

SCATTERING POLARIZATION OF THE Ca II IR TRIPLET FOR PROBING THE QUIET SOLAR CHROMOSPHERE

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ABSTRACT

The chromosphere of the quiet Sun is a very important stellar atmospheric region whose thermal and magnetic structure we need to decipher in order to unlock new discoveries in solar and stellar physics. To this end, we need to identify and exploit observables sensitive to weak magnetic fields ($B \lesssim 100$ G) and to the presence of cool and hot gas in the bulk of the solar chromosphere. Here, we report on an investigation of the Hanle effect in two semi-empirical models of the quiet solar atmosphere with different chromospheric thermal structures. Our study reveals that the linear polarization profiles produced by scattering in the Ca II IR triplet have thermal and magnetic sensitivities potentially of great diagnostic value. The linear polarization in the 8498 Å line shows a strong sensitivity to inclined magnetic fields with strengths between 0.001 and 10 G, while the emergent linear polarization in the 8542 Å and 8662 Å lines is mainly sensitive to magnetic fields with strengths between 0.001 and 0.1 G. The reason for this is that the scattering polarization of the 8542 Å and 8662 Å lines, unlike the 8498 Å line, is controlled mainly by the Hanle effect in their (metastable) lower levels. Therefore, in regions with magnetic strengths noticeably larger than 1 G, their Stokes Q and U profiles are sensitive only to the orientation of the magnetic field vector. We also find that for given magnetic field configurations the sign of the Q/I and U/I profiles of the 8542 Å and 8662 Å lines is the same in both atmospheric models, while the sign of the linear polarization profile of the 8498 Å line turns out to be very sensitive to the thermal structure of the lower chromosphere. We suggest that spectropolarimetric observations providing information on the relative scattering polarization amplitudes of the Ca II IR triplet will be very useful to improve our empirical understanding of the thermal and magnetic structure of the quiet chromosphere.

Key words: polarization – radiative transfer – scattering – stars: magnetic field – Sun: chromosphere

1. INTRODUCTION

The chromosphere of the quiet Sun is one of the most complex stellar atmospheric regions (e.g., Harvey 2006, 2009; Judge 2006, 2009). Lying between the thin photosphere and the extended 10^6 K corona, it is here where the magnetic field becomes the globally dominating factor ruling the outer solar atmosphere. Measuring the magnetic field vector in the solar chromospheric plasma is, however, notoriously difficult, especially outside sunspots and related active regions. Quantitative information can be obtained through spectropolarimetry, but the measurement and physical interpretation of the weak polarization signatures that the Hanle and Zeeman effects produce in the few chromospheric lines that can be observed from ground-based telescopes is not an easy task (see Trujillo Bueno 2010 for a recent review). It is thus important to develop novel methods of “measuring” the chromospheric magnetic field, ideally based on the action of the Hanle and Zeeman effects in spectral lines whose line-center intensity images reveal the fine-scale thermodynamic structuring of the solar chromosphere. Here, we show that the scattering polarization observed in the IR triplet of Ca II contains valuable information on the thermal and magnetic structure of the “quiet” chromosphere, from the shock-dominated region of the “lower chromosphere” to the “upper chromosphere” below the interface region to the 10^6 K solar corona.

The only way to obtain direct empirical information on the intensity and topology of the magnetic fields of the solar chromosphere is via the measurement and interpretation of polarization signals in chromospheric spectral lines. In regions with high concentrations of magnetic flux, such as in sunspots and plages, the polarization patterns are dominated by the Zeeman effect. Hence, diagnostic techniques based on this effect are quite useful (e.g., Socas-Navarro et al. 2000; Socas-Navarro 2005). However, in the quiet Sun, which covers most of the solar disk at any given time during the solar magnetic activity cycle, the Zeeman splitting between the π ($\Delta M = M_u - M_l = 0$) and $\sigma_{b,r}$ ($\Delta M = \pm 1$) components (M being the magnetic quantum number) is only a small fraction of the width of the spectral lines formed in the chromosphere. As a result, the transverse Zeeman effect makes a very small or insignificant contribution to the emergent linear polarization amplitudes. On the other hand, the circular polarization of the Zeeman effect as a diagnostic tool is of limited practical interest for the exploration of the magnetism of the upper chromosphere because the response function of the emergent Stokes V profiles of strong lines like H α and Ca II 8542 Å is significant only in the photosphere and/or lower chromosphere (Socas-Navarro & Uitenbroek 2004; Uitenbroek 2006).

There is yet another mechanism producing linear polarization in the spectral lines that can be exploited to diagnose the solar chromosphere. Atoms align due to the optical pumping caused by the incident anisotropic radiation field, i.e., the individual magnetic M sublevels of energy levels with total angular momentum $J > 1/2$ are unevenly populated (in such a way that the populations of substates with different values of $|M|$ are unequal) and coherences between them may appear. This, in turn, gives rise to a sizable amount of linear polarization in chromospheric spectral lines, either by selective emission or absorption of polarization components in the spectral line. A remarkable example is provided by the IR triplet of Ca II. Its linear polarization pattern was considered “enigmatic” for a while (Stenflo et al. 2000) because it was impossible to achieve even a qualitative fit to the observed relative polarization amplitudes considering that scattering line polarization results only from selective emission processes caused by the

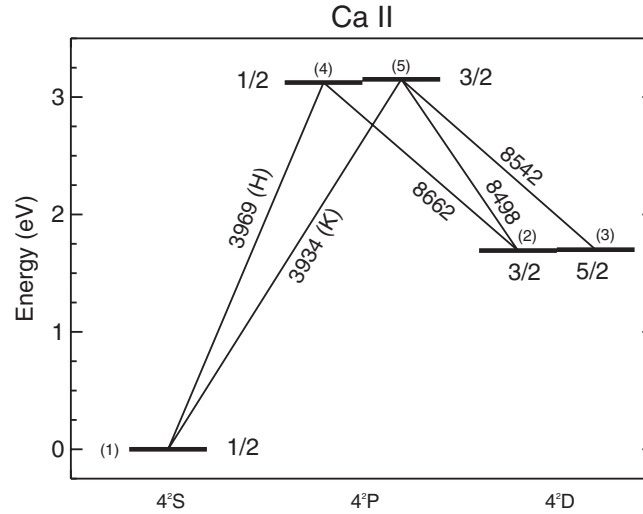


Figure 1. Atomic model for Ca II. Labels indicate the total angular momentum of the level and its numbering (between parentheses) in Equations (10)–(18). Solid lines connecting levels show allowed radiative transitions and labels show their wavelength in Å.

population imbalances of the line’s upper level. It was later realized that selective absorption of polarization components caused by the presence of populations imbalances in the line’s lower level is a very efficient mechanism for producing linear polarization in the spectral lines of a stellar atmosphere (Trujillo Bueno & Landi Degl’Innocenti 1997; Trujillo Bueno 1999; Trujillo Bueno et al. 2002). In fact, Manso Sainz & Trujillo Bueno (2003a) demonstrated quantitatively that the physical origin of the linear polarization pattern observed in the Ca II lines at 8542 and 8662 Å is indeed “zero-field” dichroism, i.e., selective absorption of polarization components caused by the presence of atomic alignment in the metastable lower levels $4^2D_{3/2}$ and $4^2D_{5/2}$ (see Figure 1), while that observed in the 8498 Å line is controlled by both dichroism and by the selective emission of polarization components that result from the population imbalances between the sublevels of its upper level $2^2P_{3/2}$.

The alignment of the atomic J levels and the ensuing emergent linear polarization is modified by the presence of a magnetic field through the Hanle effect, even if the magnetic field turns out to have a complex, tangled topology, with mixed polarities at subresolution scales (e.g., Landi Degl’Innocenti & Landolfi 2004). In the absence of collisional depolarization, the magnetic field strength B (in G) that is sufficient to produce a significant change is

$$B \approx \frac{1.137 \times 10^{-7}}{t_{\text{life}} g_J}, \quad (1)$$

where t_{life} and g_J are, respectively, the lifetime (in seconds) and the Landé factor of the J level under consideration. Interestingly enough, since the lifetimes of the metastable lower levels of the Ca II IR triplet are about 2 orders of magnitude larger than the upper level lifetimes and levels with $J = 1/2$ or $J = 0$ cannot carry atomic alignment, the linear polarization of the Ca II line at 8662 Å (whose upper level has $J_u = 1/2$) is expected to be sensitive to mG fields only, while the scattering polarization of the 8498 Å and 8542 Å lines (which share the same upper level whose $J_u = 3/2$) can in principle be sensitive also to magnetic fields in the gauss range. In the quiet solar chromosphere depolarization of the Ca II atomic levels by elastic collisions with neutral hydrogen atoms is insignificant for the upper levels of the H and K lines, while such collisions have a rather small impact on the atomic polarization of the metastable lower levels (Manso Sainz & Trujillo Bueno 2003a; Derouich et al. 2007). Actually, the mere fact that the Ca II 8662 Å line shows a clear Q/I polarization signal (see Stenflo et al. 2000) implies that its lower level (whose $J_l = 3/2$) is significantly polarized in the quiet solar chromosphere (Manso Sainz & Trujillo Bueno 2003a).

The aim of this paper is to investigate the thermal and magnetic sensitivity of the scattering polarization Q/I and U/I profiles of the Ca II IR triplet. After formulating the multilevel problem of scattering polarization and the Hanle effect in Section 2, Section 4 discusses the unmagnetized case in the “hot” and “cool” chromospheric models described in Section 3. The impact of the Hanle effect on the linear polarization profiles of the Ca II IR triplet is discussed in Section 5, showing for each solar atmospheric model detailed results for the cases of disk center (Section 5.1) and close to the limb observations (Section 5.2). The ensuing information is summarized in Section 5.3 in the form of Hanle effect diagrams for each of the spectral lines of the IR triplet. Finally, Section 6 summarizes our main conclusions with an outlook to future research.

2. FORMULATION OF THE PROBLEM AND RELEVANT EQUATIONS

The relevant radiative transfer problem here implies calculating, at each spatial point of the (generally magnetized) astrophysical plasma model under consideration, the values of the diagonal and non-diagonal elements of the atomic density matrix corresponding to each atomic level of total angular momentum J , which quantify its overall population, the population imbalances between its magnetic sublevels, and the quantum coherences between each pair of them. The values of such density-matrix elements have to be consistent with the intensity, polarization, and symmetry properties of the radiation field generated within the medium. This requires solving jointly the radiative transfer (RT) equations for the Stokes parameters and the statistical equilibrium equations for the density-matrix elements. Once such a self-consistent solution is obtained, it is straightforward to compute the emergent Stokes

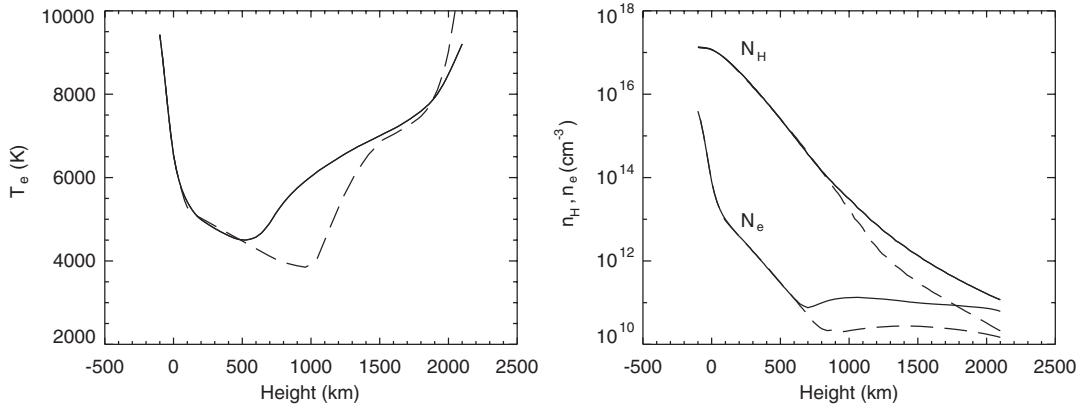


Figure 2. Atmospheric models. Variation with height of the kinetic temperature (left panel), hydrogen and electron density (right panel) in the FAL-C model (solid lines), and M-CO model (dashed lines). Note that in the left panel the “lower chromosphere” of the M-CO model is cooler than in FAL-C.

profiles via the formal solution of the Stokes-vector transfer equation for any desired line of sight (LOS). The aim of this section is to formulate such equations with emphasis on the case of the IR triplet of Ca II.

2.1. The Multipolar Components of the Atomic Density Matrix

The excitation state of the atomic system is described through the density matrix (Fano 1957). We consider the populations ($\rho_J(M, M)$), and coherences ($\rho_J(M, M')$) among the $(2J + 1)$ magnetic sublevels M of each given atomic level with total angular momentum J . Alternatively, we use the multipolar components of the atomic density matrix (Omont 1977):

$$\rho_Q^K(J) = \sum_{MM'} (-1)^{J-M} \sqrt{2K+1} \begin{pmatrix} J & J & K \\ M & -M' & -Q \end{pmatrix} \rho_J(M, M'), \quad (2)$$

where $K = 0, 1, \dots, 2J$, $Q = -K, -K+1, \dots, K-1, K$, and the expression in brackets is the Wigner $3j$ symbol (e.g., Brink & Satchler 1968). The ρ_0^K elements with $K \neq 0$ are real numbers measuring population imbalances between sublevels, while $\sqrt{2J+1}\rho_0^0$ is the total population of the level. The $\rho_{Q \neq 0}^K$ elements are combinations of the complex non-diagonal components $\rho_J(M, M')$. Hermiticity of the density matrix ($\rho_J(M, M') = \rho_J(M', M)^*$) implies $\rho_{-Q}^K = (-1)^Q [\rho_Q^K]^*$ (“*” means complex conjugate). In the absence of net circular polarization in the incident radiation $\rho_J(M, M) = \rho_J(-M, -M)$, i.e., atoms *align* but they do not *orientate*. Hence, odd- K ρ_Q^K elements vanish. Therefore, the five-level atomic model of Ca II considered here (Figure 1) requires $1 + 6 + 15 + 1 + 6 = 29$ ρ_Q^K elements to fully characterize the excitation state of the atomic system.

2.2. The Transfer Equation for the Stokes Parameters

The transfer of spectral line polarization is described by the following equations for the Stokes parameters:

$$\frac{d}{ds} I = \epsilon_I - \eta_I I - \eta_Q Q - \eta_U U - \eta_V V, \quad (3a)$$

$$\frac{d}{ds} Q = \epsilon_Q - \eta_Q I - \eta_I Q - \rho_V U + \rho_U V, \quad (3b)$$

$$\frac{d}{ds} U = \epsilon_U - \eta_U I + \rho_V Q - \eta_I U - \rho_Q V, \quad (3c)$$

$$\frac{d}{ds} V = \epsilon_V - \eta_V I - \rho_U Q + \rho_Q U - \eta_I V. \quad (3d)$$

In the Hanle effect regime, the magnetic splitting of σ and π components of the line transitions are negligible (compared with the spectral line width) and the line emissivity thus depends exclusively on the excitation state of the upper level of the transition (i.e., on the $\rho_Q^K(J_u)$ values). In particular, linearly polarized emission is due to the presence of alignment and coherences between atomic sublevels of the upper level of the transition. Moreover, since levels are not oriented, $\epsilon_V^{\text{line}} \equiv 0$. More explicitly (Landi Degl’Innocenti & Landolfi 2004; see also Manso Sainz & Trujillo Bueno 1999)

$$\begin{aligned} \epsilon_I^{\text{line}} = & \epsilon_0 \rho_0^0 + \epsilon_0 w_{J_u J_l}^{(2)} \left\{ \frac{1}{2\sqrt{2}} (3\mu^2 - 1) \rho_0^2 - \sqrt{3} \mu \sqrt{1 - \mu^2} (\cos \chi \text{Re}[\rho_1^2] - \sin \chi \text{Im}[\rho_1^2]) \right. \\ & \left. + \frac{\sqrt{3}}{2} (1 - \mu^2) (\cos 2\chi \text{Re}[\rho_2^2] - \sin 2\chi \text{Im}[\rho_2^2]) \right\}, \end{aligned} \quad (4a)$$

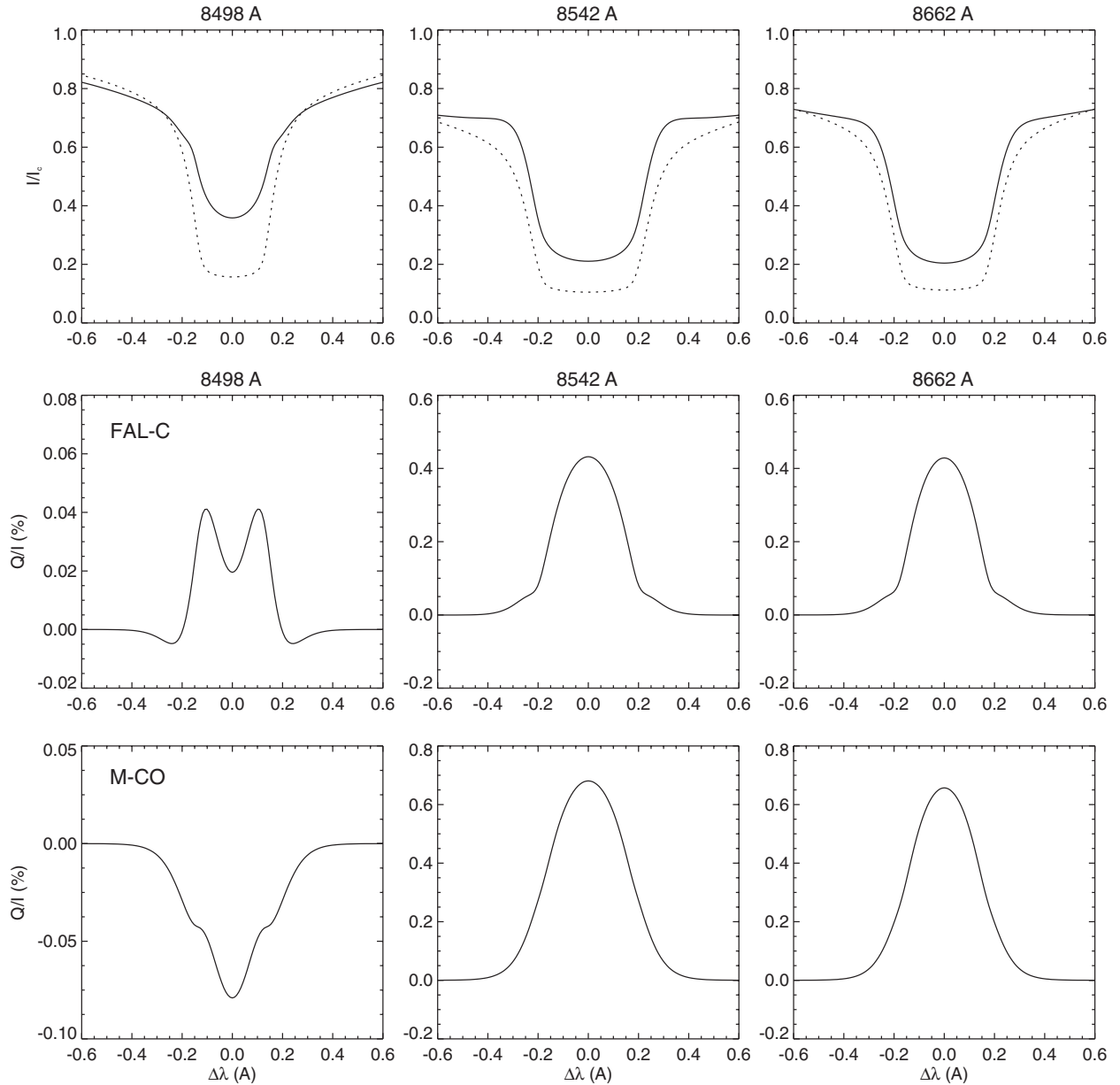


Figure 3. Unmagnetized case. Emergent Q/I profiles of the Ca II IR triplet calculated for an LOS with $\mu = 0.1$ in the “hot” FAL-C model (middle panels) and in the “cool” M-CO model (bottom panels). The positive Q -direction is parallel to the nearest limb. Note that the sign of the Q/I profile of the 8498 Å line is positive in the “hot” model and negative in the “cool” model. The uppermost panel shows the emergent intensity profiles computed in the FAL-C model (solid lines) and in the M-CO model (dotted lines).

$$\epsilon_Q^{\text{line}} = -\epsilon_0 w_{J_u J_l}^{(2)} \left\{ \frac{3}{2\sqrt{2}} (\mu^2 - 1) \rho_0^2 - \sqrt{3} \mu \sqrt{1 - \mu^2} (\cos \chi \text{Re}[\rho_1^2] - \sin \chi \text{Im}[\rho_1^2]) - \frac{\sqrt{3}}{2} (1 + \mu^2) (\cos 2\chi \text{Re}[\rho_2^2] - \sin 2\chi \text{Im}[\rho_2^2]) \right\}, \quad (4b)$$

$$\epsilon_U^{\text{line}} = -\epsilon_0 w_{J_u J_l}^{(2)} \sqrt{3} \left\{ \sqrt{1 - \mu^2} (\sin \chi \text{Re}[\rho_1^2] + \cos \chi \text{Im}[\rho_1^2]) + \mu (\sin 2\chi \text{Re}[\rho_2^2] + \cos 2\chi \text{Im}[\rho_2^2]) \right\}, \quad (4c)$$

where $\epsilon_0 = (h\nu/4\pi) A_{ul} \phi_x \mathcal{N} \sqrt{2J_u + 1}$ (with $\mathcal{N}$ as the total number of atoms per unit volume, A_{ul} as the Einstein coefficient for spontaneous emission, and ϕ_x as the Voigt profile, with $x = \nu_0 - \nu$ and ν_0 the transition’s frequency), $\theta = \arccos(\mu)$ and χ are the inclination and azimuth of the ray, and $w_{J_u J_l}^{(2)}$ is a numerical coefficient that depends only on J_u and J_l . In particular, $w_{1/2,3/2}^{(2)} = 0$, $w_{3/2,3/2}^{(2)} = -2\sqrt{2}/5$, and $w_{3/2,5/2}^{(2)} = \sqrt{2}/10$ (see Table 10.1 in Landi Degl’Innocenti & Landolfi 2004).

The absorption coefficient η_I^{line} , and the dichroism coefficients η_Q^{line} and η_U^{line} are given by analogous expressions but with $\eta_0 = (h\nu/4\pi) B_{lu} \phi_x \mathcal{N} \sqrt{2J_l + 1}$ instead of ϵ_0 , $w_{J_l J_u}^{(2)}$ instead of $w_{J_u J_l}^{(2)}$ ($w_{3/2,1/2}^{(2)} = \sqrt{2}/2$, $w_{5/2,3/2}^{(2)} = \sqrt{7}/5$), and with the ρ_Q^K

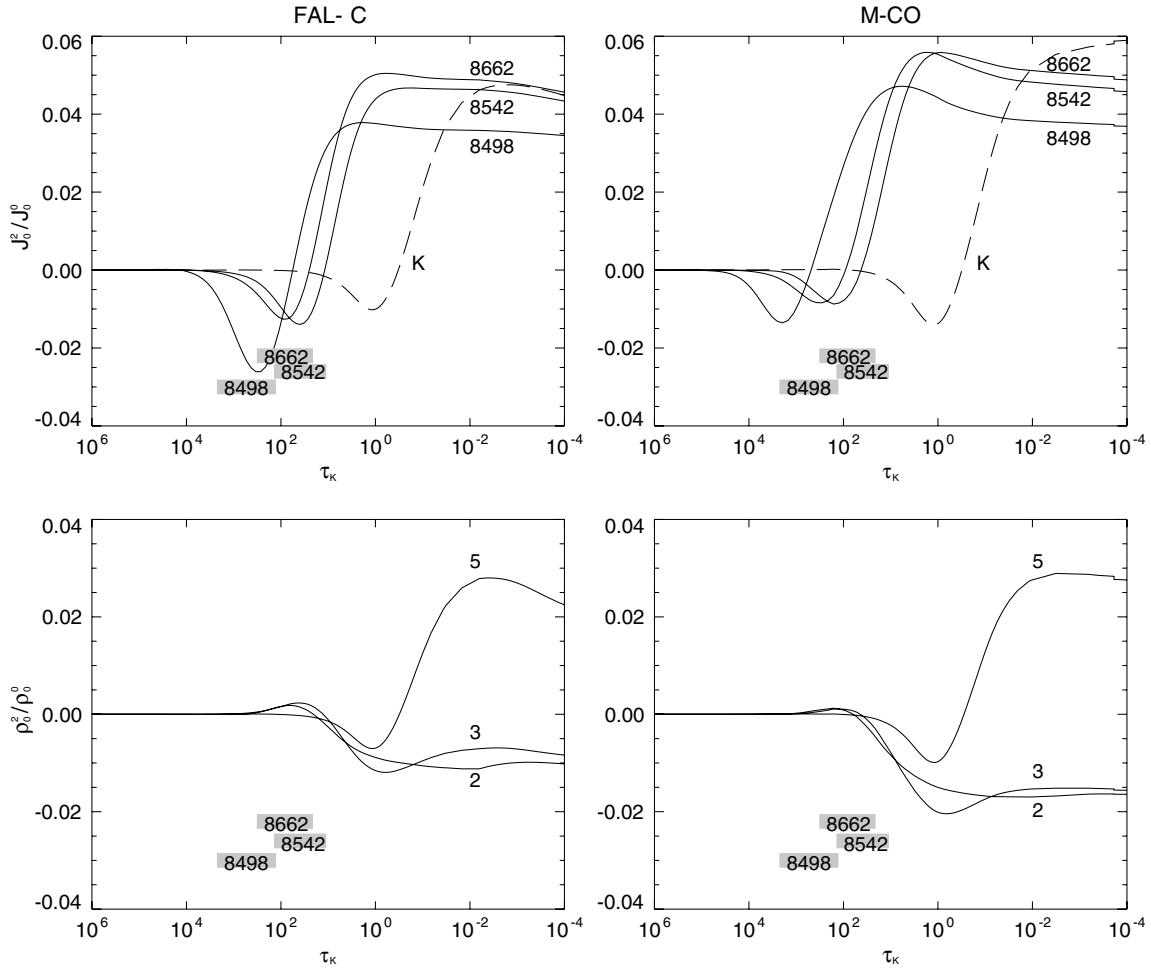


Figure 4. Variation with the (vertical) K-line-center optical depth of the degree of anisotropy in each of the lines of the Ca II model of Figure 1 after obtaining the self-consistent solution in the FAL-C model (left panel) and in the M-CO model (right panel). For each of the IR triplet lines the corresponding thick horizontal segment indicates where its line-center optical depth is unity for LOSs going from $\mu = 1$ (the lowest τ_K of each segment) to $\mu = 0.1$ (the largest τ_K of each segment).

values of the *lower* level of the line transition instead of those of the upper level (we neglect stimulated emissions). Likewise, since we are assuming that the atomic levels are not oriented, $\eta_V^{\text{line}} \equiv 0$ (when neglecting the wavelength shifts between the σ components caused by the Zeeman effect). Finally, the anomalous dispersion terms ($\rho_{Q,U,V}^{\text{line}}$) have the same functional dependence on the ρ_Q^{line} values of the *lower* level, but with an antisymmetric dispersive dependence on frequency, i.e., $\rho_{Q,U,V}^{\text{line}} = \eta_{Q,U,V}^{\text{line}} \psi_v / \phi_v$, with ψ_v as the dispersion profile. Note that $\rho_V^{\text{line}} = 0$ (for the same reason that $\eta_V^{\text{line}} = 0$).

Additionally, we consider an unpolarized background continuum providing an extra absorption coefficient η_I^{cont} and emissivity $\epsilon_I^{\text{cont}} = \eta_I^{\text{cont}} B_\nu$ (where B_ν is the Planck function), a very good approximation toward the red part of the solar spectrum (e.g., Gandorfer 2000). Therefore, in Equations (3a)–(3d), $\eta_I = \eta_I^{\text{line}} + \eta_I^{\text{cont}}$, $\epsilon_I = \epsilon_I^{\text{line}} + \epsilon_I^{\text{cont}}$, while all other emission, dichroism, and anomalous dispersion coefficients have only line contributions ($\epsilon_Q = \epsilon_Q^{\text{line}}$, $\epsilon_U = \epsilon_U^{\text{line}}$, $\eta_Q = \eta_Q^{\text{line}}$, $\eta_U = \eta_U^{\text{line}}$, $\rho_Q = \rho_Q^{\text{line}}$, $\rho_U = \rho_U^{\text{line}}$). Note that ϵ_Q and η_Q depend on both the population imbalances (ρ_Q^2) and on the coherences (ρ_Q^2 , with $Q = 1, 2$), while ϵ_U and η_U depend *only* on the quantum coherences.⁴

The ρ_0^0 elements produce the dominant contribution to the Stokes *I* profile, while the ρ_Q^2 elements (the *alignment* components) contribute to the *linear* polarization signals, which we quantify by the Stokes parameters *Q* and *U*. Although $\epsilon_V = \eta_V = 0$ (because, as mentioned above, $\rho_Q^1 = 0$) according to Equation (3d) it is possible to generate Stokes *V* in the spectral line due to the $-\rho_U Q + \rho_Q U$ (second-order) terms. This contribution is however negligible compared to the longitudinal Zeeman effect signal produced by the splitting of the σ components of the emission profile which we do not consider here.

Since the degree of anisotropy of the radiation field in solar-like atmospheres is weak, the population imbalances and coherences of each *J* level are only a small fraction of its overall population. Therefore, second-order terms can be safely neglected in the Stokes-vector transfer equation and the relevant equations for calculating *Q/I* and *U/I* are

$$\frac{d}{ds} I \approx \epsilon_I - \eta_I I, \quad (5a)$$

⁴ In these expressions, the reference direction for Stokes *Q* is the *perpendicular* to the plane formed by the propagation direction and the quantization *Z*-axis of total angular momentum.

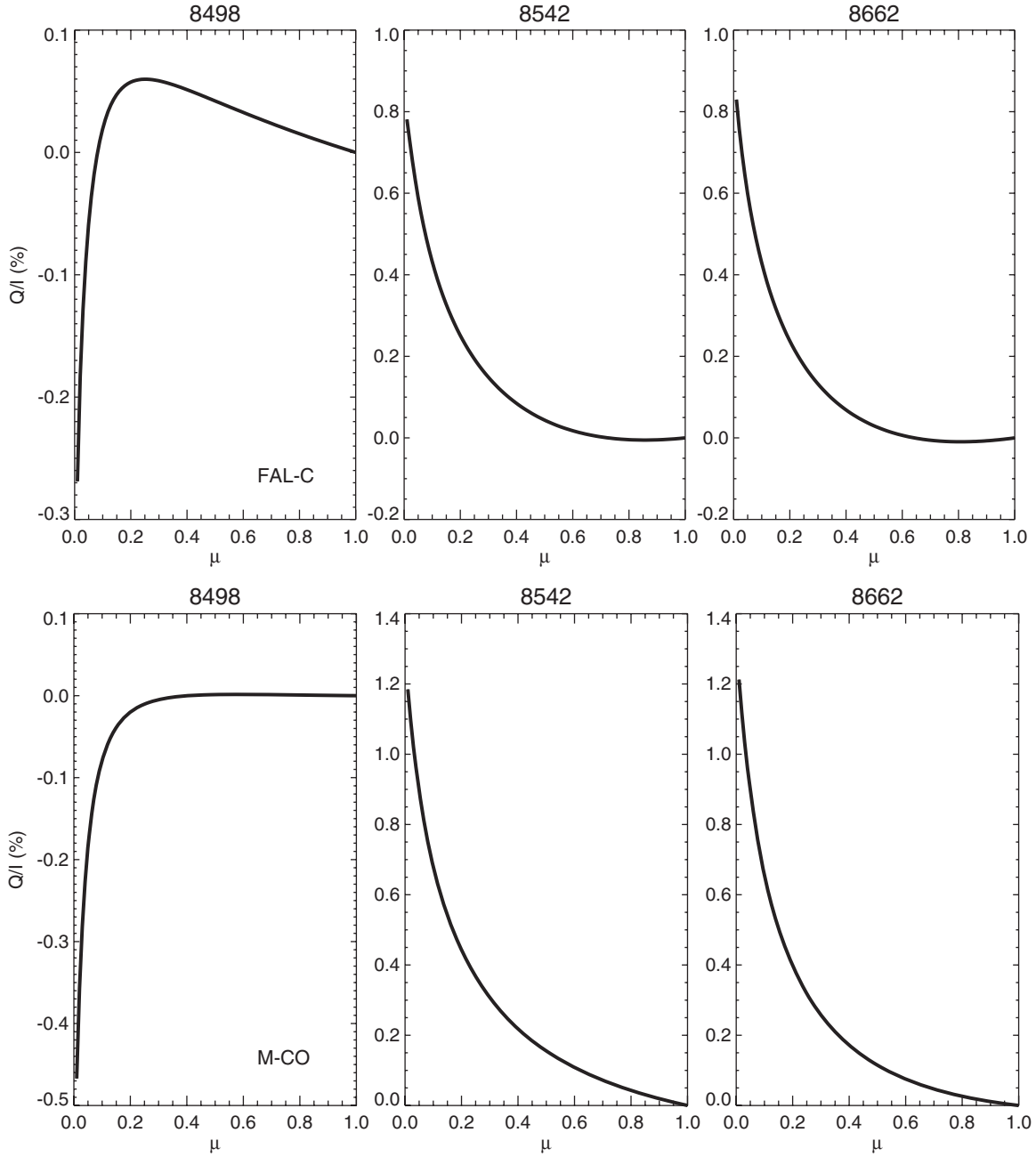


Figure 5. Unmagnetized case. Center-to-limb variation of the line-center scattering polarization amplitudes in the Ca II IR triplet produced by atomic level polarization in the “hot” FAL-C model (top panels) and in the “cool” M-CO model (bottom panels). The positive Q -direction is parallel to the nearest limb. Note that for relatively small μ values the Q/I of $\lambda 8498$ is negative for both model atmospheres.

$$\frac{d}{ds}Q \approx [\epsilon_Q - \eta_Q I] - \eta_I Q, \quad (5b)$$

$$\frac{d}{ds}U \approx [\epsilon_U - \eta_U I] - \eta_I U. \quad (5c)$$

Nevertheless, all the calculations of this paper were carried out by solving numerically the full Stokes-vector transfer Equations (3a)–(3d) through the application of the formal solver mentioned in the [Appendix](#). The complication with respect to the unmagnetized case considered by Manso Sainz & Trujillo Bueno (2003a) is that now the presence of a magnetic field of given strength B , inclination θ_B , and azimuth χ_B at each grid point of the stellar atmosphere model under consideration implies that we also have to consider the quantum coherences (ρ_Q^2 , with $Q \neq 0$) between each pair of magnetic substates pertaining to each J level, in addition to their individual populations (ρ_0^2). For this reason, not only Stokes Q but also Stokes U is now non-zero, in general.

The expressions above for the components of the emission vector and propagation matrix (Equations (4a)–(4c) and paragraphs following them) simplify considerably for several cases of practical interest. For a one-dimensional atmosphere, either unmagnetized,

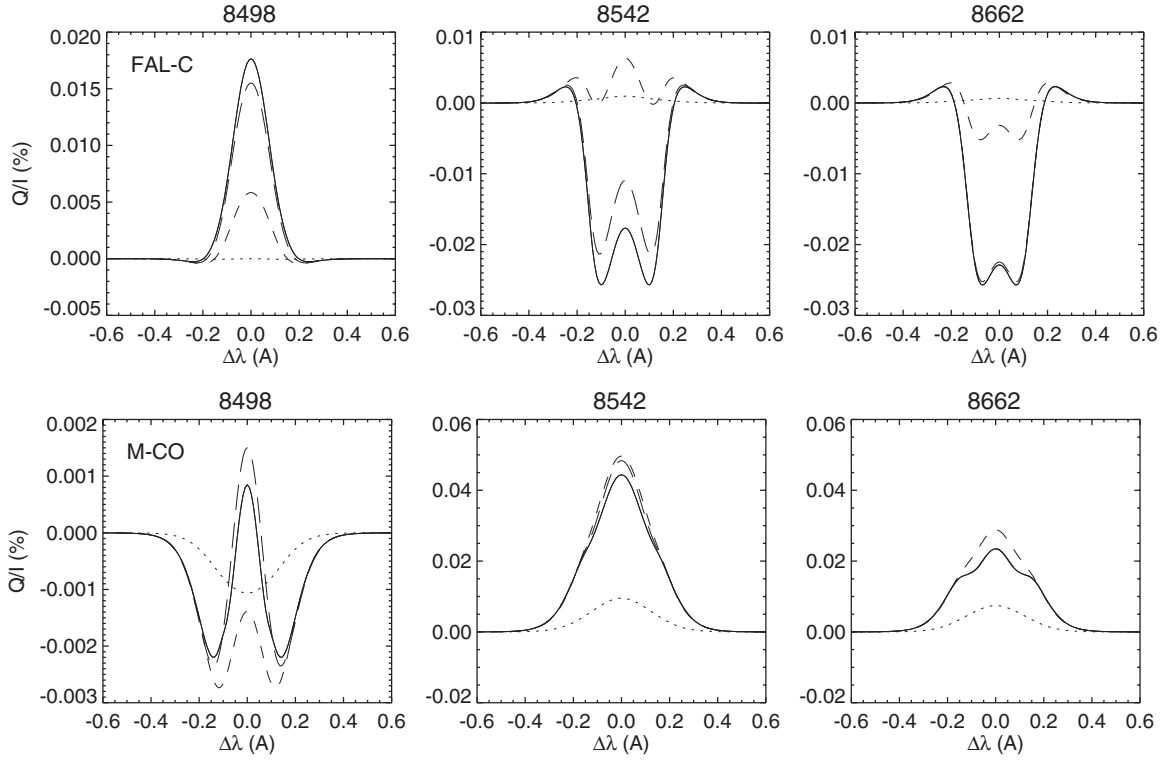


Figure 6. Hanle effect of a horizontal magnetic field in forward scattering. The emergent Q/I profiles of the Ca II IR triplet calculated for an LOS with $\mu = 1$ (i.e., disk center observation) assuming the presence of a horizontal magnetic field of 100 G (solid lines), 0.1 G (long-dashed line), 0.01 G (short-dashed line), and 0.001 G (dotted line) in the “hot” FAL-C model (top panels) and in the “cool” M-CO model (bottom panels). The positive Q -direction is along the magnetic field.

or permeated by a *microturbulent* and *isotropically distributed* magnetic field of strength B , or in the presence of a magnetic field with a fixed inclination but with a random azimuth below the line’s photon mean free path, or in the presence of a deterministic magnetic field with a fixed orientation but with a strength in the saturation regime of the Hanle effect, the quantum coherences (i.e., the real and imaginary parts of the ρ_1^2 and ρ_2^2 components) vanish in a reference system whose quantization axis must be the parallel to the magnetic field vector for the deterministic field case and the normal to the stellar surface for the three other cases:

$$\epsilon_I^{\text{line}} = \epsilon_0 \left[\rho_0^0 + w_{J_u J_l}^{(2)} \frac{1}{2\sqrt{2}} (3\mu^2 - 1) \rho_0^2 \right], \quad (6)$$

$$\epsilon_Q^{\text{line}} = \epsilon_0 w_{J_u J_l}^{(2)} \frac{3}{2\sqrt{2}} (1 - \mu^2) \rho_0^2, \quad (7)$$

$$\epsilon_U^{\text{line}} = 0, \quad (8)$$

where the ρ_0^K ($K = 0, 2$) values are those of the *upper* level of the line transition under consideration, and where the orientation of the ray is specified by the azimuthal angle χ and by $\mu = \cos\theta$ (with θ the angle between the direction of the radiation beam and the quantization axis, which must be the magnetic field direction itself for the deterministic field case and the normal to the surface of the stellar atmospheric model for the other three cases). Likewise, the only non-zero elements of the propagation matrix are η_I and η_Q which are given by identical expressions (i.e., by $\eta_I = \epsilon_I$ and $\eta_Q = \epsilon_Q$), but with η_0 instead of ϵ_0 , $w_{J_l J_u}^{(2)}$ instead of $w_{J_u J_l}^{(2)}$ and with the ρ_0^K values of the *lower* level of the line transition instead of those of the upper level (for the case in which stimulated emissions are neglected).

Under such assumptions, and taking into account the low polarization level in solar-like atmospheres (i.e., $\eta_Q/\eta_I \ll 1$ and $\epsilon_Q/\epsilon_I \ll 1$), an approximate formula can be derived to estimate the emergent fractional linear polarization amplitude at the center of a strong spectral line (Trujillo Bueno 2003):

$$Q/I \approx \frac{3}{2\sqrt{2}} (1 - \mu^2) [w_{J_u J_l}^{(2)} \sigma_0^2(J_u) - w_{J_l J_u}^{(2)} \sigma_0^2(J_l)], \quad (9)$$

where $\sigma_0^2 = \rho_0^2/\rho_0^0$ must be evaluated at height in the model atmosphere where the line-center optical distance along the LOS is unity. Its first term is due to *selective emission* of polarization components (caused by the population imbalances of the upper level), while its second term accounts for “zero-field” dichroism. Note that since $w_{1/2,3/2}^{(2)} = 0$ and $w_{3/2,1/2}^{(2)} = \sqrt{2}/2$, the scattering polarization of $\lambda 8662$ is exclusively due to “zero-field” dichroism.

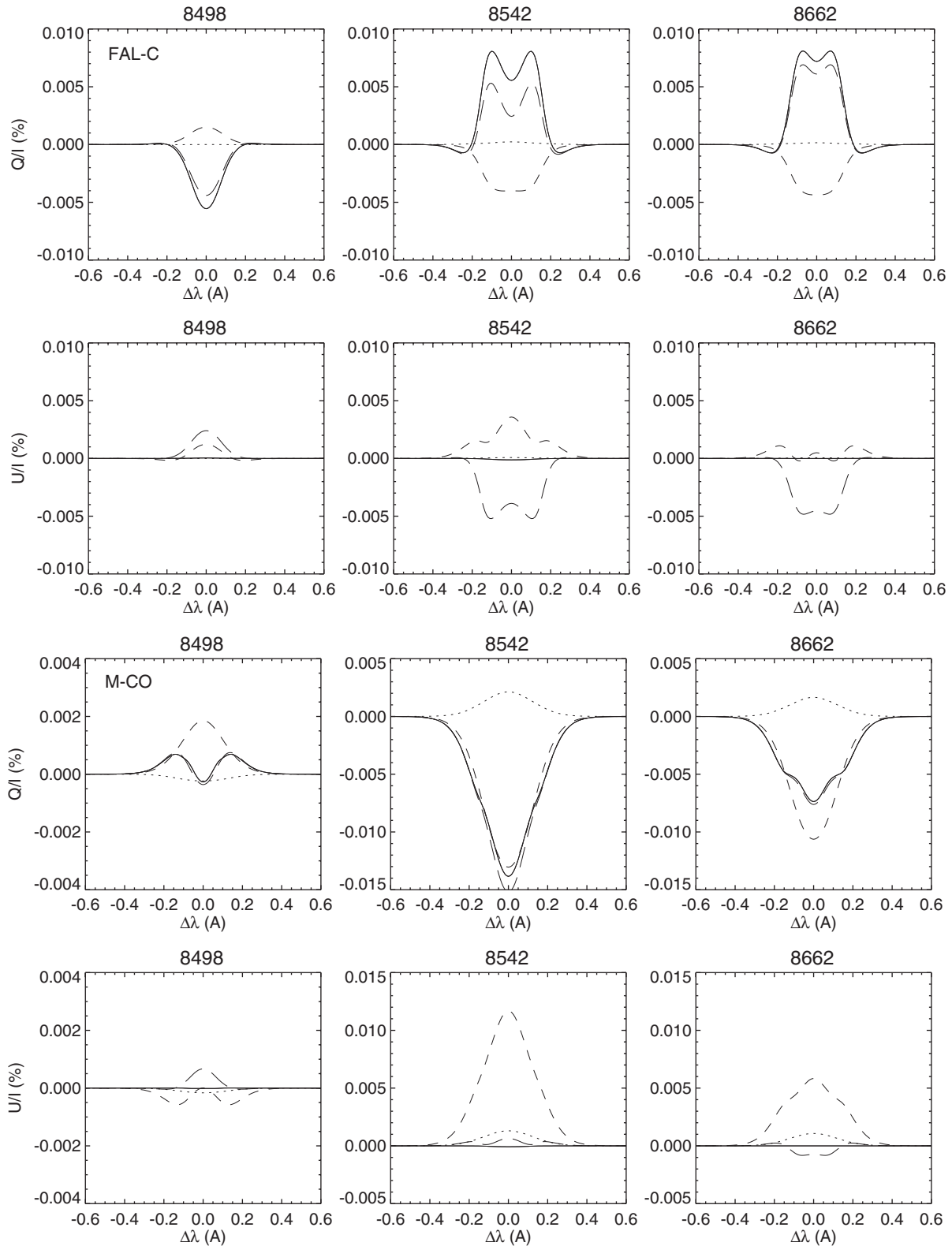


Figure 7. Hanle effect of a non-horizontal magnetic field in forward scattering as in Figure 6, but for a magnetic field inclined by 30° . Note that in this case Stokes U is not zero.

2.3. The Statistical Equilibrium Equations

General equations for the multilevel atom model which neglects quantum interferences between the sublevels pertaining to different J levels are derived in Landi Degl'Innocenti & Landolfi (2004). For the particular case of the five-level atom of Figure 1 the rate

equations for the density matrix elements are as follows:

$$\frac{d}{dt}\rho_0^0(1) = - \left[\sum_{u=4}^5 B_{1u} J_0^0(1 \rightarrow u) + \sum_{i \neq 1} C_{1i} \right] \rho_0^0(1) + A_{41} \rho_0^0(4) + \sqrt{2} A_{51} \rho_0^0(5) + \sum_{i \neq 1} C_{i1} \sqrt{\frac{2J_i+1}{2}} \rho_0^0(i), \quad (10)$$

$$\begin{aligned} \frac{d}{dt}\rho_0^0(2) = & - \left[\sum_{u=4}^5 B_{2u} J_0^0(2 \rightarrow u) + \sum_{i \neq 2} C_{2i} \right] \rho_0^0(2) - \sum_{Q'=-2}^2 \left(\frac{1}{\sqrt{2}} B_{24} J_{Q'}^2(2 \rightarrow 4) - \frac{2\sqrt{2}}{5} B_{25} J_{Q'}^2(2 \rightarrow 5) \right) \rho_{Q'}^2(2) \\ & + \frac{1}{\sqrt{2}} A_{42} \rho_0^0(4) + A_{52} \rho_0^0(5) + \sum_{i \neq 2} C_{i2} \frac{\sqrt{2J_i+1}}{2} \rho_0^0(i), \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{d}{dt}\rho_Q^2(2) = & - \left[2\pi i v_L g_2 Q + \sum_{u=4}^5 B_{2u} J_0^0(2 \rightarrow u) + \sum_{i \neq 2} C_{2i} - D_2^{(2)} \right] \rho_Q^2(2) \\ & - \left(\frac{1}{\sqrt{2}} B_{24} J_{-Q}^2(2 \rightarrow 4) - \frac{2\sqrt{2}}{5} B_{25} J_{-Q}^2(2 \rightarrow 5) \right) (-1)^Q \rho_0^0(2) + \frac{1}{5} A_{52} \rho_Q^2(5) + \sum_{i=3,5} C_{i2}^{(2)} \frac{\sqrt{2J_i+1}}{2} \rho_Q^2(i), \end{aligned} \quad (12)$$

$$\frac{d}{dt}\rho_0^0(3) = - \left[B_{35} J_0^0(3 \rightarrow 5) + \sum_{i \neq 3} C_{3i} \right] \rho_0^0(3) - \sum_{Q=-2}^2 \frac{\sqrt{7}}{5} B_{35} J_Q^2(3 \rightarrow 5) \rho_Q^2(3) + \sqrt{\frac{2}{3}} A_{53} \rho_Q^2(5) + \sum_{i \neq 3} C_{i3} \sqrt{\frac{2J_i+1}{6}} \rho_0^0(i), \quad (13)$$

$$\begin{aligned} \frac{d}{dt}\rho_Q^2(3) = & - \left[2\pi i v_L g_3 Q + B_{35} J_0^0(3 \rightarrow 5) + \sum_{i \neq 3} C_{3i} - D_3^{(2)} \right] \rho_Q^2(3) - B_{35} (-1)^Q J_{-Q}^2(3 \rightarrow 5) \frac{\sqrt{7}}{5} \rho_0^0(3) \\ & - \sum_{q, Q'=-2}^2 B_{35} J_q^2(3 \rightarrow 5) \left[-\sqrt{\frac{5}{7}} \begin{pmatrix} 2 & 2 & 2 \\ Q & -Q' & q \end{pmatrix} (-1)^{Q'} \rho_{Q'}^2(3) + \frac{9}{2} \sqrt{\frac{3}{35}} \begin{pmatrix} 2 & 4 & 2 \\ Q & -Q' & q \end{pmatrix} (-1)^{Q'} \rho_{Q'}^4(3) \right] \\ & + \frac{2}{5} \sqrt{\frac{7}{3}} A_{53} \rho_Q^2(5) + \sum_{i=2,5} C_{i3}^{(2)} \sqrt{\frac{2J_i+1}{6}} \rho_0^0(i), \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d}{dt}\rho_Q^4(3) = & - \left[2\pi i v_L g_3 Q + B_{35} J_0^0(3 \rightarrow 5) + \sum_{i \neq 3} C_{3i} - D_3^{(4)} \right] \rho_Q^4(3) \\ & - \sum_{q, Q'=-2}^2 B_{35} J_q^2(3 \rightarrow 5) \left[\frac{9}{2} \sqrt{\frac{3}{35}} \begin{pmatrix} 4 & 2 & 2 \\ Q & -Q' & q \end{pmatrix} (-1)^{Q'} \rho_{Q'}^2(3) + 3 \sqrt{\frac{11}{70}} \begin{pmatrix} 4 & 4 & 2 \\ Q & -Q' & q \end{pmatrix} (-1)^{Q'} \rho_{Q'}^4(3) \right]. \end{aligned} \quad (15)$$

$$\frac{d}{dt}\rho_0^0(4) = - \left[\sum_{\ell=1}^2 A_{4\ell} + \sum_{i \neq 4} C_{4i} \right] \rho_0^0(4) + \sum_{\ell=1}^2 B_{\ell 4} J_0^0(\ell \rightarrow 4) \sqrt{\frac{2J_\ell+1}{2}} \rho_0^0(\ell) + \sum_{Q=-2}^2 B_{24} J_Q^2(\ell \rightarrow 4) \rho_Q^2(2) + \sum_{i \neq 4} C_{i4} \sqrt{\frac{2J_i+1}{2}} \rho_0^0(i), \quad (16)$$

$$\begin{aligned} \frac{d}{dt}\rho_0^0(5) = & - \left[\sum_{\ell=1}^3 A_{5\ell} + \sum_{i \neq 5} C_{5i} \right] \rho_0^0(5) + \sum_{\ell=1}^3 B_{\ell 5} J_0^0(\ell \rightarrow 5) \frac{\sqrt{2J_\ell+1}}{2} \rho_0^0(\ell) \\ & + \sum_{Q=-2}^2 \left(-\frac{2\sqrt{2}}{5} B_{25} J_Q^2(2 \rightarrow 5) \rho_Q^2(2) + \frac{\sqrt{42}}{10} B_{35} J_Q^2(3 \rightarrow 5) \rho_Q^2(3) \right) + \sum_{i \neq 5} C_{i5} \frac{\sqrt{2J_i+1}}{2} \rho_0^0(i), \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{d}{dt}\rho_Q^2(5) = & - \left[2\pi i v_L g_5 Q + \sum_{\ell=1}^3 A_{5\ell} + \sum_{i \neq 5} C_{5i} - D_5^{(2)} \right] \rho_Q^2(5) + \frac{1}{5} B_{25} J_0^0(2 \rightarrow 5) \rho_Q^2(2) + \frac{\sqrt{21}}{5} B_{35} J_0^0(3 \rightarrow 5) \rho_Q^2(3) \\ & + -\frac{2\sqrt{2}}{5} B_{25} J_{-Q}^2(2 \rightarrow 5) \rho_0^0(2) + \sqrt{\frac{3}{100}} B_{35} J_{-Q}^2(3 \rightarrow 5) \rho_0^0(3) \end{aligned}$$

Table 1
Data for Five-level Atomic Model of Ca II

λ (Å)	u^a	ℓ^b	$A_{u\ell}$ (s $^{-1}$) ^c	$C_{u\ell}/N_e$ (s $^{-1}$ cm 3) ^d			
				3000 K	5000 K	7000 K	9000 K
Allowed transitions							
K	5	1	1.4×10^8	3.60×10^{-7}	2.92×10^{-7}	2.59×10^{-7}	2.39×10^{-7}
H	4	1	1.4×10^8	3.60×10^{-7}	2.92×10^{-7}	2.59×10^{-7}	2.39×10^{-7}
8498	5	2	1.11×10^6	3.68×10^{-7}	2.93×10^{-7}	2.51×10^{-7}	2.23×10^{-7}
8542	5	3	9.6×10^6	1.59×10^{-6}	1.29×10^{-6}	1.12×10^{-6}	1.01×10^{-6}
8662	4	2	1.06×10^7	1.61×10^{-6}	1.31×10^{-6}	1.15×10^{-6}	1.04×10^{-6}
Forbidden transitions							
...	5	4	...	3.94×10^{-7}	3.05×10^{-7}	2.58×10^{-7}	2.27×10^{-8}
...	4	3	...	2.69×10^{-7}	2.08×10^{-7}	1.76×10^{-7}	1.55×10^{-7}
...	3	1	...	2.20×10^{-7}	1.71×10^{-7}	1.44×10^{-7}	1.27×10^{-7}
...	3	2	...	5.51×10^{-7}	4.27×10^{-7}	3.61×10^{-7}	3.19×10^{-7}
...	2	1	...	2.20×10^{-7}	1.71×10^{-7}	1.44×10^{-7}	1.27×10^{-7}

Notes.

^a Upper level of the transition labeled in Figure 1.

^b Lower level of the transition in Figure 1.

^c From NIST atomic spectra database <http://www.nist.gov/physlab/data/asd.cfm>.

^d From Shine & Linsky (1974), with N_e being the electron density.

$$\begin{aligned}
& + \sum_{Q'=-2}^2 \left(2\sqrt{\frac{7}{5}} B_{25} J_Q^2 (2 \rightarrow 5) \rho_{Q+Q'}^2 (2) - \sqrt{\frac{3}{5}} B_{35} J_Q^2 (3 \rightarrow 5) \rho_{Q+Q'}^2 (3) \right) \begin{pmatrix} 2 & 2 & 2 \\ -Q & Q & -Q' \end{pmatrix} \\
& + \sum_{i=2,3} C_{i5}^{(2)} \frac{\sqrt{2J_i+1}}{2} \rho_0^2(i).
\end{aligned} \tag{18}$$

In the previous equations, $A_{u\ell}$ and $B_{\ell u}$ are the Einstein coefficients for spontaneous emission and absorption, respectively (see Table 1). $J_{Q_r}^{K_r}$ ($K_r = 0, 2$; $Q_r = -K_r, \dots, K_r$) are the radiation field tensors introduced by Landi Degl'Innocenti (1984). They are integrals over the frequency and solid angle of the Stokes parameters (for their explicit expressions appropriate for computational work, see Manso Sainz & Trujillo Bueno 1999). In particular, J_0^0 is the mean intensity integrated over the absorption profile, while

$$J_0^2 = \int dx \oint \phi_x \frac{d\bar{\Omega}}{4\pi} \frac{1}{2\sqrt{2}} [(3\mu^2 - 1)I_{x\bar{\Omega}} + 3(\mu^2 - 1)Q_{x\bar{\Omega}}]. \tag{19}$$

The contribution of other J_Q^K components is very small here, where only the magnetic field breaks the symmetry of the radiation field. Thus, $|J_{Q \neq 0}^2/J_0^0| \leq 10^{-4}$ throughout the whole outer atmosphere in the five transitions relevant for our problem.

$C_{\ell u}$ and $C_{u\ell}$ are the excitation and de-excitation collisional rates, respectively (see Table 1), between levels ℓ and u , while $C_{\ell u}^{(2)}$ and $C_{u\ell}^{(2)}$ are collisional transfer rates for alignment between polarizable levels 2, 3, and 5. Derouich et al. (2007) have computed values for these transitions in Ca II. Alternatively, one can follow the approach suggested by Landi Degl'Innocenti & Landolfi (2004), and consider $C_{u\ell}^{(2)} = C_{\ell u} p$ where p is just a numerical factor depending on the quantum numbers of the transition. $D_i^{(K)}$ is the depolarization rate of the K th multipole of level i due to elastic collisions with neutral hydrogen. Theoretical values for Ca II are given by Derouich et al. (2007), who confirmed that such $D_i^{(K)}$ terms of the statistical equilibrium equations have only a very small impact on the atomic polarization of the metastable lower levels (cf. Manso Sainz & Trujillo Bueno 2003a).

The first term on the right-hand-side of Equations (12), (14), (15), and (18) represents the relaxation of coherences in the presence of a magnetic field with Larmor frequency $\nu_L = 1.3996 \times 10^6 B$ (B expressed in gauss and ν_L in Hz), and g_J the Landé factor of the J level under consideration. Equations (10)–(18) are expressed in a reference system with the quantization axis along the magnetic field. The radiative and collisional terms remain formally invariant in any other reference system—although clearly, their actual numerical values may change due to the different values of J_Q^K in the new reference system. In an arbitrary reference system where the magnetic field has an inclination θ_B with respect to the quantization axis and an azimuth χ_B , the magnetic relaxation term reads

$$2\pi i \nu_L g_J Q \rho_Q^K \rightarrow 2\pi i \nu_L g_J \left[Q \cos \theta_B \rho_Q^K - \frac{1}{2} \sqrt{(K-Q)(K+Q+1)} \sin \theta_B e^{i\chi_B} \rho_{Q+1}^K + \frac{1}{2} \sqrt{(K+Q)(K-Q+1)} \sin \theta_B e^{-i\chi_B} \rho_{Q-1}^K \right], \tag{20}$$

where the term multiplying ρ_{Q+1}^K (ρ_{Q-1}^K) is not considered when $Q = K$ ($Q = -K$).

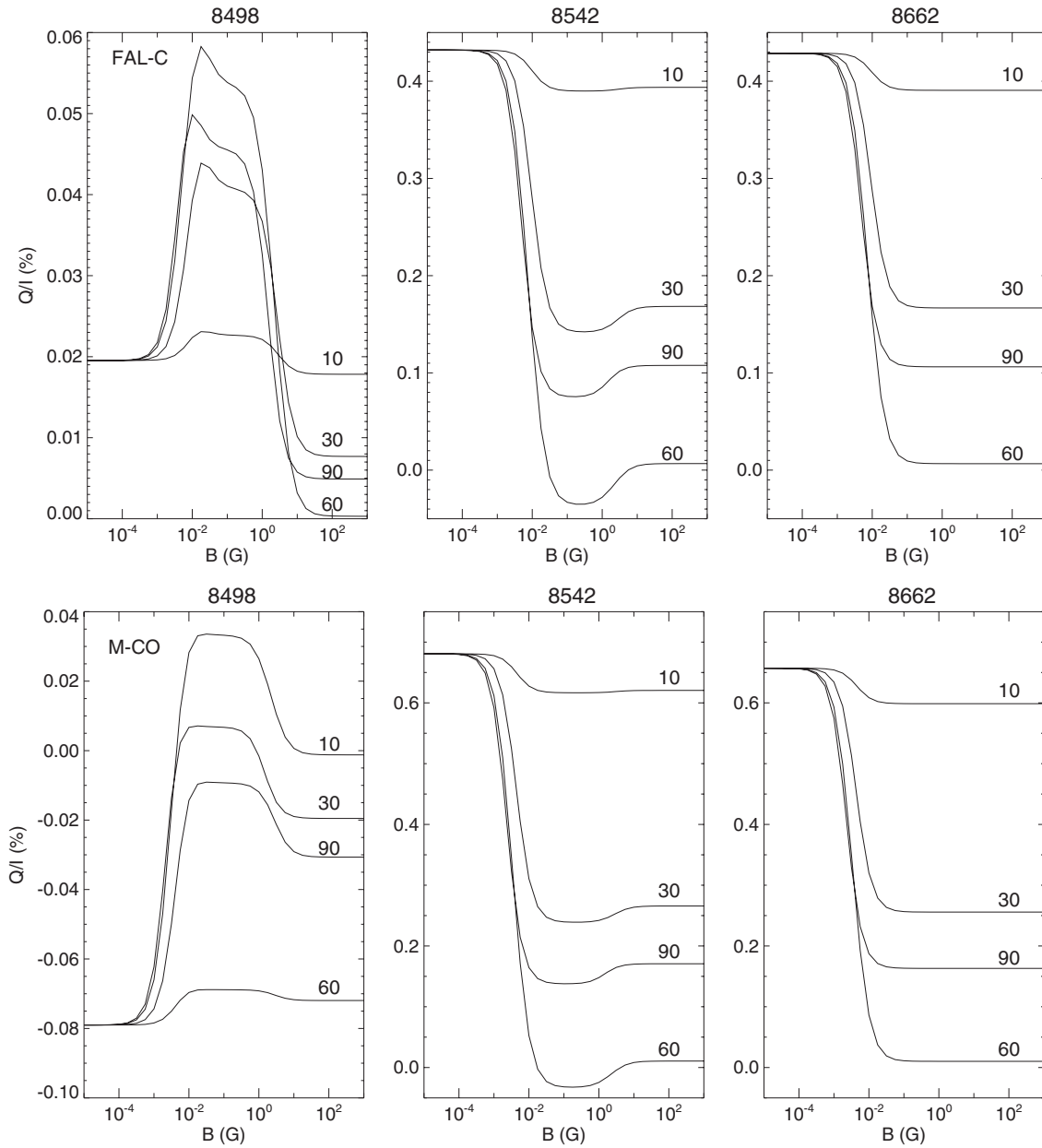


Figure 8. Hanle effect for a close-to-the-limb LOS. The emergent Q/I line-center amplitudes of the Ca II IR triplet calculated for an LOS with $\mu = 0.1$ in the “hot” FAL-C model (top panels) and in the “cool” M-CO model (bottom panels), assuming a magnetic field with the indicated inclination and a uniformly distributed azimuth within the spatio-temporal resolution element. The positive Q -direction is the parallel to the nearest limb.

The statistical equilibrium Equations (10)–(18) have been derived under several hypotheses. Collisions are considered under the impact approximation. The interaction time of the collision is negligible and hence, collisional and radiative terms can be added independently. The colliders are assumed to be isotropically distributed (they are Maxwellian), and they cannot generate atomic polarization, though they can transfer it between levels.⁵ Radiative rates have been derived within the framework of the quantum theory of spectral line polarization described in Landi Degl’Innocenti & Landolfi (2004), assuming that the radiation field is spectrally flat (i.e., lacking spectral structure), in spectral ranges of the order of the energy separation between levels with coherences. This is an excellent approximation for the case under consideration since we are considering relatively weak fields (less than ~ 100 G) and we neglect coherences between different J levels. Neglecting J -level interferences is a good approximation for modeling the Ca II IR triplet and the core of the H and K lines, but it cannot account for the general Q/I pattern of the UV doublet (see Stenflo 1980), which is beyond the scope of this work. Finally, we neglect Doppler correlations due to the thermal movement of the atoms. All these hypotheses together account for what is called complete frequency redistribution (CRD) in the laboratory frame, since correlations between the incident and scattered radiation fields are completely neglected.

⁵ See Manso Sainz & Trujillo Bueno (2009) for an interesting consequence of this fact for the case of permitted lines at EUV wavelengths (i.e., a polarization mechanism of EUV coronal lines and the possibility of mapping the magnetic fields of coronal loops).

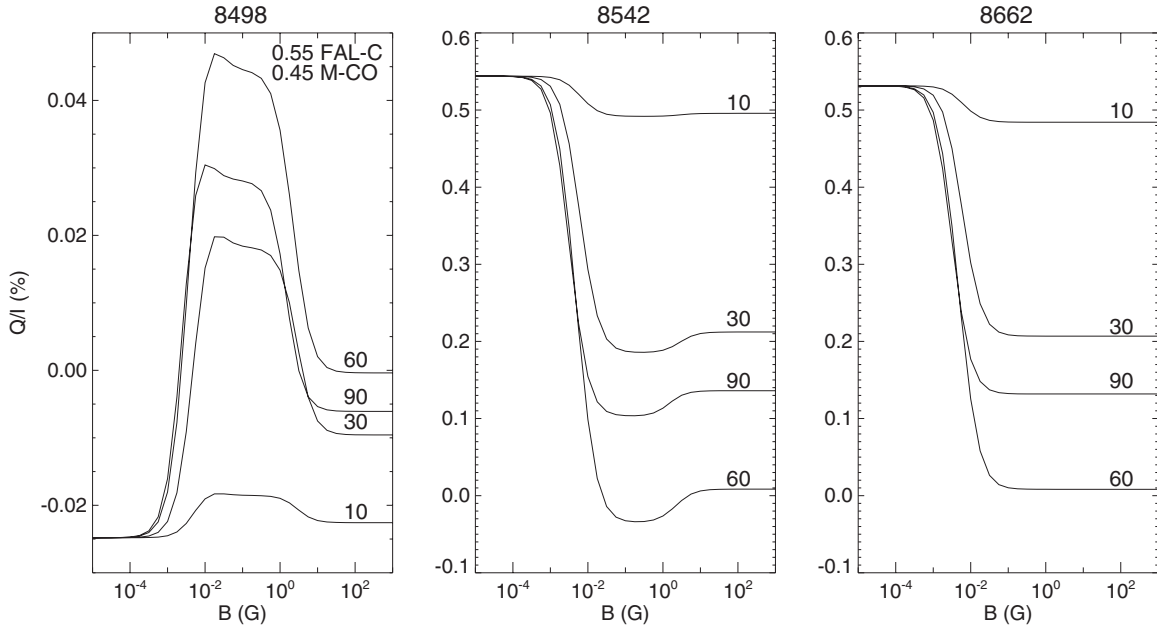


Figure 9. Hanle effect for a close-to-the-limb LOS. The emergent Q/I line-center amplitudes of the Ca II IR triplet calculated for an LOS with $\mu = 0.1$ assuming that 55% of the radiation originates in the “hot” FAL-C model and 45% in the “cool” M-CO model, and assuming a magnetic field with the indicated inclination and a uniformly distributed azimuth. The positive Q -direction is the parallel to the nearest limb.

We calculate the $29 \rho_Q^K$ unknowns at each grid point of the chosen stellar atmosphere model assuming statistical equilibrium

$$\frac{d}{dt} \rho_Q^K(J) = 0. \quad (21)$$

The resulting system of equations is linearly dependent and we substitute one of the equations, say, the one for the ground-level population, by the trace equation of the density matrix, which establishes the conservation of the number of particles:

$$\sum_{J_i} \sqrt{2J_i + 1} \rho_0^0(J_i) = 1. \quad (22)$$

We have developed a general multilevel radiative transfer computer program for the numerical solution of the non-LTE problem of the second kind, which we have applied in this investigation to obtain the self-consistent solution of the previous equations. See the [Appendix](#) for details.

3. THE ATMOSPHERIC AND THE ATOMIC MODELS

High spatial and temporal resolution intensity images taken at the line center of strong absorption lines, like H α and Ca II 8542 Å, show that the chromosphere of the quiet Sun is a highly inhomogeneous and dynamic medium (see, e.g., the review by Rutten 2007). The spatial and dynamic complexity is such that it is obvious that the two one-dimensional atmospheric models we have chosen for this investigation (i.e., the “hot” FAL-C model of Fontenla et al. 1993 and the “cool” M-CO model of Avrett 1995) should only be considered as rough representations of the stratification of the kinetic temperature and density at two different phases of inter-network oscillations (see Figure 2). Nevertheless, as we shall see below, our solution of the multilevel Hanle-effect problem for the IR triplet of Ca II in such semi-empirical models of the solar atmosphere allows us to reach the main goal of this paper, namely, to demonstrate the diagnostic potential of the linear polarization signals produced by atomic polarization in the levels of the Ca II IR triplet.

We consider a five-level atomic model of Ca II including the ground level, the metastable $^2D_{3/2,5/2}$ levels, and the $^2P_{1/2,3/2}$ upper levels of the H and K lines (see Figure 1). The excitation state in such a model is given by 29 statistical tensor elements ρ_Q^K ($K = 0, 2, 4$; $Q = -K, \dots, K$), including the total populations of the five levels. The radiative transition probabilities are those of Edlén & Risberg (1956) compiled at the NIST database.⁶ (Note that these transition probabilities for the infrared triplet are slightly larger than those used by Manso Sainz & Trujillo Bueno 2003a for the unmagnetized case; correspondingly, the Q/I values for the zero-field case are slightly larger here than in our previous work.) Inelastic collisional rates and line-broadening parameters have been taken from Shine & Linsky (1974; see Table 1). Elastic depolarizing collisions are treated according to Lamb & ter Haar (1971), which gives results similar to those obtained using the collisional rates given by Derouich et al. (2007). The collisional pumping of $K \neq 0$ statistical tensors are treated following Landi Degl’Innocenti & Landolfi (2004).

⁶ <http://physics.nist.gov/PhysRefData/ASD>

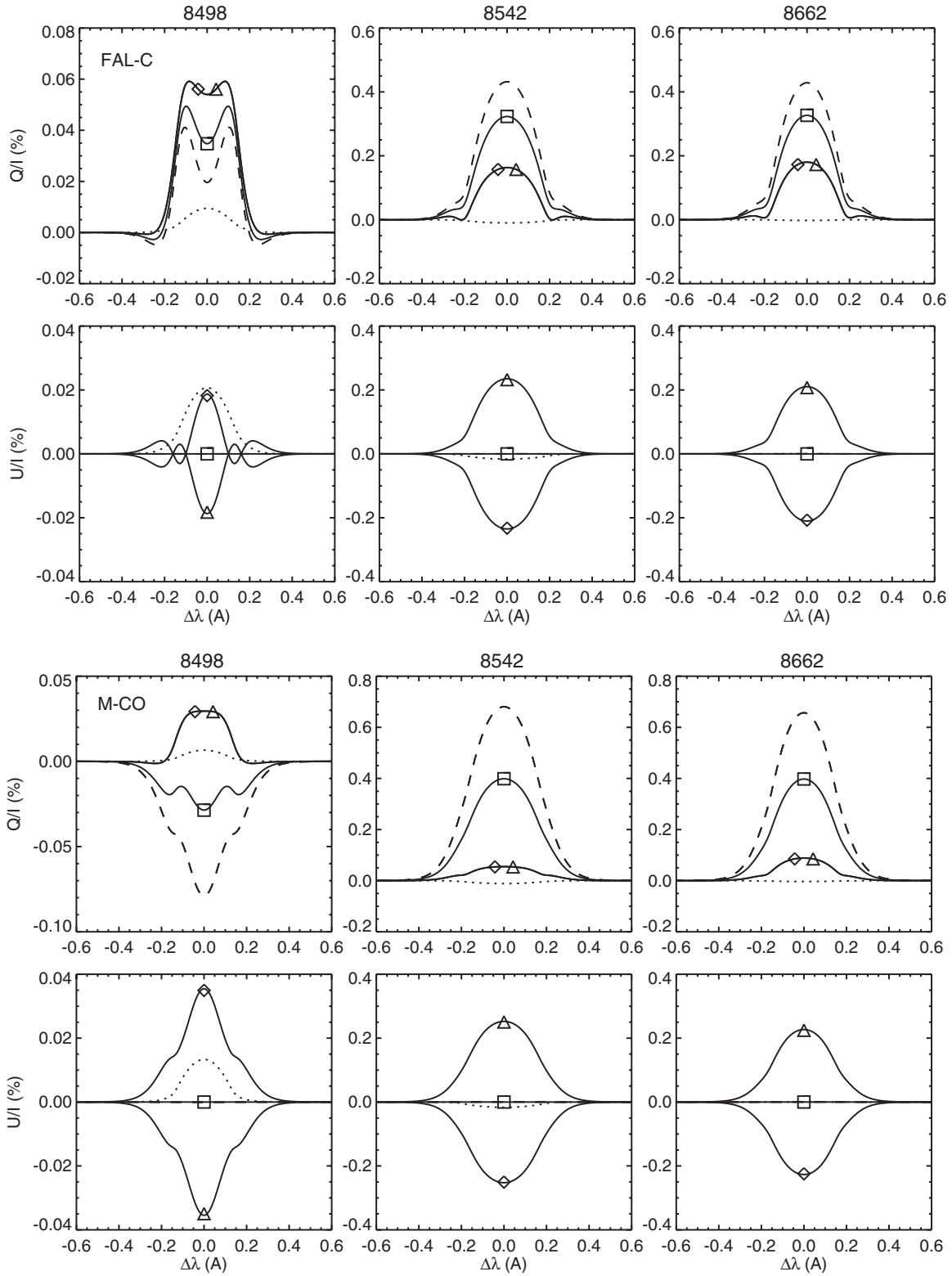


Figure 10. Emergent Q/I and U/I profiles of the Ca II IR triplet calculated for an LOS with $\mu = 0.1$ in the “hot” FAL-C model (top panels) and in the “cool” M-CO model (bottom panels), assuming a magnetic field with the following magnetic configurations: pure scattering case (dashed lines) and the case of a magnetic field pointing to the observer with $B = 0.005$ G (solid lines with the triangle symbol) and $B = 100$ G (dotted lines). Actually, for the $B = 0.005$ G case three different orientations of the LOS are shown: along the field (triangles), perpendicular to the field (squares), and away from the field (diamonds). The positive Q -direction is the parallel to the nearest limb.

We have calculated the number density of Ca II ions at each atmospheric height of the model under consideration by solving the standard non-LTE radiative transfer problem for a realistic atomic model that also includes the bound-free transitions from all the bound levels of Ca II to the ground level of Ca III. With the resulting number densities of Ca II ions we then solved the non-LTE

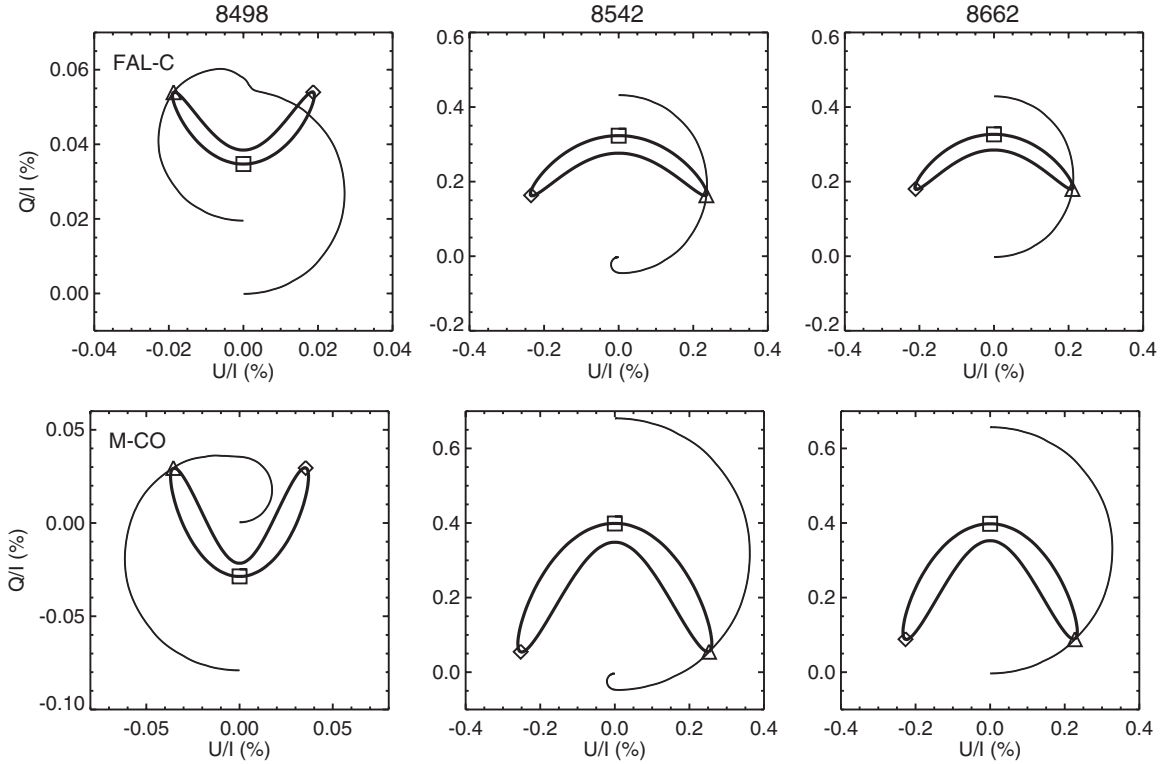


Figure 11. Evolution of the line-center signal in the Q/I - U/I space, for the FAL-C model (upper panels) and the M-CO model (bottom panels). Thin lines: horizontal magnetic field with variable strength (from $B = 0$ G to $B \rightarrow \infty$). Thick lines: $B = 0.005$ G horizontal field observed from different angles. The symbols correspond to the particular configurations shown in Figure 10.

problem of the second kind for the atomic model of Figure 1 (see the Appendix), which is sufficiently realistic for investigating the polarization signatures produced by atomic level polarization and the Hanle effect in the lines of the Ca II IR triplet. We point out that while B_H (see Equation (1)) is of the order of a few gauss for the upper level of the 8498 Å and 8542 Å lines, it lies in the milligauss range for the lower (metastable) levels of the Ca II IR triplet.

4. THE UNMAGNETIZED REFERENCE CASE

For symmetry reasons, in a one-dimensional model atmosphere without magnetic fields the only non-zero Stokes parameters are $I(\lambda)$ and $Q(\lambda)$ (choosing the polarization directions as in Equations (4a)–(4c)).

Figure 3 shows the wavelength variation of the emergent fractional linear polarization (Q/I) in the Ca II IR triplet, calculated for an LOS with $\mu = 0.1$ in the “hot” FAL-C model (middle panels) and in the “cool” M-CO model (bottom panels). While Q/I in the 8542 Å and 8662 Å lines are positive in both models, the Q/I profile of the 8498 Å line is positive in the “hot” model but negative in the “cool” model. Moreover, the 8498 Å Q/I profiles calculated in such atmospheric models have even different shapes, and their amplitudes are 1 order of magnitude smaller than those of the 8542 Å and 8662 Å lines. The (positive) Q/I amplitudes of the 8542 Å and 8662 Å lines turn out to be significantly larger in the “cool” model. This dependence of the amplitude of the Q/I profiles on the chosen model atmosphere is not at all surprising, because the atomic polarization that anisotropic pumping processes induce in the atomic energy levels sensitively depends on the temperature structure of the model. In particular, the steeper the source function gradient, the larger the anisotropy factor and hence, the induced atomic level polarization (e.g., see Figure 4 in Trujillo Bueno 2001). The upper panels of Figure 4 show the spatial variation of the “degree of anisotropy” in each of the considered atmospheric models, indicating for each LOS characterized by μ the atmospheric height where the line-center optical depth is unity along the LOS. Note that at the height corresponding to an LOS with $\mu = 0.1$ the anisotropy is larger in the M-CO model. The decrease of the anisotropy factor at very low optical depths is due to the rapid widening of the ϕ_x profile with which the radiation field is averaged (e.g., see Equation (18) for the definition of the J_0^2 radiation field tensor component). This is a consequence of the temperature rising in the upper chromosphere-transition region. The two lower panels of Figure 4 show the behavior of the fractional atomic alignment of the levels of Figure 1 having $J > 1/2$ (i.e., the two metastable levels, which are the lower levels of the Ca II IR triplet, and the upper level of the K line). The fact that the sign of the Q/I profile of $\lambda 8498$ (i.e., the weakest line of the Ca II IR triplet) is very sensitive to the thermal structure of the lower chromosphere can be easily understood by combining the information provided in the lower panels of Figure 4 and the approximate Eddington–Barbier formula (Equation (9)).

Figure 3 suggests that the scattering polarization of the Ca II IR triplet can be used as a sensitive thermometer of the “quiet” regions of the solar chromosphere, as already noted by Manso Sainz & Trujillo Bueno (2001). In this respect, a particularly useful quantity is the center-limb variation of the Q/I line-center amplitudes (see Figure 5). As seen in this figure, the most noteworthy feature is that in the “hot” FAL-C model the (positive) Q/I amplitude of the 8498 Å line is larger at $\mu \approx 0.2$ than at $\mu \approx 0.1$ and that it becomes negative for LOSs with μ values significantly smaller than 0.1.

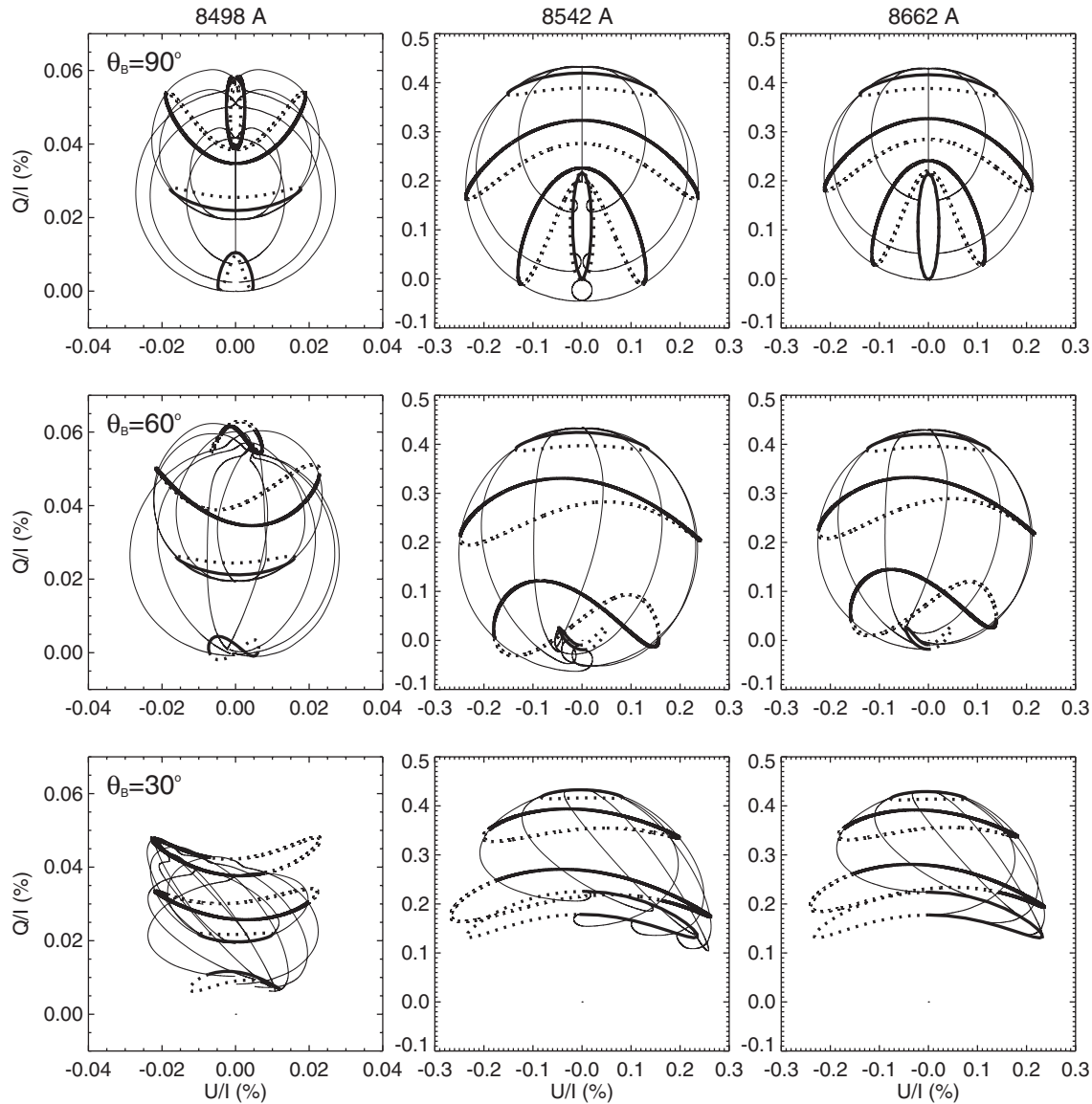


Figure 12. Hanle diagrams showing the Q/I and U/I signal at the line center of the Ca II infrared triplet lines for several LOS and magnetic field configurations in the FAL-C atmospheric model. The observation is always close to the limb ($\mu = 0.1$). Thick lines correspond to constant strength magnetic fields of 1.8×10^{-3} , 5.6×10^{-3} , 1.8×10^{-2} , and 18 G when observed with $0^\circ < \chi - \chi_B < 180^\circ$ (solid line) and $180^\circ < \chi - \chi_B < 360^\circ$ (dotted line), where $\chi - \chi_B$ is the azimuth between the LOS and the magnetic field. (These two tracks do not coincide in the $\theta_B = 90^\circ$ case because we are not in the 90° scattering or tangential observation limit, but at $\mu = 0.1$.) The thin solid lines correspond to $\chi - \chi_B = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$, and 180° . Upper panels correspond to a magnetic field inclined $\theta_B = 90^\circ$ with respect to the vertical (i.e., horizontal magnetic field), central and, lower panels correspond to inclinations of $\theta_B = 60^\circ$ and $\theta_B = 30^\circ$, respectively.

5. THE HANLE EFFECT IN THE Ca II IR TRIPLET

The aim of this section is to show how the Hanle effect modifies the emergent linear polarization of the IR triplet of Ca II. To this end, we consider the following two geometries.

5.1. Forward Scattering Geometry

In the absence of magnetic fields and horizontal atmospheric inhomogeneities, the polarization of the atomic levels do not produce any linear polarization in the spectral line radiation observed at disk center ($\mu = 1$; see Figure 5). The same is true in the presence of a vertical magnetic field, because in a one-dimensional stellar atmospheric model, the vertical direction coincides with the symmetry axis of the anisotropic radiation field that induces atomic level polarization. However, in the presence of an inclined magnetic field the symmetry of the problem is broken and, as a result, the ensuing atomic level polarization can generate linear polarization even for an LOS with $\mu = 1$. In this case, the linear polarization is created by the Hanle effect of the inclined field (see a demonstrative observational example in Trujillo Bueno et al. 2002).

Figure 6 shows the fractional linear polarization of the IR triplet of Ca II generated in forward scattering geometry by the Hanle effect of a horizontal magnetic field in the FAL-C (top panels) and M-CO (bottom panels) models. In the (“hot”) FAL-C model, the linear polarization amplitudes are at the level of 10^{-4} when the magnetic strength of the horizontal field is similar or larger than only 0.1 gauss, with $Q/I > 0$ (i.e., parallel to the horizontal magnetic field) for the 8498 Å line and with $Q/I < 0$ (i.e., perpendicular to

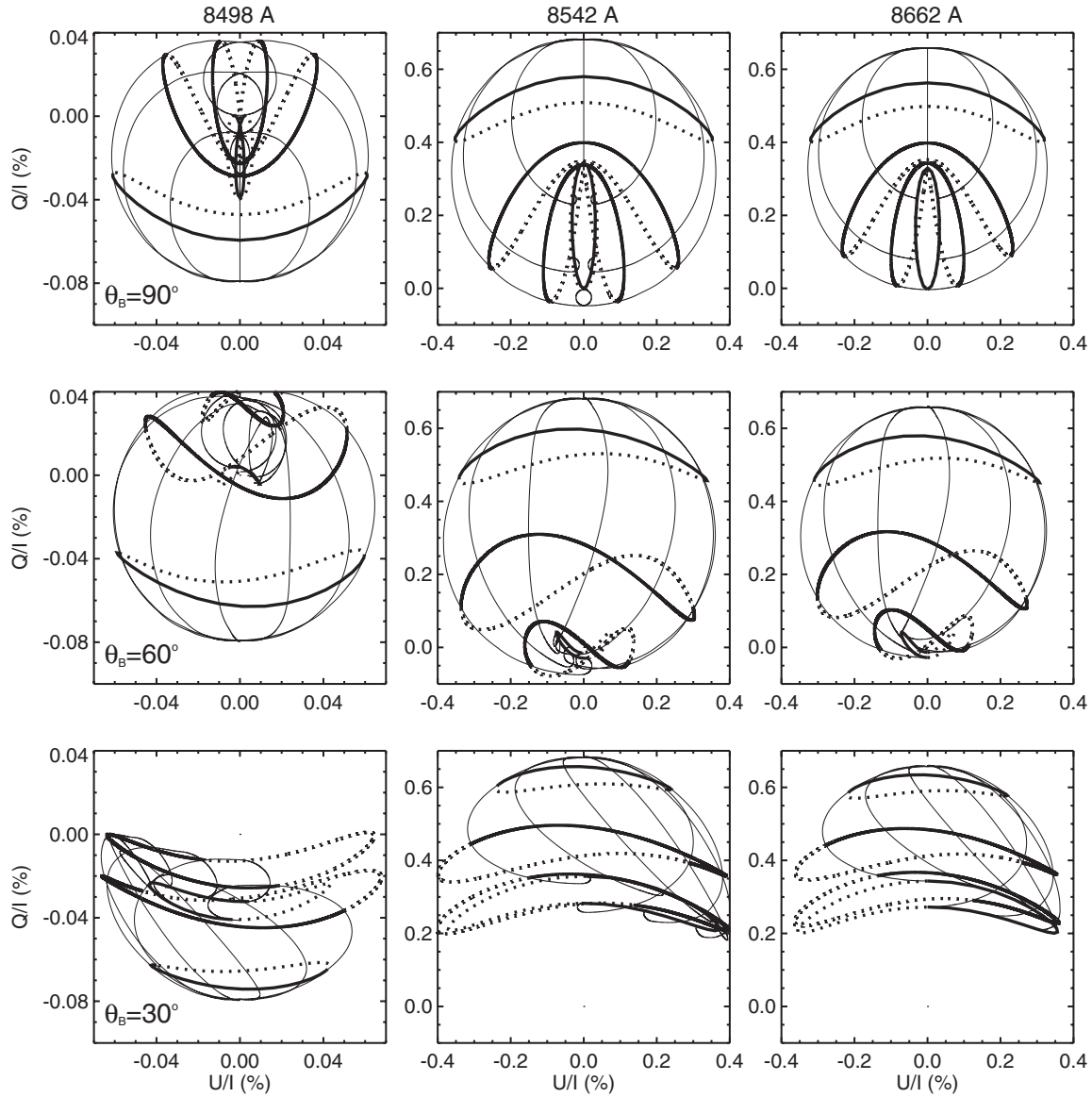


Figure 13. As in Figure 12, but for the M-CO model atmosphere.

the horizontal magnetic field) for the 8542 Å and 8662 Å lines. In the (“cool”) M-CO model $Q/I > 0$ in the 8542 Å and 8662 Å lines, with amplitudes again at the 10^{-4} level for $B \gtrsim 0.1$ gauss, while the shape of Q/I in the 8498 Å line is very peculiar (i.e., similar to the typical observational signature of the transverse Zeeman effect) and with its maximum amplitude 1 order of magnitude smaller. Figure 7 shows the results of our RT calculations for the case of a magnetic field inclined by only 30° . Note that now $U/I \neq 0$. At present, detection of these weak linear polarization signals is challenging, though feasible. But they will become straightforward with future large-aperture solar telescopes such as the Advanced Technology Solar Telescope (Keil et al. 2009) or the European Solar Telescope (Collados 2008).

5.2. Close to the Limb Geometry

In forward scattering geometry, symmetry imposes that linear polarization vanishes if the magnetic field is inclined but has a random azimuth distribution, unlike in the fixed inclination and azimuth case just considered. However, for observations away from the solar disk center ($\mu < 1$), scattering may produce linear polarization in the emergent spectral line radiation even for magnetic fields with a uniformly distributed azimuth within the spatio-temporal resolution element of the observations. This is illustrated in Figure 8, which shows how the linear polarization amplitudes of the Ca II IR triplet in the FAL-C model (top panels) and in the M-CO model (bottom panels) vary with the magnetic strength (B) and the inclination (θ_B) of the assumed random azimuth field for a close-to-the-limb observation ($\mu = 0.1$).

Figure 8 shows how sensitive the Q/I line-center amplitude of the 8498 Å line is to the model’s thermal structure, and its very peculiar behavior in the sub-gauss range. The 8542 Å and 8662 Å lines are mostly sensitive to magnetic fields between 0.001 and 0.1 gauss, although the 8542 Å line is also weakly sensitive to fields between 1 and 10 gauss. The 8542 Å line enters into the saturation regime of the upper-level Hanle effect for fields stronger than ~ 10 gauss, while just 0.1 gauss saturate the 8662 Å line. On the contrary, the 8498 Å line is highly sensitive in the whole regime 0.001–10 gauss. As seen in Figure 3, in the absence of

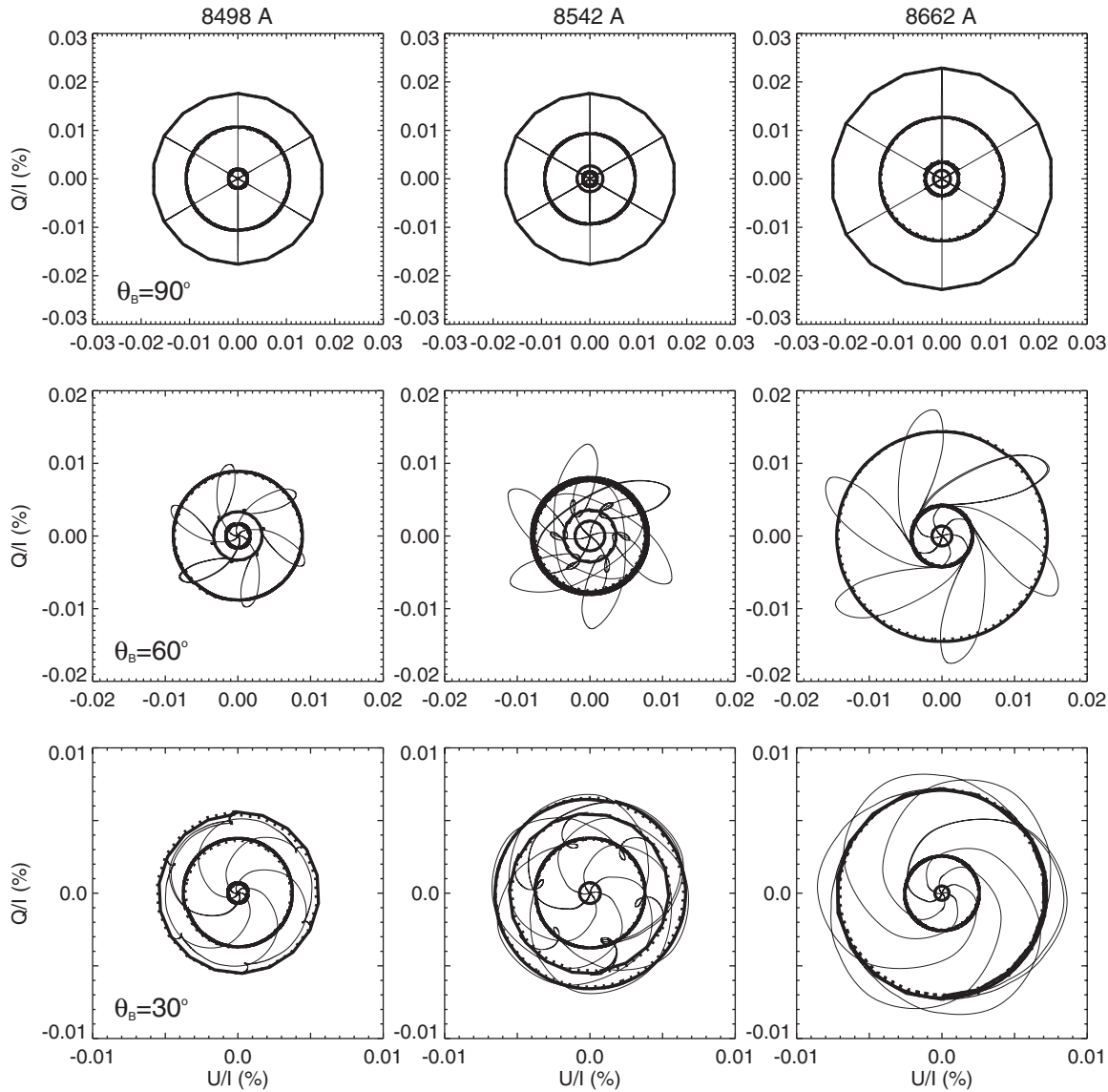


Figure 14. As in Figure 12, but in forward scattering geometry.

magnetic fields the linear polarization amplitude of the 8498 Å line for an LOS with $\mu = 0.1$ is 1 order of magnitude smaller than those corresponding to the 8542 Å and 8662 Å lines. However, the three Ca II lines may show similar Q/I amplitudes if there is a magnetic field of the order of only 0.1 gauss in the atmospheric region where the linear polarization of the 8498 Å line originates.

Figure 9 shows the emergent Q/I amplitudes averaging the Q/I profiles of both atmospheric models with a weight of 0.55 for the FAL-C profiles and 0.45 for the M-CO profiles. These weights are similar to those used by Avrett (1995) to simultaneously reproduce the intensity profiles observed in the $4.6 \mu\text{m}$ lines of CO and in the H and K lines of Ca II, and to those used by Holzreuter et al. (2006) to fit the Q/I profile of the Ca II K line observed in quiet regions very close to the solar limb. These types of fits obtained by mixing a cool and a hot component should not be considered as a realistic representation of the chromospheric conditions, but as further indication of the complexity of the (time-dependent) three-dimensional thermal structure of the solar chromosphere. Nevertheless, this two-component model shows that the Q/I observations of the IR triplet of Ca II reported by Stenflo et al. (2000) are compatible with very weak magnetic fields in the “region of formation” of the 8498 Å line (i.e., in the lower chromosphere) and with fields stronger than 10 gauss (and inclined by about 30°) in the “regions of formation” of the 8542 Å and 8662 Å lines (i.e., in the upper chromosphere).

Stokes U is zero if the magnetic field has a random azimuth distribution, and it is non-zero if the atmosphere is permeated by a magnetic field vector with a well-defined inclination and azimuth. This is illustrated in the upper panels of Figure 10, which corresponds to the FAL-C model. In the absence of magnetic fields, only Q/I is non-zero (see the dashed lines), but U/I becomes significant even at fields strengths as low as 0.005 gauss. Note that for this case ($B = 0.005$ gauss) characterized by a horizontal magnetic field pointing toward the observer (triangles), perpendicular to the LOS (squares) and away from the LOS (diamonds), the Stokes Q/I profiles of the Ca II IR triplet lines are always positive, while U/I changes its sign. The dotted lines of Figure 10 show the case of a 100 gauss horizontal field pointing toward the observer (i.e., a strength for which the three IR lines of Ca II are in the saturation regime of the upper-level Hanle effect). The bottom panels of Figure 10 show the results of similar calculations for the cool

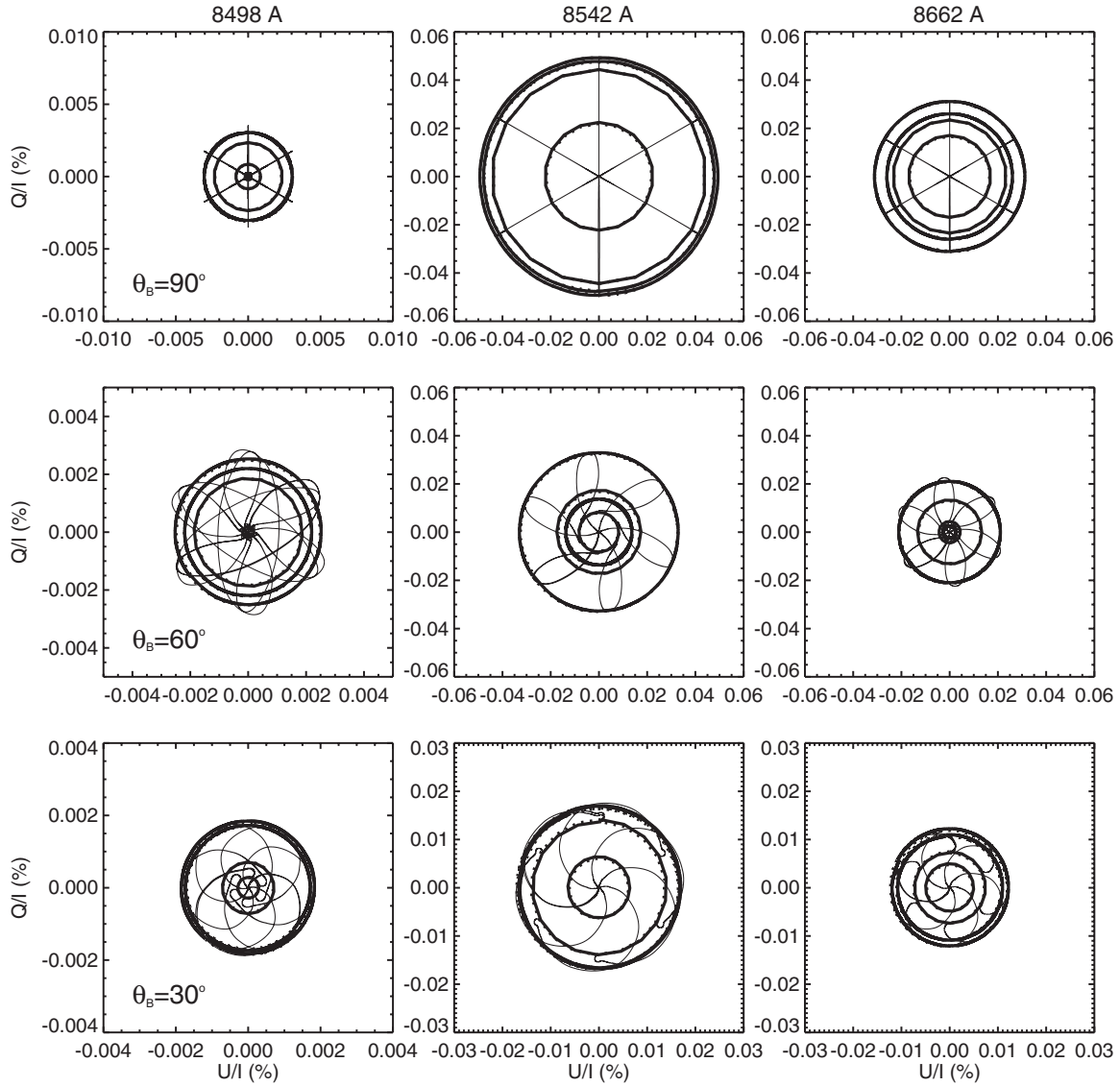


Figure 15. As in Figure 13, but in forward scattering geometry.

M-CO model. In this model, the Q/I profile of the 8498 Å line changes its sign and shape when the magnetic field azimuth changes from $\chi_B = 0^\circ$ (or $\chi_B = 180^\circ$) to $\chi_B = 90^\circ$.

5.3. Hanle-effect Diagrams

A Hanle-effect diagram shows the line-center amplitudes of Q/I and U/I varying with the strength and the azimuth of the magnetic field corresponding to a given LOS and inclination of the magnetic field vector. Figure 11 shows restricted Hanle-effect diagrams corresponding to the configurations considered in Figure 10 (close-to-the-limb $\mu = 0.1$ observation). In particular, the thin solid lines show how the linear polarization varies with the magnetic strength for a horizontal magnetic field pointing toward the observer. We recall that 50 gauss are needed to completely depolarize the 8498 Å line, while just 10 gauss are sufficient for the 8542 Å line and only 0.1 gauss for the 8662 Å line (see Figure 8). The thick solid lines of Figure 11 show, for the case of a 0.005 gauss horizontal field, how the Q/I and U/I line-center amplitudes vary as the orientation of the magnetic field vector changes from pointing toward the observer (triangles), to being perpendicular to the LOS (squares) to pointing away from the observer (diamonds). Comparing the upper and lower panels we see the differences due to the different thermal structures of the FAL-C and M-CO models.

Figures 12 and 13 show full Hanle-effect diagrams obtained from the emergent Stokes Q/I and U/I profiles calculated for a close-to-the-limb ($\mu = 0.1$) LOS in the “hot” and “cool” atmospheric models, respectively. Each panel corresponds to a fixed inclination of the magnetic field vector. Each of the four thick lines in any given panel correspond to a fixed strength of the magnetic field with azimuth ranging from $\chi_B = 0^\circ$ to $\chi_B = 180^\circ$ (solid line), and from $\chi_B = 180^\circ$ to $\chi_B = 360^\circ$ (dashed line). Thin lines correspond to magnetic fields ranging between 0 G and 1000 G for a fixed χ_B . They show the pattern of depolarization (Q/I decreases) and rotation of the direction of linear polarization (U/I appears) characteristic of the Hanle effect in a close-to-the-limb scattering geometry.

Finally, Figures 14 and 15 show Hanle-effect diagrams for a disk center observation ($\mu = 1$). Note that in this forward scattering geometry the Hanle effect of a magnetic field inclined with respect to the solar local vertical creates linear polarization.

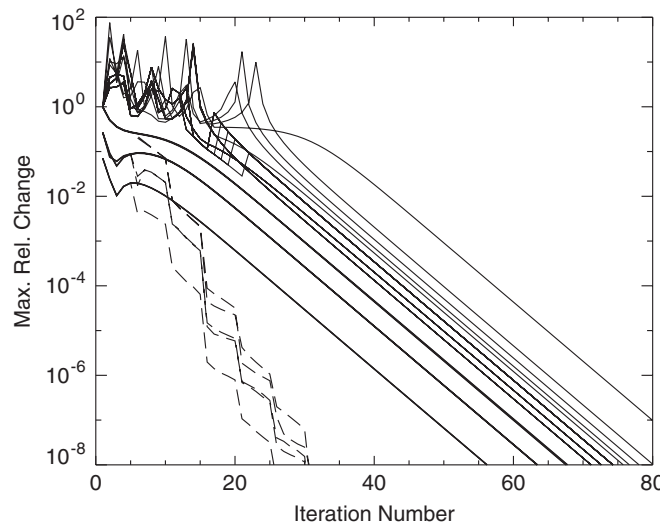


Figure 16. Maximum relative change ($R_c(it) = \text{Max}_{i=1, \dots, N_z} [|\rho_Q^K(i; it) - \rho_Q^K(i; it-1)| / |\rho_Q^K(i; it)|]$) for each of the 29 ρ_Q^K components in a radiative transfer calculation using a five-level atomic model of Ca II. Solid lines show the convergence rate of all ρ_Q^K components. Heavy solid lines correspond to ρ_0^0 . Note that due to superpositions only three lines are visible while there are five ρ_0^0 components. Dashed lines correspond to the case in which Ng acceleration is used. For clarity, in this case only the ρ_0^0 convergence rates are shown.

6. CONCLUSIONS

We have investigated theoretically the sensitivity of the scattering polarization Q/I and U/I profiles of the Ca II IR triplet to the presence of magnetic fields through the Hanle effect. To this end, we have applied a multilevel radiative transfer code (see the [Appendix](#)) for treating the transfer of spectral line polarization due to scattering in weakly magnetized stellar atmospheres. We find that the Ca II 8542 Å and 8662 Å lines are mostly sensitive to fields between 0.001 and 0.1 G, the former also being weakly sensitive to fields between 1 and 10 G. Otherwise the linear polarization signals are dependent only on the magnetic field geometry, but not to its intensity. The 8498 Å line is highly sensitive in the whole regime 0.001–10 G, but its scattering polarization amplitude is significantly lower in the absence of magnetic fields.

The most interesting line of the Ca II IR triplet because of its diagnostic potential is probably the strongest one: $\lambda 8542$. Its linear polarization is sensitive to the orientation of the magnetic field in the chromosphere of the quiet Sun, though it is practically insensitive to the field's strength unless $B < 1$ G there. Although its scattering polarization amplitude depends on the thermal structure of the solar chromosphere, the emergent Q/I and U/I profiles do not change sign between the (“hot”) FAL-C model and the (“cool”) M-CO model, unlike $\lambda 8498$, which is very sensitive to the physical conditions of the lower chromosphere. The thermal sensitivity of the $\lambda 8662$ line (which is sensitive only to milligauss field strengths) is similar to that of the 8542 Å line. Therefore, spatial fluctuations in the sign of the emergent U/I profiles of the Ca II 8542 Å and 8662 Å lines may be more safely ascribed to spatial variations in the azimuth of the chromospheric magnetic field than is the case for $\lambda 8498$.

More generally, the emergent Q/I and U/I profiles are produced by the joint action of atomic level polarization, and the Hanle and transverse Zeeman effects. In this paper, we have considered only the action of atomic level polarization and the Hanle effect. These effects dominate the emergent linear polarization profiles for inclined magnetic fields with strengths weaker than B_0 , where the B_0 value depends on the scattering geometry.

In the forward scattering geometry of a disk center observation ($\mu = 1$), the linear polarization of the Ca II IR triplet is dominated by the Hanle effect if the magnetic field is weaker than about 10 G. In fact, while the contribution of the transverse Zeeman effect is negligible for $0 \leq B < 10$ G, the Hanle effect creates fractional linear polarization signals of the order of 10^{-4} already for horizontal magnetic fields as weak as 0.1 G (see Figures 6 and 7). For magnetic strengths $10 \lesssim B \lesssim 100$ G the contribution of the transverse Zeeman effect to the linear polarization observed at the solar disk center should not be neglected. Detection of Q/I and U/I disk center signals caused either by the Hanle effect alone (if $B < 10$ G) or by the joint action of the Hanle and transverse Zeeman effects (if $10 \lesssim B \lesssim 100$ G) would require very high polarimetric sensitivity together with a spatial and temporal resolution sufficient to at least partially resolve the magnetic field azimuth.

Given the weakness of the theoretical scattering polarization signals ($\lesssim 10^{-4}$ at $\mu = 1$; $\lesssim 10^{-3}$ at $\mu = 0.1$), the most favorable geometry for observing them is in quiet regions close to the limb. In this geometry, the emergent linear polarization of the Ca II IR triplet is non-zero even if the magnetic field has a random azimuth within the spatio-temporal resolution element of the observation (see Figures 8 and 9). Moreover, for magnetic strengths noticeably weaker than 100 G the contribution of the transverse Zeeman effect to the emergent linear polarization is expected to be smaller than that caused by atomic level polarization. For stronger fields (e.g., $100 \lesssim B \lesssim 500$ G) the wings of the emergent Q/I and U/I profiles may show sizable features produced by the transverse Zeeman effect, but the line-center amplitudes would still be dominated by atomic level polarization.

Finally, we point out that information on the relative amplitudes and signs of the scattering polarization profiles of the Ca II IR triplet would be very useful to help constrain the thermal and magnetic structure of the quiet solar chromosphere. In principle, to determine the structure of the magnetic field via Stokes inversion techniques is possible, although it would be easier if one is able to first infer the atmospheric thermal structure from the observed intensity spectrum. The Q/I and U/I signals we have studied here

should also be exploited to investigate the reliability of three-dimensional models of the solar chromosphere resulting from holistic magnetohydrodynamic (MHD) simulations of the solar atmosphere. To this end, it suffices with confronting spectropolarimetric observations of the Ca II IR triplet with synthetic Stokes profiles obtained by solving the non-LTE problem of the second kind in three-dimensional snapshot models resulting from such MHD simulations.

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APPENDIX

THE NUMERICAL SOLUTION OF THE NON-LTE PROBLEM OF THE SECOND KIND

Modeling the spectral line polarization produced by atomic level polarization and its modification by the Hanle effect requires calculating, for multilevel systems, the excitation and ionization state of chemical species of given abundance that is consistent with both the intensity and polarization of the radiation field generated within the (generally magnetized) plasma under consideration. This so-called non-LTE problem of the second kind (cf. Landi Degl’Innocenti & Landolfi 2004) is a very involved non-local and non-linear radiative transfer problem which requires solving the statistical equilibrium equations for the elements of the atomic density matrix and the Stokes-vector transfer equation for each of the allowed transitions in the multilevel model. Once such a self-consistent excitation state is known throughout the medium, it is then straightforward to solve the transfer equation for any desired LOS in order to obtain the emergent Stokes profiles to be compared with spectropolarimetric observations. To this end, we developed a general computer program, which we will describe in greater detail in a future publication along with the efficient and accurate radiative transfer methods on which it is based (for a preview, see the workshop contribution by Manso Sainz & Trujillo Bueno 2003b; see also Manso Sainz 2002). In this Appendix, we present only a brief summary of the code with emphasis on its application for solving the problem of scattering polarization and the Hanle effect in the IR triplet of ionized calcium.

A summary of the numerical approach is as follows:

1. The equations are formulated within the spherical tensors representation of the density matrix and radiation field tensor. There are $(2J + 1)(J + 1)$ unknowns ρ_Q^K with K even, for each level with integer angular momentum J , and $(2J + 1)J$ unknowns for each level with semi-integer angular momentum J .
2. The radiative transfer equations for the Stokes parameters are integrated along short characteristics using the quasi-parabolic DELO method (DELOPAR; Trujillo Bueno 2003).
3. The iterative corrections for the unknowns (the statistical tensors ρ_Q^K) are calculated applying a suitable generalization to the multilevel atom case of the fast iterative methods described in Trujillo Bueno & Manso Sainz (1999). This implies writing down the statistical equilibrium equations taking explicitly into account the contribution of the diagonal components of the Λ operator and linearizing according to Equations (53) and (54) of Trujillo Bueno (2003).
4. The iterative scheme can speeded up further through polynomial Ng acceleration (e.g., Auer 1987).

The number density of the ion under consideration is computed in non-LTE. At each point in the atmosphere we solve the statistical equilibrium equations for the multipolar components of the atomic density matrix plus the conservation of particles equation to calculate the excitation state of the atomic system. We calculate the radiation field at each point in the model atmosphere for each of the allowed radiative transitions in the model atom by formal integration of the radiative transfer equations for the Stokes parameters applying the DELOPAR method. Then, radiation field tensors J_Q^K are calculated and used to work out the new excitation state of the atomic system and so on, iteratively. Figure 16 shows the convergence behavior of the above-mentioned iterative method for the numerical solution of the (Ca II) non-LTE problem of the second kind.

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