

## COLOR BIMODALITY IN M87 GLOBULAR CLUSTERS

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### ABSTRACT

We present an analysis of a 50 orbit *Hubble Space Telescope* (HST) Advanced Camera for Surveys (ACS) observation of the M87 globular cluster system. We use the extraordinary depth of this data set to test whether the colors and magnitudes show evidence for a mass–metallicity relation in globular cluster populations. We find only a weak or absent relation between the colors and magnitudes of the metal-poor subpopulation of globular clusters. The weakness or absence of a color–magnitude relation is established over a wide range of luminosity from  $M_V = -11$  to  $M_V = -6$ , encompassing most of the M87 globular clusters. The constancy of the colors of the metal-poor subpopulation seen in our 50 orbit observation is in contrast to suggestions from single orbit ACS data that the metal-poor globular clusters in M87 and several other galaxies show a “blue tilt.” The formal best fit for the mass–metallicity relation for the metal-poor subpopulation in our much deeper data is  $Z \propto M^{0.08 \pm 0.05}$ . Our analysis of these data also shows a possible small “red tilt” in the metal-rich globular cluster subpopulation. While either of these small tilts may be real, they may also illustrate the limit to which mass–metallicity relations can be determined, even in such extraordinarily deep data. We specifically test for a wide range of systematic effects and find that while small tilts cannot be confirmed or rejected, the data place a strong upper limit to any tilt of  $|0.20| \pm 0.05$ . This upper limit is much smaller than some earlier claims from single orbit data, and strongly limits self-enrichment within globular clusters. This mass–metallicity relation for globular clusters is also shallower than the relation for galaxies, suggesting that the formation mechanisms for these two types of objects are different.

**Key words:** galaxies: individual (M87) – galaxies: star clusters – globular clusters: general

**Online-only material:** color figures

### 1. INTRODUCTION

The color distribution of globular clusters in most galaxies has long been known to be bimodal (e.g., Kundu & Whitmore 2001; Larsen et al. 2001). The standard understanding of this feature is that the red and blue subpopulations for a given galaxy represent a difference in metallicity, with the blue clusters being on average more metal poor than the red clusters. The correspondence of the blue clusters with lower metallicity and red clusters with higher metallicity follows directly from studies of the Milky Way globular cluster system (e.g., Ashman & Zepf 1998; Harris 1991), and has been confirmed in a number of extragalactic globular cluster systems (e.g., Cohen et al. 2003; Strader et al. 2007; Kundu & Zepf 2007).

Recently, several studies of the globular cluster systems of early type galaxies have suggested a relation between the color and luminosity of the blue metal-poor clusters, with brighter clusters appearing more red (Harris et al. 2006; Strader et al. 2006; Mieske et al. 2006). This “blue tilt” has been taken as evidence of a relation between the mass and metallicity of these clusters. If such a trend were real, it would suggest an important role for self-enrichment in luminous globular clusters (Strader & Smith 2008). It might also suggest that globular clusters are surrounded by more massive halos when they form, in order to allow them to retain metals from multiple generations of stars. Alternatively, the absence of a mass–metallicity relation would strongly limit the degree to which self-enrichment is a factor in the observed cluster metallicities. Such an absence also emphasizes the differences between the formation histories of

globular clusters and galaxies, which are known to have a strong mass–metallicity relation.

Firmly establishing any such relation requires determining the color and luminosities of the globular clusters over a large range in brightness to detect any effect over a significant luminosity and mass scale. Unfortunately, many of the results that show evidence of this blue tilt do so using fairly shallow single orbit ACS data, with comparably low signal to noise for the remaining more distant galaxies. In this paper, we present the results of an extraordinarily deep 50 orbit ACS study of the central region of M87. These deep data allow clusters to be detected over the entire range of cluster magnitudes, providing a sample that is effectively complete. No trend between the mass and metallicity is observed for the blue globular cluster population, suggesting that the previously published results are influenced strongly by the much lower signal to noise in that data. Section 2 discusses the data and the photometric reduction used to ensure accurate cluster measurements. Section 3 reviews the KMM algorithm used to determine the peaks of the color distributions, and the sensitivity of the results to changes in the binning methods. Finally, section 4 presents the results for our sample of clusters, and section 5 discusses these results in the context of the formation history of the globular cluster system.

### 2. OBSERVATIONS AND REDUCTIONS

The data for this project come from a 50 orbit observation program with the Advanced Camera for Surveys (ACS) aboard the *Hubble Space Telescope* (HST) (PI: Baltz, Proposal ID:

10543). The images are of the central region of the giant elliptical galaxy M87, extending out to a projected radius of 8 kpc (taking  $m - M = 31.021$ ; Macri et al. (1999)). The data were taken over the course of three months, as part of a search for microlensing events, which require repeat visits to find changes in brightness. These multiple images of the same field yield data that can be combined into very deep exposures. On each observing day, four exposures in F814W were taken with dithered pointing along with a single matching exposure in F606W. The four F814W images provide a fully dithered image for each day, with the F606W yielding full coverage over the entire set of observations. The images were combined using Multidrizzle (Fruchter & Hook 2002) to a resolution of  $0''.05 \text{ pixel}^{-1}$ , the nominal resolution of ACS. Although the dither pattern for this data set is more than sufficient to allow higher resolution final images to be constructed, this is not necessary for the identification and photometry of the globular clusters. At the resolution used, the globular clusters are significantly broader than the ACS point-spread function (PSF), so retaining the nominal ACS resolution provides the highest possible signal to noise. In all, 49 F606W and 205 F814W images were combined to yield final images with exposure times of  $t_V = 24,500 \text{ s}$  and  $t_I = 73,800 \text{ s}$ , making these some of the deepest images ever taken with *HST*. In addition to the exposures used, 8 F606W and 13 F814W were taken but excluded from analysis due to a loss in the guide star tracking.

The final drizzled image contains the strongly varying galaxy light. As the main source of noise in the final image is due to the variations in this galaxy light from pixel to pixel, constructing an accurate model of the galaxy is essential to estimating the detection efficiency across the image. We use a model of the galaxy determined from isophote fitting, but to optimize this fit, it is advantageous to remove sources other than the galaxy light. In addition to the rich globular cluster population, these sources include background galaxies and the optically bright jet emanating from the galaxy core. In order to identify these sources, we first create a model of the galaxy by median filtering the image, with a box size chosen to be larger than the expected size of any features on the image (taken to be  $100 \times 100$  pixels for this resolution). This galaxy model does a poor job near the galaxy center, where the filtering box is unable to deal with the steep light profile. In addition, bright globular clusters tend to bias the model, leading to oversubtraction around such objects. Despite these issues, the model subtracted data image is significantly smoother than the initial image. This image is then used to construct a mask to remove the effect of the globular clusters and jet. All pixels that are  $3\sigma$  above the local mean (calculated again in a  $100 \times 100$  pixel box) are added to the mask image. This method does an excellent job of cleaning the image of the contribution from sources other than the galaxy light. The STSDAS ELLIPSE and BMODEL tasks were then used to create a smooth model of the galaxy, based on the original and mask images. The mask ensures that bright clusters do not bias the ELLIPSE fitting, and that the galaxy core profile is not influenced by the jet. ELLIPSE only runs for annuli which are more than 50% complete (not masked or off the frame). To rectify this, additional C code was written to calculate a continuation of the ELLIPSE model into the image corners, under the assumption that the position angle and ellipticity are fixed at the last value found by ELLIPSE. This final model was then subtracted from the original drizzled image, along with a constant background level, calculated to be the mode of the galaxy subtracted image histogram.

Source Extractor (Bertin & Arnouts 1996) was used to generate a database of objects detected on the images. To ensure that the detection efficiency was similar across the image, the light from the galaxy was subtracted and used to weight the detection process. As the noise on the final data image is highly dependent on the galaxy flux, a constant detection threshold would miss many objects near the center of the galaxy. This weighting scales the detection threshold to more accurately reflect the local image noise. A detection threshold of  $3\sigma$  was used for this project, with a minimum area for objects of 2 pixels. This area criterion helps limit small noise spikes from being detected as potential clusters. To resolve this issue further, we also set the requirement that an object must be detected on both the F606W and F814W images to be considered for further analysis.

All objects that were detected in our images were measured in ten apertures from  $r = 1$  pixel to  $r = 10$  pixels. As the clusters are resolved in our images, the differences in the sizes of clusters of equal brightness can greatly change the light detected within a fixed aperture. Therefore, a size-dependent aperture correction was constructed to accurately measure the total cluster flux. To calibrate such a correction, we created simulated globular cluster images by convolving two-dimensional King (1966) models with the instrumental PSF, calculated using the empirical ACS PSFs of Anderson & King (2006). These simulated clusters were generated with a fixed concentration  $c = 1.03$  over a grid of apparent instrumental magnitudes and tidal radii. At each grid point, 200 simulated clusters were randomly added to the galaxy subtracted images, and then processed through the same detection and measurement process as for the data images.

Based on this large sample of simulated clusters, we adopted a radius of 4 pixels to measure the cluster light, and then determine an aperture correction from this radius to infinity. This correction was parameterized by an estimate of the size of the object, defined to be the difference in the measured magnitudes within two radii:  $\mathcal{R} \equiv m_{4 \text{ pixel}} - m_{2 \text{ pixel}}$ . The logic behind this parameter is that a very small object (with a radius smaller than 2 pixels) will have approximately the same magnitude in both apertures, yielding a value of  $\mathcal{R} \sim 0$ . As an object increases in size, more light is measured in the large aperture compared to the smaller, which pushes  $\mathcal{R}$  to more negative values.

The grid of simulated clusters allows for the easy construction of an aperture correction. For each grid point in  $(m, r_i)_{\text{simulated}}$ , the median  $m_{4 \text{ pixel}}$  and  $\mathcal{R}$  are found among the set of simulated clusters detected by Source Extractor. These values are used as samples of the aperture correction:

$$\mathcal{A}(\mathcal{R}, m_{4 \text{ pixel}}) = m_{\text{simulated}} - m_{4 \text{ pixel}}. \quad (1)$$

The upper end of this correction is fixed, such that  $\mathcal{A}(0, m_{4 \text{ pixel}}) = 0$ . For each real object detected in the galaxy subtracted data image, the correct aperture correction is found by interpolating on the grid of  $(\mathcal{R}, m_{4 \text{ pixel}})$ . This aperture correction is mainly a function of  $\mathcal{R}$ , with very little dependence on the cluster magnitude. This is exactly what is expected, as any magnitude dependence would suggest that the photometry is not uniform. The median aperture corrections in our two filters are  $-0.61$  and  $-0.71$  for F606W and F814W, respectively, and vary directly with the size of the cluster.

The accuracy of these aperture corrections can be checked for the data clusters by fitting the images with PSF convolved King models (King 1966). Each of the clusters in this sample had such King models fit to their F814W image, which has

higher signal to noise than the F606W data. The exact details of this procedure will be presented in a future paper (C. Z. Waters et al. 2008, in preparation). The aperture correction correlates well with the best fitting tidal radius of the clusters

$$\begin{aligned}\mathcal{A}_{F606W} &= -0.60 - 0.0013 \times r_t, \\ \mathcal{A}_{F814W} &= -0.70 - 0.0014 \times r_t,\end{aligned}$$

which illustrates the importance of including these size effects. The magnitudes of the best fitting King models provide a more detailed measurement of a given cluster's total flux. Over the range of magnitudes where this fitting is able to faithfully fit the cluster images ( $I \lesssim 25$ ), the aperture magnitudes and the King model magnitudes agree well, with  $m_{\text{aperture}} - m_{\text{King}} = -0.02 \pm 0.12$ . This scatter is consistent with our expected photometric uncertainties, and illustrates that this aperture correction strategy is a robust way to measure the light from the clusters with a wide range of sizes.

The PSFs used in the generation of the simulated clusters extend out to  $0''.5$  from their core (Anderson & King 2006). The ACS detector is known to have large angle scattering beyond this limit, which will not be accounted for, even with this radius-dependent aperture correction. Because of this scattering, additional  $-0.088$  and  $-0.087$  mag corrections were applied to the F606W and F814W instrumental magnitudes, respectively, to correct them to an infinite aperture (Sirianni et al. 2005). To compare this data to previous studies of globular clusters, we converted the instrumental F606W and F814W magnitudes to standard  $V$  and  $I$ . This was done using the color correction formulas presented by Sirianni et al. (2005). The final magnitudes were then corrected for extinction ( $-0.074$  and  $-0.043$  for  $V$  and  $I$ ), using the extinction maps of Schlegel et al. (1998).

This ACS pointing contains the majority of the clusters imaged by the Kundu et al. (1999) and Waters et al. (2006) Wide Field Planetary Camera 2 (WFPC2) studies, allowing us to test our photometric consistency by comparing to those previously published. For the Kundu et al. (1999) sample, there are 968 clusters matched between the two samples with magnitude differences

$$\begin{aligned}V - V_{\text{Kundu}} &= -0.048 \pm 0.127, \\ I - I_{\text{Kundu}} &= 0.030 \pm 0.115, \\ (V - I) - (V - I)_{\text{Kundu}} &= -0.022 \pm 0.116.\end{aligned}$$

This comparison uses a re-analysis of the Kundu et al. (1999) sample that incorporates the most recent official WFPC2 zero-points. For the Waters et al. (2006) sample, there are 886 clusters in common, with differences

$$\begin{aligned}V - V_{\text{Waters}} &= 0.073 \pm 0.098, \\ I - I_{\text{Waters}} &= 0.082 \pm 0.084, \\ (V - I) - (V - I)_{\text{Waters}} &= -0.000 \pm 0.118.\end{aligned}$$

The original published Waters et al. (2006) measurements included a  $0.1$  mag offset in the final  $V$  magnitudes due to an inadequate PSF used in the aperture correction. The differences presented here correct this error, and shows that this sample is generally consistent with these new measurements. For both of these samples, all bright clusters that fall within the ACS field of view are detected.

Although these images are extraordinarily deep, and the detection threshold has been set at a fairly liberal level, we

still expect that we are not 100% efficient at detecting clusters. Conveniently, the simulated clusters used to construct the aperture correction also sample the completeness of objects in both filters. The fraction of clusters detected at each grid point in apparent instrumental magnitude and tidal radius was used to generate the expected completeness of the data objects. Although the detectability of an object is a function of the object's surface brightness, there is little dependence on the simulated cluster size, so for all further analysis of the completeness, we consider it only as a function of the apparent magnitude. As expected from our simple noise model of the galaxy, the detection efficiency, and hence the completeness, is a function of projected radius from the center of the galaxy. Due to the scaling of the detection threshold, clusters with low  $R_{\text{gal}}$  must have a larger flux peak to be detected at the same threshold as a cluster with larger  $R_{\text{gal}}$ . To minimize the effect this has on our calculated completeness values, we divide the total sample into two radius bins, separated at the median cluster radius,  $68''.95$ . The completeness is calculated independently for the two bins, which allows the completeness in the outer bin to extend to slightly fainter levels. Due to the requirement that objects be found on both the F606W and the F814W images, the final completeness levels are color dependent. For the range of colors we consider for our globular cluster sample, the median 50% completeness limit follows the trend  $V_{50\%} \sim 25.4 + 0.8(V - I)$ . For the following analysis of the cluster bimodality, we use a stricter  $I$  band 90% limit, which follows  $I_{90\%} \sim 24.5 - 0.1(V - I)$ .

The cluster detection method used was designed to detect as many objects as possible. Unfortunately, this also means that some fraction of the objects found are not true globular clusters. The two main sources of contaminating objects are background galaxies and noise fluctuations from the galaxy light due to random local overdensities in the stars that make up the galaxy. We have chosen to make a series of cuts in the measured parameters that serve to exclude as many of these contaminating objects as possible, while retaining the real globular clusters. In addition to the requirement that objects be detected in both filters, we also apply a color cut such that only objects with  $0.5 < V - I < 1.7$  are considered to be globular clusters. These limits are based on the observed colors of globular cluster systems and bracket the obvious feature in the color-magnitude diagram of the sample of all detected objects. Next, we only wish to consider objects that have a final completeness value greater than 50%. Due to the fact that this completeness limit is so faint, we expect that only a few real clusters will be excluded by this cut. Since globular clusters are known to be round, we can place a cut on the ellipticity of the clusters in both filters, such that  $\epsilon_{|V,I} < \frac{1}{3}$ . This cut serves as an excellent way to exclude the many background spiral galaxies that are seen edge on. Finally, we exclude objects where the average surface brightness is brighter than the peak surface brightness. Visual inspection of the objects excluded by this cut show them to be entirely high spikes in the galaxy noise.

As a check of how well these cuts function to clean our catalog of objects that are not globular clusters, we can apply these same cuts to catalogs of objects that contain no globular clusters, and only contaminating objects. If the variations in the galaxy noise were purely Gaussian, then the number of expected noise maxima on the data image should match the number of minima. By multiplying the galaxy subtracted image by  $-1$ , and searching for objects on that image, we should be able to estimate the noise contribution. Unfortunately, the asymmetry



**Table 1**  
Number of Detected Objects

| Cut                 | $N_{\text{clusters}}$ | $N_{\text{inverse}}$ | $N_{\text{UDF}}$ |
|---------------------|-----------------------|----------------------|------------------|
| Matched             | 5392                  | 136                  | 242              |
| Completeness > 50%  | 3996                  | 45                   | 231              |
| $0.5 < V - I < 1.7$ | 2832                  | 13                   | 110              |
| Ellipticity         | 2168                  | 4                    | 31               |
| Surface brightness  | 2090                  | 2                    | 31               |
| Final               | 2089                  | 2                    | 31               |

of the image histograms for these images suggest that the fluctuations may not be Gaussian distributed, but are rather biased toward the positive fluctuations, as might be expected from the incipient detection of the brightest individual stars in M87.

The contamination by background galaxies obviously contributes to the number of objects found. A quick look at the galaxy subtracted image shows a large number of background spiral galaxies that are easy to identify. The Hubble Ultra Deep Field (UDF) (Beckwith et al. 2006) provides a data set containing only background galaxies, and as such, gives a straightforward way to estimate the contribution we expect to find from these galaxies. The only complication is ensuring that the data quality matches between the two data sets. We first rebinned the UDF images from a pixel scale of  $0''.030$  to the pixel scale of our data ( $0''.05$ ). Next, the UDF was overlaid on our image footprint, and trimmed to match the field of view. With the geometry matched, we considered the image noise. The UDF count rate was scaled to match the exposure time of the M87 data. The M87 galaxy model was then used to generate a noise model, which assumes that each pixel is drawn from a Gaussian distribution with mean zero and variance equal to the galaxy model value. This noise was then scaled such that when added to the UDF image, the final image variance equaled that of the galaxy subtracted image. This method makes it certain that objects in the UDF frame that fall “near the center of the galaxy” have more noise than objects “at the edge.” The reduction for objects detected in the UDF images is identical to the M87 images, with the only exception being that the UDF filters are F606W and F775W. Since these filters can be converted to the same standard  $V$  and  $I$  system, the effect of this difference is minimal.

The number of objects detected with each of the various cuts in the M87 data, the inverse images, and the UDF sample are given in Table 1. It is clear that the various restrictions on the data remove the majority of the contaminating objects, and yield a final number of clusters consistent with what is expected based on earlier surveys. The one object that is manually excluded from the sample is the *HST*–1 knot of the jet, which due to its brightness and compact size, successfully eludes these cuts.

### 3. CLUSTER BIMODALITY

The color distribution of our extremely deep sample of globular clusters is shown in Figure 1. This distribution is clearly bimodal, as has been observed before for M87 and appears to be a common feature of globular cluster populations (e.g., Kundu & Whitmore 2001; Larsen et al. 2001). The degree of bimodality is generally quantified using the KMM test, as presented by Ashman et al. (1994). Briefly, this test determines the best fitting mixture of Gaussian distributions to a set of data points. For each Gaussian component included in the mixture model, the best fitting mean and variance are found, along with the number of points that belong to each component. This algorithm

works quickly, using a maximum likelihood method to estimate the best fitting parameters based on only simple initial values. In addition, as the data are not binned in any way before the analysis, there is no uncertainty due to the choice of bin size. Closely separated groups, such as from a sample that does not appear bimodal visually, can often be disentangled given a sufficient number of data points (see Ashman et al. 1994 for details).

The KMM test can fit any number of Gaussian components, under the assumption that all variances are the same (homoscedasticity) or that the variances are possibly different (heteroscedasticity). For our analysis of the M87 globular cluster system, we consider a maximum of two components, based on the obvious visual bimodality. However, we compare against a single component model to check the possibility that the data are better fit by this more simple model. This check is done using the likelihood ratio test which is defined as

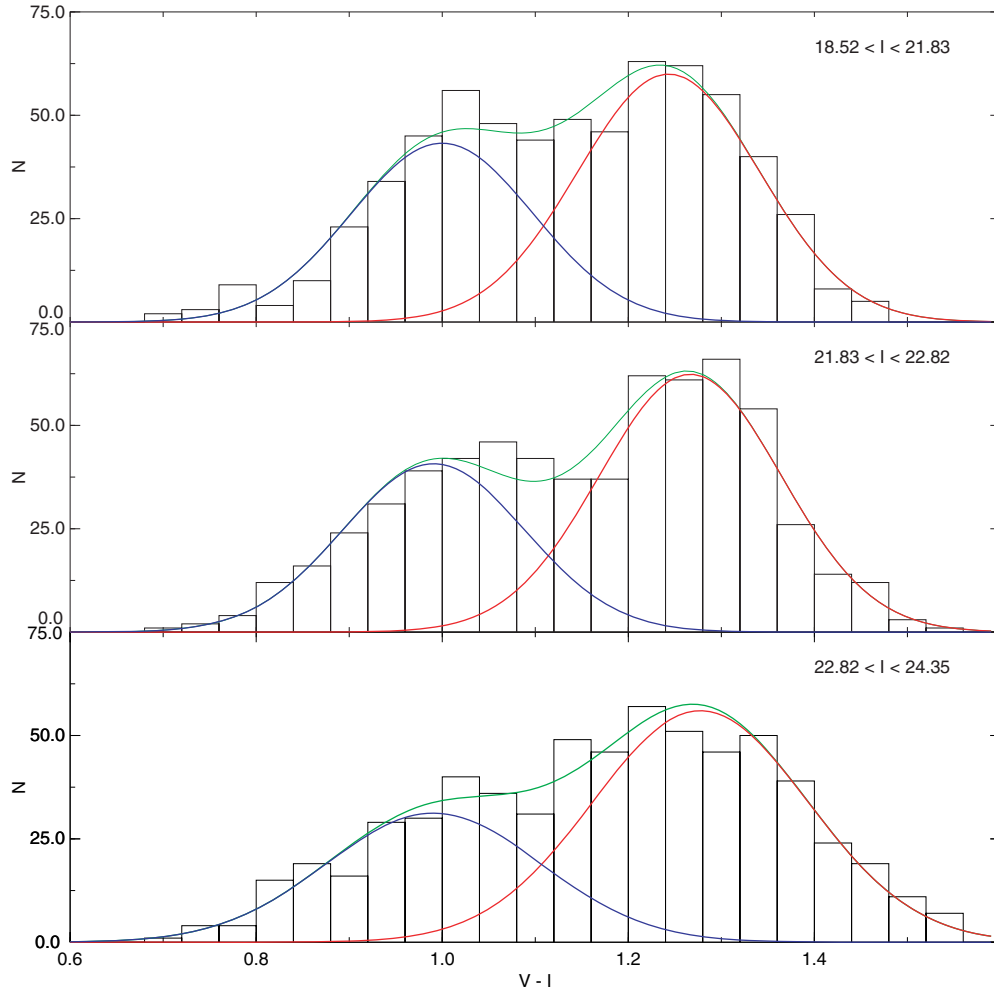
$$\lambda = \frac{L(\theta|x)_{\text{simple}}}{L(\theta|x)_{\text{complex}}}. \quad (2)$$

This provides a quantity between zero and one that tells how likely the simpler model fits the data better. By comparing  $-2 \ln \lambda$  to a  $\chi^2$  distribution with a number of degrees of freedom equal to twice the difference in the number of free parameters between the two models, we can determine the point at which the more complicated model is a sufficient improvement over the simple model to justify the additional free parameters. Therefore, using this test, we can check that our data are truly fit better by the two component model by checking that the  $p$ -value for the unimodal model is small.

One limitation of the KMM method is that outliers in the distribution can significantly bias the results, especially for the heteroscedastic fits (Ashman et al. 1994). These models are biased the most as they can adjust the individual dispersions to larger values that better accommodate the outliers. This in turn will tend to increase the separation of the means of the two modes. As the dispersions of the homoscedastic models are coupled, the weight of an individual outlier is significantly decreased, so many more outliers are needed to create such an effect.

Given that heteroscedastic fits are more influenced by outliers, it is preferred to fit any population that does not obviously have different dispersions with homoscedastic models. The KMM test does not clearly state whether the input data are better fit by a homo- or heteroscedastic model. Bootstrapping the sample can provide an answer, as the value of  $-2 \ln \lambda$  (as given above, using the likelihood of the homoscedastic and heteroscedastic models for the simple and complex cases, respectively) calculated for each bootstrapped sample approximates the underlying  $\chi^2$  distribution (Lo 2008). Therefore, the fraction of bootstrap samples with  $-2 \ln \lambda$  more extreme than that calculated from the observed data is an estimate of the  $p$ -value that the data are truly better fit by a heteroscedastic model.

To determine whether our data are better described by a homo- or heteroscedastic model, we fit the cluster sample using both models and bootstrapped as described above to estimate  $p_{\text{heteroscedastic}}$ . The sample used in this fit and subsequent analysis is restricted to those clusters with color between  $0.7 < V - I < 1.5$  to limit the effect of the largest outliers. This sample of clusters was then fit using both models and bootstrapped as described above to determine the probability that the data are more accurately described by a heteroscedastic model. The resulting dispersion for the homoscedastic model



**Figure 1.** Histograms of the cluster colors in three bins of magnitude, each containing 632 clusters, along with the best fitting KMM component models for the red and blue populations as well as the sum of these two components (shown in green). The top panel shows the brightest clusters, with  $I < 21.94$ . The middle panel shows fainter clusters, with  $21.94 < I < 23.06$ . The bottom panel shows the faintest clusters. The constancy of the peaks in the red and blue populations indicate the absence of any strong tilts in the M87 globular cluster system.

(A color version of this figure is available in the online journal.)

is  $\sigma_{\text{homoscedastic}} = 0.105$  and for the heteroscedastic model  $\sigma_{\text{blue}} = 0.126$  and  $\sigma_{\text{red}} = 0.090$ . The similarity of these dispersions is supported by our statistical test, which gives  $p_{\text{heteroscedastic}} = 0.03$ , excluding the heteroscedastic fit for the entire sample at the 95% confidence level. This does not mean that the underlying color dispersions of the red and blue subpopulations are necessarily identical, only that there is no statistical support for fitting the distributions with different dispersions. Therefore, the remainder of this paper focuses on the results for the homoscedastic models, which we again note are much more robust against the influence of outliers.

As we wish to investigate any trends in bimodality with cluster mass, we need to divide the clusters into luminosity bins. Following previous studies (Strader et al. 2006; Harris et al. 2006), we use the  $I$  magnitude as a tracer of cluster mass. Two types of magnitude bins were used for this study. The first method uses fixed width bins from  $I = 19.5$  to  $I = 24.5$ , using bin widths of 0.5 and 1.0 mag. The mean value of  $I$  is calculated for each bin, which due to the shape of the globular cluster luminosity function, does not fall exactly at the center of the bin range. Unfortunately, the KMM test as originally stated does not directly yield uncertainties for the parameters calculated. Since the algorithm converges quickly,

it is reasonable to estimate these uncertainties by bootstrapping the sample (Bastford et al. 1997). Therefore, the best fitting means, variances, and population ratios were calculated with their associated uncertainties from 100 bootstrapped samples in each magnitude bin. These values are listed in Tables 2 and 3, with the results for the 0.5 mag bins plotted in Figure 2.

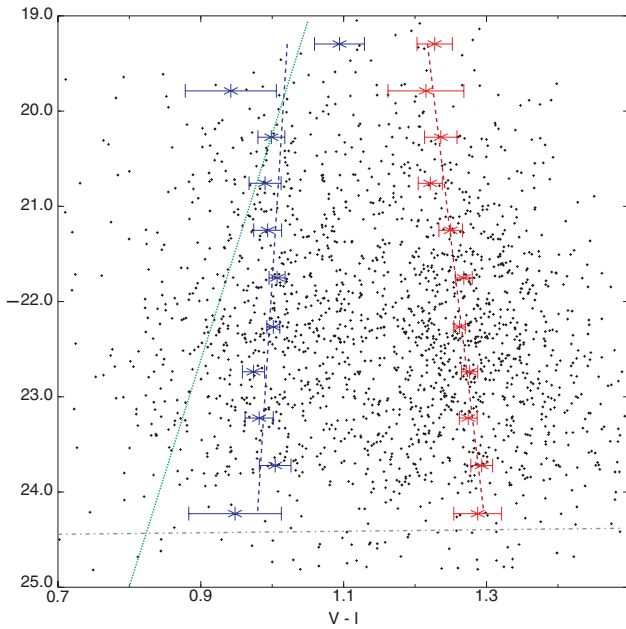
The second binning method uses running samples of 100 clusters. The clusters are sorted in  $I$  magnitude, and the KMM test run on the first 100 points. For the next bin, the brightest cluster in the sample is removed and the next cluster from the sorted list of magnitudes is included, and the process repeated. As these bins contain on average fewer clusters than any of the fixed width bins, the number of outlying points in any of the running bins should also be small. The effect of these outliers can be somewhat mitigated by bootstrapping, as not all samples will retain the discrepant points. However, taking the average values from these bootstrap results allows the outlying points to continue to influence the results. One solution to remove this influence is to use the bootstrap “bumping” procedure of Tibshirani & Knight (1999). For each set of bootstrapped results, the fit with the largest likelihood is retained and the rest discarded. This makes the fitting resistant to outliers, as the small probability of finding such a point given the model ensures that

**Table 2**  
Best Fit Values: 0.5 mag bins/homoscedastic

| $I$    | $\langle V - I \rangle_{\text{blue}}$ | $\sigma_{\text{blue}}$ | $\langle V - I \rangle_{\text{red}}$ | $\sigma_{\text{red}}$ | $\pi_{\text{blue}}$ | $\pi_{\text{red}}$ | $P_{\text{unimodal}}$ | $N_{\text{clusters}}$ |
|--------|---------------------------------------|------------------------|--------------------------------------|-----------------------|---------------------|--------------------|-----------------------|-----------------------|
| 19.295 | $1.094 \pm 0.035$                     | $0.046 \pm 0.009$      | $1.227 \pm 0.025$                    | $0.046 \pm 0.009$     | $0.460 \pm 0.196$   | $0.540 \pm 0.196$  | 0.312                 | 15                    |
| 19.788 | $0.942 \pm 0.064$                     | $0.115 \pm 0.011$      | $1.215 \pm 0.053$                    | $0.115 \pm 0.011$     | $0.419 \pm 0.187$   | $0.581 \pm 0.187$  | 0.269                 | 42                    |
| 20.274 | $0.999 \pm 0.019$                     | $0.088 \pm 0.009$      | $1.236 \pm 0.023$                    | $0.088 \pm 0.009$     | $0.535 \pm 0.082$   | $0.465 \pm 0.082$  | 0.012                 | 90                    |
| 20.758 | $0.990 \pm 0.022$                     | $0.095 \pm 0.009$      | $1.221 \pm 0.017$                    | $0.095 \pm 0.009$     | $0.430 \pm 0.067$   | $0.570 \pm 0.067$  | 0.016                 | 134                   |
| 21.252 | $0.994 \pm 0.020$                     | $0.092 \pm 0.009$      | $1.250 \pm 0.017$                    | $0.092 \pm 0.009$     | $0.432 \pm 0.068$   | $0.568 \pm 0.068$  | 0.000                 | 170                   |
| 21.752 | $1.007 \pm 0.011$                     | $0.092 \pm 0.006$      | $1.268 \pm 0.012$                    | $0.092 \pm 0.006$     | $0.415 \pm 0.043$   | $0.585 \pm 0.043$  | 0.000                 | 262                   |
| 22.267 | $1.002 \pm 0.009$                     | $0.086 \pm 0.005$      | $1.262 \pm 0.008$                    | $0.086 \pm 0.005$     | $0.424 \pm 0.035$   | $0.576 \pm 0.035$  | 0.000                 | 332                   |
| 22.737 | $0.974 \pm 0.015$                     | $0.106 \pm 0.005$      | $1.276 \pm 0.011$                    | $0.106 \pm 0.005$     | $0.325 \pm 0.036$   | $0.675 \pm 0.036$  | 0.000                 | 288                   |
| 23.223 | $0.982 \pm 0.020$                     | $0.112 \pm 0.006$      | $1.274 \pm 0.012$                    | $0.112 \pm 0.006$     | $0.370 \pm 0.042$   | $0.630 \pm 0.042$  | 0.000                 | 270                   |
| 23.722 | $1.004 \pm 0.022$                     | $0.112 \pm 0.008$      | $1.293 \pm 0.015$                    | $0.112 \pm 0.008$     | $0.384 \pm 0.063$   | $0.616 \pm 0.063$  | 0.000                 | 189                   |
| 24.227 | $0.948 \pm 0.065$                     | $0.128 \pm 0.012$      | $1.288 \pm 0.033$                    | $0.128 \pm 0.012$     | $0.262 \pm 0.128$   | $0.738 \pm 0.128$  | 0.059                 | 55                    |

**Table 3**  
Best Fit Values: 1.0 mag bins/homoscedastic

| $I$    | $\langle V - I \rangle_{\text{blue}}$ | $\sigma_{\text{blue}}$ | $\langle V - I \rangle_{\text{red}}$ | $\sigma_{\text{red}}$ | $\pi_{\text{blue}}$ | $\pi_{\text{red}}$ | $P_{\text{unimodal}}$ | $N_{\text{clusters}}$ |
|--------|---------------------------------------|------------------------|--------------------------------------|-----------------------|---------------------|--------------------|-----------------------|-----------------------|
| 19.658 | $0.888 \pm 0.073$                     | $0.111 \pm 0.012$      | $1.175 \pm 0.027$                    | $0.111 \pm 0.012$     | $0.198 \pm 0.122$   | $0.802 \pm 0.122$  | 0.074                 | 57                    |
| 20.564 | $0.995 \pm 0.012$                     | $0.093 \pm 0.007$      | $1.227 \pm 0.015$                    | $0.093 \pm 0.007$     | $0.468 \pm 0.049$   | $0.532 \pm 0.049$  | 0.001                 | 224                   |
| 21.555 | $1.001 \pm 0.010$                     | $0.093 \pm 0.005$      | $1.262 \pm 0.009$                    | $0.093 \pm 0.005$     | $0.427 \pm 0.029$   | $0.573 \pm 0.029$  | 0.000                 | 432                   |
| 22.485 | $0.991 \pm 0.009$                     | $0.097 \pm 0.004$      | $1.269 \pm 0.007$                    | $0.097 \pm 0.004$     | $0.372 \pm 0.024$   | $0.628 \pm 0.024$  | 0.000                 | 620                   |
| 23.428 | $0.990 \pm 0.014$                     | $0.113 \pm 0.005$      | $1.281 \pm 0.010$                    | $0.113 \pm 0.005$     | $0.375 \pm 0.036$   | $0.625 \pm 0.036$  | 0.000                 | 459                   |



**Figure 2.** Color trends with  $I$  magnitude for the globular clusters in these data, based on 0.5 mag width bins with homoscedastic fits. The errors are calculated via bootstrapping. The best fitting trends show effectively no blue tilt ( $Z \propto M^{0.08 \pm 0.05}$ ), and is significantly different from the previously published trend  $Z \propto M^{0.55}$  shown as a dotted line. For reference, the median 90% completeness line is shown as a dot-dashed line.

(A color version of this figure is available in the online journal.)

any sample that contains the outlying point will naturally have a much lower likelihood than a sample that has excluded that point. Each bin was bootstrapped in this way with 200 samples, which should allow the fitting to be resistant to the influence of up to five outliers (Tibshirani & Knight 1999).

The best fitting models for the fixed width bins are shown in Tables 2 and 3. We can see that based on these models, the unimodal description of the cluster colors is strongly ruled out over most of the range in magnitude considered. The only deviations from this trend occur at the very brightest and faintest

bins. These deviations are not surprising, as these bins have a smaller number of clusters. Simulations by Ashman et al. (1994) showed for cases like these, with a small number of objects ( $N \sim 50$ ) and a modest ratio of component mean separation to component variance ( $\Delta\mu = \frac{\mu_2 - \mu_1}{\sigma} \sim 2$ ), there is a high probability of a truly bimodal distribution being unrecognized by the KKM algorithm

Based on these fits, we can see that as we look at bins containing fainter clusters, the variances in the best fitting models increase. This is true for both subpopulations, and shows the effect the decrease in signal to noise has on the scatter in the measured colors. The population fractions for the red and blue fits show that as the bins move to fainter magnitudes, the blue population fraction falls. This is a result of the differences in the luminosity functions of the red and blue clusters and will be discussed in a future paper on the globular cluster luminosity function of this very deep data (C. Z. Waters et al. 2008, in preparation).

#### 4. COLOR–METALLICITY AND MASS–METALLICITY RELATIONS

We use the means determined by KMM for the red and blue clusters as a function of  $I$  magnitude to investigate the existence of any color–magnitude trends in both populations. For each binning method, the best fitting trend is calculated for both the red and blue subpopulations. For the fixed width bins that have errors from bootstrapping, these errors are used to weight the fits. This helps limit the influence from the brightest and faintest bins, which tend to have large errors due to the low number of clusters. The running bins constrain the fit with equal weight, as the large number of these bins reduces the relative influence of any individual bin.

These fits yield the best trend  $(V - I) = a + b \cdot I$ , which can be converted to a mass–metallicity relation of the form  $[\text{Fe}/\text{H}] = k + \alpha \log_{10} \frac{M}{M_{\odot}}$ . We follow Harris (1996) in creating a relation

$$[\text{Fe}/\text{H}] = 5.2267(V - I) - 6.2613 \quad (3)$$

**Table 4**  
Best Fit Trends

| Model                  | $a_{\text{blue}}$ | $b_{\text{blue}}$ | $k_{\text{blue}}$ | $\alpha_{\text{blue}}$ | $a_{\text{red}}$ | $b_{\text{red}}$ | $k_{\text{red}}$ | $\alpha_{\text{red}}$ |
|------------------------|-------------------|-------------------|-------------------|------------------------|------------------|------------------|------------------|-----------------------|
| Fixed 0.5 mag          | 1.184             | −0.008            | −1.749            | 0.110                  | 0.908            | 0.016            | 1.666            | −0.209                |
| Fixed 1.0 mag          | 1.029             | −0.002            | −1.197            | 0.021                  | 0.824            | 0.020            | 1.972            | −0.258                |
| 100 Point running bins | 1.199             | −0.009            | −1.728            | 0.114                  | 1.015            | 0.011            | 1.231            | −0.144                |

**Table 5**  
Best Fit Trends Above Turnover

| Model                  | $a_{\text{blue}}$ | $b_{\text{blue}}$ | $k_{\text{blue}}$ | $\alpha_{\text{blue}}$ | $a_{\text{red}}$ | $b_{\text{red}}$ | $k_{\text{red}}$ | $\alpha_{\text{red}}$ |
|------------------------|-------------------|-------------------|-------------------|------------------------|------------------|------------------|------------------|-----------------------|
| Fixed 0.5 mag          | 1.135             | −0.006            | −1.538            | 0.080                  | 0.906            | 0.016            | 1.672            | −0.210                |
| Fixed 1.0 mag          | 1.009             | −0.001            | −1.118            | 0.009                  | 0.733            | 0.024            | 2.338            | −0.314                |
| 100 Point running bins | 0.928             | 0.004             | −0.596            | −0.054                 | 0.794            | 0.021            | 2.118            | −0.278                |

based on observations of Milky Way globular clusters, and use a constant mass-to-light ratio of  $M/L = 3$  for all clusters. Table 4 shows the best fitting values for these trends for each of the binning methods. It is clear that for the blue subpopulation, all three binning methods yield similar best fits. To determine our formal best fit models, we average the three binning methods, and take the scatter in these values as our expected uncertainty in the fit due to the binning. Over the full range of clusters, we find an average best fit for the blue clusters of  $Z \propto M^{0.08 \pm 0.05}$ . To offer a more direct comparison with the tilts measured in previous studies, we have also fit only those clusters brighter than the turnover ( $I \sim 22.5$ ). These fits are given in Table 5. We find a similarly small  $Z \propto M^{0.01 \pm 0.07}$  relation with these limits, and as they are only marginally different from the fits over the full magnitude range, we do not plot these trends separately.

It is essential to characterize how well the formal uncertainties in the fit reflect the true uncertainty in the slope of the mass–metallicity relation. In order to test the accuracy of the calculated fits, we constructed simulated color–magnitude diagrams with a known tilt in the blue clusters. Each cluster in our data sample was checked against the 0.5 mag homoscedastic fixed width bins to determine if it was a likely member of the blue subpopulation. All of these blue clusters then had their color shifted, such that

$$(V - I)_{\text{new}} = (V - I)_{\text{old}} + (a_{\text{trend}} - a_{\text{blue}}) + I(b_{\text{trend}} - b_{\text{blue}}), \quad (4)$$

where  $a_{\text{blue}}$  and  $b_{\text{blue}}$  are the best fitting trend in the blue clusters, and  $a_{\text{trend}}$  and  $b_{\text{trend}}$  define the trend we are adding. This method therefore preserves the cluster  $I$  magnitude, as well as the distribution of points around the best fit trend. These simulated color–magnitude diagrams are then run through the KMM test, and the best calculated trend found. By comparing the difference in the input and output slopes, we can estimate how large we can expect the errors to be in our mass–metallicity fits.

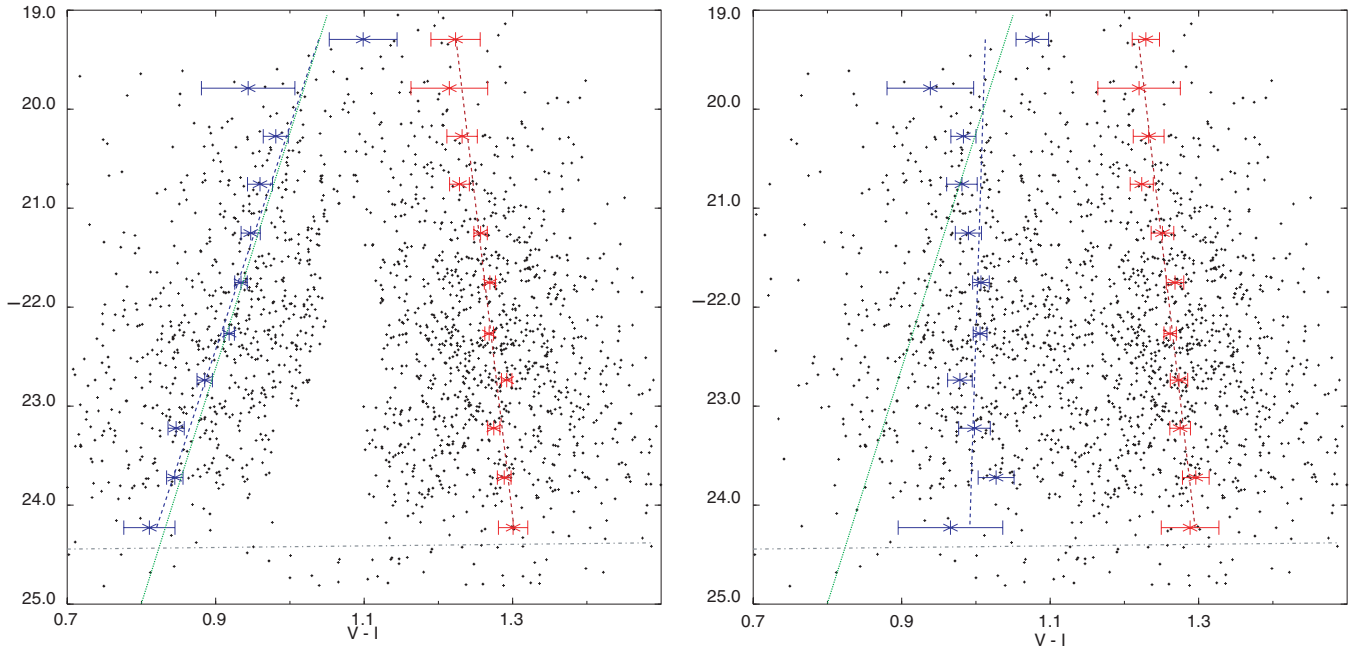
Two simulated trends were used for this test, representing both extremes in the blue tilt. First, a strong tilt of  $Z \propto M^{0.55}$  like that claimed in some analyses of shallower data was added, the results of which are shown in Figure 3. The best fitting trend for this simulation is  $Z \propto M^{0.58 \pm 0.05}$  ( $Z \propto M^{0.44 \pm 0.05}$  including only those clusters brighter than the turnover), suggesting that the small formal errors calculated are not significant underestimates. As we recover this large input trend with high accuracy, it is clear that were such a trend truly present in our data, we would have no difficulty in detecting it. The main reason that such a large trend is so easy to recover is that it must have a large separation between the means of the red and blue

subpopulations at faint magnitudes. This separation is much larger than the color dispersion of the clusters in our sample, which creates a gap between the two groups. Such a gap allows the KMM test to easily group the clusters, providing an excellent fit.

The creation of a large gap is not an issue for the second simulation, in which the slight blue trend is completely removed, with a simulated trend of  $Z \propto M^{0.00}$ . As there is only a slight shift in the cluster colors, the degree of blending between the red and blue clusters should be nearly identical to the real cluster data. The calculated trend for this simulation is  $Z \propto M^{-0.06 \pm 0.06}$  ( $Z \propto M^{0.07 \pm 0.06}$  including only those clusters brighter than the turnover). These tests on simulated data indicate that our fits to the color–magnitude trends in the globular cluster subpopulations accurately reflect the underlying trend in the data with small well-understood uncertainties. The specific fitting uncertainties in the fit to the slope have a maximum of 0.06, which occurs for small real slopes. This uncertainty decreases to even smaller values when the input slope is large. However, our method does not directly address any possible systematic issues in the measurement of the colors and magnitudes. While our measurements are based on a careful analysis of this extraordinarily deep data, it is important to consider all possible systematic effects in the measurement.

Our tests with artificial data sets indicate that we accurately recover the simulated slopes with a maximum uncertainty of 0.06. These tests provide an excellent way to assess the uncertainties in the fits to the data. However, this approach does not directly address potential systematic biases in the measurements of the colors and magnitudes. The most likely source of any systematic error in these is due to the aperture corrections. The extraordinary depth of our data allows a much more accurate determination of the sizes of the globular clusters, which directly leads to a much more accurate determination of the total magnitude of the globular clusters. These sizes are influenced by the choice of PSF used, and we simulated the effects of different PSFs on the resulting aperture corrections. Our simulations show that in this well sampled data, in which the globular clusters are spatially resolved, large differences in the magnitudes (of up to 0.1 mag) only occurred with PSFs that were inconsistent with our Anderson & King (2006) PSFs and were not good fits to the single unsaturated star in the image. Using such poor PSFs can create tilts of up to  $\pm 0.2$  in the final mass–metallicity relation for both subpopulations on top of the modest tilts seen in our fitting. This extreme allows us to note that any possible systematic effect due to the aperture correction must be smaller than this level, and that both of the slopes found in our data fall within this range of  $\alpha < |0.2| \pm 0.06$ .





**Figure 3.** Color–magnitude diagram of a set of simulated clusters with a mass–metallicity trend defined to be  $Z \propto M^{0.55}$  (left). As shown by the best fitting line ( $Z \propto M^{0.58 \pm 0.05}$ ), if the clusters truly had such a trend, it would be easily identifiable in our data. The right panel shows the results of fitting simulated clusters with no mass–metallicity trend at all. The best fitting line for this data is also shown ( $Z \propto M^{0.06 \pm 0.06}$ ), which falls within the uncertainty of our fits. (A color version of this figure is available in the online journal.)

## 5. DISCUSSION

Our 50 orbit ACS observation of M87 shows no significant relation between the colors of the blue metal-poor clusters and their luminosity, with a formal best fit of  $Z \propto M^{0.08}$  from  $19.5 < I < 24.5$ , and a conservative upper limit on the slope of any relation in the M87 globular cluster population of  $\alpha < |0.20| \pm 0.06$  including systematic effects. A similarly small trend is found when the fitting is restricted to only the bright clusters above the luminosity function turnover ( $19.5 < I < 22.5$ ). This absence of any significant mass–metallicity relation rules out some earlier claims of such a trend from much shallower data. Some earlier work investigated the mass–metallicity relation of metal-poor globular cluster populations in much shallower images of nearly ellipticals, including single orbit data for M87 and several other bright Virgo ellipticals (Strader et al. 2006; Mieske et al. 2006), single orbit pointings of NGC 4594 (Spitler et al. 2006), and several distant ellipticals with longer exposures and thus similar signal to noise as the nearby single orbit observations (Harris et al. 2006). These shallower studies suggested that the metal-poor globular cluster populations of some of these galaxies, including M87, had a blue tilt and inferred a mass–metallicity relation of about  $Z \propto M^{0.55}$ . These results are clearly not confirmed in our 50 orbit data, which places an upper limit on any mass–metallicity relation that is much smaller than this strong trend. There are also several ground-based studies of the color–magnitude trends in globular cluster systems that find a variety of results, including both blue tilts (Forte et al. 2007; Wehner et al. 2008) as well as red tilts (Bassino et al. 2008; Lee et al. 2008). Forte et al. (2007) examined M87, and found evidence for mass–metallicity relation of  $Z \propto M^{0.44}$ , which although smaller than the claims from space-based studies, is still inconsistent with the lack of a tilt in our much deeper data. This emphasizes the difficulty in accurately determining color tilts from data with low signal to noise and poor spatial resolution.

The clear absence of a significant mass–metallicity effect in our data contrary to the very strong effect in single orbit data for galaxies like M87 and in data with similar signal to noise at larger distance, highlights the need for very deep observations to address this question. Kundu (2008) has investigated this question in detail. We do not reproduce this extensive work here, but note that Kundu (2008) identifies two major issues that arise in single orbit data or data with similar signal to noise for the bulk of the clusters. First, such data lacks the depth necessary to accurately follow the sizes of globular clusters with magnitude. This lack of size discrimination will cause size-dependent photometric errors. Second, the error bars for detections are not symmetric in color and magnitude. These two effects are shown to be able to produce apparent color–magnitude trends of the level seen in the lower signal to noise data from an underlying distribution with no trend at all (Kundu 2008).

The absence of a significant mass–metallicity relation for globular clusters also suggests a fundamental difference between globular clusters and galaxies, as galaxies have a well-known mass–metallicity relation. Specifically, using Sloan Digital Sky Survey (SDSS) data, Tremonti et al. (2004) found a mass–metallicity relation of  $Z \propto M^{0.3}$  for a very large sample of galaxies. This was extended to nearby dwarf irregular galaxies by Lee et al. (2006), who found a mass–metallicity relation consistent with that for the more massive galaxies. This dwarf galaxy sample extends down to  $M \sim 10^6$ – $10^7$ , the range where the most massive globular clusters are found, and thus is suggestive of a fundamental difference between globular clusters and galaxies in their mass–metallicity relations. Such a difference likely reflects differences in the formation histories of galaxies and globular clusters. If globular clusters experience significant self-enrichment, then the more massive clusters will appear more metal-rich, since their greater mass will enable them to retain more metals from earlier generations of stars.



Any self-enrichment of metals that affect the broadband colors will make the more massive clusters redder, and produce color-luminosity and mass-metallicity relations within a globular cluster system. Therefore, the weakness or absence of the observed color-luminosity and mass-metallicity relations for globular clusters thus sets a limit on the role of self-enrichment, and is a key target for future models of globular cluster formation.

A natural explanation for the difference between the mass-metallicity relations of globular clusters and galaxies is that globular clusters form without extensive mass distributions or dark matter halos. Without such halos, globular clusters are unable to retain the material produced by their massive stars, preventing the formation of subsequent generations of metal-enriched stars. Such a picture is consistent with the compact, dense nature of globular clusters which implies short formation timescales, and with models in which globular cluster formation is a rapid, dynamic process in a high pressure starburst environment as suggested by observations of globular cluster formation in the local universe (Ashman & Zepf 2001; Elmegreen & Efremov 1997). In contrast then, galaxies tend to form over time within larger dark-matter dominated structures that help retain metals to be incorporated in subsequent generations to produce the observed galaxy mass-metallicity relation.

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