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ON THEORIES FOR STOCHASTIC ACCELERATION IN THE SOLAR WIND

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ABSTRACT

The suprathermal tails on the distribution functions of ions in the solar wind are observed to have a common spectral shape in many different circumstances: a power law in particle speed with spectral index of −5. Three possible approaches for explaining these observations are considered: (1) the acceleration mechanism of Fisk & Gloeckler in which energy is redistributed from a core particle population into the suprathermal tail; (2) traditional stochastic acceleration in which particles are accelerated by damping turbulence; and (3) the statistical approach introduced by Schwadron et al. in which the −5 spectrum is formed by averaging over individual spectra. The acceleration mechanism of Fisk & Gloeckler has advantages: (1) it appears to occur in conditions that are readily satisfied: compressive turbulence that is thermally isolated (no large-scale spatial gradients), with a core distribution of particles with a sharp initial cutoff in particle speed, above which particles can spatially diffuse; and (2) it yields spectra that are consistent with observations. Traditional stochastic acceleration has the disadvantage that it is unlikely to yield spectral shapes consistent with observations, and the spectra are dependent upon the plasma conditions and thus unlikely to be the same in different circumstances. The statistical approach of Schwadron et al. can yield the −5 spectrum and is consistent with the results of Fisk & Gloeckler when the assumed distribution functions for individual events and the averaging technique are taken to be compatible with the assumptions and averaging in Fisk & Gloeckler.

Key words: acceleration of particles – interplanetary medium – turbulence

1. INTRODUCTION

One of the more interesting observations in heliospheric physics in recent years has been the discovery that the suprathermal tails on the distribution functions of ions in the solar wind tend to have the same spectral shape: a power law in particle speed with spectral index of −5 (Gloeckler et al. 2000; Gloeckler 2003; Simunac & Armstrong 2004). Ongoing studies have revealed that the common spectral shape is observed in the quiet solar wind and in the solar wind from the polar coronal holes, where no local shocks are observed; in disturbed conditions; in upstream and especially downstream of shocks; and particularly in the heliosheath (e.g., Decker et al. 2005); and when adequate statistics are available the common spectral shape is observed on relatively short timescales, such as one hour, corresponding to turbulent correlation lengths. Deviations from the common spectral shape are observed particularly in disturbed conditions. However, in all cases that can be studied in detail, the deviations are accompanied by strong anisotropies, even at low energies, suggesting that the particles are accelerated elsewhere and propagate to the observing site. In all cases, the spectra roll over at higher particle speeds, often as a relatively gradual exponential. The location and shape of the exponential rollover vary widely, and they clearly depend upon the plasma conditions in the acceleration region and/or the time for acceleration. However, the spectral index of −5 at low particle speeds is common to spectra in many circumstances, and thus largely independent of the plasma conditions.

These observations demand an explanation, which should have the following features: the spectral index of −5 should arise naturally and be largely independent of the conditions where the acceleration is occurring. Any form of shock acceleration is unlikely, for the simple reason that the common spectra can be observed far from shocks. We then need to focus on some form of stochastic acceleration.

Fisk & Gloeckler, in a series of papers, have constructed an acceleration theory, which has as its purpose to yield the −5 spectrum (Fisk & Gloeckler 2006, 2007, 2008, 2009). They consider a core distribution of particles in compressive turbulence, where the core undergoes adiabatic compressions and expansions and the total system is thermally isolated. Particles above some threshold speed can undergo spatial diffusion. In this situation, particles can be pumped up in energy and out of the core, to form a suprathermal tail. Since the system is thermally isolated, the tail will tend toward a state of maximum entropy, which Fisk & Gloeckler show is a spectrum with spectral index of −5. The unique feature of this acceleration mechanism is that the particles in both the core and the tail experience only adiabatic compressions and expansions, and there is no damping of the turbulence. The acceleration mechanism simply redistributes energy from the core to the tail.

An alternative acceleration mechanism is traditional stochastic acceleration, in which particles diffuse in velocity space due to their random interactions with turbulence. Countless papers have been written on this mechanism, and it is routinely presented in basic plasma physics books (e.g., Bellan 2006). The problem is that traditional stochastic acceleration does not naturally yield a spectrum with spectral index of −5. Indeed, power-law spectra are not common, and the spectral index that results depends upon the plasma parameters.

Jokipii & Lee (2010) have recently drawn a similar conclusion about traditional stochastic acceleration. In a thorough derivation of traditional stochastic acceleration in compressive turbulence, they conclude that it is not possible through this mechanism to have spectral indices steeper than −3, as opposed to the required spectral index of −5. Jokipii & Lee (2010), however, argue that density is not properly treated in the equation governing the acceleration mechanism of Fisk & Gloeckler, a conclusion we will demonstrate is not correct.

Recently, Schwadron et al. (2010), have introduced an interesting concept. They point out that for reasonable choices of the form of the distribution function for an individual acceleration event, and for reasonable choices for the probability that an acceleration event will occur with a given state, e.g., temperature, it is possible to average the individual spectra together and yield a kappa-function spectrum, which has a spectral index of −5 for speeds well above the characteristic thermal speeds. In this case, the exact value of the spectral index critically depends upon the plasma properties of the individual acceleration events. We show here that these properties are quite consistent with the physical conditions required for the Fisk & Gloeckler acceleration mechanism.

It is the purpose of this paper to compare and to contrast the various stochastic acceleration theories that can be applied to understand the common power-law spectrum with spectral index of −5. We begin by describing, in more detail than has previously been published, the acceleration mechanism of Fisk & Gloeckler. We provide a simple qualitative explanation of how this acceleration mechanism works, and we re-derive the basic equations, showing all of the key assumptions. We then use comparable techniques to derive the governing diffusion equation for traditional stochastic acceleration, and discuss the fundamental differences between the acceleration mechanism of Fisk & Gloeckler and traditional stochastic acceleration. Finally, we show that one of the cases considered by Schwadron et al. (2010), involving Gaussian distributions, can be interpreted as being compatible with the situation considered by Fisk & Gloeckler; and thus these two approaches yield consistent results. Finally, we discuss how these different physical mechanisms can be tested observationally.

2. THE STOCHASTIC ACCELERATION MECHANISM OF FISK & GLOECKLER

2.1. *A Qualitative Description of the Acceleration Mechanism of Fisk & Gloeckler*

Figure 1 provides a schematic description of the basic principles behind the acceleration mechanism of Fisk & Gloeckler. Particle speed is plotted on the vertical axis and position on the horizontal axis. There is a core distribution of particles with speeds greater than the thermal speed of the bulk plasma, which contains the mass, and with less than an upper threshold speed, $v \leqslant v_{\rm th}$. The bulk thermal plasma contains random compressions and expansions, which randomly and adiabatically compress and expand the particles shown in Figure 1. Particles with speeds above the threshold speed $v_{\rm th}$ are the tail particles. The distinction between the core and the tail particles is that the tail particles can diffuse spatially.

Consider then what happens in the compression shown in the center of the figure. The core particles are compressed adiabatically and energy and particles flow across the threshold boundary from the core into the tail. The tail particles are also compressed adiabatically, and raised in energy, as illustrated by the extension in the compression region to higher particle speeds.

The opposite behavior occurs in the two expansion regions on either side of the compression region. In the expansion regions, particles and energy flow from the tail back into the core and the energy of the tail particles is reduced, as is illustrated by the reductions in the number of particles at higher particle speeds in the two expansion regions.

Note the large spatial gradients that result at higher particle speeds between the compression and the surrounding expansion regions. Tail particles are able to spatially diffuse, and so at higher particle speeds, particles will diffuse in response to these gradients out of the compression region into the surrounding expansion regions.

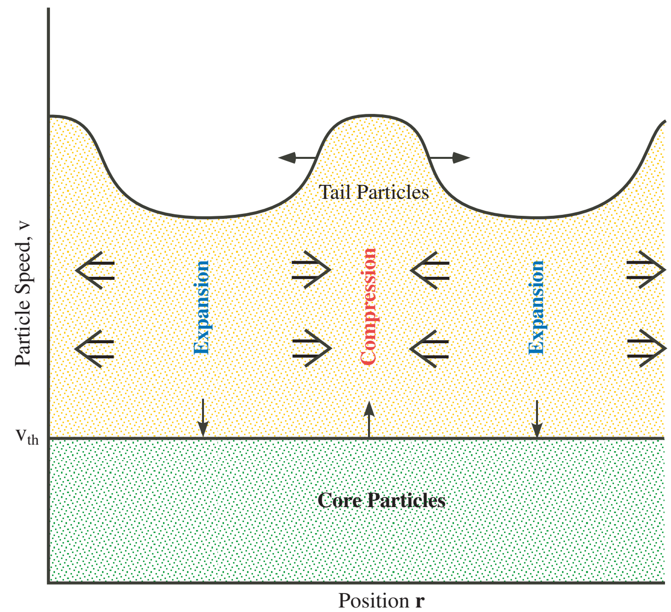

Figure 1. Schematic description of the basic principles behind the acceleration mechanism of Fisk & Gloeckler. Particle speed is plotted on the vertical axis and position on the horizontal axis. There is a core distribution of particles with speeds greater than the thermal speed of the bulk plasma and less than an upper threshold speed, $v \leqslant v_{\rm th}$. The bulk thermal plasma contains random compressions and expansions, which randomly and adiabatically compress and expand the particles as shown. Particles with speeds above the threshold speed $v_{\rm th}$ are the tail particles. The distinction between the core and the tail particles is that the tail particles can diffuse spatially.

Subsequently, the compression region will become an expansion region, and the process will be reversed. Particles and energy will flow back into the core from the tail. However, since particles have escaped from the tail by diffusion when it was a compression region, there are fewer particles and less energy to return to the core.

If the process of compressions and expansions is repeated sequentially, then a suprathermal tail will form. The particles in the tail and the energy they contain will systematically increase in time. This is a classic pump mechanism. The combination of adiabatic compressions and expansions, and spatial diffusion of the tail particles, will pump particles out of the core to form a suprathermal tail.

Note, in particular, that the basic mechanism of providing energy to the tail is an adiabatic compression. The compressive turbulence is not damped in this mechanism. Energy is simply redistributed from the core into the tail.

2.2. *Derivation of the Equations Describing the Acceleration Mechanism of Fisk & Gloeckler*

Based upon the physical description of the acceleration mechanism of Fisk & Gloeckler in the previous section, as illustrated in Figure 1, it is straightforward to derive an equation that describes the time evolution of the suprathermal tail that is being pumped out of the core distribution. This derivation follows the general principles employed in Fisk & Gloeckler (2008) but is simpler in its presentation.

We solve for the particle distribution function of the suprathermal tail, $f(\mathbf{r}, v, t)$, where $\mathbf{r}$ is position, v is particle speed, and

t is time. We assume that at $t = 0$ there are only core particles below a threshold speed, v_{th}; the suprathermal tail develops at $t > 0$. The definition of v_{th} is when the tail begins to develop at $t > 0$, core particles that are compressed and attain speeds above v_{th} are able to spatially diffuse out of a compression region on a characteristic timescale τ that is less than the characteristic time of a compression, i.e., less than $\lambda/\delta u$, where λ is the characteristic scale size of a compression and δu is the average magnitude of the velocity of the turbulence, $\delta\mathbf{u}$. As we discuss in Section 2.4, the speed of the effective threshold will increase as the tail develops to higher particle speeds.

We separate the distribution function $f(\mathbf{r}, v, t)$ of the tail particles into two parts:

$$f(\mathbf{r}, v, t) = f_o(\mathbf{r}, v, t) + \delta f(\mathbf{r}, v, t). \tag{1}$$

Here, δf is the portion of the distribution function that is subject to spatial diffusion. Note that both f_o and δf each have a dependence on position, particle speed, and time that will have to be determined by the governing equations.

We assume that in each compression and expansion f behaves according to the standard Parker transport equation, in which we approximate the effects of spatial diffusion as a loss (gain) term, $-\delta f/\tau$, where again τ is the characteristic time to diffuse out of (into) a compression (expansion), or

$$\frac{\partial f}{\partial t} + \delta\mathbf{u}\cdot\nabla f = \frac{(\nabla\cdot\delta\mathbf{u})}{3} v\frac{\partial f}{\partial v} - \frac{\delta f}{\tau}. \tag{2}$$

We then substitute Equation (1) into Equation (2), or

$$\begin{aligned}&\left[\frac{\partial f_o}{\partial t} - \frac{(\nabla\cdot\delta\mathbf{u})}{3v^4}\frac{\partial}{\partial v}(v^5\delta f) + \delta\mathbf{u}\cdot\nabla f_o + \frac{5}{3}(\nabla\cdot\delta\mathbf{u}) f_o\right]\\ &\quad + \left[\frac{\delta f}{\tau} - \frac{(\nabla\cdot\delta\mathbf{u})}{3v^4}\frac{\partial}{\partial v}\left(v^5 f_0\right)\right]\\ &= -\left[\frac{\partial\delta f}{\partial t} + \delta\mathbf{u}\cdot\nabla\delta f + \frac{5}{3}(\nabla\cdot\delta\mathbf{u})\,\delta f\right],\end{aligned} \tag{3}$$

where we have regrouped and rewritten the terms.

The basic physics of the acceleration mechanism described in the previous section and illustrated in Figure 1 is as follows: each compression and expansion of the tail is adiabatic, which requires that to first order the energy that flows in from the core in a compression escapes by spatial diffusion, or, in an expansion, the energy that flows in by spatial diffusion flows back into the core. To achieve this physics, the second term in square brackets in Equation (3) must be set to zero, or

$$\frac{\delta f}{\tau} = \frac{(\nabla\cdot\delta\mathbf{u})}{3v^4}\frac{\partial}{\partial v}\left(v^5 f_0\right). \tag{4}$$

To show that Equation (4) ensures that the flow of energy into and out of a compression or expansion balances to first order, we integrate to find the behavior of the pressure of the tail

$$P_t = \frac{4\pi m}{3}\int_{v_{\mathrm{th}}}^{\infty} v^4 f\,dv, \tag{5}$$

where m is particle mass, or, Equation (4) becomes

$$-\frac{4\pi m}{9}(\nabla\cdot\delta\mathbf{u})\, v^5 f_0\big|_{v=v_{\mathrm{th}}} = \frac{4\pi m}{3}\int_{v_{\mathrm{th}}}^{\infty} v^4\frac{\delta f}{\tau}dv. \tag{6}$$

The term on the left is evaluated at $v = v_{\mathrm{th}}$ and represents the first order flow of energy across the threshold boundary. The term on the right is the escape (source) of energy due to spatial diffusion.

The pump mechanism described in the previous section and illustrated in Figure 1 results in a second-order growth in the tail, i.e., in addition to the tail particles undergoing first-order adiabatic compressions and expansions, the tails will grow to second order. To achieve this physics, the first term in square brackets in Equation (3) must be set equal to zero, or substituting in Equation (4),

$$\frac{\partial f_o}{\partial t} = \frac{1}{v^4}\frac{\partial}{\partial v}\left(\frac{\delta u^2}{9\kappa}v\frac{\partial}{\partial v}\left(v^5 f_0\right)\right) - \delta\mathbf{u}\cdot\nabla f_o - \frac{5}{3}(\nabla\cdot\delta\mathbf{u}) f_o, \tag{7}$$

where we have assumed that $(\nabla\cdot\delta\mathbf{u})^2\tau$ does not depend strongly on position, and have replaced it with its average value, which we define as in Fisk & Gloeckler (2008),

$$\left\langle(\nabla\cdot\delta\mathbf{u})^2\tau\right\rangle \approx \frac{\delta u^2}{\lambda^2}\tau \equiv \frac{\delta u^2}{\kappa}. \tag{8}$$

Here, λ is the characteristic length scale for a compression or expansion, and κ is the spatial diffusion coefficient for diffusion in the same direction as λ is measured, i.e., κ is the cross-field diffusion coefficient. This replacement of τ with κ is strictly valid only when the spatial gradients in the distribution function between a compression and an expansion region have a characteristic length scale of λ, which, as can be seen in Figure 1, occurs only in the high particle speed extensions of the tail. At particle speeds below this, the spatial gradients will be smaller or equivalently the characteristic length scales longer than λ. As we shall see in Sections 2.3 and 2.4, τ and κ are in fact important for the solutions of Equation (7) only in the high particle speed extensions of the tail.

Equation (7) is the governing equation for the time evolution of the tail that was derived by Fisk & Gloeckler (2008). To show that it ensures that the tail undergoes adiabatic compressions and expansions, and grows to second order by the pump mechanism, we can again integrate to form an equation for the behavior of the tail pressure (Equation (5)), or Equation (7) becomes

$$\frac{\partial P_{t,o}}{\partial t} + \delta\mathbf{u}\cdot\nabla P_{t,o} + \frac{5}{3}(\nabla\cdot\delta\mathbf{u}) P_{t,o} = -\frac{4\pi m}{3}\frac{\delta u^2}{9\kappa}v\frac{\partial}{\partial v}\left(v^5 f_0\right)\bigg|_{v=v_{\mathrm{th}}}. \tag{9}$$

The left side is the behavior of the pressure $P_{t,o}$ associated with f_o in an adiabatic compression or expansion. The right side is evaluated at the threshold $v = v_{\mathrm{th}}$ and represents the average, second-order flow of energy from the core to the tail.

With the first two terms in square brackets in Equation (3) zero, we must also require that the third term in square brackets be zero. This cannot result from setting $(\nabla\cdot\delta\mathbf{u})\,\delta f$ to be zero since this term is the same as the basic acceleration term in Equation (7). Rather, we need to require that

$$\frac{\partial\delta f}{\partial t} + \delta\mathbf{u}\cdot\nabla\delta f + \frac{5}{3}(\nabla\cdot\delta\mathbf{u})\,\delta f = 0. \tag{10}$$

Or, with Equation (4), and expressing Equation (10) as a continuity of density equation,

$$\frac{\partial\delta f}{\partial t} + \nabla\cdot(\delta\mathbf{u}\delta f) = -\frac{2}{9}\frac{(\nabla\cdot\delta\mathbf{u})^2\tau}{v^4}\frac{\partial}{\partial v}(v^5 f_o). \tag{11}$$

Thus, moving with the plasma, there is an inherent source or sink term of particles. Note that δf can be either positive or negative, i.e., this can be a source or a sink term.

The presence of this source or sink term of particles is necessary to ensure that the behavior of the density and the behavior of the energy are consistent with each other. According to Equation (4), the energy that flows out of (into) the core is balanced by the energy that flows out of (is provided to) a volume by spatial diffusion. In an adiabatic compression, the energy that flows in from the core is in the form of low-energy particles; the energy that escapes by spatial diffusion is in the form of higher energy particles. Thus, in an adiabatic compression, more particles flow in from the core than escape by spatial diffusion. Conversely, in an adiabatic expansion, more particles flow out to the core than enter by spatial diffusion.

The time-averaged behavior of the density of the tail will thus be the result of two processes: the average of the flow of particles into or out of the core and the average of the flow of particles into or out of the volume by spatial diffusion. The effects of these two processes can be seen by integrating the first term on the right of Equation (7) to determine the long-term behavior of the density of the tail,

$$n_{t,o} = 4\pi \int_{v_{\rm th}}^{\infty} f_o v^2 dv, \tag{12}$$

or,

$$\begin{aligned}\frac{dn_{t,o}}{dt} &= 4\pi \int_{v_{\rm th}}^{\infty} \frac{dv}{v^2}\frac{\partial}{\partial v}\left(\frac{\delta u^2}{9\kappa} v \frac{\partial}{\partial v}(v^5 f_o)\right)\\ &= -4\pi \frac{1}{v}\frac{\delta u^2}{9\kappa}\frac{\partial}{\partial v}(v^5 f_o)\Big|_{v=v_{\rm th}} + 8\pi \int_{v_{\rm th}}^{\infty} \frac{dv}{v^2}\frac{\delta u^2}{9\kappa}\frac{\partial}{\partial v}(v^5 f_o).\end{aligned} \tag{13}$$

If we then average the source or sink term in Equation (11), averaged in the same sense as the averaging to form $\langle(\nabla\cdot\delta\mathbf{u})^2\tau\rangle$, or

$$\nabla\cdot\langle\delta\mathbf{u}\delta f\rangle = -\frac{2}{9}\frac{\delta u^2}{\kappa}\frac{1}{v^4}\frac{\partial}{\partial v}\left(v^5 f_o\right), \tag{14}$$

we find that

$$\frac{dn_{t,o}}{dt} = -4\pi \frac{1}{v}\frac{\delta u^2}{9\kappa}\frac{\partial}{\partial v}(v^5 f_o)\Big|_{v=v_{\rm th}} - 4\pi \int_{v_{\rm th}}^{\infty} v^2 dv \nabla\cdot\langle\delta\mathbf{u}\delta f\rangle. \tag{15}$$

As can be seen from Equation (9), the first term on the right of Equation (15) is the average rate of increase in particles in the tail due to the average flow of energy from the core to the tail. The second term is the average rate of loss of particles due to spatial diffusion.

It is required in any valid derivation that both energy and number of particles be conserved. Since the energy is carried by the particles, the requirement that energy be properly treated in Equations (4) and (7) will ensure that the density is properly treated. Equation (10) results from Equations (4) and (7), and its average form, Equation (14), ensures that the behavior of the density is consistent with the behavior of the energy.

Jokipii & Lee (2010) have criticized the presence of the average source term in Equation (14) arguing that it shows that particles are not properly conserved in the acceleration mechanism of Fisk & Gloeckler (2008). Quite the contrary: this term is necessary so that both particles and energy are conserved.

2.3. Solutions to the Acceleration Equation of Fisk & Gloeckler

The governing equation of the acceleration mechanism of Fisk & Gloeckler, Equation (7),

$$\frac{\partial f_o}{\partial t} = \frac{1}{v^4}\frac{\partial}{\partial v}\left(\frac{\delta u^2}{9\kappa} v \frac{\partial}{\partial v}(v^5 f_o)\right) - \delta\mathbf{u}\cdot\nabla f_o - \frac{5}{3}(\nabla\cdot\delta\mathbf{u}) f_o \tag{7}$$

can be readily solved for the case where the spatial diffusion coefficient is of the form

$$\kappa = \kappa_o v^{1+\alpha}. \tag{16}$$

Here, the mean free path is a power law in rigidity, with exponent α; κ_o contains the appropriate mass-to-charge dependence. The resulting solution to Equation (7) is then

$$f_o = f_{o,\rm th}\left(\frac{v}{v_{\rm th}}\right)^{-5} \exp\left[-\frac{9\kappa}{(1+\alpha)^2\,\delta u^2}\frac{1}{t}\right], \tag{17}$$

where, $f_{o,\rm th}$ is a normalization factor at $v = v_{\rm th}$ and satisfies

$$\frac{\partial f_{o,\rm th}}{\partial t} + \delta\mathbf{u}\cdot\nabla f_{o,\rm th} + \frac{5}{3}(\nabla\cdot\delta\mathbf{u}) f_{o,\rm th} = 0. \tag{18}$$

The suprathermal tail described by the solution is the result of multiple compressions and expansions and is thus valid for times long compared to a single compression or expansion, i.e., for $t \gg \lambda/\delta u$. It is also valid only for the tail, i.e., for $v > v_{\rm th}$.

The solution in Equation (17) meets our expectations. The tail undergoes adiabatic compressions and expansions (Equation (18)) and the spectrum at low particle speeds is a power law with spectral index of -5, independent of the plasma parameters. There is an exponential rollover at higher particle speeds, where the rollover depends upon the plasma parameters δu^2 or κ, as well as the time required to accomplish the acceleration. The solution in Equation (17) is consistent with the many observations of suprathermal tails that are observed throughout the heliosphere.

Note that in this treatment the tail in Equation (17), and the particles and energy it contains, grow without limit in time. At some point, a thermodynamic constraint will be encountered. As discussed in Fisk & Gloeckler (2007), the energy of the core and the tail is conserved, and as the tail develops the system tends toward a state of maximum entropy. Assuming that additional energy is not provided to the core, there must be a thermodynamic constraint on how much energy and how many particles can be pumped out of the core, without violating the assumption that the entropy of the system must increase. Fisk & Gloeckler (2007), in fact, develop a formula that relates the maximum pressure in the tail to the pressure in the core and find good agreement with observations.

Note also that there is a second solution to Equation (7) for the form of κ in Equation (16): $f_o \propto v^{-4+\alpha}$. This solution, however, is not physical and can be ignored. For all values of $\alpha + 1 \geqslant 0$, i.e., for other than an inverse dependence of the diffusion coefficient on v, the distribution contains infinite energy and cannot be created by pumping energy out of the core, as can be seen in Equation (9).

2.4. Conditions in Which the Acceleration Mechanism of Fisk & Gloeckler Should Apply

The suprathermal tail in the acceleration mechanism of Fisk & Gloeckler is created by a series of adiabatic compressions and expansions, in which energy is pumped from a core distribution of particles into the tail. Energy is conserved and thus the sum of the energy in the core particles and the tail particles must be constant. The acceleration mechanism of Fisk & Gloeckler is thus a redistribution mechanism in which energy is redistributed from the core to the tail.

The compressive turbulence, which is created by the thermal plasma that has the mass, and which is responsible for the adiabatic compressions and expansions of the tail, could in principle be damped due to a back-pressure exerted by the core and/or the tail. Suppose that the turbulence is not damped by the back-pressure from the core, before the tail is created. In the acceleration mechanism of Fisk & Gloeckler, it will then not be damped by the development of the tail, since as far as the thermal plasma is concerned, the pressure of the core plus the tail at any location is the same as was the pressure in the initial core. Thus, the creation of the tail in the acceleration mechanism of Fisk & Gloeckler does not damp the turbulence.

Note also in Figure 1 that the spatial gradients are largest at the higher particle speeds and that this is where the principal loss or gain due to spatial diffusion occurs. This effect can be seen in Equation (4) where the diffusion is strongest when the spectrum deviates from $f_o \propto v^{-5}$, i.e., in the rollover region at higher particle speeds for the solution in Equation (17). At particle speeds below the rollover, where $f_o \propto v^{-5}$, the spatial gradients between compression and expansion regions will be relatively small, there is little loss of particles or energy due to spatial diffusion, and the particles undergo adiabatic compressions and expansions. Fisk & Gloeckler (2007) showed that a spectrum of $f_o \propto v^{-5}$ should be expected if the tail tends toward a state of maximum entropy and is undergoing adiabatic compressions and expansions.

At $t = 0$, there is only a core distribution, and no tail, i.e., there is a sharp cutoff at the higher particle speeds, as, for example, for interstellar pickup ions which have a distribution that does not extend beyond the solar wind speed in the frame moving with the solar wind. When the core distribution is compressed, the core particles are raised in energy above the sharp cutoff, and conversely, when they are expanded, the core particles are lowered in energy below the sharp cutoff. The presence then of an initial sharp cutoff in the core distribution, and compressions and expansions, will introduce strong spatial gradients, and spatial diffusion at particle speeds above the sharp cutoff will occur. The initial threshold on the acceleration mechanism of Fisk & Gloeckler, v_{th}, above which particles can spatially diffuse, should thus coincide with the initial sharp cutoff of the core distribution.

As the tail develops, for $t > 0$, the particle speeds at which the large spatial gradients occur, and spatial diffusion is important, move to higher and higher values, into the rollover region of the solution in Equation (17). The $f_o \propto v^{-5}$ portion of the tail thus becomes an extension of the core and undergoes only adiabatic compressions and expansions, and limited spatial diffusion. In that sense, the threshold speed, above which particles can spatially diffuse, moves to higher particle speeds.

The conditions then for the acceleration mechanism of Fisk & Gloeckler to work are a core distribution of particles with an initial sharp cutoff in the presence of compressive turbulence. It is also necessary, as discussed in Fisk & Gloeckler (2007, 2008), that the compressive turbulence is thermally isolated in the sense that there are no large-scale spatial gradients that would cause particles to spatially diffuse into the accelerating region from elsewhere; i.e., it is assumed in the Fisk & Gloeckler acceleration mechanism that the acceleration is being produced by local compressive turbulence.

These conditions seem to be remarkably easy conditions to satisfy. Plasmas are rarely in equilibrium. Indeed, as shown by Parker (1972), it is not possible to have a plasma with a braided and tangled magnetic field that has no unbalanced forces. That being the case, in the absence of extensive reconnection, we should expect a certain minimum level of compressive turbulence. There are ample examples of core populations with sharp initial cutoffs (e.g., interstellar pickup ions). And there are many circumstances in the heliosphere where large-scale spatial gradients are not significant, e.g., in the heliosheath where the $f_o \propto v^{-5}$ spectrum is most pronounced (e.g., Decker et al. 2005). It is perhaps not surprising then that spectra of the form $f_o \propto v^{-5}$ with a rollover at higher particle speeds occur so commonly in the heliosphere.

3. THE TRADITIONAL FORM OF STOCHASTIC ACCELERATION

We now apply comparable techniques, as in the previous section, to derive the governing equation for traditional stochastic acceleration, in which particles diffuse in velocity space. We do this to compare and contrast the basic assumptions between traditional stochastic acceleration and the acceleration mechanism of Fisk & Gloeckler.

We begin with the standard Parker transport equation for the isotropic distribution function, f, in Equation (2):

$$\frac{\partial f}{\partial t} + \delta \mathbf{u} \cdot \nabla f = \frac{(\nabla \cdot \delta \mathbf{u})}{3} v \frac{\partial f}{\partial v} - \frac{\delta f}{\tau}. \qquad (2)$$

We then rearrange the terms as follows:

$$\frac{\partial f}{\partial t} + \nabla \cdot (\delta \mathbf{u} f) = \frac{(\nabla \cdot \delta \mathbf{u})}{3v^2} \frac{\partial}{\partial v} \left(v^3 f\right) - \frac{\delta f}{\tau}. \qquad (19)$$

We take f to have an ensemble-averaged value, f_o, and a deviation from the average of δf, or $f = f_o + \delta f$, and we ensemble average Equation (19) to find that

$$\frac{\partial f_o}{\partial t} = \frac{1}{3v^2} \frac{\partial}{\partial v} \left(v^3 \left\langle (\nabla \cdot \delta \mathbf{u})\, \delta f \right\rangle\right) - \nabla \cdot \langle \delta \mathbf{u} \delta f \rangle , \qquad (20)$$

where the angular brackets denote ensemble average.

We then make the key assumption in the derivation of the governing equation for traditional stochastic acceleration and require that

$$\nabla \cdot \langle \delta \mathbf{u} \delta f \rangle = 0, \qquad (21)$$

on the grounds that the ensemble average is spatially homogeneous and there are no source/sink terms in the ensemble average. Thus,

$$\frac{\partial f_o}{\partial t} = \frac{1}{3v^2} \frac{\partial}{\partial v} (v^3 \langle (\nabla \cdot \delta \mathbf{u}) \delta f \rangle). \qquad (22)$$

We then subtract Equation (22) from Equation (19) and keep only first-order terms, or

$$\frac{\partial \delta f}{\partial t} + \nabla \cdot (\delta \mathbf{u} f_o) = \frac{(\nabla \cdot \delta \mathbf{u})}{3v^2} \frac{\partial}{\partial v} \left(v^3 f_o\right) - \frac{\delta f}{\tau}. \qquad (23)$$

We assume that

$$\frac{1}{\tau} \gg \left| \frac{\nabla \cdot \delta \mathbf{u}}{3} \right|. \qquad (24)$$

In which case,

$$\frac{\delta f}{\tau} \approx \frac{(\nabla \cdot \delta \mathbf{u})}{3} v \frac{\partial f_o}{\partial v}. \tag{25}$$

Finally, we substitute Equation (25) into Equation (22), with the result that

$$\frac{\partial f_o}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{\delta u^2}{9\kappa} v^4 \frac{\partial f_o}{\partial v}\right), \tag{26}$$

where we have used the same definitions as in Equation (8).

Equation (26) describes the diffusion of particles in velocity space, where the velocity diffusion coefficient is $(\delta u^2 v^2)/(9\kappa)$. The solutions to Equation (26) are not naturally power-law spectra with spectral indices of -5, as required by the observations (e.g., Jokipii & Lee 2010). Also, the energy provided to the particles to form the suprathermal tail, in traditional stochastic acceleration, must be extracted from and damp the compressive turbulence.

4. COMPARING THE ACCELERATION MECHANISM OF FISK & GLOECKLER WITH TRADITIONAL STOCHASTIC ACCELERATION

The governing equation of the acceleration mechanism of Fisk & Gloeckler, Equation (7), is clearly different from the traditional stochastic acceleration equation, Equation (26). There are a number of reasons for these differences.

First, the distribution function, $f_o(\mathbf{r}, v, t)$, that Fisk & Gloeckler solve for is valid in an individual compression or expansion. The only averaging that occurs is that the quantity $(\nabla \cdot \delta \mathbf{u})^2\, \tau$ is averaged in Equation (7), under the assumption that this quantity, by itself, does not have strong spatial variations. The spatial variations in *f* between compressions and expansions are explicitly retained. In contrast, in traditional stochastic acceleration, the distribution function that is solved for is an ensemble average, which is taken to be a spatially homogeneous average over multiple compressions and expansions, and no spatial gradients in *f* are retained.

Further, because Fisk & Gloeckler retain the spatial gradients, which are responsible for the spatial diffusion, there will be apparent source terms in their equations, $\nabla \cdot \langle \delta \mathbf{u} \delta f\rangle \neq 0$ (Equation (14)), which describe the average loss of particles to a volume due to spatial diffusion. As is discussed in Section 2.2, these source terms are necessary so that both the energy and the number of particles are conserved. In contrast, in traditional stochastic acceleration, $\nabla \cdot \langle \delta \mathbf{u} \delta f\rangle = 0$, Equation (21), under the assumption that the distribution function that is being solved for is spatially homogenous.

Of most importance, the acceleration mechanism of Fisk & Gloeckler, in contrast to traditional stochastic acceleration, is not a Markovian process. Traditional stochastic acceleration is a Markovian process in which it is assumed that the past history of a particle does not matter. Each particle thus behaves separately from any other particle, and in particular the behavior of particles at different energies is unrelated. In contrast, the basic principle of the acceleration mechanism of Fisk & Gloeckler is that to first order each compression or expansion of the tail is adiabatic, and that the energy that flows in from the core at low particle speeds escapes by spatial diffusion at high particle speeds. Thus, to conserve energy, the behavior of the particles at different energies must be related, and the process is inherently non-Markovian. The non-Markovian nature of the acceleration mechanism of Fisk & Gloeckler is discussed in more detail in Fisk & Gloeckler (2008).

4.1. Perturbations in the Solutions of the Fisk & Gloeckler Acceleration Mechanism

It is interesting to consider perturbations in the solutions of Fisk & Gloeckler (Equation (17)), and ask how such perturbations will behave in time; i.e., are there any issues with stability in this solution? A perturbation in f_o at a specific energy could result from particles being accelerated elsewhere and flowing into the local region of compressive turbulence, or simply be the result of some turbulence to which only particles at the specific energy respond. Such a perturbation is not introduced by, nor will its subsequent time behavior be governed by the non-Markovian acceleration process of Fisk & Gloeckler. The governing equation of Fisk & Gloeckler, Equation (7), is explicitly derived under the assumption that the behavior of particles at different particle speeds are related, whereas the perturbation particles are explicitly behaving independently of other particles.

Rather, the subsequent time evolution of the perturbation should be described by a standard stochastic acceleration equation (Equation (26)). The perturbation is introduced by a Markovian process, and the time evolution of the perturbation should be described by a Markovian equation. We should expect then that any perturbation in the spectra that result from the acceleration mechanism of Fisk & Gloeckler will dissipate by diffusion in particle speed and not grow in time. Indeed, the observations support this position, since the spectra $f_o \propto v^{-5}$ are observed to occur quite stably in the solar wind.

5. THE FORMATION OF −5 SPECTRA USING THE APPROACH OF SCHWADRON ET AL.

Schwadron et al. (2010) point out that for reasonable choices of the form of the distribution function for an individual acceleration event, and for reasonable choices for the probability that an acceleration event will occur, it is possible to average the individual spectra together and yield a spectrum with spectral index of -5. The spectra created in the Fisk & Gloeckler acceleration mechanism are average spectra. Thus, if the choice of the distribution function and the probability of occurrence correspond to the circumstances and assumptions made in the Fisk & Gloeckler acceleration mechanism, we can show that the two approaches yield a consistent result.

One of the choices for the distribution function for an event considered in Schwadron et al. (2010) is a Gaussian distribution, which they note arises "by maximizing entropy subject to the constraint of a fixed average energy of the distribution, which is determined by the temperature." The Gaussian distribution is expressed in Equation (9) of Schwadron et al. as

$$f(v) = n\left(\frac{\lambda}{\pi}\right)^{3/2} \exp\left(-\lambda v^2\right). \tag{27}$$

Here, n is density and $\lambda = m\big/(2kT)$, where T is temperature. Schwadron et al. then require that the density n is proportional to the temperature *T*, or $n = n_o\lambda_o/\lambda$, where λ_o is the average value of λ and n_o is the reference density at λ_o.

Consider that Equation (27) applies only to the tail particles, which we define as having particle speeds, $\lambda v^2 \gg 1$. With this restriction, and with the density proportional to the temperature, the distribution function in Equation (27) has features similar to the tails created in the mechanism of Fisk & Gloeckler. In particular, the tail described by Equation (27) is always in a state of maximum entropy, which is one of the key assumptions of Fisk & Gloeckler. Also, density proportional to temperature is consistent with the pumping mechanism of Fisk & Gloeckler. Suppose that at time $t = t_i$, there is an initial Gaussian distribution with temperature T_o. Then through a series of compressions and expansions, a suprathermal tail is formed at times $t > t_i$. As a result of the repeated pumping, both the temperature and the density of the tail should increase linearly in proportion to time. The density of tail particles is

$$n_t = 4\pi n \left(\frac{\lambda}{\pi}\right)^{3/2} \int_{v_{\mathrm{th}}}^{\infty} \exp(-\lambda v^2) v^2 dv = 0.572n, \tag{28}$$

where $v_{\mathrm{th}} = 1/\sqrt{\lambda}$. Thus, for the distribution function in Equation (27) to be consistent with a pumping mechanism of Fisk & Gloeckler, n must increase in time, as does T, and thus n is proportional to T.

Schwadron et al. (2010) argue that the probability of an individual acceleration event behaving according to a Poisson distribution is

$$P(\lambda) = \frac{\lambda}{\lambda_o^2} \exp\left(-\frac{\lambda}{\lambda_o}\right). \tag{29}$$

The dependence on λ in Equation (29) is justified on the grounds that the occurrence rate of an acceleration event is proportional to $1/T$, or equivalently λ. "The higher the temperature, the lower the rate of the process that produced that temperature."

The choice of the probability of an event occurrence in Equation (29) is also consistent with Fisk & Gloeckler. They define an acceleration event as starting at $t = t_i$, and running until time t. The temperature increases in proportion to time. Thus, the occurrence rate varies inversely with the duration of the event, and as a result inversely with temperature.

Finally, following Schwadron et al. (2010), the resulting expression for the ensemble-averaged distribution function is

$$\begin{aligned}\bar{f}(v) &= \int_0^{\infty} P(\lambda) f(v, \lambda)\, d\lambda \\ &= \frac{n_o}{\lambda_o (\pi)^{3/2}} \int_0^{\infty} \lambda^{3/2} \exp\left(-\frac{\lambda}{\lambda_o}\right) \exp\left(-\lambda v^2\right) d\lambda \\ &= \frac{3n_o}{2\pi} \frac{\lambda_o^{3/2}}{\left(1 + \lambda_o v^2\right)^{5/2}},\end{aligned} \tag{30}$$

where $n_t \propto T \propto 1/\lambda$, which yields a spectrum $\bar{f} \propto v^{-5}$, for $\lambda_o v^2 \gg 1$, which is well in the tail region, where Equation (27) is assumed to be valid. Note that Equation (30) has the form of a kappa distribution (Vasyliunas 1968; Livadiotis & McComas 2009), where the kappa is $3/2$, and contains infinite energy when extended to high particle speeds.

Schwadron et al. (2010) are concerned with the possibility that the -5 spectrum results from an average over individual events, each of which is presumably observable on a fine timescale. The average performed is then effectively a spatial average, over multiple regions of space. The method, however, should apply equally well to a temporal average. In the statistical treatment of Schwadron et al., the ergodic theorem should apply. For sufficiently long periods of time, a spatial average over the volume should be the same as a temporal average in one location.

A temporal average is more appropriate for comparison with the approach of Fisk & Gloeckler in which particles are spatially diffusing into and out of a given volume. The particles present in the volume thus have a variety of time histories, and so the spectrum present in any volume is the average spectrum given in Equation (30).

6. CONCLUDING REMARKS

We have considered three possible approaches for explaining the observation that suprathermal tails on the distribution functions of ions in the solar wind tend to have the same spectral shape, a power law with spectral index of -5: (1) the acceleration mechanism of Fisk & Gloeckler; (2) traditional stochastic acceleration in which particles are accelerated by damping turbulence; and (3) the statistical approach of Schwadron et al. (2010) in which the -5 spectrum is formed by averaging over individual spectra.

The acceleration mechanism of Fisk & Gloeckler has a number of advantages: (1) it appears to occur in conditions that are readily satisfied: a plasma that is thermally isolated (no large-scale spatial gradients), that is not in equilibrium (and thus contains compressive turbulence), and that includes a core distribution with an initial sharp cutoff, above which particles can spatially diffuse; and (2) it yields spectra that are consistent with observations, a spectral index of -5 independent of the conditions in the plasma and an exponential rollover that is dependent on the conditions. The acceleration mechanism of Fisk & Gloeckler, in which energy is redistributed from a core particle population into the tail without damping the turbulence, has particular advantages in the heliosheath, where the energy in the turbulence is observed to be small compared to the energy in the interstellar pickup ions (Fisk & Gloeckler 2009).

A traditional stochastic acceleration mechanism, in which particles diffuse and damp the turbulence, appears ill suited to explain the -5 spectra, since such spectral shapes are either unlikely to occur in this mechanism, or are dependent upon the plasma conditions.

The statistical approach of Schwadron et al. (2010) can yield the -5 spectra and is consistent with the results of Fisk & Gloeckler when the assumed distribution functions for individual acceleration events and the averaging technique are taken to be compatible with the assumptions and averaging in Fisk & Gloeckler.

The statistical approach of Schwadron et al. (2010) yields a -5 spectrum in other cases besides the one where there is an analog to the acceleration mechanism of Fisk & Gloeckler. They find a -5 spectrum for different forms of the distribution function of individual events and for averages over individual events, i.e., spatial averages, as opposed to the temporal averages of Fisk & Gloeckler. As we noted above, the spatial averages in the statistical approach of Schwadron et al. (2010) should be the equivalent of temporal averages. In fact, observations made directly in the acceleration region appear to show that the -5 spectrum is observed on the smallest relevant timescale. This suggests that the -5 spectrum is not a spatial average over many distinct events, but rather a temporal average in a single location. If the appropriate average is temporal, it will be difficult to determine what the spectra for the underlying individual events were, and thus to evaluate whether there is a physical mechanism, applicable to the conditions in the solar wind, which is capable of generating such individual spectra.

There is an intriguing observation that may lend itself to the approach of Schwadron et al. (2010). At the termination shock of the solar wind observed by *Voyager 1*, the downstream suprathermal tails have the common spectral shape, a spectral index of −5. Upstream the spectra are complex, but when averaged yield the −5 spectral shape. Gloeckler & Fisk (2006) demonstrate that if the downstream particles leak upstream, and velocity dispersion modifies the spectra, then an average of the spectra of the leaking particles will be a −5 spectrum. The average upstream from the termination shock is essentially a spatial average over individual events, in which the −5 spectrum results. A proper treatment of this average may need to follow the approach of Schwadron et al. (2010).

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