAFs surrounding rotating black holes in the general relativistic frame showed that the value of *s* is in the range of ~0.2–1.1 depending on the value of black hole spin parameter *a* (Xu & Cao 2010). In all our calculations, we adopt s = 0.5 (e.g., Merloni & Heinz 2008; Draper & Ballantyne 2010). The mean mass accretion rate for the black holes with $M_{\rm bh}$ at redshift *z* can be calculated with

$$N(M_{\rm bh}, z)\langle M(M_{\rm bh}, z)\rangle = \int_{\lambda} \frac{\zeta(\lambda) N_{\rm AGN}(M_{\rm bh}, z)(M_{\rm bh}/M_{\odot})\lambda L_{\rm Edd,\odot}(1 - \eta_{\rm rad})}{\eta_{\rm rad}c^2} d\log\lambda,$$
(14)

where $N_{AGN}(M_{bh}, z)$ is the AGN BHMF at z, and the radiative efficiency η_{rad} is given by Equation (13).

The black hole evolution equation can be rewritten as

$$\frac{1}{M_{\rm bh}} \frac{\partial N(M_{\rm bh}, z)}{\partial z} = -\frac{dt}{dz} \frac{\partial}{\partial M_{\rm bh}} \left[\frac{N(M_{\rm bh}, t) \langle \dot{M}(M_{\rm bh}, t) \rangle}{M_{\rm bh}} \right].$$
(15)

As described in Section 2, the AGN BHMF $N_{AGN}(M_{bh}, z)$ can be calculated with a given Eddington ratio distribution (2) and the AGN LF (6), and the mean mass accretion rate can be calculated with Equation (14). Integrating Equation (15) over z from z_{max} with derived AGN BHMF $N_{AGN}(M_{bh}, z)$ and suitable initial conditions at z_{max} , the cosmological evolution of massive black holes is available. The duty cycle δ of AGN is defined as

$$\delta(M_{\rm bh}, z) = \frac{N_{\rm AGN}(M_{\rm bh}, z)}{N(M_{\rm bh}, z)},\tag{16}$$

which is required to be less than unity. In all our calculations, we assume that the duty cycle $\delta = 0.5$ at $z_{max} = 5$. There are three free parameters, $\eta_{rad,0}$, β_l , and λ_{peak} , in our calculations, which are tuned to let the BHMF of the AGN relics at z = 0 fit the measured local BHMF given in Shankar et al. (2009). This local BHMF encompasses the range of several estimates of the BHMF with different methods (Bell et al. 2003; Marconi et al. 2004; Shankar et al. 2004; Ferrarese & Ford 2005; Tundo et al. 2007; Hopkins et al. 2007a). We find that the final results are insensitive to the initial conditions at z_{max} , because the fraction of local black hole mass accreted at very high redshifts can be neglected.

The results for the evolution of the total BHMFs and AGN BHMFs with redshift *z* are plotted in Figure 4 for different values of the model parameters. In Figure 5, the duty cycles δ of AGNs are plotted as functions of black hole mass $M_{\rm bh}$ at different



Figure 3. Local BHMFs at z = 0. The shaded area encompasses several estimates of the local BHMF (see Figure 5 in Shankar et al. 2009, and references therein). The lines represent the BHMF of the AGN relics calculated with AGN life timescale $\tau_{\rm Q} = 5 \times 10^8$ years. The red solid lines are the results with $\beta_l = 0.3$ and $\lambda_{\rm peak} = 2.5$, while the red dashed and dash-dotted lines represent the results with $\lambda_{\rm peak} = 1$ and 3.5, respectively ($\beta_l = 0.3$). The blue dashed and dash-dotted lines represent the results with $\beta_l = 0.2$ and 0.4, respectively ($\lambda_{\rm peak} = 2.5$).

redshifts z. In this work, the AGNs accreting at $\ge m_{crit}$ are referred to as bright AGNs, in which radiative efficient accretion disks are present. We plot the radiative efficiency evolving with redshift in Figure 6.

4. DISCUSSION

Our estimates of the lower limits on the Eddington ratios of AGNs show that λ_{\min} increases with redshift *z*, which implies that the mean Eddington ratio is higher at high redshifts. At high redshifts, the quasar life timescale τ_Q is comparable with (or even shorter than) the age of the universe at redshift *z*, and most of the AGNs are therefore still very luminous (i.e., accreting at high rates). The mean Eddington ratios for these AGNs are relatively higher than those at low redshifts. The estimates of the Eddington ratios for AGNs show that the mean Eddington ratio increases with *z* (e.g., McLure & Dunlop 2004; Warner et al. 2004), which is qualitatively consistent with our results.

Unlike most of the previous works (e.g., Yu & Tremaine 2002; Marconi et al. 2004; Shankar et al. 2004), in which a single mean Eddington ratio is adopted as a free parameter, we use an Eddington ratio distribution in our calculations. Such a power-law Eddington ratio distribution for AGNs is expected by the self-regulated black hole growth model (Silk & Rees 1998; Hopkins et al. 2005a), which is also supported by the Eddington ratio estimates for AGNs (e.g., Heckman et al. 2004; Yu et al. 2005; Hopkins & Hernquist 2009; Kauffmann & Heckman 2009).

There are three free parameters, $\eta_{rad,0}$, β_l , and λ_{peak} , in our calculations on the evolution of massive black holes. The resultant local BHMF for the AGN relics is sensitive to the value of the peak luminosity of AGNs. It is found that the measured local BHMF can be well reproduced by the BHMF of the AGN relics at z = 0 calculated in this work, if the three parameters, $\eta_{rad,0} = 0.11$, $\beta_l = 0.3$, and $\lambda_{peak} = 2.5$, are adopted (see Figure 3). Hopkins & Hernquist (2009) suggested that $\beta_l \simeq 0.3-0.8$ based on their self-regulated black hole growth model calculations, and our calculations also provide



Figure 4. BHMFs at different redshifts: z = 0 (red), 1 (green), 2 (blue), 3 (yellow), and 4 (black). The solid lines represent the total BHMFs, while the dashed lines are for AGN BHMFs. The parameters for the Eddington ratio distribution $\beta_l = 0.3$ and $\lambda_{peak} = 2.5$ are adopted. The shaded area is the measured local BHMF. In the upper panel, the results are derived with AGN life timescale $\tau_Q = 2.5 \times 10^8$ years, while the results with $\tau_Q = 5 \times 10^8$ and 7.5×10^8 years are plotted in the middle and lower panels, respectively.

a useful constraint on the value of β_l . Our results show that the peak Eddington ratio of AGNs ~ 2.5 is required for modeling the local BHMF, which implies that a small fraction of AGNs are accreting at slightly super-Eddington rates. This is consistent with the estimates for different samples of AGNs (e.g., Warner et al. 2004; Wu 2009; Ai et al. 2010; Willott et al. 2010).

The AGN BHMFs can be calculated from the AGN LF either with a distribution of Eddington ratios or a single mean Eddington ratio. The derived BHMF of AGN relics at redshift z = 0 is required to match the measured local BHMF, which usually leads to almost all AGNs to be accreting close to the Eddington limit in the calculations with a single mean Eddington ratio. This is inconsistent with the observations of AGN samples (e.g., Ho 2002; McLure & Dunlop 2004; Warner et al. 2004; Hopkins et al. 2006; Kollmeier et al. 2006; Kelly et al. 2010). The main difference between this work and the previous works is that a power-law Eddington ratio distribution instead of a single mean Eddington ratio is adopted in this work. The AGN BHMFs derived in this work are larger than those calculated with a single Eddington ratio close to unity adopted in those previous works (e.g., Yu & Tremaine 2002; Marconi et al. 2004; Shankar et al. 2004) especially at low redshifts, because the mean ratios for the power-law Eddington ratio distributions used in this work are significantly lower than unity. We find that the AGN BHMFs are larger than the total BHMFs for relatively small black holes at low redshifts if the quasar life timescale $\tau_{\rm Q}~\lesssim~5~\times~10^8$ years (see Figure 4), i.e., the duty cycle for AGNs $\delta > 1$ (see Figure 5), which is unphysical. These results imply that the quasar life timescale $\tau_Q \gtrsim 5 \times 10^8$ years, which is qualitatively consistent with the results derived either from theoretical calculations or observational data (Di Matteo et al.



Figure 5. AGN duty cycles as functions of black hole mass M_{bh} at different redshifts: z = 0(red), 1(green), 2(blue), 3(yellow), 4(black), and 5(cyan). The solid lines represent the duty cycles of all AGNs (i.e., the sources with $\lambda \ge \lambda_{min}$), while the dashed lines represent the duty cycles of bright AGNs (i.e., the sources accreting with $\dot{m} \ge \dot{m}_{crit}$). The parameters for the Eddington ratio distribution, $\beta_l = 0.3$ and $\lambda_{peak} = 2.5$, are adopted in the calculations.

2005; Hopkins & Hernquist 2009; Kelly et al. 2010). As an Eddington ratio distribution is adopted in our calculations, the AGNs with black hole mass M_{bh} may be accreting at very low rates. We also calculate the cosmological evolution of the duty cycle δ_b for the bright AGNs accreting at $\dot{m} > \dot{m}_{crit}$ as functions of $M_{\rm bh}$ (see Figure 5). We find that the bright AGN duty cycles δ_b are significantly higher than δ_b only at low redshifts, while they converge at high redshifts ($z \gtrsim 2$), which is caused by the value of λ_{\min} increasing with redshift z (see Figure 1). It is found that the bright AGN duty cycles are higher at high redshifts, and they decrease with increasing black hole mass at low redshifts, which are qualitatively consistent with those obtained in the previous works (e.g., Marconi et al. 2004; Merloni 2004; Wang et al. 2006; Shankar et al. 2009). Our results show that the massive black holes were grown earlier than their less-massive counterparts (see Figure 4), which is similar to what was found by Merloni (2004). The black holes with $M_{\rm bh} \lesssim 10^8 M_{\odot}$ were dominantly grown at redshifts $z \leq 1$ (see Figure 4). The Eddington ratios derived with bright AGN samples

The Eddington ratios derived with bright AGN samples usually exhibit a log-normal distribution (Kollmeier et al. 2006; Shen et al. 2008; Kelly et al. 2010), while those derived with the samples containing fainter sources show a power-law distribution (Heckman et al. 2004; Yu et al. 2005; Hopkins & Hernquist 2009), or a power-law distribution with an additional log-normal component at high Eddington ratios (Kauffmann & Heckman 2009). The peak and the dispersion of the lognormal Eddington ratio distribution were found to be almost independent of the black hole mass and redshift (Kollmeier et al. 2006), or they have very weak dependence on the black hole mass and redshift for a large AGN sample selected from the SDSS (Shen et al. 2008). Cao & Li (2008) calculated the evolution of massive black holes using a log-normal Eddington



Figure 6. Radiative efficiencies as functions of redshift *z*. The lines are derived with different quasar life timescales $\tau_Q = 2.5 \times 10^8$ (green), 5×10^8 (blue), and 7.5×10^8 years (black), respectively. The solid lines are the results calculated with s = 0.5, while the dashed lines are for s = 1.

ratio distribution derived by Kollmeier et al. (2006), and found that the measured local BHMF cannot always be reproduced by their model calculations unless a black hole mass-dependent radiative efficiency is assumed (see Cao & Li 2008, for the details). The sources in a large sample of nearby galaxies are separated into two populations by the age of galaxies, which show a power law and log-normal Eddington ratio distributions, respectively (Kauffmann & Heckman 2009). The power-law Eddington ratio distribution for the subsample exhibits a similar slope, ~ -0.8 , which is independent of black hole mass (see Kauffmann & Heckman 2009, for the details). In this work, a power-law Eddington ratio distribution independent of black hole mass and redshift is assumed in our calculations, which seems to be a reasonable assumption. We note that the slope of the power-law Eddington ratio distribution derived in this work is flatter than that of the distribution derived with a subsample of nearby galaxies in Kauffmann & Heckman (2009). The observed Eddington ratio distribution of all sources in their sample is a mixture of a power-law and log-normal distributions, the slope of which in the range of $\lambda \sim 10^{-4}$ – 10^{-2} becomes flatter than that of the power-law distribution derived with the subsample (see Figure 5 in Kauffmann & Heckman 2009) (note $\log L[O_{\rm III}]/M_{\rm bh} \sim 1.7$ corresponding to the Eddington rate). In this work, we use a power-law Eddington ratio distribution with an exponential cutoff at a high ratio in all our calculations in order to avoid inducing additional parameters, which can simulate the observed Eddington ratio distributions quite well (see Hopkins & Hernquist 2009, for the details). This distribution can also describe the main feature of the observed Eddington ratio distribution for the whole sample given in Kauffmann & Heckman (2009). The investigation on the evolution of massive black holes by adopting more realistic Eddington ratio distribution (e.g., a power-law+lognormal Eddington ratio distribution) will be carried out in our future work.

In this work, we adopt a \dot{m} -dependent radiative efficiency η_{rad} , in which the accretion mode transition with mass accretion rate \dot{m} is considered. We perform calculations with different values of *s* (from s = 0, i.e., an \dot{m} -independent η_{rad} , to s = 1), and find that the final results are insensitive to the value of *s*. It is found that the radiative efficiency evolves little with redshift (see Figure 6). This is because the fraction of mass accreted in

ADAF phases is small compared with that in bright AGN phase (Cao 2005; Hopkins et al. 2006; Cao 2007; Xu & Cao 2010). Watarai et al. (2000) calculations on the slim disks showed that the radiative efficiency will not deviate significantly from that for standard thin disks if $L_{bol}/L_{Edd} \leq 2$, which implies that the present adopted radiative efficiency independent of Eddington ratio (13) is indeed a good assumption for the sources with $\dot{m} \ge \dot{m}_{crit}$.

The Eddington ratio distribution is derived from a powerlaw quasar light curve in this work. The situation becomes complicated at high redshifts when the switch-on of AGN activity is not balanced with the switch-off of AGN activity, and the Eddington ratio distribution (2) may not be valid for the AGNs at high redshifts. However, the black holes were dominantly grown up at $z \leq 3$ (see Figure 4) when a powerlaw Eddington ratio distribution is believed to be a good approximation. It has therefore not much affected our final results.

The comparison of the calculated BHMF of the AGN relics at z = 0 with the local BHMF gives a lower limit on the quasar life timescale τ_Q . The BHMFs of the AGN relics at z = 0are insensitive to the adopted value of τ_Q , while they become significantly different at relatively high redshifts (e.g., z = 1, see Figure 4). They can be compared with the measured BHMF at z, which will set a further constraint on the quasar life timescale τ_Q , provided the BHMFs are available at relatively high redshift z. This is beyond the scope of this work.

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