THE EFFECT OF COMPOSITION ON NOVA IGNITIONS

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ABSTRACT

The accretion of hydrogen-rich matter onto C/O and O/Ne white dwarfs (WDs) in binary systems may lead to unstable thermonuclear ignition of the accreted envelope, triggering a convective thermonuclear runaway and a subsequent classical, recurrent, or symbiotic nova. Prompted by uncertainties in the composition at the base of the accreted envelope at the onset of convection, as well as the range of abundances detected in nova ejecta, we examine the effects of varying the composition of the accreted material. For carbon mass fractions $< 2 \times 10^{-3}$ and the high accretion rates $\ge 10^{-9} M_{\odot}$ yr⁻¹ that we consider, we find that carbon, which is usually assumed to trigger the runaway via proton captures, is instead depleted and converted to ¹⁴N. Additionally, we quantify the importance of ³He, finding that convection is triggered by ³He+³He reactions for ³He mass fractions $> 2 \times 10^{-3}$. These different triggering mechanisms, which occur for critical abundances relevant to many nova systems, alter the amount of mass that is accreted prior to a nova, causing the nova rate to depend on the composition of the material accreted from the companion. Upcoming deep optical surveys such as Pan-STARRS-1, Pan-STARRS-4, and the Large Synoptic Survey Telescope may allow us to detect the dependence of nova rates on accreted composition. Furthermore, the burning and depletion of 3 He with a mass fraction of 10^{-3} , which is lower than necessary for triggering convection, still has an observable effect, resulting in a pre-outburst brightening in disk quiescence to > L_{\odot} and an increase in effective temperature to 6.5×10^4 K for a 1.0 M_{\odot} WD accreting at $10^{-8} M_{\odot}$ yr⁻¹.

Key words: accretion, accretion disks – binaries: close – instabilities – novae, cataclysmic variables – nuclear reactions, nucleosynthesis, abundances - white dwarfs

1. INTRODUCTION

White dwarfs (WDs) in cataclysmic variable (CV) and symbiotic binary systems accrete H-rich material from main sequence and red giant donors, respectively, with accretion rates of $\dot{M} = 10^{-11}$ to $10^{-7} M_{\odot} \text{ yr}^{-1}$ (Warner 1995). As the accreted envelope gains mass, compression of the material at the base of the layer leads to a temperature increase, eventually triggering H-burning. Depending on M, the WD mass, and the composition of the accreted material, the eventual outcome will either be steady thermally stable H-burning (Sienkiewicz 1975, 1980; Fujimoto 1982b; Iben 1982; Paczynski 1983; Shen & Bildsten 2007; Nomoto et al. 2007), or a convective thermonuclear runaway (Rose 1968; Starrfield et al. 1972; Fujimoto 1982a; Prialnik 1986; José et al. 1993; Cassisi et al. 1998; Truran 2002; Yaron et al. 2005), observable as a classical, recurrent, or symbiotic nova when the convective zone nears the WD photosphere. In this work, we define the ignition of the nova as the first instance of convection, and we will refer to them interchangeably hereafter. Note that significant nuclear burning can occur prior to convection; e.g., the very low metallicity models of José et al. (2007) have envelope temperatures > 3×10^7 K for ~ 10^4 yr before nova ignition as we define it.

It was first recognized from numerical simulations (e.g., Starrfield et al. 1972) that the mass fraction of CNO isotopes, Z_{CNO} , in the burning layers during a nova outburst must be 10-50 times higher than the solar value in order to eject sufficient mass and produce a light curve similar to those observed for fast novae. Abundance measurements confirm this metal enhancement in the ejecta of fast novae (for observational summaries, see Gehrz et al. 1998; Hachisu & Kato 2006). This degree of enrichment cannot be explained by accretion

from an evolved donor or nucleosynthesis during the nova outburst (e.g., Truran & Livio 1986), thus some mechanism of core dredge-up and mixing with the envelope must be invoked.

There is ongoing debate over the effectiveness of proposed mixing mechanisms, which we detail in Section 2. In particular, it is unclear whether convection is initiated above or below the core-envelope interface in C-poor or C-rich material. With this in mind, our work examines the effect of the accreted composition on the pre-ignition characteristics of nova systems, assuming no CNO enrichment prior to convection. We calculate ignition masses and pre-ignition luminosities for models with a large range of metallicities centered around solar (Z =0.1–5.0 Z_{\odot}), nonzero ³He mass fractions ($X_3 = 10^{-4}$ to 0.005), high accretion rates ($\dot{M} = 10^{-9}$ to $3 \times 10^{-7} M_{\odot}$ yr⁻¹), and a core mass range of $M = 1.0-1.35 M_{\odot}$.

In the Appendix, Section 3, and Section 4, we use analytical approximations to motivate the more exact numerical study described in Section 5. Below a critical accreted carbon mass fraction $X_{12} \simeq 2 \times 10^{-3}$, which is coincidentally near the solar value, we find that ¹²C is depleted and converted to ¹⁴N prior to unstable ignition, so that convection is triggered by proton captures onto ¹⁴N. We also find that convection is triggered by ³He+³He reactions for ³He mass fractions $X_3 \gtrsim 2 \times 10^{-3}$. These different nova triggers change the pre-outburst luminosity of the nova system, which we detail in Section 5.5. The ignition mass is also affected by the triggering mechanism, which results in a previously unconsidered dependence of galactic nova rates on composition. This effect is especially relevant given the upcoming flood of data from optical transient surveys such as Pan-STARRS-1, Pan-STARRS-4, and the Large Synoptic Survey Telescope, which will measure nova rates in external

galaxies with greater accuracy than currently available. We speculate on the observational consequences and summarize our work in Section 6.

2. MOTIVATION AND JUSTIFICATION FOR OUR STUDY

Core-envelope mixing models differ on the mechanism by which core material is brought into the accreted layer. Chemical diffusion (Prialnik & Kovetz 1984; Kovetz & Prialnik 1985; Prialnik & Kovetz 1995; Yaron et al. 2005; Iben et al. 1992) and shear mixing caused by differential rotation of the accreted material (Kippenhahn & Thomas 1978; Livio & Truran 1987; Kutter & Sparks 1989; Alexakis et al. 2004; MacDonald 1984) result in pre-convective penetration of a small amount of hydrogen into the underlying material and vice versa. If the material below the layer is C-rich, $p+{}^{12}C$ reactions trigger convection, which homogenizes the envelope and the entrained core material. These pre-convective enrichment models differ from mechanisms in which convection is triggered above the core in accreted material whose composition is determined by that of the donor star and is thus relatively C-poor. The introduction of core material into the envelope for these convective enrichment models is caused by the convective motion itself, either via convective overshoot into the core (Woosley 1986) or shear mixing induced by the convective eddies (Glasner et al. 1997; Rosner et al. 2001).

As of yet, no mixing mechanism has definitively proven itself successful in explaining the enrichments of all novae. For example, the multicycle diffusion studies of Prialnik & Kovetz (1995) and Yaron et al. (2005) are initiated with matter accreted directly onto naked C/O cores, yet the accreting WD may be O/Ne in as many as 1/3 of all observed novae (Truran & Livio 1986; Ritter et al. 1991; Livio & Truran 1994; Gil-Pons et al. 2003). For these systems, diffusion would not lead to the initiation of convection below the accreted layer and subsequent core dredge-up because the underlying material is not C-rich. The studies of Livio & Truran (1987), Fujimoto (1988, 1993), and Piro & Bildsten (2004) rule out any significant differential rotation between the accreted layer and the core, which casts doubt on accretion-induced shear mixing mechanisms. Kercek et al. (1998, 1999) find that convective overshoot and shearing do not sufficiently enrich the envelope to produce a fast nova, although possible problems with their boundary conditions are pointed out by Glasner et al. (2005). Moreover, some recurrent novae, which are novae with recurrence times ~ 30 vr. do not show overabundances of metals in their ejecta (Williams et al. 1981; Williams 1982; Warner 1995; Hachisu & Kato 2001), possibly due to a large helium buffer above the core resulting from the burning of unejected material from previous novae.

Thus, it is unclear how much the envelope will be enriched in metals prior to nova ignition for $\dot{M} \ge 10^{-9} M_{\odot} \text{ yr}^{-1}$. Some nova studies that include the accretion phase assume that the accreting envelope is pre-enriched by the core and consists of up to 50% core material by mass (e.g., José & Hernanz 1998; Starrfield et al. 1998). However, the uncertainties involved in the mixing mechanisms coupled with the lack of observed metal enrichment in recurrent novae lead us to examine the consequences of assuming no C-enrichment in the accreted envelope prior to the onset of convection. There are several previous studies that also follow this treatment, but none sufficiently samples the full parameter space in which we are interested: Starrfield et al. (1985, 1988) and Truran et al. (1988) consider Z = 0.02 accretion onto $M \ge 1.35 M_{\odot}$ WDs; the models of Starrfield et al. (2000) have metallicity $Z = 10^{-3}$ or 0.02, as motivated by novae in the Large Magellanic Cloud (LMC), with lower accretion rates $\leq 10^{-9} M_{\odot} \text{ yr}^{-1}$ than what we study; Piersanti et al. (2000) examine accretion with three metallicities ($Z = 0.02, 10^{-3}$, and 10^{-4}) onto WDs with masses $< 0.68 M_{\odot}$, lower than our parameter range; and José et al. (2007), in their study of novae in primordial binaries, have only solar and very sub-solar metallicity $1.35 M_{\odot}$ models with $Z = 0.02, 2 \times 10^{-6}$, or 10^{-7} , with $\dot{M} = 2 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$, which is lower than our range. Moreover, none of these studies consider the effect of ³He, which can play a dominant role in triggering the nova (Shara 1980; Townsley & Bildsten 2004).

In our study, we assume that the effect of chemical diffusion is negligible. This assumption, and thus our results, are invalid if the material directly below the accreted layer is C-rich. However, as we have described above, many nova systems exist in which the underlying material is C-poor and diffusion is indeed negligible. Our results only apply to these systems.

3. CARBON DEPLETION PRIOR TO UNSTABLE IGNITION

If the accretion rate in a CV is lower than the minimum rate for stability, the result will be a hydrogen shell flash. In this section and Section 4, we use the results of analytical approximations detailed in the Appendix to estimate the effect of composition on the ignition conditions for these thermonuclear novae. These rough calculations serve to motivate and to aid in explaining the results of the more exact numerical study described in Section 5.

With the assumptions of an ideal gas equation of state, roughly solar abundances, radiative diffusion with Kramers' opacity, and a constant luminosity atmosphere, the temperature at the base of the accreting layer is

$$T_b = 1.4 \times 10^7 \,\mathrm{K} \left(\frac{\dot{M}_{-8}\rho_3^2}{M_1}\right)^{2/11},$$
 (1)

where ρ_3 is the density at the base of the envelope ρ_b in units of 10^3 g cm⁻³, \dot{M}_{-8} is the accretion rate in units of 10^{-8} M_{\odot} yr⁻¹, and $M_1 = M/M_{\odot}$. The bottom of the layer follows the trajectory given by Equation (1) until nuclear burning becomes comparable to compressional heating, i.e., when $L_{\rm nuc} \sim L_{\rm comp}$, where $L_{\rm nuc}$ is the luminosity produced by nuclear burning, and L_{comp} is the luminosity released by compression of the accreted material. For high accretion rates $\ge 10^{-9} M_{\odot} \text{ yr}^{-1}$, H-burning occurs via CNO reactions when base conditions reach $T_b \simeq 2 \times 10^7$ K and $\rho_b \simeq 10^3$ g cm⁻³ (ignoring ³He-burning). If the accreting material has near-solar isotopic ratios, the most relevant isotope is ¹²C, since proton captures onto ¹⁴N are slower than onto ¹²C, and ¹⁶O does not participate in the CNO cycle at these temperatures. Moreover, p + p reactions are unimportant at $T_b \simeq 2 \times 10^7$ K because the lifetime of a proton with respect to self-burning is $\simeq 10$ times longer than with respect to consumption by ¹²C nuclei. Thus, proton captures onto ¹²C will be the first non-negligible reaction. These are quickly followed by the β -decay of ¹³N (with a half-life of $\tau_{1/2} = 603$ s) and proton captures onto ${}^{13}C$ ($\simeq 4$ times more rapid than onto 12 C), so that we approximate the first nuclear reactions of interest as the conversion of $^{12}\mathrm{C}$ to $^{14}\mathrm{N}$ at the $^{12}\mathrm{C}$ proton capture rate. This reaction chain releases a specific energy $X_{12}E_{12} = 8.8 \times 10^{14} (X_{12}/10^{-3}) \text{ erg g}^{-1}.$

It is often stated that the energy released by the conversion of 12 C to 14 N triggers the thermonuclear runaway in novae (e.g.,

Fujimoto 1982b; Townsley & Bildsten 2004; José et al. 2003; José 2005; José & Hernanz 2007). However, we show here that under some conditions this reaction will not release enough heat to trigger unstable ignition conditions before the ¹²C is depleted. In this case, all available ¹²C converts to ¹⁴N, which will ignite later at a larger pressure and temperature. Carbon depletion occurs when the accretion and burning timescales are comparable, with the C-burning timescale given by $t_{12} = X_{12}E_{12}/\epsilon_{12}$, where ϵ_{12} is the rate of energy generation from conversion of ¹²C to ¹⁴N. The condition for stable ¹²C depletion can be approximated as

$$X_{12} < 3 \times 10^{-3} \frac{T_b}{2 \times 10^7 \,\mathrm{K}} \frac{\chi_{\mathrm{cool}}}{8.5} \frac{15}{\chi_{12}},$$
 (2)

where the logarithmic temperature dependence of the burning or cooling rate at constant pressure is $\chi \equiv \partial \ln \epsilon / \partial \ln T |_P$, and the cooling rate is approximated as $\epsilon_{\text{cool}} \sim L_{\text{comp}}/M_{\text{env}}$, where M_{env} is the envelope mass.

Thus, for mass fractions below a critical value, coincidentally near the solar mass fraction of 2.2×10^{-3} , carbon will deplete before triggering a nova. This value is certainly relevant for novae in systems with low-metallicity donors. Furthermore, low ¹²C/¹⁴N ratios can occur when mass transfer has revealed a CNO-processed core (Schenker et al. 2002 and references therein). In these cases, the carbon mass fraction of the accreted material will be well below the solar value because proton captures onto ¹⁴N are the slow step of the CNO cycle, and thus the donor's CNO nuclei are mostly in the form of ¹⁴N, resulting in accreted carbon mass fractions $\sim 10^{-4}$ or lower (Schenker et al. 2002). For high M CV systems, evolved donors such as these are likely common. A population synthesis calculation by Podsiadlowski et al. (2003) finds that the majority of CVs with orbital periods $P_{\rm orb} > 5$ hr have an evolved secondary. Observationally, a study of UV line flux ratios of CVs both above and below the period gap (Gänsicke et al. 2003) concludes that as much as $10\overline{\sqrt{-15\%}}$ of their sample might have evolved donors with anomalously low ¹²C/¹⁴N abundance ratios due to CNO processing.

Figure 1 shows conditions for carbon depletion (short-dashed line) and stable versus unstable C-burning with varying accreted metallicities (dotted lines) for a 1.0 M_{\odot} WD. The region to the left of the dashed-dotted line has Fermi energy $E_F < 3k_BT$, so that it is mildly degenerate or nondegenerate. Free-free opacity dominates in the region of interest. Also shown are the rising temperature and density at the base of accreting layers given by Equation (1) with $\dot{M} = 10^{-7}$ and $10^{-8} M_{\odot} \text{ yr}^{-1}$ (solid lines). For mass fractions lower than near-solar, carbon depletes before it ignites unstably and is instead converted to ¹⁴N. In these cases, the base of the layer continues to become hotter and denser until it reaches burning conditions for proton captures onto ¹⁴N (longdashed line). At the temperatures and densities corresponding to these Ms, the CNO cycle as a whole is thermally unstable, and ignition is inevitable. However, because the layer must be hotter and denser to burn the 14N, more material must be accreted before ignition than in the case of carbon ignition.

4. THE SIGNIFICANCE OF ³He

The possible importance of ³He for novae was first studied by Schatzmann (1951) in the context of a theory of novae powered by thermonuclear detonations. MacDonald (1983) and Iben & Tutukov (1984) noted that the presence of ³He can decrease the envelope mass needed for nova ignition, and Shara (1980) and



Figure 1. Analytical ¹²C ignition and depletion curves for a 1.0 M_{\odot} WD. The dotted lines demarcate the boundary between stable $p+^{12}$ C burning (lower-left region) and unstable ignition (upper-right region) for the given accreted metallicities. The short-dashed line, which is independent of M and Z, shows where the accretion time $t_{acc} = M_{env}/\dot{M}$ is equal to t_{12} ; above this line, ¹²C has been depleted. The solid lines follow the trajectories at the base of envelopes accreting at $\dot{M} = 10^{-7}$ and $10^{-8} M_{\odot}$ yr⁻¹; as more material accumulates, the base becomes hotter and denser, and the lines move to the upper-right, where carbon either ignites unstably or depletes. If carbon is depleted, the base continues to heat and compress until ¹⁴N begins to burn and the full CNO cycle is ignited (long-dashed line). To the left of the dashed-dotted line, the base is mildly degenerate or nondegenerate.

Townsley & Bildsten (2004) looked more closely at its role in triggering novae. Although these studies found that ³He can play a large role in the onset of a nova, its effects have not yet been quantified in detail. Note that no observations of ³He in a CV or nova system have been performed, although measurements of planetary nebulae have found the number abundance ratio of ³He to hydrogen to be as high as 2×10^{-3} (Balser et al. 1997, 2006).

For typical nova conditions, ³He-burning takes place via ³He(³He, 2*p*)⁴He. The next-fastest reaction that consumes ³He is ³He+⁴He, which has an unscreened reaction rate that is a factor of ~ 10³ slower for $X_3 = 10^{-3}$, and thus ³He+³He is the only reaction to consider. To gauge the importance of ³He-burning for nova ignitions, we compare its relevant characteristics to proton captures onto ¹²C, the assumed trigger for nova systems with $\dot{M} \ge 10^{-9} M_{\odot} \text{ yr}^{-1}$. The energy per ³He self-reaction is 12.9 MeV, slightly larger than the 11.0 MeV released by the reaction chain that converts ¹²C to ¹⁴N. Moreover, the ³He unscreened reaction timescale, i.e., the e-folding lifetime for ³He nuclei, defined for a generic isotope *i* as $n_i/t_i = dn_i/dt_i$, is much shorter than that of ¹²C. The ratio of timescales is

$$\frac{t_{12}}{t_3} \sim 10^3 \, \frac{X_3}{X_{\rm H}} \, \exp\left(\frac{6.56}{T_7^{1/3}}\right),\tag{3}$$

which is roughly a factor of 200 for 2×10^7 K, $X_3 = 10^{-3}$, and hydrogen mass fraction $X_{\rm H} = 0.75$. Thus, for $X_3 > 10^{-5}$, ³He nuclei will begin burning via self-reactions before ¹²C nuclei have a chance to capture protons.

Since the ³He reaction releases a similar amount of energy and has a similar temperature dependence to that of the ¹²C proton capture, we must also consider the possibility of stable ³He depletion prior to nova ignition. An analysis like that of the previous section yields the same critical mass fraction $\simeq 3 \times 10^{-3}$ as in Equation (2). Again coincidentally, this critical mass fraction for ³He depletion is in the neighborhood of the expected value for mass-transferring binary systems. As mass loss uncovers deeper parts of the donor star, material that has been processed by H-burning can make its way to the surface. D'Antona & Mazzitelli (1982) and Iben & Tutukov (1984) find that the mass fraction of accreted ³He can reach values as high as 4×10^{-3} during the evolution of systems with lowmass donors, and thus the possibility exists for ³He to trigger a nova prior to depleting. For this reason, Townsley & Bildsten (2004) included calculations with $X_3 = 0.001$ and 0.005. For $\dot{M} > 3 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$, they found that ignitions were triggered by ³He, with the ignition mass M_{ign} roughly decreasing by a factor of 2 when increasing X_3 from 0.001 to 0.005.

5. NUMERICAL SIMULATION

The approximations that we have made in the analytical work of the Appendix, Sections 3 and 4, and the proximity of the critical ¹²C and ³He abundances to relevant solar and CV abundances motivate a more exact analysis. In particular, electron degeneracy pressure, other opacities, the exchange of heat between the envelope and core, and the term $\partial s/\partial t|_P$ that is neglected in Equation (A3) must be included. Moreover, the one-zone approximation of setting $\partial L/\partial M \sim L/M_{env}$ is problematic, because burning occurs in a narrow layer and is certainly not a linear function of the whole atmosphere.

5.1. Model

We utilize a time-dependent explicit Runge–Kutta code for a one-dimensional grid of 100 zones covering a pressure range of 3×10^{17} to 3×10^{20} dyne cm⁻². Since the base of the envelope at ignition is typically between 3×10^{18} and 3×10^{19} dyne cm⁻², this choice of zoning gives greater than a factor of 10 range in pressure above and below the region of interest. For the 1.35 M_{\odot} model with $\dot{M} = 10^{-9} M_{\odot}$ yr⁻¹, the pressure boundaries were changed to 10^{18} to 10^{21} dyne cm⁻² because the accreted layer reached a depth that was too close to 3×10^{20} dyne cm⁻². The zones are spaced logarithmically in pressure (thus there are roughly 33 zones per decade of pressure) to better resolve the accreted layer and to avoid over-resolution of the bottom zones. The layer is spherically symmetric, appropriate for the depths of interest if there is negligible differential rotation (Fujimoto 1988, 1993; Piro & Bildsten 2004), and plane-parallel, a good approximation as discussed in the Appendix.

The thermal evolution of the core during CN cycles was considered in detail by Townsley & Bildsten (2004), who found that heating and cooling during the nova cycle results in an equilibrium core temperature, $T_{c,eq}$.⁴ Thus, for our models, the initial thermal profile prior to accretion is assumed to be a radiative-zero solution that gives a core temperature $T_c = T_{c,eq}$. The equilibrium T_c depends on the accreted composition, but, in order to limit the parameters of this study, a single representative T_c is used for each M and \dot{M} model; these are extrapolated from Townsley & Bildsten (2004) and D. M. Townsley (2007, private communication) and are shown in Table 1. Fortunately,

Table 1	
Model Core Temperatures	

Mass (M_{\odot})	$\dot{M} (M_{\odot} \text{ yr}^{-1})$	T_c (K)
1.0	10 ⁻⁹	8.99×10^{6}
1.0	10^{-8}	1.10×10^{7}
1.0	10^{-7}	1.45×10^{7}
1.0	3×10^{-7}	1.70×10^{7}
1.2	10^{-9}	9.18×10^{6}
1.2	10^{-8}	1.20×10^{7}
1.2	10^{-7}	1.80×10^{7}
1.2	3×10^{-7}	2.29×10^{7}
1.35	10^{-9}	9.25×10^{6}
1.35	10^{-8}	1.29×10^{7}
1.35	10^{-7}	2.18×10^{7}
1.35	3×10^{-7}	3.00×10^{7}

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WD Core Properties for $T_c = 10^7$ K and Outer Boundary Condition $P_b = 10^{18}$ dyne cm⁻²

Mass (M_{\odot})	Radius (cm)	$g ({\rm cm}~{\rm s}^{-2})$
1.0	5.40×10^{8}	4.55×10^{8}
1.2	3.83×10^{8}	1.09×10^{9}
1.35	2.26×10^{8}	3.52×10^{9}

the properties of the accreting layer are relatively insensitive to T_c for these high \dot{M} s. For example, increasing T_c by a factor of 2 decreases M_{ign} by only < 10%.

At each time step, the temperature for the top-most zone is set with respect to that of the zone directly below it according to a power-law solution obtained by assuming a radiative-zero atmosphere above our grid.⁵ The thermal boundary condition for the bottom-most zone is such that the flux there is equal to that of the zone directly above it. While locally incorrect, this bottom boundary condition has no effect on the thermal properties of the accreted envelope as the thermal time at the bottom of our grid is much longer than the accretion time prior to ignition. Thus, any inaccuracies in the bottom-most zones will have no effect on the region of interest.

The WD core structure is assumed to be constant during the nova cycle. This is an excellent approximation, as the WD's central pressure, 10^{24} to 10^{27} dyne cm⁻² for $M = 1.0-1.35 M_{\odot}$, is much larger than that of the accreted layer. The WD core radius is calculated for an isothermal core that is half ¹²C and half ¹⁶O by mass with outer boundary condition $P_b = 10^{18}$ dyne cm⁻². The radius is relatively insensitive to T_c for $T_c < 2 \times 10^7$ K, so all radii are calculated with $T_c = 10^7$ K. The resulting radii and gravitational accelerations for our three models are shown in Table 2. Calculations for O/Ne WDs give the same radii to within $\simeq 1\%$, because ¹²C, ¹⁶O, and ²⁰Ne have the same charge-to-mass ratio.

The equation of state (Saumon et al. 1995; Timmes & Swesty 2000; Rogers & Nayfonov 2002), opacity (Iglesias & Rogers 1993, 1996), nuclear burning network (Timmes 1999), neutrino cooling (Itoh et al. 1996), and electron screening (Graboske et al. 1973; Alastuey & Jancovici 1978; Itoh et al. 1979) are calculated using the MESA code package.⁶ The MESA basic

⁴ CVs spend only a short amount of time at $\dot{M} > 10^{-8} M_{\odot} \text{ yr}^{-1}$ (Howell et al. 2001), and so it is unclear if the core temperature will have time to reach equilibrium before the system has evolved appreciably (Epelstain et al. 2007). However, as mentioned later, the results of our study are largely unaffected by the core temperature.

⁵ Changing the upper boundary condition to a constant temperature has a negligible effect on the long-term evolution of the layer. This is unsurprising because, for the pressure at the top of the grid, P_{top} , the ratio of the thermal time to the accretion time at the base is P_{top}/P_b . For the grid we use and typical ignition pressures, this ratio is < 0.1, and so any differences in thermal conditions at the top of the grid are radiated away and not carried deeper into the star.

⁶ http://mesa.sourceforge.net/



Figure 2. Time series of envelope profiles for a 1.0 M_{\odot} WD accreting at $\dot{M} = 10^{-8} M_{\odot} \text{ yr}^{-1}$ with $Z = 5.0 Z_{\odot}$ and $X_3 = 10^{-4}$. Each profile in each panel is separated by 317 yr. The thick line is the final profile just prior to convection, 1988 yr after the onset of accretion. Thus, the first profiles shown represent the layer after 1037 yr of accretion. From top to bottom, the panels show the temperature, the energy generation rate (in cgs units of erg g⁻¹ s⁻¹), the accreted ¹²C and ¹⁴N mass fractions, and the ³He mass fraction. The solid lines in the third panel show X_{12} , and the dashed lines show X_{14} . The vertical dotted line shows the pressure of the envelope base just prior to convection.

Table 3	
Solar Mass Fractions from Lodders (2003)

Element	Mass Fraction
¹ H	0.749
³ He	2.95×10^{-5}
⁴ He	0.237
¹² C	2.21×10^{-3}
¹⁴ N	0.71×10^{-3}
¹⁶ O	5.87×10^{-3}
²⁰ Ne	1.12×10^{-3}
²⁴ Mg	0.64×10^{-3}

nuclear network, which tracks the abundances of ¹H, ⁴He, ¹²C, ¹⁴N, ¹⁶O, ²⁰Ne, and ²⁴Mg, has been modified to explicitly follow the ³He reactions ³He+³He and ³He+⁴He. The nuclear reactions consist of the p - p chains (including the *pep* reaction, although for our high \dot{M} s $\ge 10^{-9} M_{\odot} \text{ yr}^{-1}$, it typically has a negligible contribution; Starrfield et al. 2007), the CNO cycles, α -burning up to ²⁴Mg, and C/O burning. Isotopes that are not explicitly tracked are assumed to have equilibrium abundances.

Solar composition of the accreted material is defined by the recommended solar photosphere abundances in Lodders (2003), except for X_3 , which is derived from Table 6 of Lodders (2003). The relevant elemental abundances by mass fraction are shown in Table 3; note the reduction in solar metallicity as compared to previous studies, such as Grevesse & Sauval (1998). For models with non-solar metallicities, the mass fractions of ¹²C, ¹⁴N, ¹⁶O, ²⁰Ne, and ²⁴Mg are adjusted by the same multiplicative factor and the difference made up in $X_{\rm H}$.



Figure 3. Same as Figure 2, except with $Z = 0.5 Z_{\odot}$, time between profiles of 634 yr, and $t_{ign} = 6337$ yr, so the earliest profiles show the layer after 4434 yr of accretion.

A monotonic transport first-order advection scheme (van Leer 1974; Hawley et al. 1984), modified for logarithmic coordinates, is utilized to simulate accretion. While certainly more accurate than zeroth-order donor cell advection, it is still subject to numerical advection that smoothes out what should be a step-function accretion front in the absence of diffusion, which we have neglected for the reasons given in Section 2. If unaccounted for, this nonphysical excess advection into C-rich material leads to premature burning of the small amount of hydrogen that precedes the accretion front. Thus, when calculating the burning rate and compositional changes, the core C/O is treated as ²⁴Mg, which is essentially inert in the nuclear reaction network at these temperatures. For all other calculations, the core remains half ¹²C and half ¹⁶O by mass.

The code is evolved until any local thermal gradient is steeper than the local adiabatic gradient and convection sets in, at which point we define ignition to have occurred. We define the ignition mass to be the total accreted mass at the onset of convection, $M_{ign} = \dot{M}t_{ign}$, taking into account the time required for the accretion front to reach the top of our grid prior to the beginning of the code run.

5.2. Representative Results

Figures 2 and 3 show envelope profiles for a 1.0 M_{\odot} WD accreting at $\dot{M} = 10^{-8} M_{\odot} \text{ yr}^{-1}$ with $X_3 = 10^{-4}$. Figure 2 has $Z = 5.0 Z_{\odot}$, and Figure 3 has $Z = 0.5 Z_{\odot}$. Each panel shows profiles at four different times, each separated by 317 yr in Figure 2 and by 634 yr in Figure 3. The thick solid lines in both figures show the envelope profile just prior to convection.

Figure 2 shows a typical $p+{}^{12}C$ ignition. The cause of the ignition is the onset of $p+{}^{12}C$ burning, which releases more heat than can be radiatively transported away. At ignition, the accreted carbon mass fraction (solid line in the third panel) at the envelope base (vertical dotted line) is essentially the same as in the rest of the accreted layer because it has not had a chance to deplete before ignition conditions are met. Thus, the

⁷ As noted in Section 3, systems do exist where the mass fractions of accreted ¹²C and ¹⁴N are non-solar while the other metals still have their solar values. However, for the sake of consistency and convenience, we scale all the metals by the same value in each model. Furthermore, the hydrogen mass fraction is always assumed to be the solar value, modulo the change due to metallicity. This is a fair assumption, because reducing X_H to 0.25 only increases M_{ign} by

^{~ 5%.} For very evolved donors with $X_H < 0.1$, the decreased hydrogen mass fraction will compound the effect of low X_{12} , further increasing M_{ign} .



Figure 4. Same as Figure 2, except with $Z = Z_{\odot}$, $X_3 = 2 \times 10^{-3}$, and $t_{\text{ign}} = 1082$ yr, and thus the earliest time shown is 448 yr after accretion begins.

¹⁴N mass fraction (dashed line in the third panel) in the layer is also unchanged, except near the base, where the small amount of C-burning has slightly raised the ¹⁴N mass fraction.

Contrast this sequence of events with the 0.5 Z_{\odot} case shown in Figure 3. Here, carbon has already been depleted prior to ignition. At the time of ignition, the carbon mass fraction at the base is several orders of magnitude lower than in the rest of the layer (solid line in the third panel), and the difference has been added to the ¹⁴N mass fraction (dashed line in the third panel). The energy generation profile (second panel) clearly shows two peaks in the envelope: the shallower occurs where ¹²C is burned and depleted, and the deeper peak is due to full CNO cycle burning. It is at the deeper location that ϵ_{nuc} begins to spike and convection occurs. Since the layer must become hotter and denser to burn ¹⁴N, more mass accumulates prior to ignition. The ignition mass in this case is $6.3 \times 10^{-5} M_{\odot}$, three times higher than the C-triggered case, which has $M_{ign} = 2.0 \times 10^{-5} M_{\odot}$.

The disparity in M_{ign} is even greater when compared to a case with a non-negligible amount of ³He, because as shown in Section 4, ³He always begins burning prior to proton captures onto ¹²C. Figure 4 shows the outcome of a model with $X_3 = 2 \times 10^{-3}$. This ignition is triggered by ³He-burning after only $1.1 \times 10^{-5} M_{\odot}$ has been accreted.

5.3. Ignition Masses

In this section, we show the resulting M_{ign} for a range of M, \dot{M} , and composition. Figure 5 shows M_{ign} versus accreted metallicity with $X_3 = 10^{-4}$, and Figure 6 shows M_{ign} versus ³He mass fraction with solar metallicity. The dotted, dashed, and solid lines represent M = 1.0, 1.2, and 1.35 M_{\odot} WDs, respectively. The top (higher M_{ign}) curve for each WD mass has $\dot{M} = 10^{-9} M_{\odot} \text{ yr}^{-1}$, and the bottom (lower M_{ign}) curve has $\dot{M} = 10^{-7} M_{\odot} \text{ yr}^{-1}$.

Some general trends are clear. Increasing the WD mass and the accretion rate lower the ignition mass. A larger mass is equivalent to a smaller radius, both of which contribute to a higher value of g, resulting in a higher temperature and density for a given envelope mass, and thus ignition conditions are reached at lower envelope masses. Higher accretion rates



Figure 5. Ignition mass vs. metallicity for $X_3 = 10^{-4}$ and different WD masses and \dot{M} s. The dotted, short-dashed, and solid lines are for $M = 1.0, 1.2, \text{ and } 1.35 \ M_{\odot}$ WDs, respectively. The top (higher M_{ign}) curve for each WD mass has $\dot{M} = 10^{-9} \ M_{\odot} \ \text{yr}^{-1}$, and the bottom (lower M_{ign}) curve has $\dot{M} = 10^{-7} \ M_{\odot} \ \text{yr}^{-1}$. The long-dashed line shows the analytical scaling of $M_{\text{ign}} \propto Z^{-1/5}$, as discussed in Section 5.3.



Figure 6. Same as Figure 5 except vs. X_3 . The metallicity is the solar value. The analytical scaling from Section 5.3 of $M_{\text{ign}} \propto X_3^{-2/5}$ is displayed.

translate into higher compressional luminosities and higher temperatures, also resulting in ignition for smaller envelope masses. The basic trend of lower ignition mass with higher values of ¹²C and ³He is also sensible: more fuel means more burning and quicker buildup to ignition.

However, as we have discussed previously, changing the composition does not just lead to a change in the energy generation rate. If the ³He or ¹²C is depleted, conditions for burning the next fuel will have to be reached for ignition. This is the cause of the inflections that are present in most of the M_{ign} curves in Figure 5, and for the abrupt changes in M_{ign} that are seen in Figure 6. Once a critical ¹²C or ³He mass fraction is reached, the ignition becomes qualitatively different.



Figure 7. Lines of constant M_{ign} in $M - \dot{M}$ space for $Z = Z_{\odot}$ and $X_3 = 10^{-4}$ (solid lines). Contours are evenly spaced with 0.2 decades of M_{ign} between each line. The thick solid lines have $M_{ign} = 10^{-5}$ and $10^{-4} M_{\odot}$ as labeled. Also shown are lines of constant recurrence time (dotted lines), labeled in years. The range of stable H-burning (hatched region) is obtained from Nomoto et al. (2007).

Note that carbon depletion does not have as drastic an effect as ³He depletion because depleted carbon is converted to another burning catalyst, ¹⁴N, whereas ³He is depleted to hydrogen and ⁴He, negligibly increasing the amount of hydrogen fuel.

The long-dashed lines in Figures 5 and 6 show the analytical power-law scaling of M_{ign} with Z and X_3 as derived from Equation (A7). Assuming that the constant pressure logarithmic temperature dependences of the ¹²C- and ³He-burning rates are $\chi_{12} \simeq \chi_3 \simeq 15$ yields $M_{ign} \propto Z^{-1/5}$ for ¹²C-triggered novae and $M_{ign} \propto X_3^{-2/5}$ for ³He-triggered novae. The difference in the exponent is due to ³He burning via self-reactions, as opposed to ¹²C burning via proton captures. The numerical results of Figure 5 match the analytical scaling fairly well. The high X_3 models of Figure 6 also match the analytical expectation well. However, for lower values of X_3 , ³He is depleted and does not release enough heat to trigger the nova. These novae are instead triggered by ¹²C or ¹⁴N, and since the metallicity is constant along each curve, M_{ign} is also roughly constant below a critical value of X_3 .

Figure 7 shows lines of constant M_{ign} as a function of M and \dot{M} (solid lines) for $Z = Z_{\odot}$ and $X_3 = 10^{-4}$, obtained by quadratically fitting the numerical results detailed above. The contours are evenly spaced with 0.2 dex of M_{ign} between each line. The thick solid lines have $M_{ign} = 10^{-5}$ and $10^{-4} M_{\odot}$ as labeled. Also shown are contours of constant recurrence time (dotted lines), labeled in years. The hatched region where hydrogen burns stably in a steady state is taken from Nomoto et al. (2007). It is clear that a recurrent nova system accreting solar metallicity material with low ³He mass fraction must have a WD > 1.2 M_{\odot} to achieve a recurrence time < 30 yr.

Our ignition masses are a factor of 1.5–3 times higher than the analytical estimates of Fujimoto (1982b) and 1.1–2 times lower than the work of MacDonald (1983). Given the differences in method and updates in equations of state and opacity, these discrepancies are not significant. The work of Yaron et al. (2005) assumes diffusive accretion onto initially bare C/O cores, which invalidates our assumption of accretion onto C-poor material,



Figure 8. Ignition pressure vs. metallicity for $X_3 = 10^{-4}$ and a range of M and \dot{M} . The dotted, dashed, and solid lines are for $M = 1.0, 1.2, \text{ and } 1.35 M_{\odot}$, respectively. The lower P_{ign} curve for each WD mass has $\dot{M} = 10^{-7} M_{\odot} \text{ yr}^{-1}$, and the higher P_{ign} curves have $10^{-9} M_{\odot} \text{ yr}^{-1}$.

so the fact that their $M_{ign}s$ are 2–4 times lower than ours is not surprising. A comparison of our $X_3 = 10^{-3}$ results with those of Townsley & Bildsten (2004; Figure 1 of Townsley & Bildsten 2005) reveals a large discrepancy, with our $M_{ign}s$ higher by a factor of 3–10. The difference is predominantly due to their lack of abundance evolution, which eliminates the possibility of ³He depletion. Theirs is a valid assumption for higher mass fractions $X_3 \gtrsim 2 \times 10^{-3}$, but is not correct for $X_3 = 10^{-3}$, and leads to premature nova ignitions.

Throughout this study, we have assumed that M_{ign} and t_{ign} are the accreted mass and accretion time prior to the onset of convection. However, the convective zone takes some time to grow and reach the surface, at which point the nova outburst becomes observable. It is conceivable that a significant amount of mass is accreted during the convective phase, which would cause the t_{ign} we have calculated to be lower than the actual time between novae. To quantify this error, we consider the growth time of the convective zone, which is $t_{growth} \sim c_P T_b/\epsilon$, where the temperature and burning rate are evaluated at the onset of convection, and c_P is the specific heat at constant pressure. For each model, we find that t_{growth} is always 20–100 times shorter than the time to the onset of convection, so that the ignition times and masses we have calculated are only smaller than the actual values by a few percent, and our assumption is justified.

5.4. Ignition Pressures and the Misuse of "Pcrit"

Fujimoto (1982a) is often cited to support the claim of a critical ignition pressure for nova ignition, $P_{\rm crit}$, which, depending on the study making the claim, has a value between $10^{19}-10^{20}$ dyne cm⁻². However, the ignition pressure is clearly not constant, as we show in Figures 8 and 9. These figures are identical to Figures 5 and 6 but with $P_{\rm ign}$ along the ordinate axis instead of $M_{\rm ign}$; the ignition pressure is related to $M_{\rm ign}$ through Equation (A2). The ignition pressure varies by a factor of 100 from one extreme of high accretion rate, mass, and X_3 to the other extreme and is not constant. Moreover, the original paper actually makes no such assertion. Instead, Fujimoto (1982a)



Figure 9. Same as Figure 8, except vs. X_3 with solar metallicity.



Figure 10. Surface luminosity, excluding the accretion luminosity, and effective temperature as a function of time for a 1.0 M_{\odot} WD with $\dot{M} = 10^{-8} M_{\odot}$ yr⁻¹, $X_3 = 10^{-4}$, and various metallicities. The metallicities shown are 0.1, 0.2, 0.5, 1.0, 2.0, and 5.0 Z_{\odot} , increasing from right to left.

states that there is a minimum ignition pressure necessary to produce a strong nova-like outburst powered by hydrostatic shell expansion. The ignition pressure is left as a free parameter and is calculated in Fujimoto (1982b) as a function of WD masses and accretion rates.

5.5. Pre-ignition Luminosities

The depletion of fuel will increase the WD surface luminosity above the compressional luminosity. The detection of a WD that is brighter than expected for its time-averaged \dot{M} could be a sign that the carbon abundance is not enhanced, or that the ³He abundance is non-negligible. There are several energy scales to keep in mind: the accretion, thermal, ¹²C-burning, and ³He-burning energies per mass are given, respectively, as

$$E_{\rm acc} = \frac{GM}{R} = 1.3 \times 10^{17} \, \frac{M_1}{R_9} \frac{\rm erg}{\rm g}$$



Figure 11. Same as Figure 10, but with $Z = Z_{\odot}$ and varying X_3 . The ³He mass fractions shown are 0.1, 0.2, 0.5, 1.0, 2.0, and 5.0×10^{-3} , increasing from right to left.

$$E_{\text{therm}} = \frac{kT}{\mu m_p} = 1.4 \times 10^{15} T_7 \frac{\text{erg}}{\text{g}}$$
$$X_{12}E_{12} = 8.8 \times 10^{14} \frac{X_{12}}{10^{-3}} \frac{\text{erg}}{\text{g}}$$
$$X_3E_3 = 2.1 \times 10^{15} \frac{X_3}{10^{-3}} \frac{\text{erg}}{\text{g}}.$$
 (4)

The luminosities associated with these energies can be obtained by multiplying them by the accretion rate, with an additional multiplicative factor of 1.75 for the thermal/compressional luminosity. The accretion energy is roughly 2 orders of magnitude larger than the thermal or nuclear energy available during the accretion phase; however, the accretion luminosity from the disk is variable and greatly reduced in disk quiescence. Thus, it is still possible to observe the effective temperature of the WD, as demonstrated by Winters & Sion (2003) and references therein, and ascertain the luminosity produced from the interior of the accreted envelope.

Figure 10 shows the surface luminosity, ignoring the accretion luminosity, and effective temperature as a function of time for a 1.0 M_{\odot} WD with $\dot{M} = 10^{-8} M_{\odot} \text{ yr}^{-1}$, $X_3 = 10^{-4}$, and metallicities of 0.1, 0.2, 0.5, 1.0, 2.0, and 5.0 Z_{\odot} , increasing from right to left. The 0.5 Z_{\odot} model is interesting in that there is enough carbon to power significant nuclear luminosity, but not enough to trigger a nova. For this case, the compressional luminosity is < 0.5 L_{\odot} and $T_{\text{eff}} < 5.5 \times 10^4$ K for ~ 50% of the nova cycle. Four thousand years after the onset of accretion, carbon is burned and depleted, causing the luminosity from inside the envelope to double within a span of only 1000 yr. The burning of ¹⁴N begins ~ 1000 yr after C-depletion, and the luminosity and effective temperature rise to 1.3 L_{\odot} and 8 × 10⁴ K just prior to the CNO cycle-triggered ignition.

Changing X_3 also has an effect on the surface luminosity and T_{eff} . Figure 11 shows the surface luminosity and effective temperature as in Figure 10, but with fixed metallicity $Z = Z_{\odot}$ and varying ³He mass fractions of 0.1, 0.2, 0.5, 1.0, 2.0, and 5.0 × 10⁻³, increasing from right to left. Here, the interesting case is $X_3 = 10^{-3}$, which has enough ³He to produce significant energy, but not enough to trigger the nova. The surface luminosity reaches L_{\odot} and T_{eff} rises to 6.5×10^4 K after only 1500 yr of accretion, but the surface then cools as the envelope succeeds in depleting the accumulated ³He. The surface brightens when C-burning commences, and again dims when carbon is depleted, finally rising to 2 L_{\odot} and 8 × 10⁴ K when ¹⁴N begins burning.

Note that for classical novae, this pre-outburst brightening takes place over thousand-year timescales, and thus the evolution of the luminosity is not expected to be detectable. However, the observation of a WD in a CV that is hotter than what its time-averaged M predicts may indicate stable fuel depletion within the accreted layer. Townsley & Bildsten (2003) find evidence for WDs that are hotter than expected, but these are in CVs below the period gap with Ms lower than we consider in our study. Moreover, it may be possible to observe this preoutburst brightening in recurrent novae, which have recurrence times of ~ 30 yr.

6. CONCLUSIONS

Motivated by uncertainties in classical nova core-mixing mechanisms and the lack of metal enhancements in some nova ejecta, we have quantified the effects of composition on nova ignitions (see Figure 5) under the assumption that the underlying material is C-poor and diffusion thus unimportant, as appropriate for accretors with large helium buffers or O/Ne cores. We have found that for carbon mass fractions $\lesssim 2 \times 10^{-3}$, ¹²C is depleted and converted to ¹⁴N without releasing enough heat to trigger a nuclear instability. The layer continues to accrete until ¹⁴N can capture protons, leading to a nova triggered by the full CNO cycle and an ignition mass larger than the carbonignited case. Thus, the ignition mass increases by a factor of \sim 3 as the metallicity is decreased from 5.0 Z_{\odot} to 0.1 Z_{\odot} . The critical carbon mass fraction is near-solar and is thus relevant to sub-solar metallicity systems as well as systems with evolved secondaries that have undergone CNO processing of ¹²C to ¹⁴N. We have also examined the effect of accreted 3 He (see Figure 6), which can reach mass fractions of 4×10^{-3} as an evolved donor's interior is uncovered by mass transfer. For $X_3 \gtrsim 2 \times 10^{-3}$, ³He+³He reactions trigger novae with M_{ign} a factor of ~ 3 times smaller than the C-triggered case.

The dependence of M_{ign} on accreted composition will affect population-averaged nova rates: naively, high-metallicity environments will have higher nova rates than systems with subsolar metallicities (such as novae in globular clusters; Shafter & Quimby 2007) or evolved donors that have undergone CNO processing. However, the existence of ³He would have to be taken into account, because old systems with donors that have undergone significant mass loss could have $X_3 \gtrsim 2 \times 10^{-3}$ and would thus have high nova rates, regardless of the accreted metallicity. A proper prediction of the effect of donor composition requires a population-synthesis calculation that includes further complications such as binary and donor evolution. We leave this exercise for future work.

To date, most observations only report galaxy-averaged nova rates, although some studies of M31 (Ciardullo et al. 1987; Capaccioli et al. 1989; Darnley et al. 2006) and M81 (Neill & Shara 2005) have found that their bulges produce more novae per stellar luminosity than their disk by a factor of 5–10. On the other hand, galaxy-averaged nova rates do not see any morphology dependence, finding instead that the luminosity-specific nova rate (LSNR) is roughly constant across all galaxy types at a value of 2 ± 1 yr⁻¹ ($10^{10} L_{\odot, K}$)⁻¹ (Williams & Shafter 2004), where $L_{\odot, K}$ is the *K*-band solar luminosity. The LMC, Small Magellanic Cloud (SMC), and possibly Virgo dwarf elliptical galaxies Neill & Shara 2005), which have LSNRs higher by a factor of 3, are exceptions. These measurements have large error bars due to small number statistics and issues of completion caused by both extinction and infrequent observations. A better measurement of nova rates will come with new deep optical surveys with high cadences such as Pan-STARRS-1, Pan-STARRS-4, and the Large Synoptic Survey Telescope, which will see thousands of novae every year. These will reduce the nova rate error bars and also possibly allow us to measure rates in different populations within other galaxies besides M31.

In addition to this population-averaged observable, the composition could also have a detectable effect on individual systems. In particular, the depletion of fuel can significantly increase the surface luminosity above the baseline set by the entropy released during compression of the accreted layer (see Figures 10 and 11). While this increase is still well below the accretion luminosity associated with gravitational energy release, it would be visible while the system was in disk quiescence. Recurrent novae, in particular, would be ideal systems in which to observe this brightening in quiescent luminosity due to their short recurrence times of ~ 30 yr.

These observables are dependent on the assumption that convection is not initiated in C-enhanced material. If instead H-rich envelope material penetrates into C-rich material and triggers convection there, the accreted composition will have little effect. Thus, if these composition-dependent effects are observed, they will provide evidence that convection for many novae is initiated in C-poor material, and that CNO enrichment for these novae is due to convective shear mixing or overshoot.

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APPENDIX

ANALYTICAL ESTIMATE OF IGNITION CONDITIONS

Throughout this study, we make the assumption that the accreted layer is thin, with a pressure scale height at the envelope base, $h = P_b/\rho_b g$, much less than the WD core radius, R. The subscript b refers to the base of the accreted envelope, $g = GM/R^2$ is the gravitational acceleration, assumed constant because $h \ll R$, and M is the WD core mass. The ratio of the scale height to the WD radius for an ideal gas equation of state is the ratio of the thermal energy to gravitational energy,

$$\frac{h}{R} = \frac{k_B T_b / \mu m_p}{GM/R} = 10^{-2} \frac{T_7 R_9}{M_1},$$
 (A1)

where T_7 is the base temperature in units of 10^7 K, M_1 is the WD core mass in units of M_{\odot} , $R_9 = R/10^9$ cm, the proton mass is m_p , and the atomic mass per particle is $\mu = 0.6$ for solar composition. For typical ignition conditions, $T_b \simeq 2 \times 10^7$ K, so the shell is very thin for the entire accretion phase. The base pressure is independent of the temperature in the thin-shell limit and is

$$P_b = \frac{GMM_{\rm env}}{4\pi R^4},\tag{A2}$$

where $M_{\rm env}$ is the envelope mass. This assumption will not be valid once the temperature rises during the thermonuclear runaway. For a 1.0 M_{\odot} WD, h = 0.25 R when $T_b \simeq 5 \times 10^8$ K.

To derive the conditions at the base of the envelope, we first estimate the luminosity in the accreting layer following Nomoto (1982) and Townsley & Bildsten (2004). When material accretes onto the WD surface, the gravitational energy, GM/R, is released and radiated by the spreading boundary layer (Piro & Bildsten 2004) and is not carried into the star, because the thermal timescale at the photosphere for luminosities of order the accretion luminosity is far shorter than the accretion timescale. Instead, prior to the onset of nuclear burning, the pre-ignition luminosity exiting the deep accreting layer is produced by entropy released as the material accumulates. The entropy equation yields the compressional luminosity at the surface,

$$L_{\rm comp} = \dot{M} \int_{P'}^{0} T \frac{ds}{dP} \, dP, \tag{A3}$$

where *s* is the specific entropy, and we have neglected the term $\partial s/\partial t|_P$. The lower bound *P'* is the depth at which the thermal time is equal to the time for which accretion has been ongoing, so that the luminosity produced there has had time to make its way through the envelope. For illustration, we assume Kramers' opacity ($\kappa \propto \rho T^{-7/2}$), ideal gas ($P \propto \rho T$), and a constant luminosity above *P'*, so that $P(r)^2 \propto T(r)^{17/2}$. An ideal gas has $s = k_B \ln(T^{3/2}/\rho)/\mu m_p$, which yields $ds/dP = -7k_B/17\mu m_p P$. This gives

$$L_{\rm comp} = \frac{7}{4} \dot{M} \frac{k_B T'}{\mu m_p} = 0.4 \ L_{\odot} \ \dot{M}_{-8} \left(\frac{T'}{10^7 \ \rm K} \right), \qquad (A4)$$

where T' is the temperature at P', \dot{M}_{-8} is the mass accretion rate in units of $10^{-8} M_{\odot}$ yr⁻¹, and we have set $\mu = 0.6$. If the opacity is due to electron scattering, the prefactor becomes 3/2instead of 7/4, so the exact relation is only weakly dependent on the form of radiative opacity.

The thermal time at P' is $t'_{\text{therm}} \equiv c_P T' M'_{\text{env}} / L_{\text{comp}}$, where $c_P = 5k_B/2\mu m_p$ is the specific heat at constant pressure for an ideal gas, M'_{env} is the mass in the layer above P', and we use a one-zone approximation, $dL/dM \sim L_{\text{comp}}/M'$ (e.g., Paczynski 1983). The time to accrete an envelope mass M_{env} is $t_{\text{acc}} \equiv M_{\text{env}}/\dot{M}$. To find the depth from which luminosity is able to escape during accretion, we set the two timescales equal and use Equation (A2), yielding $P' \simeq P_b$; i.e., most of the luminosity in the envelope comes from only the envelope itself, and so we neglect the compressional luminosity from the core (see the Appendix of Townsley & Bildsten 2004 for further discussion of the core's role). Thus, the compressional luminosity is given by Equation (A4), with $T' = T_b$. Setting this equal to the luminosity given by radiative diffusion with Kramers' opacity,

$$\kappa = \kappa_0 \frac{\rho}{\mathrm{g}\,\mathrm{cm}^{-3}} \left(\frac{T}{\mathrm{K}}\right)^{-7/2},\tag{A5}$$

where $\kappa_0 \simeq 10^{22}$ cm² g⁻¹ from fitting to OPAL opacities (Iglesias & Rogers 1993, 1996) for solar composition around $T = 10^7$ K and $\rho = 10^3$ g cm⁻³, yields the base temperature as a function of density (Equation (1)),

$$T_b = 1.4 \times 10^7 \,\mathrm{K} \left(\frac{\dot{M}_{-8}\rho_3^2}{M_1}\right)^{2/11}.$$
 (A6)

We have assumed solar metallicity, but this result is nearly independent of composition.

We now solve for the condition of stable fuel depletion in a thin shell. Linear stability analysis (Fujimoto et al. 1981) shows that nuclear burning is unstable in a constant-pressure thin shell if

$$\left. \frac{\partial \epsilon_{\text{nuc}}}{\partial T} \right|_{P} > \left. \frac{\partial \epsilon_{\text{cool}}}{\partial T} \right|_{P}, \tag{A7}$$

or $\epsilon_{\text{nuc}}\chi_{\text{nuc}} > \epsilon_{\text{cool}}\chi_{\text{cool}}$, where ϵ_{nuc} is the nuclear energy generation rate, the one-zone approximation to the cooling rate is $\epsilon_{\text{cool}} \sim L/M_{\text{env}}$, and $\chi \equiv \partial \ln \epsilon / \partial \ln T|_P$. For Kramers' opacity and ideal gas, $\chi_{\text{cool}} = 17/2$. The cooling rate is rewritten as

$$\epsilon_{\rm cool} = \frac{7}{4} \frac{k_B T_b}{\mu m_p} \frac{1}{t_{\rm acc}} \tag{A8}$$

after substituting the expression for compressional luminosity (Equation (A4)). Fuel depletion occurs when the burning timescale for a given isotope, $t_i = X_i E_i / \epsilon_i$, becomes equal to t_{acc} , where X_i is the mass fraction of the isotope, E_i is the specific energy released upon burning the isotope, and ϵ_i is the isotope's energy generation rate. Substituting $t_i = t_{acc}$ into Equation (A8) and demanding that $\epsilon_i \chi_i < \epsilon_{cool} \chi_{cool}$ yields the condition for stable depletion,

$$X_i < \frac{7}{4} \frac{k_B T_b / \mu m_p}{E_i} \frac{\chi_{\text{cool}}}{\chi_i},\tag{A9}$$

a more general form of Equation (2).

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