The Astrophysical Journal, 706:1605–1608, 2009 December 1 doi:10.1088/0004-637X/706/2/1605

PHASE CORRELATION METHOD FOR THE ALIGNMENT OF TOTAL SOLAR ECLIPSE IMAGES

M. Druckmüller
Institute of Mathematics, Faculty of Mechanical Engineering, Brno University of Technology, 616 69 Brno, Czech Republic; druckmuller@fme.vutbr.cz
Received 2009 July 9; accepted 2009 October 19; published 2009 November 16

ABSTRACT

A modified phase correlation method, based on Fourier transform, which enables the alignment of solar coronal images taken during the total solar eclipses, is presented. The method enables the measurement of translation, rotation, and scaling factor between two images. With the application of this technique, pairs of images with different exposure times, different brightness scale, such as linear for CCD and nonlinear for images taken with photographic film, and even images from different emission lines can be aligned with sub-pixel precision.

Key words: eclipses – Sun: corona

1. INTRODUCTION

Total solar eclipses provide a unique opportunity to obtain high-resolution and high-quality images of the solar corona. In particular, the finest details of density structures in the innermost corona, as captured in white-light images, cannot be attained without total solar eclipses even from space-based observations. However, the limiting factor in capturing the finest details which only the eye can see is the very sharp gradient of the coronal brightness.

A sequence of images with different exposure times is then needed to produce an image where the details of coronal structure can be revealed with acceptable signal-to-noise ratio. The key problem, however, is image alignment. This factor significantly influences the resolution of the resulting image. Methods based on finding corresponding points to align images are generally not adequate because of the absence of contrasty features in the images suitable for that purpose. Only stars and the details of the lunar edge can be used as reference points. However, these reference points move during a total solar eclipse. Although the parameters of these motions are known and can be compensated for, no stars are detectable on images with short exposure times, and for images with long exposure times, the lunar edge cannot be used because of pixel saturation.

The modified phase correlation technique, presented here, is based on Fourier transform. It works in frequency domain and does not require any reference point for image alignment. The phase correlation is currently a proven and widely used technique for the alignment of images. However, the alignment of total solar eclipse images has a number of specific problems which necessitates the modification of this method. We show in this paper how the application of the proposed modification leads to unprecedented resolution in images of the corona taken during total solar eclipses.

2. PHASE CORRELATION METHOD

The mathematical principles of the phase correlation alignment method for measuring translation, rotation, and scaling were described by Reddy & Chatterji (1996). Let $a(x, y)$ and $b(x, y)$ be two images (represented by their pixel matrices) which are related by the formula $b(x, y) = a(x - x_0, y - y_0)$. Let us suppose that there are no periodic structures in these images, i.e., $b(x, y) = a(x - x_1, y - y_1)$ implies that $(x_1, y_1) = (x_0, y_0)$. Let us denote by $A(\xi, \eta)$ and $B(\xi, \eta)$ the corresponding Fourier transforms of these images, by $C(\xi, \eta)$ the cross-power spectrum, and by $N(\xi, \eta)$ the normalized cross-power spectrum. These spectra are defined by the formulae

$$C(\xi, \eta) = A(\xi, \eta)B^{\star}(\xi, \eta) \tag{1}$$

$$N(\xi, \eta) = C(\xi, \eta)|A(\xi, \eta)B(\xi, \eta)|^{-1}, \tag{2}$$

where $B^{\star}$ is the complex conjugate of B. Finally, let us define an image

$$n(x, y) = F^{-1}(N(\xi, \eta)),$$

where F^{-1} denotes the inverse Fourier transform. Then $n(x, y) = 0$ for all points (x, y) except (x_0, y_0). It means that the task of finding the shift between two images is transformed to the task of finding the coordinate of the nonzero element in the image matrix $n(x, y)$.

Now, we suppose that $b(x, y)$ is a rotated replica of $a(x, y)$. Let α be the angle of rotation. The rotation can be changed to a translation by transforming images from the Cartesian coordinate system (x, y) to the polar coordinate system (r, φ). This enables the use of the described method and to measure the angle α as a shift along the φ-axis.

The problem is that the center of rotation must be known, which is usually not the case. This problem can be overcome by using the Fourier transform amplitudes $|A(\xi, \eta)|$ and $|B(\xi, \eta)|$ instead of images $a(x, y)$ and $b(x, y)$. The angle of rotation for these Fourier transform amplitudes is identical to that for images $a(x, y)$ and $b(x, y)$. However, the center of rotation is now known and is the point $(0, 0)$. If $b(x, y)$ is not only rotated but also a scaled replica of $a(x, y)$, then we use the logarithm scale for the r-axis. The scaling factor k is transformed to the shift along the r-axis in this case.

For the most general case when images are of different scale, the resulting algorithm then consists of the following steps.

1. Computing the Fourier transform amplitudes of images $a(x, y)$ and $b(x, y)$.
2. Transforming the Fourier transform amplitudes to the polar coordinate system with logarithm scale on the r-axis.
3. Measuring k and α by means of phase correlation.
4. Applying the scaling factor k^{-1} and rotation $-\alpha$ on image $b(x, y)$, resulting in $b_1(x, y)$.
5. Measuring the shift (x_0, y_0) between images $a(x, y)$ and $b_1(x, y)$.
6. Applying the shift $(-x_0, -y_0)$ on image $b_1(x, y)$ to yield $b_2(x, y)$.

In total, eight Fourier transforms (six forward and two inverse) and two transforms from the Cartesian to the log–polar coordinate system are necessary to compute all the geometrical parameters for image alignment.

3. MODIFIED PHASE CORRELATION METHOD

The previous section described a hypothetical case. The situation is more complicated in reality because the images $a(x, y)$ and $b(x, y)$ contain noise, they are of different brightness scale and may contain other dissimilarities. These factors cause the image $n(x, y)$ to contain random noise and for the point (x_0, y_0) to be a local maximum of $n(x, y)$ only. If this maximum is global it is easy to find it, and no filtration of the noise is necessary prior to the search for the maximum. However, this rarely occurs. The filtration then must be done in two steps because the noise has different sources. The first one is the low amplitude of several spatial frequencies which causes the cross-power spectrum to be divided by values close to zero. The solution is to replace the normalized cross-power spectrum (Equation (2)) by

$$N_{p,q}(\xi, \eta) = C(\xi, \eta)[(|A(\xi, \eta)| + p)(|B(\xi, \eta)| + q)]^{-1},$$

where p and q are positive constants. (If images $a(x, y)$ and $b(x, y)$ were taken with the same equipment, we may assume that $p = q$). The second source is the additive noise contained in images $a(x, y)$ and $b(x, y)$. It is possible to filter this type of noise by means of low-pass filter applied in the frequency domain, i.e., by multiplying the cross-power spectrum by a suitable weight function. We use the Gaussian function

$$G_\sigma(\xi, \eta) = \exp(-0.5(\xi^2 + \eta^2)\sigma^{-2})$$

for that purpose. Let us denote by $n_{p,q,\sigma}(x, y)$ the resulting image on which both filtrations were applied, namely,

$$n_{p,q,\sigma}(x, y) = F^{-1}\left(N_{p,q}(\xi, \eta)G_\sigma(\xi, \eta)\right). \tag{3}$$

Another problem is the presence of very low frequencies in the cross-power spectrum. These frequencies may cause misalignment, because they represent mainly information about diffuse light in the optical system, which changes significantly during the total solar eclipse. It might seem easy to remove these low frequencies again in the frequency domain by changing the low-pass filter to bandpass filter. However, many experiments made with both digital and classical film data led to the conclusion that this type of filter applied to the cross-power spectrum gives unreliable results. The problem is caused by the extreme radial gradient of the coronal brightness. Any nonlinearity in brightness scale (chip saturation, non-homogenous diffuse light, film properties, etc.) highly influences spatial frequencies in the radial direction and makes them unreliable for image alignment. The solution is to remove these frequencies and to use frequencies in the tangential direction only. However, it is not possible to perform this type of filtration in the frequency domain. It is done in space domain by a filter H_ϱ defined by the formula

$$H_\varrho(a(x, y)) = a(x, y) - \int_{\omega_1}^{\omega_2} a(r + \cos(\varphi + \omega), r + \sin(\varphi + \omega))\, e^{-\frac{\omega^2}{2\varrho^2}} d\omega, \tag{4}$$

where r and φ denote the polar coordinates with the origin in the solar center of point (x, y). Since it is not easy to find this point,

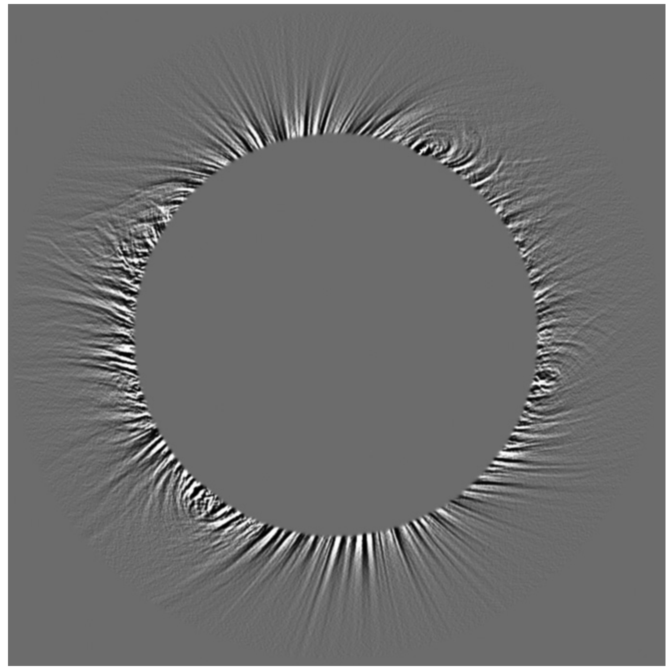

Figure 1. Example of image (Equation (5)) with $\varrho = 8$. The image was taken with a Canon EOS 5D, 1250 mm lens, during the 2008 total solar eclipse from Mongolia.

the lunar center is used as an approximation with no impact on the precision of the alignment. The integral in formula (4) may be understood as an unsharp mask created by means of a one-dimensional Gaussian low-pass filter applied to the circle centered at the center of the Sun. The limits ω_1 and ω_2 are usually set to $\pm 2\varrho$. Because $a(x, y)$ is defined for integer values (x, y) only, interpolation must be used for the computation of the integral.

The last but very important step is to remove all parts of the image which move during the total solar eclipse relative to solar corona, such as the Moon, bright stars, dust particles on the chip, and defective pixels. Leaving these parts in the image may cause misalignment, because they have different alignment parameters from the solar corona. For the same reason, it is necessary to treat the edge of the image. The removal is done by multiplying the image $a(x, y)$ by an unsharp mask $m_a(x, y)$. This mask is created by applying the Gaussian low-pass filter on the image $\overline{m}_a(x, y)$ defined by the formula

$$\overline{m}_a(x, y) = \begin{cases} 1 & \text{if}(x, y) \in C_a \text{ and } r_{x,y} > r_1 \\ & \text{and } r_{x,y} < r_2 \\ 0 & \text{otherwise,} \end{cases}$$

where C_a denotes the set of all pixels which belong to the corona in image $a(x, y)$, $r_{x,y}$ denotes the distance of the point (x, y) from the solar (resp. lunar) center (s_x, s_y), r_1 is a constant greater than the lunar radius, and r_2 is a constant lower than the radius of the incircle with the center (s_x, s_y).

Finally, we replace the input images $a(x, y)$ and $b(x, y)$ with images $a_\varrho(x, y)$ and $b_\varrho(x, y)$ defined by formulae

$$a_\varrho(x, y) = H_\varrho(a(x, y))m_a(x, y) \tag{5}$$

$$b_\varrho(x, y) = H_\varrho(b(x, y))m_b(x, y), \tag{6}$$

and apply formula (3). Let us denote the result as $n_{p,q,\sigma,\varrho}(x, y)$. This is the final image for the maximum search.

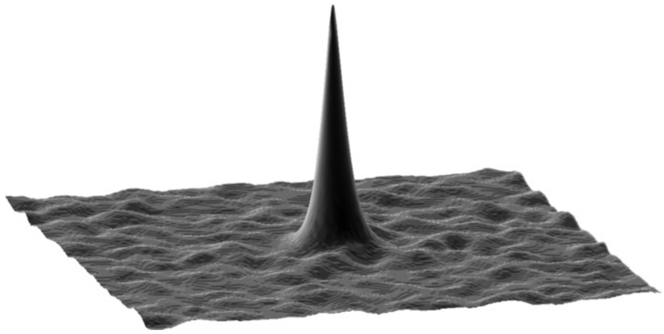

Figure 2. Example of the searched peak in the image $n_{p,q,\sigma,\varrho}(x, y)$ in a three-dimensional representation (15 × 15 pixels, bicubic interpolation used for magnification).

We have four parameters p, q, σ, and ϱ which must be determined experimentally in order to obtain a well-defined global maximum (Figure 2) in the image $n_{p,q,\sigma,\varrho}(x, y)$. Fortunately, the sensitivity of the approach to the choice p, q, and ϱ is not very high, so it is easy to find parameters which are satisfactory for the majority of images. Typical values of p and q are in the interval $\langle 0.01, 0.1\rangle$ percent of the maximal Fourier transform amplitudes, and ϱ is usually in the interval $\langle 8, 16\rangle$. On the other hand, the value of σ must be varied over a wide range which makes it impossible to automate the alignment of the images. Setting the value of σ too high may cause incorrect maximum identification and incorrect alignment. Setting the value of σ too low decreases the alignment precision because of strong suppression of high spatial frequencies. Typical values of σ are in the interval $\langle 0.01n, n\rangle$, where n denotes the width of the image in pixels.

4. SUB-PIXEL EXTENSION OF THE METHOD

The image $n_{p,q,\sigma,\varrho}(x, y)$ is represented by a pixel matrix, therefore the searched maximum coordinates (x_0, y_0) are of integer values. Several possibilities exist for how to extend the precision to sub-pixel level. These methods are based either on interpolation or on statistical moment characteristics. The second approach is more robust and more suitable for the alignment of solar corona images. We use the following formula for sub-pixel coordinate $(\overline{x}_0, \overline{y}_0)$ finding:

$$(\overline{x}_0, \overline{y}_0) = \left(M_{1,0}M_{0,0}^{-1}, M_{0,1}M_{0,0}^{-1}\right), \tag{7}$$

where

$$M_{r,s} = \sum\sum\nolimits_{k^2+l^2<\varepsilon^2} k^r l^s \; n_{p,q,\sigma}(x_0 + k, y_0 + l). \tag{8}$$

We may consider the point $(\overline{x}_0, \overline{y}_0)$ as the center of gravity of the peak and its neighborhood with radius ε. Typical values of ε range from 3 to 5.

5. TESTING OF THE PRECISION OF THE METHOD

An experimental program, written in Borland–Delphi, was developed to implement the described alignment method. The program was first tested on simulated data. The total solar eclipse image shown in Figure 1 was resized ($k \in \langle 0.9, 1.1\rangle$), rotated ($\alpha \in \langle 0, 2\pi\rangle$), and shifted ($x_0$, $y_0 \in \langle -100, 100\rangle$ pixels) by means of bilinear interpolation. The known parameters of these transforms were measured. The Fourier transforms were computed on a 4096 × 4096 element matrix. The values of k, α, x_0, and y_0 were generated randomly and 50 trials were made.

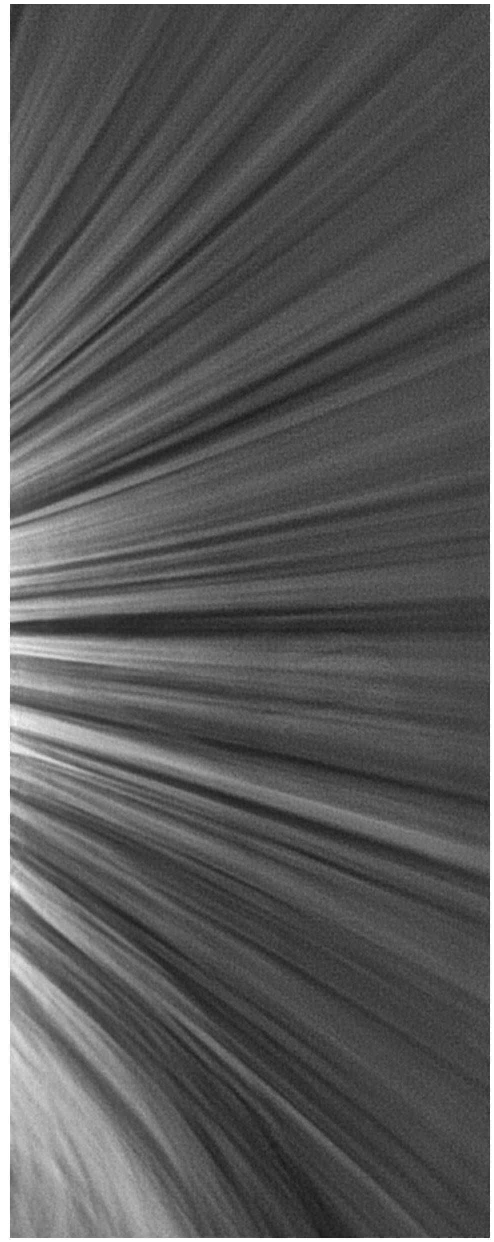

Figure 3. Details of the 12.5 MPix image composed from 25 images aligned by means of phase correlation. Exposure times range from 1/4000 s to 8 s. The resolution is 0.89 arcsec pixel^{-1}. The image shows the southern polar region of the corona on 2008 August 1, at 11:04 UT. The equipment was the same as that used to produce Figure 1.

For all these trials, the alignment error was less than 0.01 pixel near the lunar edge and less than 0.1 pixel near the corners of the image. The alignment error was less than 0.002 pixel for $k = 1$ and $\alpha = 0$. Therefore, the alignment error of the method itself may be considered negligible.

The verification of this method with real eclipse data is problematic because no other method exists with comparable precision. So, the measurement of the lunar centroid was used as the second independent method. Images with very short exposures only were used because the lunar edge is saturated and the motion blurred on longer exposure images. That is why the amount of suitable images was limited. The lunar centroid $C_M(0)$ in the master (fixed) image a_M was used as the reference point and its position was moved to position $C_M(t_i)$ as expected in the aligned image a_i according to the time t_i at which the image

Table 1
Difference d_i Between the Lunar Centroid Alignment Method and the Phase Correlation Method

i	Exposure (s)	t_i (s)	d_i (pixels)
1	1/250	−6.0	0.13
2	1/250	−3.0	0.29
3	1/125	0.0	0.00 master
4	1/125	3.0	0.38
5	1/60	6.0	0.05
6	1/60	9.0	0.11
7	1/60	12.0	0.20
8	1/30	16.0	0.40
9	1/30	18.0	0.14
10	1/30	21.0	0.35

a_i was taken. Let $C_i(t_i)$ denote the position of the lunar centroid in the aligned image a_i. The distance d_i between $C_M(t_i)$ and $C_i(t_i)$ was used for testing the precision. The results are given in Table 1.

The images used for testing the method were taken with the same equipment as the image shown in Figure 1. The following parameters for the alignment method were used: $p = q = 0.01\%$ of the maximal value of the Fourier transform amplitudes, $\sigma = 0.1n$, $\varrho = 8$, and $\varepsilon = 3$. Since a parallactic mount with precise polar alignment was used in the acquisition of the eclipse images, the rotation α was considered to be 0 for this experiment. The total computing time (Pentuim D, 3.2 GHz) for the alignment of one image was 205 s.

6. SUMMARY

The modified phase correlation method described in this paper enabled the alignment of a series of total solar eclipse images without degradation of resolution by alignment errors (see Figure 3). The method gives reliable results even for images taken with different equipment. The combined total solar eclipse images aligned by means of this method were published by Pasachoff et al. (2007, 2008). The only disadvantage of the method, at present, is that it is not possible to automate it. Finding the algorithm for automatic estimation of parameters p, q, σ, and ϱ is the subject of future research.

This work was supported by grant 205/09/1469 of the Czech Science Foundation.

REFERENCES

Pasachoff, J. M., Rušin, V., Druckmüller, M., & Saniga, M. 2007, ApJ, 665, 824
Pasachoff, J. M., et al. 2008, ApJ, 682, 638
Reddy, S. B., & Chatterji, B. N. 1996, IEEE Trans. Image Process., 5, 1266