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PROPERTIES OF GRAVITOTURBULENT ACCRETION DISKS

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ABSTRACT

We explore the properties of cold gravitoturbulent accretion disks—non-fragmenting disks hovering on the verge of gravitational instability (GI)—using a realistic prescription for the effective viscosity caused by gravitational torques. This prescription is based on a direct relationship between the angular momentum transport in a thin accretion disk and the disk cooling in a steady state. Assuming that opacity is dominated by dust we are able to self-consistently derive disk properties for a given $\dot{M}$ assuming marginal gravitational stability. We also allow external irradiation of the disk and account for a non-zero background viscosity, which can be due to the magneto-rotational instability. Spatial transitions between different co-existing disk states (e.g., between irradiated and self-luminous or between gravitoturbulent and viscous) are described and the location of the boundary at which the disk must fragment is determined in a variety of situations. We demonstrate in particular that at low enough $\dot{M}$ external irradiation stabilizes the gravitoturbulent disk against fragmentation to very large distances thus providing means of steady mass transport to the central object. Implications of our results for the possibility of planet formation by GI in protoplanetary disks and star formation in the Galactic center and for the problem of feeding supermassive black holes in galactic nuclei are discussed.

Key words: accretion, accretion disks – instabilities – planetary systems: protoplanetary disks – quasars: general

1. INTRODUCTION

Gravitational instability (GI) in astrophysical disks has been a subject of investigation for more than 50 years since the seminal work by Safronov (1960) and Toomre (1964). Originally it was studied predominantly in the context of driving the spiral structure in galaxies. Later it has been suggested that GI may play important role in planet formation (Cameron 1978; Boss 1998), and its significance for the properties of compact nuclear disks around supermassive black holes in the centers of galaxies has also been recognized (Paczynski 1978a; Goodman 2003).

It is generally accepted that the GI sets in when the so-called Toomre Q satisfies the following condition:

$$Q \equiv \frac{\Omega c_s}{\pi G \Sigma} < Q_0, \tag{1}$$

where Σ, Ω, and c_s are the local surface density, angular frequency, and the sound speed in the disk which we consider to be made of gas and having a Keplerian rotational profile. Q_0 is a constant of order unity, its precise value determining the instability threshold ranges from 0.7 to 1.7 according to different authors (Kim et al. 2002; Boss 2002). Nonlinear development of the GI sensitively depends on the thermodynamical properties of the gas as has been first shown by Gammie (2001): if gas can cool on a timescale shorter than the local dynamical timescale Ω^{-1} then the disk fragments into bound, self-gravitating objects which may grow further by accreting the surrounding gas and colliding with each other. But if the cooling timescale $t_{\rm cool}$ is longer than Ω^{-1} then the disk settles into a state of the so-called *gravitoturbulence* in which surface density can experience significant fluctuations but the disk is stable against fragmentation in the long run and maintains itself on the brink of instability with $Q \approx Q_0$. Stability against fragmentation arises because the restoring action of the thermal pressure resisting the self-gravity of overdensities is not sufficiently reduced by cooling when $t_{\rm cool} \gtrsim \Omega^{-1}$.

Torques produced by the nonaxisymmetric density perturbations in gravitoturbulent disks give rise to angular momentum transport. Considering the disk to be in a steady state on time shorter than its viscous evolution timescale one can directly relate "effective viscosity" $\alpha_{\rm GI}$ produced by the gravitational torques to the cooling time of the disk. Gammie (2001) has demonstrated that the dimensionless α-parameter characterizing the disk viscosity (Shakura & Sunyaev 1973) is

$$\alpha \sim (\Omega t_{\rm cool})^{-1}. \tag{2}$$

It is instructive to show where this relation comes from. First, in a steady state the rate of energy dissipation per unit surface area of the disk $\sim\Omega^2 \dot{M}$ has to equal the energy flux F emitted from the disk surface. Second, the accretion rate is $\dot{M} = 3\pi\nu\Sigma$, where $\nu \equiv \alpha c_s^2/\Omega$ (Pringle 1981). Combining these relations one immediately obtains Equation (2) with

$$t_{\rm cool} \approx \Sigma c_s^2/F. \tag{3}$$

Relation (2) makes it possible to interpret disk fragmentation occurring at $t_{\rm cool} \sim \Omega^{-1}$ as the inability of the disk to sustain gravitational stress at $\alpha \gtrsim 1$ (Rice et al. 2005).

In this paper, we investigate the structure and evolution of gravitoturbulent disks in which angular momentum is transferred predominantly by the gravitational torques. This problem has been previously investigated by Lin & Pringle (1987) but with a rather naive prescription for the effective viscosity. Also, some efforts have been devoted to understanding the structure of the gravitoturbulent disks which are unstable to fragmentation on large scale, i.e. disks having $Q \approx Q_0$ and $\alpha_{\rm GI} \sim 1$ everywhere (Rafikov 2005, 2007; Matzner & Levin 2005). In this work, viscous evolution of the disk is explored according to the prescription (2) without fixing the value of $\alpha_{\rm GI}$—instead it is calculated self-consistently based on the physical properties of the gas. We concentrate our attention on rather cool disks in which opacity is due to dust grains thus focusing on the GI in the outer parts of protostellar disks and disks around supermassive black holes.

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2. GENERAL CONSIDERATIONS

We consider a gravitoturbulent disk in which the dissipation of transient density waves excited by GI is capable of maintaining $Q = Q_0$, and the cooling time $t_{\rm cool}$ is longer than Ω^{-1}. The mass of the disk is assumed to be smaller than the mass of the central star to avoid complications arising from the non-local effects and the back reaction of the gravitational instability on the central star. Because of this our results are only strictly applicable to the case of low mass disks.

The cooling time of the disk is

$$t_{\rm cool} \approx \frac{\Sigma c_s^2}{\sigma T^4} f(\tau), \tag{4}$$

where Σ is the surface density of the disk, $c_s \equiv (k_B T/\mu)$ is the isothermal sound speed determined by the midplane temperature T, and $f(\tau)$ is a function of the optical depth $\tau = \int \kappa\rho dz$ (κ and ρ are the gas opacity and density, z is the vertical coordinate) which links the emitted flux F to T: $F = \sigma T^4/f(\tau)$. A specific form of $f(\tau)$ depends on the way in which energy is transported from the midplane of an optically thick disk to its photosphere where it is radiated to space. Rafikov (2007) has calculated $f(\tau)$ in the case of efficiently convecting disks. However, in this work we assume (as was previously done in Rafikov 2005) for simplicity that energy is carried from the disk midplane to its surface solely by radiation in which case $f(\tau)$ can be reasonably well approximated by

$$f(\tau) \approx \tau + \frac{1}{\tau}. \tag{5}$$

This expression smoothly interpolates between the cooling rates applicable in the optically thick ($\tau \gg 1$) and optically thin ($\tau \ll 1$) regimes.

We assume a temperature-dependent opacity in the form

$$\kappa = \kappa_0 T^\beta, \tag{6}$$

which is appropriate at low temperatures when κ is dominated by dust grains. At very low temperatures, $T < 150$ K, it is generally found (Bell & Lin 1994; Semenov et al. 2003) that opacity is due to the icy grains and is characterized by

$$\beta = 2 \text{ and } \kappa_0 \approx 5 \times 10^{-4} \text{ cm}^2 \text{ g}^{-1} \text{ K}^{-2} \tag{7}$$

within a factor of 2 or so. At higher temperatures ices evaporate and opacity behavior can be crudely described as $\kappa \approx 0.1 T^{1/2}$ cm^2 g^{-1} (Bell & Lin 1994). For simplicity in this work we do not distinguish between the Rosseland mean and the Planck mean opacities (appropriate for $\tau \gg 1$ and $\tau \ll 1$ correspondingly) as they have similar values at low T.

Now, using definition (1) and the condition $Q = Q_0$ we find that

$$c_s = \frac{\pi G Q_0 \Sigma}{\Omega}, \tag{8}$$

$$T = \frac{\mu}{k_B}\left(\frac{\pi G Q_0 \Sigma}{\Omega}\right)^2 \tag{9}$$

in a gravitoturbulent disk. In the optically thick regime total optical depth is dominated by the midplane layers of the disk in which most of the mass is concentrated, so that up to factors of order unity $\tau \approx \kappa(T)\Sigma$. Clearly, this approximation also works in the optically thin case. Thus, using Equation (9) one rather generally finds that

$$\tau \approx \kappa_0 \Sigma^{2\beta+1}\left(\frac{\mu}{k_B}\right)^\beta \left(\frac{\pi G Q_0}{\Omega}\right)^{2\beta}. \tag{10}$$

We can also calculate $\alpha_{\rm GI}$ characterizing angular momentum transport caused by the non-axisymmetric surface density perturbations. Using Equations (2), (4), and (9) one finds that

$$\alpha_{\rm GI} = \zeta \frac{\sigma(\pi G Q_0)^6}{f(\tau)}\left(\frac{\mu}{k_B}\right)^4 \frac{\Sigma^5}{\Omega^7}. \tag{11}$$

The kinematic viscosity $\nu \equiv \alpha_{\rm GI} c_s^2/\Omega$ is then given by the following expression:

$$\nu_{\rm GI} = \zeta \frac{\sigma(\pi G Q_0)^8}{f(\tau)}\left(\frac{\mu}{k_B}\right)^4 \frac{\Sigma^7}{\Omega^{10}}. \tag{12}$$

Parameter $\zeta \sim 1$ appearing in these equations absorbs our ignorance of the exact values of constant factors in Equations (2) and (4).

Equations (10)–(12) provide us with the desired viscosity prescription needed for determining the physical structure and evolution of the gravitoturbulent disk. The only two essential ingredients that went into deriving this viscosity recipe are (1) requirement that disk maintains itself in a state of marginal stability with respect to GI and (2) prescription (2) for $\alpha_{\rm GI}$ which arises from reasonable assumption that disk is in thermal equilibrium on timescales shorter than the viscous timescale.

3. CONSTANT $\dot{M}$ DISKS

In this section, we consider the structure of the gravitoturbulent disk with a specified mass accretion rate $\dot{M}$. In a steady state $\dot{M} = 3\pi\nu\Sigma$, which with the aid of Equation (12) can be manipulated into the following general relation:

$$\dot{M} = 3\pi\zeta \frac{\sigma(\pi G Q_0)^8}{f(\tau)}\left(\frac{\mu}{k_B}\right)^4 \frac{\Sigma^8}{\Omega^{10}}. \tag{13}$$

Function $f(\tau)$ entering this expression depends on Σ and Ω through Equation (10). This allows us to uniquely express Σ as a function of Ω for a given $\dot{M}$.

We should note here that although in the following we will mainly discuss disks with constant $\dot{M}$ our results are also directly applicable to disks in which $\dot{M}$ varies with distance. Indeed, as long as one knows $\dot{M}$ at a particular distance (or Ω) in the disk Equation (13) uniquely determines the value of Σ in this location.[2] Thus, all our subsequent numerical estimates would apply also to the case of non-constant $\dot{M}$ disks as long as $\dot{M}$ is specified at a location of interest.

We separately consider the cases of optically thick and optically thin gravitoturbulent disks. Before we do this we note that there are two important transitions characterizing such a disk. One is the

$$\tau = 1 \tag{14}$$

[2] Strictly speaking the relation $\dot{M} \sim \nu\Sigma$ used in deriving (13) is valid only in disks with smoothly varying Σ (e.g., in disks with power-law dependence of $\dot{M}$ on r); constant factor in this relation is in general different from 3π.

transition between the optically thick and optically thin regions. Another is the point at which $t_{\rm cool}$ becomes comparable to Ω^{-1} and disk fragments. This transition is defined by condition

$$\alpha_{\rm GI} = \chi \sim 1, \tag{15}$$

where χ is the constant of order unity, its precise value has been determined by Gammie (2001) and Rice et al. (2003, 2005) in a variety of circumstances.

According to Equations (10) and (11) each of these two relations sets a unique constraint between Σ and Ω. However, if we demand that both of them are fulfilled *simultaneously* (i.e., disk fragments exactly at the $\tau = 1$ transition) then these relations hold only for specific values of Σ and Ω, which we denote Σ_f and Ω_f. Equations (10), (11), (14), and (15) then yield the following values of these parameters:

$$\Sigma_f \approx \left[\left(\frac{\zeta\sigma}{\pi G Q_0 \chi}\right)^{2\beta}\left(\frac{\mu}{k_B}\right)^{\beta}\kappa_0^{-7}\right]^{(4\beta+7)^{-1}}, \tag{16}$$

$$\Omega_f \approx \left[\left(\frac{\zeta\sigma}{\chi}\right)^{2\beta+1}\frac{(\pi G Q_0)^{2\beta+6}}{\kappa_0^5}\left(\frac{\mu}{k_B}\right)^{3\beta+4}\right]^{(4\beta+7)^{-1}} \tag{17}$$

(we set $f(\tau) \sim 1$ at $\tau = 1$). Through Equations (8), (9), (13) Σ_f and Ω_f (with $f(\tau) \sim 1$) also determine fiducial values of the mass accretion rate $\dot{M}_f$, sound speed $c_{s,f}$, and midplane temperature T_f:

$$\dot{M}_f \approx 3\pi\chi \times \left[\left(\frac{\zeta\sigma\kappa_0^2}{\chi}\right)^3 (\pi G Q_0)^{4(\beta+1)}\left(\frac{\mu}{k_B}\right)^{6(\beta+2)}\right]^{-(4\beta+7)^{-1}}, \tag{18}$$

$$c_{s,f} \approx \left[\frac{\pi G Q_0 \chi}{\zeta\sigma\kappa_0^2}\left(\frac{\mu}{k_B}\right)^{-2(\beta+2)}\right]^{(4\beta+7)^{-1}}, \tag{19}$$

$$T_f \approx \left[\left(\frac{\pi G Q_0 \chi}{\zeta\sigma\kappa_0^2}\right)^2 \frac{k_B}{\mu}\right]^{(4\beta+7)^{-1}}. \tag{20}$$

Note that all these fiducial quantities depend only on physical constants and opacity parametrization. For κ given by Equation (7) we find the following values of these parameters (assuming $Q_0 \approx 1$, $\chi, \zeta \sim 1$):

$$\Sigma_f \approx 15 \text{ g cm}^{-2},\ c_{s,f} \approx 0.22 \text{ km s}^{-1},\ T_f \approx 11.6 \text{ K},$$
$$\Omega_f \approx 1.4 \times 10^{-10} \text{ s}^{-1},\ \dot{M}_f \approx 7 \times 10^{-6}\ M_\odot \text{ yr}^{-1}. \tag{21}$$

The numerical value of T_f conveniently falls into the regime of κ dominated by icy dust grains.

For a given mass of a central object $M_\star$ angular frequency Ω_f determines a fiducial distance r_f according to the formula

$$r_f = \left(\frac{G M_\star}{\Omega_f^2}\right)^{1/3} \approx 130 \text{ AU}\left(\frac{M_\star}{M_\odot}\right)^{1/3}. \tag{22}$$

In the case of a disk around $M_\star = 10^6\ M_\odot$ black hole one finds $r_f \approx 0.06$ pc.

We now consider the disk structure in the constant $\dot{M}$ case separately for the optically thick and the optically thin regimes, as well as the effects of external irradiation and the background viscosity in the disk.

3.1. *Optically Thick Case*

In the optically thick case $f(\tau) \approx \tau$. Plugging this into Equation (13) and using expression (10) we find the following scalings:

$$\Sigma = \Sigma_f(\dot{m}\omega^{10-2\beta})^{(7-2\beta)^{-1}} = \Sigma_f \dot{m}^{1/3}\omega^2, \tag{23}$$

$$T = T_f(\dot{m}^2\omega^6)^{(7-2\beta)^{-1}} = T_f \dot{m}^{2/3}\omega^2, \tag{24}$$

where second equalities are for $\beta = 2$ and we have defined the following dimensionless quantities:

$$\dot{m} \equiv \frac{\dot{M}}{\dot{M}_f}, \quad \omega \equiv \frac{\Omega}{\Omega_f}.$$

Using Equations (10) and (11) we also find that in the optically thick regime

$$\alpha_{\rm GI} = \chi(\dot{m}^{4-2\beta}\omega^{-9})^{(7-2\beta)^{-1}} = \chi\omega^{-3}, \tag{25}$$

$$\tau = (\dot{m}^{2\beta+1}\omega^{10+4\beta})^{(7-2\beta)^{-1}} = \dot{m}^{5/3}\omega^6. \tag{26}$$

According to Equation (26) our assumption of $\tau > 1$ is self-consistent only if

$$\omega > \omega_1 = \dot{m}^{-(2\beta+1)/(10+4\beta)} \tag{27}$$

(we assume $\beta < 7/2$ as expected for dust opacity). Thus, the gravitoturbulent disk is going to be optically thick for all $\Omega > \Omega_f$ (or $\omega > 1$) if $\dot{M} > \dot{M}_f$ (or $\dot{m} > 1$).

Note that Equations (23)–(26) predict very rapid variation of disk properties with radius in the optically thick regime. Indeed, for $\beta = 2$ one finds $\Sigma, T \propto r^{-3}$, while $\tau \propto r^{-9}$. It is clear that even a moderate increase in r would lead to the disk becoming optically thin. Also, $\alpha_{\rm GI} \propto r^{9/2}$ (analogous scaling has been found by Goodman (2003) under different assumptions) and given this rapid variation it is obvious that an optically thick gravitoturbulent region can exist only within a limited range of radii (see Sections 3.4 and 3.5).

3.2. *Optically Thin Case*

In the optically thin regime, we need to use $f(\tau) \approx \tau^{-1}$ in Equation (13) which combined with Equation (10) gives the following scalings:

$$\Sigma = \Sigma_f(\dot{m}\omega^{10+2\beta})^{(9+2\beta)^{-1}} = \Sigma_f \dot{m}^{1/13}\omega^{14/13}, \tag{28}$$

$$T = T_f(\dot{m}\omega)^{2/(9+2\beta)} = T_f \dot{m}^{2/13}\omega^{2/13}, \tag{29}$$

$$\alpha_{\rm GI} = \chi(\dot{m}^{2\beta+6}\omega^{-3})^{(9+2\beta)^{-1}} = \chi\dot{m}^{10/13}\omega^{-3/13}, \tag{30}$$

$$\tau = (\dot{m}^{2\beta+1}\omega^{10+4\beta})^{(9+2\beta)^{-1}} = \dot{m}^{5/13}\omega^{18/13}. \tag{31}$$

Assumption of an optically thin gravitoturbulent disk is valid provided that a condition opposite to (27) is satisfied. In particular, $\tau < 1$ for all $\Omega < \Omega_f$ (or $\omega < 1$) only if $\dot{M} < \dot{M}_f$ (or $\dot{m} < 1$).

Properties of optically thin gravitoturbulent disks exhibit more moderate variation with r than in the optically thick case. Indeed, when $\tau < 1$ and $\beta = 2$ the disk temperature and $\alpha_{\rm GI}$ vary with distance quite slowly, $T \propto r^{-3/13}$ and $\alpha_{\rm GI} \propto r^{9/26}$.

As a result, the optically thin gravitoturbulent regime can be realized in a rather extended region of the disk.

Disks in which $\dot{M}$ varies with distance can also be described by expressions (23)–(26) and (28)–(31): in this situation $\dot{m} = \dot{m}(\omega)$ and the dependence of various disk properties on ω can be directly obtained by plugging $\dot{m}(\omega)$ into these equations.

3.3. External Irradiation

According to Equations (24) and (29) the temperature of a gravitoturbulent disk decreases with r. At some point external irradiation becomes more important for the thermal balance than the internal gravitoturbulent dissipation. Here we want to investigate a transition from a self-luminous disk (heated only by internal dissipation) to an irradiated regime. We assume that the disk is illuminated by external radiation at spatially constant temperature T_0. In many situations, the irradiation temperature is an explicit function of radius (e.g., if irradiation is due to illumination by the central star), and our constant T_0 treatment may be trivially extended to this more general situation.

Given that most of the disk material is concentrated in its midplane region the gravitational stability of the disk is going to be determined by the temperature T near the midplane and the condition for marginal stability is still given by $Q \approx Q_0$ with Q defined in Equation (1). Below we consider separately the optically thick and optically thin cases.

3.3.1. Optically Thick Case

Irradiated optically thick disk appears as an extension of a self-luminous optically thick disk. A specific location at which such transition occurs can be found by equating T given by Equation (24) to T_0. The angular frequency corresponding to this transition is

$$\omega_T = \dot{m}^{-1/3}\left(\frac{T_0}{T_f}\right)^{(7-2\beta)/6}. \tag{32}$$

By construction τ at this location must be greater than unity, which with the aid of Equation (26) yields the following constraint on the optically thick transition to an irradiated disk:

$$\frac{T_0}{T_f} > \dot{m}^{(5+2\beta)^{-1}}. \tag{33}$$

If $\tau \gtrsim 1$ then the midplane temperature of the disk is given by the solution of the vertical radiative transfer in diffusion approximation as

$$T^4 \approx T_{\rm ph}^4 + \eta\tau\frac{\dot{M}\Omega^2}{\sigma}, \tag{34}$$

where η is a factor of order unity which absorbs our ignorance of how the dissipation rate is distributed across the vertical thickness of the disk. Also, from energy conservation the temperature at the disk surface (photospheric temperature $T_{\rm ph}$) is

$$T_{\rm ph}^4 = T_0^4 + \frac{3\dot{M}\Omega^2}{8\pi\sigma}. \tag{35}$$

These two relations allow us to distinguish three levels of the importance of irradiation.

First, when

$$\sigma T_0^4 \lesssim \dot{M}\Omega^2 \tag{36}$$

irradiation is so weak that it does not play any significant role even in the photosphere of the disk. This corresponds to the case of a self-luminous disk which was covered in Section 3.

Second, when

$$\dot{M}\Omega^2 \lesssim \sigma T_0^4 \lesssim \tau\dot{M}\Omega^2 \tag{37}$$

external irradiation keeps disk surface temperature at the level of T_0 and creates a roughly isothermal gas layer underneath the surface. Nevertheless, the midplane temperature is still set predominantly by the internal dissipation and the outgoing flux $\sim\dot{M}\Omega^2$ is still given by $F = \sigma T^4/\tau$, according to Equations (34) and (37). For that reason this irradiation regime also corresponds to the case of self-luminous disks explored in Section 3 with some minor corrections having to do with the fact that the surface disk layers are hotter than they would have been in the absence of irradiation.

Finally, when

$$\tau\dot{M}\Omega^2 \lesssim \sigma T_0^4 \tag{38}$$

the irradiation is so strong that even the midplane temperature reaches T_0 and the disk becomes vertically isothermal. This regime is different from the case of a self-luminous disk considered before since the disk can no longer regulate its thermal state—its midplane temperature is fixed at $T \approx T_0$. Assuming that the disk is gravitoturbulent under strong irradiation we find from Equation (8) that its surface density must scale linearly with Ω:

$$\Sigma = \frac{c_{s,0}\Omega}{\pi G Q_0} \approx \Sigma_f\left(\frac{T_0}{T_f}\right)^{1/2}\omega, \tag{39}$$

where $c_{s,0}$ is the sound speed corresponding to temperature T_0. According to this result Σ is independent of $\dot{M}$ in irradiated gravitoturbulent regions.

Under strong irradiation one can no longer use the expression (4) to derive $\alpha_{\rm GI}$. However, we know Σ from (39), so from the steady-state condition $\dot{M} = 3\pi\nu\Sigma$ and definition $\nu = \alpha c_0^2/\Omega$ one finds that

$$\alpha_{\rm GI} = \frac{Q_0}{3}\frac{G\dot{M}}{c_0^3} \approx \dot{m}\left(\frac{T_0}{T_f}\right)^{-3/2}. \tag{40}$$

This result implies that $\alpha_{\rm GI}$ is *constant* in the strongly irradiated part of the disk. Note that an analogous result has been previously obtained in Goodman (2003).

We can also calculate the run of the optical depth

$$\tau = \frac{c_0\kappa_0 T_0^\beta}{\pi G Q_0}\Omega \approx \left(\frac{T_0}{T_f}\right)^{\beta+1/2}\omega, \tag{41}$$

which shows that τ decreases as $r^{-3/2}$. This result combined with Equation (34) also demonstrates that as r increases the relative contribution of the internal dissipation to the midplane temperature rapidly goes down since it is proportional to $\tau\Omega^2 \propto \Omega^3$.

3.3.2. Optically Thin Case

An optically thin irradiated disk can emerge as an extension of an optically thin self-luminous disk, which happens when T given by Equation (29) becomes equal to T_0, or at

$$\omega_T = \dot{m}^{-1}\left(\frac{T_0}{T_f}\right)^{(9+2\beta)/2}. \tag{42}$$

Such an optically thin transition is possible when an inequality opposite to (33) is satisfied.

An irradiated optically thin disk can also appear as a continuation of the irradiated optically thick disk considered in Section 3.3.1. According to Equation (41) this $\tau = 1$ transition occurs at

$$\omega_T = \left(\frac{T_0}{T_f}\right)^{-(\beta+1/2)}. \tag{43}$$

An optically thin disk is roughly isothermal vertically and its thermal balance requires

$$T^4 \approx T_0^4 + \eta \frac{\dot{M}\Omega^2}{\tau\sigma}, \tag{44}$$

which is different from Equation (34) by a factor τ^{-2} in the second term on the right-hand side accounting for the inefficiency of radiative cooling and absorption in the optically thin disk. There are two obvious regimes to consider. First, when

$$\sigma T_0^4 \lesssim \dot{M}\Omega^2/\tau, \tag{45}$$

irradiation does not affect disk properties and we go back to the case studied in Section 3.2. Second, when the condition opposite to (45) is satisfied irradiation sets the disk temperature. In this case, all results (except for Equation (44), different from (34)) obtained in the optically thick irradiated case—Equations (39)–(41)—remain valid since in deriving them we did not use any assumptions about the value of τ.

The most important result regarding externally irradiated constant $\dot{M}$ disks is that they can remain gravitoturbulent independent of their optical depth and that their effective viscosity $\alpha_{\rm GI}$ remains constant. If the background viscosity does not dominate angular momentum transport at least in some self-luminous parts of the gravitoturbulent disk it will not dominate the transport in the irradiated part either since $\alpha_{\rm GI}$ is constant there. Thus, torque needed for transporting mass through the disk must be due to the GI.

3.4. Fragmentation Limit

When $t_{\rm cool}$ becomes comparable to Ω^{-1} the disk can no longer sustain the gravitoturbulence and has to fragment into bound objects (Gammie 2001). As mentioned before fragmentation condition $\Omega t_{\rm cool} \lesssim 1$ can be recast in terms of the $\alpha_{\rm GI}$ threshold according to Equation (15). This formulation now allows us to directly apply our results for $\alpha_{\rm GI}$ derived in previous sections.

In particular, a self-luminous optically thick gravitoturbulent disk starts fragmenting at

$$\omega < \omega_{\rm frag} = \dot{m}^{(4-2\beta)/9}, \tag{46}$$

which follows from demanding $\alpha_{\rm GI}$ given by Equation (25) to be larger than χ—the critical value of α needed for fragmentation. The radius $r_{\rm frag}$ at which fragmentation first occurs is given by $r_{\rm frag} = r_f \dot{m}^{-4(2-\beta)/27}$ and is rather insensitive to either β or $\dot{m}$, so that fragmentation always occurs not too far from r_f. It is rather interesting that for $\beta = 2$, corresponding to the low temperature dust opacity the location of the fragmentation edge in the optically thick limit is completely independent of $\dot{m}$: fragmentation occurs exactly at $\Omega = \Omega_f$. This fact has been first noticed by Matzner & Levin (2005). Clearly, the constant $\dot{M}$ self-luminous gravitoturbulent disk cannot be fed by a source located outside $r_{\rm frag}$.

According to Equation (27) $\omega_{\rm frag}$ corresponds to the optically thick part of the disk only if $\dot{m} > 1$. Thus, whenever $\dot{m} > 1$ the gravitoturbulent disk stays optically thick all the way to the fragmentation edge located inside r_f. Then external disk feeding must necessarily occur interior to r_f.

In the optically thin case one finds from Equation (30) that

$$\omega_{\rm frag} = \dot{m}^{(2\beta+6)/3}. \tag{47}$$

The fragmentation boundary lies in the optically thin part of the disk only if $\dot{m} < 1$, in which case it is located outside the fiducial radius r_f. The fragmentation radius $r_{\rm frag} = r_f \dot{m}^{-(2\beta+6)/9}$ is a rather sensitive function of $\dot{m}$: for $\beta = 2$ and $\dot{M} = 10^{-7}$ $M_\odot$ yr^{-1} $\approx 0.014\dot{M}_f$ (rather typical value of $\dot{M}$ in protoplanetary disks) one finds $r_{\rm frag} \approx 10^2 r_f$. Thus, in the optically thin regime fragmentation can be pushed out to large distances by reducing $\dot{m}$ (for the just used values of β and $\dot{M}$ and $M_\star = M_\odot$ one finds $r_{\rm frag} \approx 10^4$ AU), but it still cannot be avoided if the disk is self-luminous.

Everything we said before regarding fragmentation applied to self-luminous disks. If the disk is stable against fragmentation all the way to the point where its temperature is determined by external irradiation then in the irradiated region $\alpha_{\rm GI}$ is constant and given by Equation (40). Since $\alpha_{\rm GI}$ must attain this value somewhere near the transition to irradiated regime and the disk is assumed to be non-fragmenting there (i.e., $\alpha_{\rm GI} \lesssim \chi$ at $\omega \sim \omega_T$) we may conclude from Equation (40) that the disk is going to remain in a gravitoturbulent state stable against fragmentation as long as it is externally irradiated and

$$\dot{m} \lesssim \chi \left(\frac{T_0}{T_f}\right)^{3/2}. \tag{48}$$

This is a rather interesting conclusion since it implies that a sufficiently low $\dot{M}$ disk can in principle be stably fed by a source of mass located at very large distance. In particular, for $T_0 \approx 10^2$ K, when the low-temperature opacity (Equation (7)) still applies, one finds using estimates (21) that $\dot{M} \lesssim 10^{-4}$ $M_\odot$ yr^{-1} satisfies condition (48) in which we set $\chi \sim 1$ for simplicity. Thus, disks obeying the condition (48) can transfer mass at a constant rate from very large distances despite being gravitationally unstable, unlike the high $\dot{M}$ disks around quasars. Note that the criterion (48) is independent of either the opacity behavior or the optical depth of the disk. A qualitatively similar conclusion about the stabilizing role of irradiation has been reached in Matzner & Levin (2005) and Cai et al. (2008).

Real irradiated gravitoturbulent disks cannot extend to infinity for one of the following reasons. First, surface density scaling given by Equation (39) implies that the disk mass diverges at large radii as $r^{1/2}$ while all our results assume that the disk mass is small compared to the mass of the central star. One can easily show that this constrains the applicability of our results to radii

$$r < r_f \frac{T_f}{T_0}\left(\frac{M_\star}{M_f}\right)^{2/3}, \tag{49}$$

where

$$M_f = \left[\kappa_0 \left(\frac{\zeta\sigma}{\chi}\right)^{2\beta+4} (\pi G Q_0)^{6\beta+10} \left(\frac{\mu}{k_B}\right)^{9\beta+16}\right]^{-(4\beta+7)^{-1}}$$
$$\sim \frac{\dot{M}_f}{\Omega_f} \sim \Sigma_f \left(\frac{c_f}{\Omega_f}\right)^2 \sim 10^{-3}\ M_\odot \tag{50}$$

is the fiducial mass scale relevant for our problem. For a stellar mass central object ($M_\star \sim M_\odot$) the disk mass becomes comparable to $M_\star$ only at $r \sim 10^2 r_f \sim 10^4$ AU. For more massive objects (like a supermassive black hole) the boundary at which the disk mass equals $M_\star$ is pushed even further in units of r_f.

Second, in the irradiated disk radiation pressure at large radii becomes more important than the gas pressure: the former is constant with r while the latter drops as Ω^2. This lowers the effective adiabatic index of the disk fluid making it susceptible to fragmentation even when the cooling time is longer than Ω^{-1}. The radiation pressure is negligible compared to gas pressure and fragmentation does not happen as long as

$$r < r_f \frac{T_f}{T_0}\left(\frac{c}{c_f}\right)^{1/3} \approx 110 r_f \frac{T_f}{T_0}, \tag{51}$$

where c is the speed of light. Depending on the mass of the central object one of the criteria (49) or (51) determines the extent of the region of applicability of our results.

The argument based on Equation (48) may not be without caveats. The value of $\alpha_{\rm GI}$ given by Equation (40) is derived based on the cooling rate equal to the energy production rate $\sim \dot{M}\Omega^2$ due to the dissipation of gravitoturbulence. This rate is much lower than the (external heating) rate $\sigma T_0^4 \times \min(1, \tau)$ at which the disk would cool if irradiation were suddenly switched off or if the GI in irradiated disks were capable of producing surface density perturbations of order unity. One can easily show that $\Omega t_{\rm cool}$ based on such a fast cooling rate does not stay constant in irradiated part of the disk but steadily increases. The tricky question is the following: which cooling rate should determine the ability of the disk to fragment? This issue can be settled satisfactorily via careful numerical simulations of strongly irradiated gravitoturbulent disks, something that has not yet been done. But given that in accretion disks which are close to the steady state, angular momentum transfer (determining $\dot{M}$) must be uniquely related to the energy dissipation rate, we feel that it is more likely for the fragmentation condition to be determined by the criterion (48) rather than by the much shorter cooling time set by the irradiation heating rate. Our subsequent consideration will be based on this assumption.

3.5. Background Viscosity

If the angular momentum transport caused by gravitational torques becomes weak some other mechanisms may start providing effective viscosity. Here we assume that in addition to gravitational torques disk also possesses some background viscosity α_ν due to, e.g., the magneto-rotational instability (MRI). Normally one expects $\alpha_\nu \ll 1$ so that this background viscosity would become significant only when $\alpha_{\rm GI}$ gets rather small.

If, as expected for the dust opacity, β is between 0 and 2 then both in the optically thick and optically thin regimes $\alpha_{\rm GI}$ decreases as ω increases. In other words, gravitoturbulent disk becomes less "viscous" as the distance to the center decreases. With this in mind we find that in the optically thick case the background viscosity would dominate over the gravitoturbulent torque (i.e., $\alpha_\nu \gtrsim \alpha_{\rm GI}$) at

$$\omega \gtrsim \omega_\nu \approx \left(\alpha_\nu^{2\beta-7}\dot{m}^{4-2\beta}\right)^{1/9}, \ \tau > 1. \tag{52}$$

According to Equation (26) this critical angular frequency corresponds to the optically thick regime only if $\dot{m} \gtrsim \alpha_\nu^{(10+4\beta)/(7+4\beta)}$.

In the optically thin case, the background viscosity regulates the disk at

$$\omega \gtrsim \omega_\nu \approx \left(\alpha_\nu^{-2\beta-9}\dot{m}^{2\beta+6}\right)^{1/3}, \ \tau < 1, \tag{53}$$

and this ω_ν corresponds to $\tau < 1$ provided that $\dot{m} \lesssim \alpha_\nu^{(10+4\beta)/(7+4\beta)}$.

Inside the region where the background viscosity dominates (at $\omega > \omega_\nu$), equations governing the disk structure change. Previously, when considering the gravitoturbulent transport, we did not have a constraint on α but instead had a relationship between T and Σ in the form of Equation (9), arising from the requirement of the marginal gravitational instability. However, with the background viscosity dominating the angular momentum transport α is constrained to be equal to α_ν, which leads to an overdetermined system of equations if we also try to keep the condition $Q = Q_0$. This contradiction is naturally avoided by dropping the latter constraint, i.e., allowing the disk not to be marginally gravitationally unstable when $\alpha = \alpha_\nu$.

In Appendix A, we present Σ, T, and τ behaviors in accretion disk with the dominant background viscosity α_ν and opacity in the form (6). If we now use these expressions (Equations (A1), (A2), (A5), and (A6)) to calculate Toomre Q in the self-luminous viscous part of the disk at $\omega > \omega_\nu$ we find that

$$Q/Q_0 = (\omega/\omega_\nu)^{9/(10-2\beta)}, \ \tau > 1 \tag{54}$$

in the optically thick case and

$$Q/Q_0 = (\omega/\omega_\nu)^{3/(6+2\beta)}, \ \tau < 1 \tag{55}$$

in the optically thin case. Apparently, for any reasonable dust opacity behavior Q starts to deviate from its marginal stability value Q_0 toward higher values at the transition from the gravitoturbulent to viscous regime (at $\omega = \omega_\nu$). Thus, the accretion disk cannot be gravitationally unstable if its angular momentum transport is not dominated by gravitational torques but is rather due to some other form of effective viscosity. This consideration demonstrates explicitly how the transition between the gravitoturbulent and viscous parts of the disk occurs.

Note that $\alpha_{\rm GI}$ given by Equation (40) is independent of ω. This implies that if

$$\dot{m} \lesssim \alpha_\nu \left(\frac{T_0}{T_f}\right)^{3/2}, \tag{56}$$

the accretion disk remains viscous everywhere and does not transition into the gravitoturbulent state at all. Thus, at low enough $\dot{m}$ the disk is gravitationally stable at large radii and can transport matter all the way to the central object from very large distances. Condition (56) is more restrictive than the constraint (48) which delineates the regime in which the gravitoturbulent constant $\dot{M}$ disk can transfer material from very large distances without fragmentation.

4. DISCUSSION

Our results derived in Section 3 apply to a variety of situations in which gravitoturbulent disks can exist: they can be optically thick or thin, fragmenting or not. Here we classify different states in which accretion disks can be found according to the values of $\dot{M}$, background viscosity α_ν, and irradiation temperature T_0. We also describe applications of these results to real astrophysical systems and discuss their connection with the work of others.

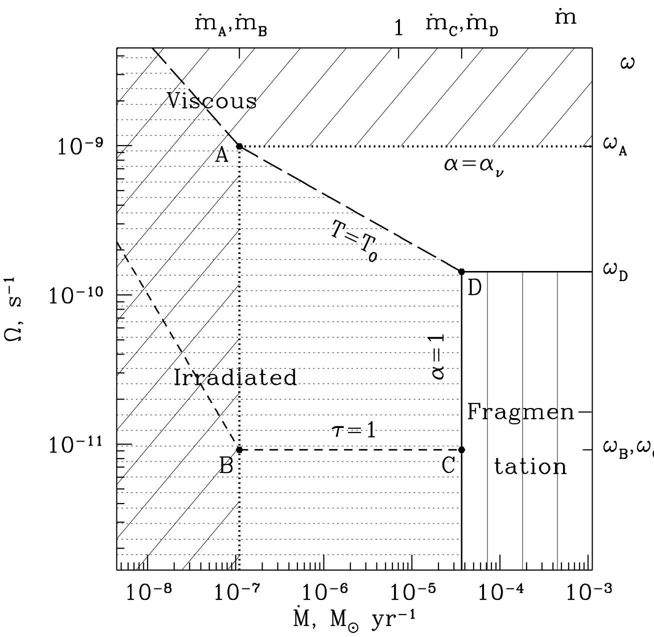

Figure 1. Plot of different possible states in which an accretion disk can be found as a function of mass accretion rate $\dot{M}$ and angular frequency Ω (the corresponding dimensionless quantities $\dot{m} = \dot{M}/\dot{M}_f$ and $\omega = \Omega/\Omega_f$ are shown on the upper and right axes). The disk has a non-zero background viscosity $\alpha_\nu = 0.003$ and is externally irradiated at a spatially constant temperature $T_0 = 3T_f \approx 35$ K, corresponding to condition (57). Shading indicates viscous regions (slant solid shading), irradiated regions (horizontal dotted shading), and region where the disk must fragment (vertical solid shading). The gravitoturbulent, self-luminous region is unshaded. Different curves separate regions with distinct physical conditions and are marked on the plot (see the text for more details). Coordinates of the points where these curves cross in the $(\dot{m}, \omega)$ coordinates are given by expressions (B1).

4.1. *Separation of Different Regimes*

We consider a particular case when external irradiation at constant temperature T_0 is strong,

$$T_0/T_f > 1, \tag{57}$$

in which case the phase space of possible regimes can be represented by Figure 1. Given that in the case of opacity dominated by cold dust T_f is just slightly higher than 10 K (see Equation (21)) it is clear that condition (57) should apply to virtually all accretion disks in the universe as even the lowest measured temperatures encountered in some dense molecular cores are not very different from 10 K (Di Francesco et al. 2007). For that reason in the following we consider only disks for which condition (57) is fulfilled.

In Figure 1, one can examine different regimes (indicated by shading) which are relevant for a given $\dot{M}$. Curves[3] separating different regimes correspond to various critical transitions: $\alpha = \alpha_\nu$ (dotted lines) implies a transition from viscous (above and to the left of this curve, slant solid shading) to gravitoturbulent state, $\alpha = 1$ (solid lines) describes the onset of fragmentation (below and to the right of this curve, vertical solid shading), $T = T_0$ is a (long-dashed) line below which external irradiation starts to dominate disk structure (region with horizontal dotted shading), $\tau = 1$ curve (short-dashed) shows a transition from an optically thick (above this curve) to an optically thin disk. Coordinates (in phase space $\dot{m}, \omega$, upper and right axes) of the points A, B, C, D where these curves cross are given by expressions (B1) in Appendix B. Unshaded region corresponds to a self-luminous, gravitoturbulent disk. A constant $\dot{M}$ disk corresponds to a straight vertical line in Figure 1 cutting through different regimes.

The numerical values on the left and lower axes of this figure correspond to a particular choice of $\alpha_\nu = 0.003$, $T_0/T_f = 3$ ($T_0 = 35$ K), and the opacity in form (7). As described in Section 3.5, the very low $\dot{M} \lesssim 10^{-7}\ M_\odot$ yr^{-1} disk is always viscous and gravitationally stable, even at very large distances from the central object. Above this value of $\dot{M}$ the disk must be gravitoturbulent within some range of distances. For $10^{-7}\ M_\odot$ yr$^{-1} \lesssim \dot{M} \lesssim 4\times10^{-5} M_\odot$ yr^{-1}, the gravitoturbulent region extends from $\Omega \sim 10^{-10}$–10^{-9} s^{-1} all the way to very large distances (where $\Omega \to 0$), limited only by conditions (49) or (51). However, for $\dot{M}$ above $\dot{M} \gtrsim 4\times10^{-5}\ M_\odot$ yr^{-1}, the gravitoturbulent disk fragments at $\Omega \approx 1.4\times10^{-10}$ s^{-1} (in the optically thick case with opacity characterized by $\beta = 2$ fragmentation occurs exactly at Ω_f, see Section 3.4) so that a constant $\dot{M}$ disk can be maintained only interior to this point. For any $\dot{M} \gtrsim 10^{-7}\ M_\odot$ yr^{-1}, the gravitoturbulent angular momentum transport becomes weak at large enough Ω so that the background viscosity starts to determine disk properties at small radii (the upper part of the plot, at $\Omega > 10^{-9}$ s^{-1} in this particular case).

As an example, let us use Figure 1 to figure out where the transitions between different regimes occur in a disk with $\dot{M} = 10^{-5}\ M_\odot$ yr^{-1}, $\alpha_\nu = 0.003$, $T_0/T_f = 3$ and opacity in the form (7). At $\Omega > 10^{-9}$ s^{-1}, the disk is gravitationally stable and self-luminous, angular momentum is transported through the disk by background viscosity. For 2×10^{-10} s$^{-1} < \Omega < 10^{-9}$ s^{-1}, the disk is gravitoturbulent and self-luminous. For $\Omega < 2\times10^{-10}$ s^{-1}, the disk is gravitoturbulent and externally irradiated. The disk is optically thick for all $\Omega \gtrsim 10^{-11}$ s^{-1} and optically thin for smaller Ω. This example clearly shows the complexity of possible states in which a given accretion disk can be found at different distances from the central object.

Figure 1 exhibits four invariant properties of accretion disks: (1) the disk is dominated by background viscosity for all Ω at very low $\dot{M}$ (for $\dot{m} < \alpha_\nu(T_0/T_f)^{3/2}$, see Appendix B) and non-zero T_0; (2) for all $\dot{M}$ the disk is dominated by background viscosity at high enough Ω; (3) at intermediate values of $\dot{M}$ (for $\alpha_\nu(T_0/T_f)^{3/2} < \dot{m} < (T_0/T_f)^{3/2}$), the disk possesses a gravitoturbulent, externally irradiated region that extends to large distances; (4) at large values of $\dot{M}$ (for $\dot{m} > (T_0/T_f)^{3/2}$), the disk has a gravitoturbulent region within a finite range of distances but must inevitably fragment at some large distance. Similar qualitative conclusions hold also for opacity behaviors different from (7).

We should note here that calculations presented in this section rely on our use of opacity (6) and (7) throughout the whole region of $\dot{M}, \Omega$ phase space that we consider. In reality, at high $\dot{M}$ and Ω the disk temperature should exceed 10^2 K at which point icy grains sublimate leaving metal-silicate grains as a source of opacity. This latter opacity source while still being in form (6) is characterized by smaller values of κ at the same temperature and $\beta \approx 1/2$. This is likely to quantitatively (but not qualitatively) affect results presented in Figure 1 at high $\dot{M}$ and Ω. To not complicate things further here we do not attempt to self-consistently describe transitions between different opacity regimes but rather display a qualitative picture for a single opacity law.

[3] Equations for these curves can be found in Sections 3.1–3.5 and Appendix A.

4.2. *Applications*

Our results can be applied to understanding the properties of the outer, cold parts of realistic accretion disks. In particular, we address three important issues.

First, a possibility of giant planet formation by GI in protoplanetary disks has been discussed since Cameron (1978). In this context, it is interesting to ask under which conditions a constant $\dot{M}$ protoplanetary disk would be prone to fragmentation into gravitationally bound, self-gravitating objects. Our results described in Section 4.1 can directly address this issue. Indeed, conditions used in producing Figure 1, namely $\alpha = 0.003$ and $T_0 \approx 35$ K, are quite typical for the outer regions of protoplanetary disks, beyond $\sim$100 AU from the central star. On one hand, the disk surface density there is low enough (see below) for the cosmic ray ionization to stimulate MRI operation which gives rise to α_ν at the level of $\sim 10^{-3}$–10^{-2} (e.g., Fleming & Stone 2003). On the other hand, outer regions of protoplanetary disks are warmed up by radiation of either the parent star or the neighboring stars at the level of several tens of K. Thus, the situation represented in Figure 1 can be directly used for understanding the properties of external parts of protoplanetary disks.

What is obvious from this figure is that giant planet formation by gravitational instability in constant $\dot{M}$ disks can take place only beyond $\approx$ 120 AU which is the distance from a 1 $M_\odot$ star at which $\Omega = \Omega_f$—remember that in the optically thick[4] gravitoturbulent disks fragmentation occurs at this specific value of Ω (Matzner & Levin 2005; see Equation (46)). Thus, planets produced by gravitational instability should be born far from their parent stars although one cannot exclude their subsequent migration to shorter periods.

Another obvious constraint on planet formation that follows from Figure 1 is that $\dot{M}$ must be pretty high at the location where disk fragments and planets form. Indeed, one can easily see that fragmentation is possible only if $\dot{M}$ locally exceeds $10^{-5} M_\odot$ yr^{-1}. At lower $\dot{M}$, the disk maintains itself in a gravitoturbulent state (or even being kept gravitationally stable by its own background viscosity at very low $\dot{M}$) without fragmentation even very far from the star.

Accretion rates in excess of 10^{-5} $M_\odot$ yr^{-1} are atypical for mature T Tauri disks (Gullbring et al. 1998). However, they may have been present at the very earliest stages of star and disk formation when the material from collapsing protostellar envelope rains down onto the disk at a very high rate, possibly exceeding 10^{-5} $M_\odot$ yr^{-1} in some locations. Such disks are likely not to have $\dot{M}$ constant through their whole extent but as we discussed in Section 3 our results are still applicable[5] with some modifications even to this more complicated case.

The second practical issue that we are going to address has to do with the feeding of supermassive black holes in centers of galaxies. It has been known for a long time that the outer parts of quasar disks must be gravitationally unstable which was always raising a question of how gas is transported to the black hole from large distances. Our results demonstrate that as long as $\dot{M}$ is not very high the GI is not going to impede mass transfer through the disk since for low enough $\dot{M}$, namely for $\dot{M} \lesssim \dot{M}_f (T_0/T_f)^{3/2}$, see Equation (48), a gravitationally unstable disk can persist in a gravitoturbulent state out to very large distances from the central object. How high $\dot{M}$ can be carried through a gravitoturbulent disk globally thus depends only on the level of external irradiation.

Radiation fields in galactic nuclei due to circumnuclear stars are expected to be quite intense giving rise to T_0 at the level of tens to hundreds of K. Assuming $T_0 = 100$ K (which is a radiation field slightly more intense than that expected in the Galactic center), we find that the gravitoturbulent disk can transport mass from large distance to the black hole as long as $\dot{M} \lesssim 10^3 \dot{M}_f \approx 10^{-2} M_\odot$ yr^{-1}. This is about 10% of the Eddington rate (for radiative efficiency of 10%) for the 4×10^6 $M_\odot$ black hole in the center of our Galaxy, which is quite significant given that the Bondi accretion rate of this object is $\lesssim 10^{-5} M_\odot$ yr^{-1} (Baganoff et al. 2003). Thus, irradiated gravitoturbulent accretion disks provide a natural way of continuous feeding at least some (not too massive) central black holes at reasonable rates by gas transported from very large distances.

However, it is also clear from our results and Figure 1 that accretion disks around more massive black holes ($>10^6$ $M_\odot$) consuming mass at rates close to Eddington cannot remain gravitoturbulent out to very large distances—for any reasonable level of external irradiation the disk must fragment at some point, and the most distant possible location of the fragmentation boundary corresponds to $\Omega = \Omega_f \approx 1.4 \times 10^{-10}$ s^{-1} (for the case of dust opacity in the form [7]). Transport of gas from beyond this distance is still an open issue discussed by Goodman (2003).

Finally, we briefly discuss the origin of the young stellar disk around a supermassive black hole in the center of our Galaxy. Inner parsec of the Galaxy is known (Paumard et al. 2006; Lu et al. 2009) to contain at least one disk of young (6 ± 2 Myr old) massive stars spread between 0.04 pc and 0.5 pc. To explain formation of these stars so close to the black hole where they are subject to action of its strong tidal field a fragmentation of a gravitationally unstable disk has been proposed (Levin & Beloborodov 2003; Levin 2007). Such an event can in principle happen both in a (quasi-)steady state disk like the one we considered in this work or in a short-lived promptly fragmenting (on a dynamical time scale) disk-like structure that may arise as a result of molecular cloud collision with the black hole (Wardle & Yusef-Zadeh 2008; Bonnell & Rice 2008; Hobbs & Nayakshin 2009). Here we try to constrain the first possibility. We will assume that the disk was illuminated by surrounding stars which kept T_0 at the level of tens of K so that its opacity law was given by Equation (7). Distance r_f at which fragmentation would occur in an optically thick disk around the 4×10^6 $M_\odot$ black hole is ≈ 0.1 pc which is within the span of the observed stellar disk.

For a long-lived disk to start fragmenting, a variability of some of its properties must take place. One possibility is an increase of $\dot{M}$ which can bring a gravitoturbulent irradiated disk extending out to large distances across a fragmentation threshold (see Figure 1). But then the outer regions of the disk (at $r \gg r_f$) where $\dot{M}$ has increased would immediately fragment and it is not at all obvious that mass would be transferred inward increasing $\dot{M}$ at small radii (at $r \sim r_f$) where stellar disks are observed. In principle, the disk could be not an accretion but a spreading (Pringle 1991) disk formed as a result of a dense molecular cloud disruption at very small distances. However, in this case an *increase* of $\dot{M}$ is unlikely while a decrease of $\dot{M}$ typical for spreading disks would only stabilize the disk against gravitational fragmentation.

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[4] That the disk with the assumed values of α_ν and T_0 is optically thick at the fragmentation boundary is evident from Figure 1.

[5] A disk with $\dot{M}$ varying with distance would not correspond to a straight vertical line in Figure 1 like a constant $\dot{M}$ disk would but must follow a more complicated path determined by a specific dependence of $\dot{M}$ on r (or, alternatively, Ω).

Another (probably less likely) possibility is a reduction of external irradiation which can cause fragmentation of even a constant in time $\dot{M}$ disk as soon as the condition (48) gets violated. The problem with this scenario is that it may then be difficult to explain the existence of stars at 0.04 pc which is significantly closer to the black hole than the minimum radius r_f at which fragmentation occurs in a cold disk. In principle, Ω_f can be increased by lowering disk metallicity which affects κ_0. However, according to Equations (17) and (22) moving r_f from 0.1 pc to 0.04 pc would require reducing κ_0 by a factor of 60 compared to the value in (7), implying an extremely subsolar metallicity in the disk.

More generally, star formation in a nearly Keplerian disk does not naturally explain rather significant eccentricities of disk stars (Bartko et al. 2009) and the possible presence of a second disk component. Thus, it seems unlikely (although not completely impossible) that stellar disks were formed by gravitational fragmentation of a long-lived gaseous disk. Scenario of a prompt fragmentation of a tidally disrupted molecular cloud advanced by Wardle & Yusef-Zadeh (2008), Bonnell & Rice (2008), and Hobbs & Nayakshin (2009) presents a more attractive possibility.

4.3. Comparison with Previous Studies

The first investigation of self-gravitating accretion disks has been undertaken by Paczynski (1978a), later followed by Paczynski (1978b) and Kozlowski et al. (1979). These early numerical modeling efforts concentrated on studying hot quasar disks around the supermassive black holes with accretion rates close to the Eddington rate. Some of the important ingredients of these models have been the inclusion of the radiation pressure (neglected in our case) and use of high-T opacities, which makes comparison of these calculations to the results of our study rather difficult.

Another investigation of a quasi-viscous evolution of a self-gravitating disk driven by gravitational torques has been done by Lin & Pringle (1987). In their work, based on rather general arguments, a specific model for the viscosity due to the disk self-gravity has been assumed, namely $\nu \propto \Sigma^2 r^6 \Omega$ (or $\alpha \sim Q^{-2} > 1$). The correct prescription (2) is in general quite different from this naive ansatz precluding direct comparisons with our results. Also, the assumption $Q \sim 1$ has been relaxed in Lin & Pringle (1987) allowing Q to drop significantly below unity.

More recently, Goodman (2003) has analytically investigated properties of $Q \approx 1$ regions of constant $\dot{M}$ quasar disks, again taking into account the radiation pressure. This study has a lot in common with our work with the major difference being that Goodman (2003) left α to be a free parameter while we self-consistently calculate its value using a prescription (2). Some of the results derived in Goodman (2003) have been retrieved in our study.

Subsequently, Rafikov (2005, 2007) has looked at the properties of gravitationally unstable disks, which are capable of forming giant planets by direct fragmentation. Such disks must simultaneously fulfill two constraints: $Q = Q_0$ and $\alpha_{\rm GI} = \chi$. These assumptions were used to set stringent constraints on the properties of disks that are able to form planets by GI. Clearly, such disks do not have constant $\dot{M}$ in general. Our current assumptions are different in that we assume only $Q = Q_0$ and fix $\dot{M}$ at some value (not necessarily constant, see the discussion after Equation (13)) at every point in the disk, but $\alpha_{\rm GI}$ is then calculated self-consistently. Matzner & Levin (2005) and Levin (2007) have also studied cold $Q = 1$ disks with opacity dominated by dust specifically looking at the conditions necessary for fragmentation. We successfully reproduce some of their results such as the location of the fragmentation boundary in the optically thick regime and the importance of irradiation for stabilizing the gravitoturbulent disk against fragmentation at large distances.

Kratter et al. (2008) have looked at the properties and conditions for fragmentation in massive, self-gravitating disks with continuing infall. In their study, they have adopted a prescription for the angular momentum transport different from ours: their effective $\alpha_{\rm GI}$ depends on Toomre Q (which is fixed at the level of Q_0 in our case) and the disk-to-star mass ratio (which is close to zero in our case), which allowed them to cover the massive disk case. For that reason (and because of a different physical setup, e.g., absence of infall, in our case) a direct comparison of our results with theirs even in the limit of small disk mass is not trivial.

Finally, Terquem (2008) and Zhu et al. (2009) have constructed global numerical models of protoplanetary disks accounting for the possibility of GI in some parts of the disk. These studies pay special attention to the presence of the so-called "dead zones" (Gammie 1996)—disk regions where MRI cannot operate because of low ionization. Our analytical calculations do not account for the existence of such regions, nevertheless they provide good foundation for understanding the numerical results of Terquem (2008) and Zhu et al. (2009) in the outermost regions of their disks.

5. CONCLUSIONS

We have explored the properties of marginally gravitationally unstable accretion disks using a realistic prescription for the angular momentum transfer driven by the gravitational torques. We self-consistently derived scalings of important disk variables such as surface density and temperature in both the optically thick and thin regimes. We also accounted for the possibility of disk having some background viscosity, e.g., due to MRI, and demonstrated that in this case a gravitoturbulent disk inevitably switches to an ordinary viscous disk at small radii. Another important ingredient of our study is the inclusion of possible external irradiation of the disk (e.g., by central object or nearby stars). We have demonstrated that for low enough $\dot{M}$ external irradiation helps to stabilize the disk against fragmentation out to very large distances providing means of external mass feeding of the central object. At extremely low $\dot{M}$, disks have been shown to never become gravitationally unstable because of background viscosity. At high $\dot{M}$ (the exact threshold depends on the irradiation temperature) the fragmentation of the disk is inevitable at large distances. Results of this work apply to our understanding of the possibility of giant planet formation by GI and star formation in the Galactic center, and to the problem of feeding the quasars.

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APPENDIX A

PROPERTIES OF VISCOUS ACCRETION DISKS

When the angular momentum transport in the disk is dominated by some internal source of viscosity characterized by parameter α_ν one finds that in the optically thick case,

$$\Sigma_\nu \approx \left[\frac{\sigma}{\kappa_0}\frac{\Omega^{2-\beta}\dot{M}^{3-\beta}}{\alpha_\nu^{4-\beta}}\left(\frac{k_B}{\mu}\right)^{\beta-4}\right]^{(5-\beta)^{-1}} \approx \Sigma_f\left(\frac{\omega^{2-\beta}\dot{m}^{3-\beta}}{\alpha_\nu^{4-\beta}}\right)^{(5-\beta)^{-1}}, \tag{A1}$$

$$T_\nu \approx \left[\Omega^3\frac{\kappa_0\dot{M}^2}{\sigma\alpha_\nu}\frac{\mu}{k_B}\right]^{(5-\beta)^{-1}} \approx T_f\left(\frac{\omega^3\dot{m}^2}{\alpha_\nu}\right)^{(5-\beta)^{-1}}, \tau > 1, \tag{A2}$$

where we have dropped constant numerical factors of order unity. With these expressions one finds

$$\tau = \left(\frac{\omega^{2\beta+2}\dot{m}^{\beta+3}}{\alpha_\nu^4}\right)^{(5-\beta)^{-1}}, \tag{A3}$$

meaning that the viscous disk is optically thick only when

$$\omega > \left(\frac{\alpha_\nu^4}{\dot{m}^{\beta+3}}\right)^{(2+2\beta)^{-1}}. \tag{A4}$$

When condition (A4) is violated, the disk becomes optically thin and we find

$$\Sigma_\nu \approx \left[\sigma\kappa_0\frac{\Omega^{2+\beta}\dot{M}^{3+\beta}}{\alpha_\nu^{4+\beta}}\left(\frac{\mu}{k_B}\right)^{4+\beta}\right]^{(3+\beta)^{-1}} \approx \Sigma_f\dot{m}\left(\frac{\omega^{2+\beta}}{\alpha_\nu^{4+\beta}}\right)^{(3+\beta)^{-1}}, \tag{A5}$$

$$T_\nu \approx \left[\Omega\frac{\alpha_\nu}{\sigma\kappa_0}\frac{k_B}{\mu}\right]^{(3+\beta)^{-1}} \approx T_f\,(\alpha_\nu\omega)^{(3+\beta)^{-1}}, \tag{A6}$$

$$\tau = \dot{m}\left(\frac{\omega^{1+\beta}}{\alpha_\nu^2}\right)^{2/(3+\beta)}, \tau < 1. \tag{A7}$$

Derivation of Equations (A1)–(A7) assumes that it is the internal viscous dissipation in the disk that sets its midplane temperature. However, analogous to the case studied in Section 3.3 one can consider a possibility that the disk temperature is set by external irradiation at the level of $T = T_0$ even when its angular momentum transport has non-gravitational nature. From Equation (A2) we find that an optically thick self-luminous viscous disk changes to an externally irradiated disk at a radius where

$$\omega = \omega_T = \left(\frac{\alpha_\nu}{\dot{m}^2}\right)^{1/3}\left(\frac{T_0}{T_f}\right)^{(5-3)/3}. \tag{A8}$$

Analogously, Equation (A6) predicts that an optically thin transition from a self-luminous to an externally irradiated disk occurs at the point where

$$\omega = \omega_T = \alpha_\nu^{-1}\left(\frac{T_0}{T_f}\right)^{(3+\beta)}. \tag{A9}$$

In those regions where the disk temperature is fixed at the constant level T_0 one can easily show that

$$\Sigma = \frac{\dot{m}\omega}{\alpha_\nu}\frac{T_0}{T_f}, \tag{A10}$$

$$\tau = \frac{\dot{m}\omega}{\alpha_\nu}\left(\frac{T_0}{T_f}\right)^{(\beta-1)}. \tag{A11}$$

$$Q/Q_0 = \frac{\alpha_\nu}{\dot{m}}\left(\frac{T_0}{T_f}\right)^{3/2}. \tag{A12}$$

The last equation implies that Toomre Q is constant in the irradiated viscous part of the constant $\dot{M}$ disk. That $Q > Q_0$ there can easily be seen from Equation (40), which allows us to rewrite Equation (A12) as $Q/Q_0 = \alpha_\nu/\alpha_{\rm GI}$ and this ratio is > 1 since for α_ν to dominate over the gravitoturbulent torques $\alpha_\nu > \alpha_{\rm GI}$ must be fulfilled. It also follows from Equation (A11) that the $\tau = 1$ transition, if it occurs in the externally irradiated region of viscous disk, takes place at

$$\omega = \omega_1 = \frac{\alpha_\nu}{\dot{m}}\left(\frac{T_0}{T_f}\right)^{1-\beta}. \tag{A13}$$

APPENDIX B

PHASE SPACE PLOT

The $(\dot{m}, \omega)$ coordinates of various critical points in Figure 1 corresponding to $T_0/T_f > 1$ are

$$\begin{aligned}
A &= \left(\alpha_\nu\left(\frac{T_0}{T_f}\right)^{3/2}, \alpha_\nu^{-1/3}\left(\frac{T_0}{T_f}\right)^{(2-\beta)/3}\right),\\
B &= \left(\alpha_\nu\left(\frac{T_0}{T_f}\right)^{3/2}, \left(\frac{T_0}{T_f}\right)^{-(\beta+1/2)}\right),\\
C &= \left(\left(\frac{T_0}{T_f}\right)^{3/2}, \left(\frac{T_0}{T_f}\right)^{-(\beta+1/2)}\right),\\
D &= \left(\left(\frac{T_0}{T_f}\right)^{3/2}, \left(\frac{T_0}{T_f}\right)^{(2-\beta)/3}\right).
\end{aligned} \tag{B1}$$

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