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GIANT CONVECTION CELL TURNOVER AS AN EXPLANATION OF THE LONG SECONDARY PERIODS IN SEMIREGULAR RED VARIABLE STARS

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ABSTRACT

Giant convection cells in the envelopes of massive red supergiants turn over in a time comparable in order of magnitude with the observed long secondary periods in these stars, according to a theory proposed some years ago by Stothers & Leung. This idea is developed further here by using improved theoretical data, especially a more accurate convective mixing length and a simple calculation of the expected radial-velocity variations at the stellar surface. The theory is applied to the two best-observed red supergiants, Betelgeuse and Antares, with more success than in the earlier study. The theory can also explain the long secondary periods seen in the low-mass red giants, thus providing a uniform and coherent picture for all of the semiregular red variables. How the turnover of a giant convection cell might account for the observed slow light and radial-velocity variations, their relative phasing, and the absence of these variations in certain stars is discussed here in a qualitative way, but follows naturally from the theory.

Key words: convection – stars: interiors – stars: late-type – stars: oscillations – stars: variables: general

1. INTRODUCTION

A heterogeneous class of stars known as the semiregular red variables consists of moderately unstable M-type giants and supergiants that occupy a broad mass range of $\sim$1–30 $M_\odot$. These objects share some properties in common, however. Their amplitudes are typically small (a few tenths of a magnitude in visual light and a few km s^{-1} in radial velocity), and many of them display two prominent periods, both semiregular in character. The period ratios, P_2/P_1, are 7 ± 4 among the red supergiants (Stothers & Leung 1971; Kiss et al. 2006) and 9 ± 4 among the red giants (Wood et al. 1999). It is believed that the primary period, P_1, reflects radial pulsation that is driven by the classical opacity and ionization mechanisms (Stothers & Schwarzschild 1961; Stothers 1969; Keeley 1970; Wood 1974; Tuchman et al. 1979; Fox & Wood 1982; Wood et al. 1983; Li & Gong 1994; Heger et al. 1997; Bono & Panagia 2000; Guo & Li 2002; Olivier & Wood 2005; Nicholls et al. 2009) and/or stochastically by solar-like supergranular convective motions in the envelope (Schwarzschild 1975; Antia et al. 1984; Lebzelter et al. 2000; Christensen-Dalsgaard et al. 2001; Bedding 2003; Kiss et al. 2006; Gray 2008). The observationally favored modes for P_1 are the fundamental mode or the first overtone in the case of the red supergiants (Stothers 1969, 1972; Stothers & Leung 1971; Li & Gong 1994; Guo & Li 2002; Kiss et al. 2006) and the first or second overtone in the case of the red giants (Wood et al. 1983, 1999).

A long-standing puzzle about these stars is the origin of their very long secondary periods, P_2. If these are due to radial pulsation in the fundamental mode, their length is inexplicable since the theoretically computed ratio of fundamental period to first overtone period is typically $\sim$2 (Stothers 1972; Tuchman et al. 1979; Fox & Wood 1982; Lovy et al. 1984; Li & Gong 1994; Guo & Li 2002). Any larger ratios can only be achieved for unrealistically low effective temperatures (Stothers 1972; Fox & Wood 1982), very bright luminosities characteristic of the last few years of stellar evolution (Tuchman et al. 1979; Li & Gong 1994; Heger et al. 1997), or very high initial stellar masses (Guo & Li 2002). If the long secondary periods happen to arise from a beat between two normal modes of radial pulsation, the two beating periods would have to exist in the tight ratio $\sim$1.1. Such close periods, however, occur in the models only for very high overtones. Therefore on the grounds of period alone, radial pulsation is unlikely to explain P_2.

For the red supergiants, it was proposed long ago that P_2 represents the overturning time of giant convection cells in the convective envelope (Stothers & Leung 1971). This idea derived from the hypothesis of giant-cell turnover occurring in the Sun made by Simon & Weiss (1968) using a convective efficiency argument (but afterward retracted; Simon & Weiss 1991). Later, Petrovay (1990) showed that a very deep convection zone in a star would tend to generate large cells due simply to the morphological properties of convection. Numerical simulations for the solar envelope have also found some tentative evidence for giant cells (Miesch et al. 2008). Although faint surface manifestations of giant cells have been observed in magnetic networks and Doppler velocity patterns on the Sun (Bumba 1987; Beck et al. 1998; Lisle et al. 2004; Williams & Cuntz 2009), these observations can also be interpreted in terms of supergranular motions alone—not necessarily of supergranules advected by giant cells (Ulrich 2001; Schou 2003). In the case of the red supergiants, several bright spots have been directly imaged on the surface of Betelgeuse, beginning with the work of Buscher et al. (1990); however, the sizes and lifetimes of the spots are very uncertain (see the recent review by Haubois et al. 2009). Thus, it is unclear whether these spots could represent the giant cells of Stothers & Leung (1971), with overturning times of several years, or the supergranulation cells of Schwarzschild (1975), for which he predicted $\sim$90 at the surface of a red supergiant, with overturning times of several months. The numerical simulations of Chiavassa et al. (2009) appear to show both of the predicted scales of cellular convection occurring at the supergiant surface (see also Freytag et al. 2002; Dorch 2004).

In view of this uncertainty and of the fact that the theoretical overturning times of giant convection cells have been found to agree only crudely with the observed P_2 (Stothers & Leung 1971), the present study focuses in more detail on the two best-observed red supergiants, Betelgeuse and Antares, using improved theoretical and observational data. The same theory of

Table 1
Periods of Betelgeuse

P_1 (days)	P_2 (days)	Ref.
~200	2110	1,2
...	1980	3
200–500	2110	4
250	2070	5
200–400	2200	6
380	2100	7
420	...	8
380	...	9
436	...	10
...	1500	11
200[a]	2000	12
400	1500	13
388	2050	14
400	2000	15

Note. [a] Or 290 or 450 days.
References. (1) Spencer Jones 1928; (2) Goldberg 1984; (3) Stebbins 1931; (4) Sanford 1933; (5) Palmér 1939; (6) Stothers & Leung 1971; (7) Karovska 1987; (8) Dupree et al. 1987; (9) Smith et al. 1989; (10) Dupree et al. 1990; (11) Smith et al. 1995; (12) Percy et al. 1996; (13) Wasatonic & Guinan 1998; (14) Kiss et al. 2006; (15) Gray 2008.

giant cells is then applied to the semiregular M-type giants. The instability of these red giants has previously been interpreted in terms of radial pulsation, nonradial pulsation, axial rotation effects, magnetic dynamo action, and orbiting companions, but none of these mechanisms works very well (Wood et al. 2004; Nicholls et al. 2009). We show here that the theory of giant convection cells can explain both P_2 and the observed surface radial velocities for both classes of semiregular red variables.

2. OBSERVATIONAL DATA

2.1. Betelgeuse

Betelgeuse (α Ori) is a red supergiant whose mean spectral type is M2 Iab. The star is variable in both light and radial velocity on a number of timescales, but two stand out prominently, with periods of roughly 400 and 2100 days (Table 1). The radial-velocity amplitude is 4–6 km s^{-1} for both the long period (Spencer Jones 1928; Sanford 1933; Goldberg 1984) and the short period (Dupree et al. 1987; Smith et al. 1989; Gray 2008). The cycles are semiregular.

Harper et al. (2008) have derived a mean radius of 950 $R_\odot$ for Betelgeuse by using an accurate new distance of 197 pc and an angular diameter of 45 mas. This agrees well with the radius determined from the Stefan–Boltzmann law, $L = 4\pi R^2 \sigma T_e{}^4$, as those authors have also shown. The mass then follows from theoretical evolutionary tracks fitted to the measured absolute bolometric magnitude, $M_{\rm bol} = -8.03$; the mass comes out to be 20 $M_\odot$ (Stothers & Chin 1997) or perhaps 18 $M_\odot$, allowing for some mass loss (Harper et al. 2008). Harper et al. (2008) adopted an effective temperature of 3650 K from Levesque et al. (2005), but this value is close to the mean of many other recent determinations as reviewed by Lobel & Dupree (2000).

2.2. Antares

Antares (α Sco) is an M1.5 Iab supergiant showing two dominant periods of light and radial-velocity variability (Table 2).

Table 2
Periods of Antares

P_1 (days)	P_2 (days)	Ref.
...	2120	1
...	2680	2
...	1733	3
260	2150	4
...	2200	5
350	...	6
...	1650	7

References. (1) Lunt 1916; (2) Spencer Jones 1928; (3) Stothers & Leung 1971; (4) Smith et al. 1989; (5) Smith et al. 1995; (6) Percy et al. 1996; (7) Kiss et al. 2006.

These periods are not strictly regular, but are roughly equal to 300 and 1700 days if the longest records are carefully examined. The slow period is associated with a radial-velocity amplitude of ~4 km s^{-1} (Halm 1909; Spencer Jones 1928; Smith et al. 1989), which seems to characterize the fast period as well (Smith et al. 1989).

The distance modulus of Antares has been determined by three independent methods. The *Hipparcos* catalog (SIMBAD 1997) provides a trigonometric parallax that converts to a distance modulus of $(m - M)_0 = 6.34$, while the moving cluster method gives 6.11 (Bertiau 1958) and 5.88 (de Bruijne 1999), and the color–magnitude diagram of the Upper Scorpius region yields values of 6.20 (Eggen 1983), 6.00 (de Geus et al. 1989), and 6.15 (Eggen 1998). An average value of these seven different determinations is 6.11, which yields a distance of 170 pc. Using the most recently measured angular diameter of the star, 41.3 mas (Richichi & Lisi 1990), which agrees fairly well with older measurements, we find a radius of 750 $R_\odot$ for Antares.

An independent way of deriving the radius utilizes the Stefan–Boltzmann law. We need to first evaluate the absolute bolometric magnitude from

$$M_{\rm bol} = V - 3E_{B-V} - (m - M)_0 + {\rm BC}. \qquad (1)$$

Adopting $V = 1.06$ and $B–V = 1.86$ (de Bruijne 1999), $(B–V)_0 = 1.70$ (Lee 1970), and BC $= -1.43$ (Levesque et al. 2005), we obtain $M_{\rm bol} = -6.98$. With $M_{\rm bol\odot} = 4.72$ and $T_e = 3710$ K (Levesque et al. 2005), the radius comes out to be 530 $R_\odot$. We shall simply adopt an average of our two radius determinations, $R = 640\ R_\odot$. Using the derived absolute bolometric magnitude, we infer a mass of 15 $M_\odot$ (Stothers & Chin 1997).

2.3. M-type Giants

Red (M-type) giants that are fainter than classical Mira variables show a weaker variability than those stars do. Cycles occur on at least two timescales (Wood et al. 1999; Olivier & Wood 2003). The shorter cycle has a typical length of ~80 days, while the longer cycle is characteristically ~700 days long; the cosmic scatter around both cycle times is ~40%. These stars are all semiregular. Radial velocity varies during the long cycle by 3–6 km s^{-1} (Hinkle et al. 2002; Wood et al. 2004). The radial-velocity amplitude for the short cycle cannot be larger than this and is likely to be comparable in size.

For typical M4 giants, Dyck et al. (1996, 1998) and van Belle et al. (1999) derived empirically a mean effective temperature of 3500 K and a mean radius of 100 $R_\odot$. Subsequently, by fitting theoretical nonlinear pulsational models to observations of red semiregular variables, Nicholls et al. (2009) inferred 3700 K

Table 3
Observed and Predicted Surface Velocities and Long Secondary Periods

Star Name	T_e (K)	$M/M_\odot$	$R/R_\odot$	P_2 (days)	$2R/P_2$ (km s^{-1})	ΔV_2 (km s^{-1})	α	$2\tau_{mix}$ (days)
Betelgeuse	3650	20	950	2100	7.3	5	1.3	2500
Antares	3710	15	640	1700	6.1	4	1.5	2100
M-type Giant	3600	1.5	100	700	2.3	4.5	2.0	940

and $\sim$130 $R_\odot$. Masses of most of these stars are low, 1–2 $M_\odot$ (Hinkle et al. 2002; Nicholls et al. 2009) with a scattering of values up to $\sim$9 $M_\odot$ (Wood et al. 1983). The latter group of more massive stars can be identified by their high luminosities in the Magellanic Clouds, but open clusters of intermediate age in the Galaxy also contain a few luminous M-type giants that likewise populate the asymptotic giant branch (Harris 1976; Stothers & Chin 1995). Their variability has not yet been studied.

3. THEORY OF GIANT CONVECTION CELLS

A giant cell is assumed here to range over the full depth of the convection zone, regardless of its shape and its filling factor. The cell is presumed to move upward and downward at a mean velocity, v, given by standard mixing-length theory (Böhm-Vitense 1958) modified to include radiation pressure:

$$v = [\alpha(\Gamma_3 - 1)\, H_{\rm conv}/2\rho]^{1/3}. \tag{2}$$

Here α is the ratio of mixing length to pressure scale height, Γ_3 is the third generalized adiabatic exponent, $H_{\rm conv}$ is the convective flux, and ρ is the density. A good approximation is to take $H_{\rm conv} = \sigma T_e{}^4 = L/4\pi r^2$ throughout the bulk of the convection zone, where T_e is the local effective temperature. Since the radial extent of the convection zone is essentially equal to the radius of the star for such an extended object as a red giant or supergiant, the time required to move between the base and the top is just equal to the integral of v^{-1} over the radius. This turns out to be (Stothers & Leung 1971):

$$\tau_{\rm mix} = I\left(2M/4\pi\sigma T_e^4\alpha\right)^{1/3}, \tag{3}$$

where T_e is the effective temperature at the stellar surface and I is a constant of order unity. Since I depends on Γ_3 only weakly—as $\langle\Gamma_3 - 1\rangle^{-1/3}$—a lower limit on I follows from taking $\Gamma_3 = 5/3$ for a perfect gas and adopting a uniform-density model ($\rho =$ constant) which yields $I = 0.99$; the centrally concentrated Roche model [$M(r) = M$, $\rho \sim r^{-2}$] provides an upper limit of $I = 1.65$. More realistic models of red supergiants yield $I \approx 1.4$ (Stothers & Leung 1971); in view of the restrictive limits just stated this value is provisionally assumed to hold approximately also for red giants.

In our previous work (Stothers & Leung 1971), we adopted $\alpha = 0.5$ (incorrectly referred to there as the ratio of mixing length to density scale height). This value is now known to be too small. Fits to red giants and red supergiants in the Hertzsprung–Russell diagram show that the observed effective temperatures are matched very well by stellar models with $\alpha = 2.0$ for red giants (Salaris & Cassisi 1996; Ferraro et al. 2006) and $\alpha = 1.3$–1.5 for red supergiants (Stothers & Chin 1997).

Since models of red giant and supergiant envelopes possess a high central condensation, their density distribution closely follows the Roche distribution (see, e.g., the red giant models of Tuchman et al. 1978). Thus the local mean convective velocity, as given by Equation (2), remains approximately constant throughout most of the envelope (Antia et al. 1984). Even for the Sun, v stays close to 1–2 km s^{-1} except very near the radiative boundaries at the top and bottom (Baker & Temesvary 1966). A near constancy of the mean velocity is also ensured by the continuity equation:

$$4\pi r^2\rho v = \text{constant}. \tag{4}$$

Furthermore, detailed three-dimensional numerical simulations of deep convective envelopes confirm the adequacy of mixing-length theory except near the boundary layers (Chan & Sofia 1987; Cattaneo et al. 1991; Kim et al. 1996) and show in particular the rough constancy of the upward and downward mean velocities (Chan & Sofia 1986). This constancy assures a high degree of coherence of the giant cells. It is probably the large turbulent eddy viscosity that maintains the predicted laminar structure of cells of all sizes (Schwarzschild 1959; Stothers 2000; Canuto 2000; Olivier & Wood 2005). Observationally, unlike the Sun, the upper radiative boundary lies so high up within the atmosphere that the giant cells ought to appear much more visibly than in the Sun.

4. COMPARISON OF THEORY AND OBSERVATION

Two tests of our giant convection cell idea will be performed with the observational data of Table 3. First, the radial-velocity amplitudes, ΔV_2, for the long secondary cycles are seen to be typically 4–5 km s^{-1}. If these cycles were due to some kind of radial pulsation, the total excursions of radius would exceed the stars' mean radii by an order of magnitude, as has long been recognized. Furthermore, the enormous changes of radius would induce equally large changes of the primary periods of pulsation (via the period–mean density relation), which are not observed (Wood et al. 1999, 2004; Nicholls et al. 2009). Thus, the radial velocities probably reflect a slow cyclical upwelling and downwelling of material. If this motion is due to the turnover of a few giant convection cells, and if these cells have a coherent structure as was argued above, the surface radial-velocity amplitude should be closely related to the mean convective turnover velocity, which is given by $2R/P_2$. According to Table 3, the values of $2R/P_2$ are 2–7 km s^{-1}, supporting this idea.

Next, the computed values of the convective overturning time, $2\tau_{\rm mix}$, are obtained from Equation (3) and are entered in Table 3. The agreement with the long secondary periods, P_2, is striking, considering the observational and theoretical uncertainties. Observationally, P_2 closely tracks the stellar masses in the way predicted by Equation (3). This is expected because the dispersions of T_e and α among all these stars are comparatively small.

It was found many years ago that the observed long secondary periods of red supergiants increase with luminosity (i.e., mass) and with advancing spectral type (i.e., decreasing effective temperature; Stothers & Leung 1971). Although these trends were at the time qualitatively predicted by the proposed theory, the theoretical values of the periods turned out to be, on the average, $\sim$40% too large. This discrepancy, as we now see, almost disappears when α is raised from the former value of 0.5 to a more realistic 1.5.

The situation is not so clear for the red giants, however. The full range of long secondary periods for these stars extends over a factor of two or three (Wood et al. 1999; Olivier & Wood 2003), but taking Equation (1) at face value, this would imply a range of masses extending over a factor of $\sim$10–30. In fact, the actual masses cover, for the most part, only a factor of $\sim$2. We suspect that the fault lies in our adopted value of $I = 1.4$. This value is appropriate for red supergiants in which the ionization zones of hydrogen and helium lie relatively close to the surface (Stothers 1972), so that throughout most of the envelope Γ_3 falls between 5/3 and 4/3, leading to little variation in $(\Gamma_3 - 1)^{1/3}$. Red giants, however, are much cooler stars and have deeper ionization zones, where Γ_3 can drop to values almost as low as 1 over significant portions of the envelope (Tuchman et al. 1978). One would therefore expect I to increase somewhat with decreasing effective temperature. This would in turn lead to longer secondary periods for the brighter of these variables, which possess cooler effective temperatures. The consequence would be a period–luminosity relation. Although this trend is actually observed (Wood et al. 1999), we cannot predict it quantitatively, since detailed models of red giants are needed to obtain I in particular cases. Note that only modest changes of I seem to be necessary since the simple value adopted here works well for a typical red giant (Table 3). The present conjecture is reinforced by the fact that extremely long secondary periods are observed for the low-mass carbon (C-type) stars, which are even cooler than the M giants (Olivier & Wood 2003). Although the true dependence of v on the adiabatic exponents is somewhat more complicated than $(\Gamma_3 - 1)^{1/3}$, which holds (with a minor simplification) for a fully ionized gas and radiation, the qualitative arguments made here remain unaffected.

There remains the problem of how the turnover of a giant convection cell or of a very small number of such cells can produce the observed light and radial-velocity variations at the surface. The numerical simulations of Chiavassa et al. (2009), as applied to the convective envelope of Betelgeuse, cover effectively only about half of a long cycle and so cannot shed light on this question. We conjecture that, in analogy with available observational and theoretical data for the granular and supergranular motions at the surface of the Sun, a giant cell consists of an upwelling inner part surrounded by downflows. During half of the long cycle, the upflowing part is very broad and dominates the integrated stellar disk. Since the uprising material is also hotter than its surroundings, the star appears brighter when the radial velocity is directed outward. During the other half of the cycle, we suppose that the downflows predominate and cause the star to appear fainter and to display an inward-directed radial velocity. In the Sun, the two halves of the granular and supergranular cycles are observed to be less symmetric, but these solar phenomena are very superficial and are tied to the underlying convection. A giant cell in a supergiant star could well undergo a more regular cycle. Why the convective turnover is cyclical rather than random and sporadic has to do with the internal dynamics of convection, as one giant cell is replaced at the envelope bottom after its predecessor dissolves in a time equal to $2\tau_{\mathrm{mix}}$—conservation of mass will dictate a certain quasi-periodicity. Observational evidence lends some supports to this idea. Goldberg (1984) has found that maximum outward radial velocity coincides very nearly with light maximum (and, more generally, that the radial velocity and light vary together) during the long cycle of Betelgeuse.

Not all semiregular variables would necessarily display long secondary periods, and the fainter ones tend not to. Convection has to be vigorous enough to generate noticeable giant cells, and this would occur preferentially at higher luminosities. Christensen-Dalsgaard & Frandsen (1983) and Christensen-Dalsgaard et al. (2001) showed theoretically that convection intensifies greatly in going up the red giant branch.

Although rotation should be ineffectually slow in the upper part of the envelope of cool giants and supergiants, this condition would probably not obtain in the inner part where the giant cells start forming. These low-lying layers, close to the stellar core, could well be swiftly rotating in some cases. As main-sequence stars show a range of rotational velocities, so would the deep inner envelopes of their cool descendants owing to the local conservation of angular momentum. Fast rotation might thus prevent the formation of coherent giant cells or at least render them irregular and therefore not easily visible at the stellar surface. This could account for the fact that many red giants and supergiants do not show long secondary periods, or at least prominent ones.

5. CONCLUSION

Giant convection cell turnover can potentially explain the long secondary periods as well as the surface radial-velocity amplitudes that are seen in many semiregular red variable stars, as exemplified by Betelgeuse, Antares, and typical M-type giants. Therefore, the present study supports the original giant-cell conjecture of Stothers & Leung (1971), who only treated the problem statistically and also exclusively for the class of red supergiants. Their rough results for P_2 have been considerably improved here by using an updated value for the ratio of convective mixing length to pressure scale height. Red supergiants displaying very large amplitudes for their primary periods, like S Per, might not easily reveal their low-amplitude secondary periods if the latter cycles do exist (Kiss et al. 2006). The same difficulty of detection would exist for the bright M-type giants that show large primary-period amplitudes, like Mira (Wood et al. 2004).

It is a fact that the primary periods of the red supergiants are associated with moderate surface radial-velocity amplitudes that are nearly identical to those for the long secondary periods. This suggests that both periods have a convective origin and therefore supports the conjecture of Schwarzschild (1975) that the smaller and more numerous supergranular cells either drive the primary cycles or make them somewhat irregular if they are actually excited by the classical opacity and ionization mechanisms, which seems more likely (Nicholls et al. 2009). The ratio of the depths of the giant cells and supergranular cells would then be expected to be approximately equal to the ratio of the secondary and primary periods, or $\sim$7. In fact, Schwarzschild (1975) estimated that the supergranular cells in a red supergiant model extend over roughly 14% of the stellar radius. If their horizontal extent should happen to be proportionately much larger than in the Sun, they might even look superficially like giant cells, as Schwarzschild tentatively suggested. In contrast, Stothers & Leung (1971) provisionally suggested that the giant cells themselves might induce irregularity in the primary periods. Both of these two additional suggestions now seem to be unnecessary.

Each type of convection cell probably overshoots into the weakly radiative atmosphere (Boesgaard 1979; Josselin & Plez

2007; Gray 2008) and creates the chromospheric activity and dust formation that are observed in both the red supergiants (Dupree et al. 1987; Bester et al. 1996; Uitenbroek et al. 1998; Lim et al. 1998; Lobel & Dupree 2000) and the red giants (Wood et al. 2004; Nicholls et al. 2009; Wood & Nicholls 2009). The rising cells would essentially push material outward into the upper atmosphere, whereupon most of it would fall back inward as the cells turn over. Therefore, the same cellular convective mechanisms could explain some important aspects of both the low-mass and high-mass semiregular red variables.

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