

THE BIFURCATION PERIODS IN LOW-MASS X-RAY BINARIES: THE EFFECT OF MAGNETIC BRAKING AND MASS LOSS

BO MA AND XIANG-DONG LI

Department of Astronomy, Nanjing University, Nanjing 210093, China; xiaomabo@gmail.com, lixid@nju.edu.cn
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ABSTRACT

The bifurcation period in low-mass X-ray binaries is the initial orbital period which separates the formation of converging systems (which evolve with decreasing orbital periods until the donor becomes degenerate) from the diverging systems (which evolve with increasing orbital periods until the donor star loses its envelope and a wide detached binary is formed). We systematically calculate the bifurcation periods of binary systems with a $1.4 M_{\odot}$ neutron star and a $0.5\text{--}2 M_{\odot}$ donor star, taking into account different kinds of magnetic braking (MB) and mass-loss mechanisms. Our results show that the saturated MB can considerably decrease the values of bifurcation period compared to the traditional MB, while the influence of mass-loss mechanisms on bifurcation periods is quite weak. We also develop a semianalytical method to compute the bifurcation period, the result of which agrees well with the numerical method in the leading order.

Key words: binaries: close – stars: evolution – X-rays: binaries

1. INTRODUCTION

One interesting and important topic in the secular evolution of low-mass X-ray binaries (LMXBs) is the so-called “bifurcation period” P_{bif} , the initial binary orbital period which separates the formation of converging systems (which evolve with decreasing orbital periods until the donor becomes degenerate) from the diverging systems (which evolve with increasing orbital periods until the donor star loses its envelope and a wide detached binary is formed; Tutukov et al. 1985). The first systematic investigations on the bifurcation period were done by Pylyser & Savonije (1988, 1989). Neglecting mass loss from the binary system and assuming angular momentum loss due to magnetic braking (MB; Verbunt & Zwaan 1981) and gravitational radiation (GR; Landau & Lifshitz 1975), these authors found that the bifurcation period is in the range $P_{\text{bif}} \sim 0.4\text{--}0.7$ days for LMXBs, and strongly depends on MB efficiency. Ergma et al. (1998) included mass loss from the binary system and recalculated the bifurcation period for two mass configurations $(M_1/M_{\odot}, M_2/M_{\odot}) = (1.4, 1)$ and $(1.4, 1.5)$ and two chemical compositions ($Z = 0.003, 0.03$). They pointed out that the mass loss from the binary system also plays an important role besides MB in determining the value of P_{bif} , while the chemical composition could only cause small change in P_{bif} . Their bifurcation periods are $P_{\text{bif}} \sim 0.85\text{--}1.05$ days under conservative mass transfer, and 1.6–1.7 times larger if moderate nonconservative mass transfer is assumed. Podsiadlowski et al. (2002) found a bifurcation period around 18 hr for a $1.4 M_{\odot}$ neutron star (NS) and a $1 M_{\odot}$ companion star, where they defined the bifurcation period as the orbital period when the Roche lobe overflow (RLOF) just began, instead of the initial orbital period.

van der Sluys et al. (2005a, 2005b) also investigated the bifurcation period in LMXBs focusing the formation of ultra-compact X-ray binaries (UCXBs), and specified the bifurcation period as “the longest initial period that leads to UCXBs within a Hubble time (13.7 Gyr). (van der Sluys et al. 2005a, p. 649)” UCXBs are bright X-ray sources with very short orbital periods ($P \lesssim 1$ hr). The donor has to be a compact source like a white dwarf or a compact core of an evolved giant star to fit in the small Roche lobe size. Such sources may be formed

through dynamical processes including stellar collisions and common envelope evolution (Clark et al. 1975; Rasio et al. 2000; Lombardi et al. 2006). An alternative scenario for the formation of such sources is through stable mass transfer in X-ray binaries with a low- or intermediate-mass donor star, which may explain the negative derivative of the 11 minute source in NGC 6624 (van der Klis et al. 1993; Chou & Grindlay 2001). It has been found that systems with initial orbital period just below the bifurcation period may form UCXBs (Nelson et al. 1986; Tutukov et al. 1987; Pylyser & Savonije 1988; Podsiadlowski et al. 2002; van der Sluys et al. 2005a). Podsiadlowski et al. (2002) showed that the closer the initial orbital period to the bifurcation period from below, the smaller the minimum orbital period that will be achieved. So the value of bifurcation period is crucial to understanding the formation of UCXBs (van der Sluys et al. 2005b).

In this paper, we make a systematic investigation of the bifurcation period for binary systems containing an NS with a main-sequence (MS) companion of mass from $0.5 M_{\odot}$ to $2 M_{\odot}$. This work was motivated by recent progress in studies on mass and angular momentum loss mechanisms in LMXB evolution. In previous works the MB law originally postulated by Verbunt & Zwaan (1981) and Rappaport et al. (1983) was usually adopted. However, this law predicts too fast spin-down of low-mass MS stars, which contradicted with the observation of rapid rotators in young open clusters (Sills et al. 2000; Andronov et al. 2003). Obviously a modification of the MB law will have significant influence on the period evolution (van der Sluys et al. 2005b). Additionally, there is strong evidence that during LMXB evolution the mass transfer is highly nonconservative. Recent measurements of the masses of binary and millisecond pulsars indicate that a large fraction of the transferred mass may be lost from the systems rather accreted by the NS (Bassa et al. 2006; Steeghs & Jonker 2007, and references therein).¹ Theoretically possible ways of mass loss have been suggested, including “evaporation” of the donor (Ruderman et al. 1989) or “radio-ejection” of the transferred material (Burderi et al.

¹ The massive ($\sim 1.9 M_{\odot}$) NS discovered in the globular cluster M5 (Freire et al. 2008) may reflect a bimodal distribution of the initial masses of NSs (rather heavy accretion during the previous LMXB evolution), as already predicted by hydrodynamical core collapse simulations (Timmes et al. 1996).

2001, 2002; D’Antona et al. 2006) due to the pulsar radiation/wind impinging on. In the latter case, the matter is lost from the system at the inner Lagrangian (L_1) point, carrying away angular momentum and altering the period evolution.

This paper is organized as follows. Section 2 briefly describes the stellar evolution code, the binary models, and the physical assumptions, especially the MB laws and the mass-loss mechanisms. Then we present the calculated results in Section 3. Our discussion and conclusions are given in Section 4.

2. EVOLUTION CODE AND BINARY MODEL

2.1. The Stellar Evolution Code

We use an updated version of the stellar evolution code originally developed by Eggleton (1971, 1972; see also Han et al. 2004; Pols et al. 1995) to calculate the evolutions of binaries consisting of an NS (of mass M_1) and an MS secondary (of mass M_2). For the secondary star we assume a solar chemical composition ($X = 0.70$, $Y = 0.28$, and $Z = 0.02$), the ratio of mixing length to pressure scale height $\alpha = 2.0$, and the convective overshooting parameter to be 0.12. The opacity table is from Rogers & Iglesias (1992), Alexander & Ferguson (1994), and Hubbard & Lampe (1969). The effective radius of the Roche lobe for the secondary is taken from Eggleton (1983),

$$R_{L,2} = \frac{0.49q^{-2/3}}{0.6q^{-2/3} + \ln(1 + q^{-1/3})}a, \quad (1)$$

where $q = M_2/M_1$ is the mass ratio, and a is the orbital separation. Mass transfer rate via RLOF is evaluated as $-\dot{M}_2 = \text{RMT} \cdot \max(0, (R_2/R_{L,2} - 1)^3) M_\odot \text{ yr}^{-1}$, and we adopt $\text{RMT} = 10^3$ in the calculations.

2.2. Mass and Angular Momentum Loss Mechanisms

For LMXBs the timescale of tidal synchronization is much shorter than the characteristic evolutionary timescale of the binary, so we can assume that the spin of the secondary star and the binary orbital revolution are always synchronized. Assuming rigid body rotation of the secondary star and neglecting the spin angular momentum of the neutron star, the total angular momentum of the binary system can be expressed as

$$J = I_2\omega + J_{\text{orb}} = I_2\omega + G^{2/3}M_1M_2(M_1 + M_2)^{-1/3}\omega^{-1/3}, \quad (2)$$

where I_2 is the moment of inertia of the secondary star and ω is the angular velocity of the binary.

We consider three kinds of mechanisms of angular momentum loss. The first is the angular momentum loss due to GR (Landau & Lifshitz 1975)

$$\frac{dJ_{\text{GR}}}{dt} = -\frac{32}{5} \frac{G^{7/2}}{c^5} \frac{M_1^2 M_2^2 (M_1 + M_2)^{1/2}}{a^{7/2}}, \quad (3)$$

where c is the light speed. This mechanism is important only in very short period binary systems.

The second angular momentum loss mechanism is for non-conservative mass transfer. We assume that a fraction α of the transferred mass is accreted by the NS, and the remaining mass is ejected out of the binary as isotropic winds from the NS, carrying away the specific angular momentum of the NS,

$$\frac{dJ_{\text{ML}}}{dt} = -(1 - \alpha)\dot{M}_2 \left(\frac{q}{1 + q} \right)^2 a^2 \omega. \quad (4)$$

In our numerical calculations we have set $\alpha = 0$. Alternatively, if the NS is spun up to be a millisecond pulsar, its radiation pressure may be strong enough to halt the transferred matter at the L_1 point and quench the accretion. This “radio ejection” may cause almost all the matter from the secondary to be lost from the binary (Burderi et al. 2001, 2002). The corresponding rate of angular momentum loss is

$$\frac{dJ_{\text{ML}}}{dt} = -\dot{M}_2 a_{L1}^2 \omega, \quad (5)$$

where a_{L1} is the distance from the L_1 point to the center of mass of the binary system.

The third angular momentum loss mechanism is MB. For a low-mass MS star with deep convection zone, stellar winds which are magnetically coupled with the star can decelerate the stellar spin efficiently, thus carrying away the orbital angular momentum because of tidal synchronization. The widely used formula for such an MB effect was postulated by Verbunt & Zwaan (1981) and Rappaport et al. (1983) as

$$\frac{dJ_{\text{MB}}}{dt} = -3.8 \times 10^{-30} M_2 R_2^4 \omega^3 \text{ dyn cm}. \quad (6)$$

However, observations of rapid rotators in young open clusters suggest a modification of the MB law at high rotation rate (Sills et al. 2000),

$$\begin{aligned} \frac{dJ_{\text{MB}}}{dt} &= -K \omega^3 \left(\frac{R_2}{R_\odot} \right)^{0.5} \left(\frac{M_2}{M_\odot} \right)^{-0.5}, & \omega \leq \omega_{\text{crit}}, \\ \frac{dJ_{\text{MB}}}{dt} &= -K \omega_{\text{crit}}^2 \omega \left(\frac{R_2}{R_\odot} \right)^{0.5} \left(\frac{M_2}{M_\odot} \right)^{-0.5}, & \omega > \omega_{\text{crit}}, \end{aligned} \quad (7)$$

where $K = 2.7 \times 10^{47} \text{ g cm}^2 \text{ s}$ (Andronov et al. 2003), ω_{crit} is the critical angular velocity at which the angular momentum loss rate reaches a saturated state, given by (Krishnamurthi et al. 1997)

$$\omega_{\text{crit}}(t) = \omega_{\text{crit}\odot} \frac{\tau_{t_0,\odot}}{\tau_t}, \quad (8)$$

where $\tau_{t_0,\odot}$ and τ_t are the global turnover timescales for the convective envelope of the Sun at its current age and of the secondary star at age t , respectively. They can be calculated by integrating the inverse local convective velocity over the surface convective envelope (Kim & Demarque 1996):

$$\tau_t = \int_{R_b}^{R_2} \frac{dr}{v}, \quad (9)$$

where R_b is the radial distance from the center of the star to the bottom of the surface convective envelope, and v is the local convective velocity from mixing-length theory (Böhm-Vitense 1958). Our calculation gives $\tau_{t_0,\odot} \simeq 28.4$ days, slightly larger than $\tau_{t_0,\odot} \simeq 13.8$ days in van der Sluys et al. (2005b), but consistent with the results of Kim & Demarque (1996) and Jung & Kim (2007). See Eggleton (2006, p. 46) for the discussion of a possible reason for different values of $\tau_{t_0,\odot}$ calculated.

Following the suggestion of Podsiadlowski et al. (2002), we also add an ad hoc factor

$$\exp(-0.02/q_{\text{conv}} + 1) \text{ if } q_{\text{conv}} < 0.02$$

in Equations (6) and (7), where q_{conv} is the mass fraction of the surface convective envelope, to reduce the MB effect when the convective envelope becomes too small.

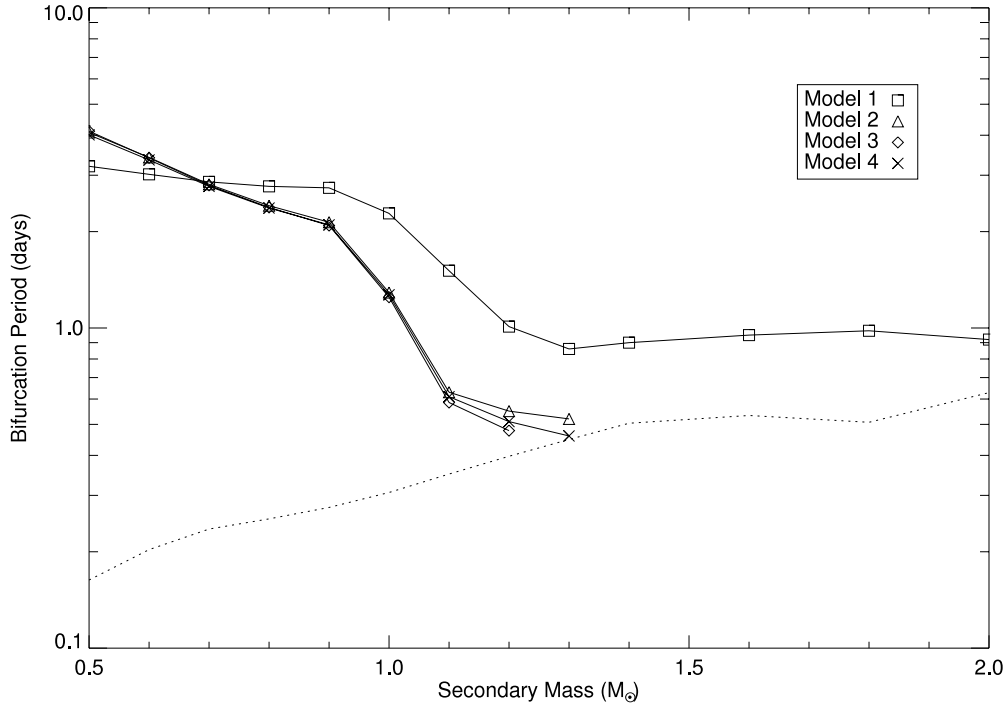


Figure 1. Bifurcation periods as a function of the secondary mass in an LMXB for the four kinds of models described in Section 2.3. The dotted line shows the minimum initial period P_{ZAMS} that corresponds to a Roche lobe filling ZAMS secondary star.

2.3. Binary Models

To examine the influence of mass and angular momentum loss mechanisms on the period evolution, we construct four models with various mass and angular momentum loss combinations—model 1: conservative mass transfer with traditional MB law (Equation (6)); model 2: conservative mass transfer with saturated MB law (Equation (7)); model 3: nonconservative mass transfer with mass loss from L_1 point (Equation (5)) and saturated MB law (Equation (7)); and model 4: nonconservative mass transfer with mass loss from the NS (Equation (4)) and saturated MB law (Equation (7)). In all four models, the initial NS mass is set to be $M_{1,i} = 1.4 M_\odot$, and the initial mass of the secondary $M_{2,i}$ ranges from 0.5 to $2.0 M_\odot$.

3. NUMERIC RESULTS

3.1. The Bifurcation Periods

Throughout this paper we define the bifurcation period P_{bif} as the initial binary orbital period P_i with a zero-age main-sequence (ZAMS) companion star that separates converging from diverging systems. We use P_f to denote the final orbital period after the mass transfer. Another definition of the bifurcation period used by Podsiadlowski et al. (2002) is the orbital period when the RLOF just begins, which is expressed as P_{rlof} in this paper.

The results of the bifurcation periods for the four models described in Section 2.3 are summarized in Figure 1 and Table 1. We also draw the minimum initial period P_{ZAMS} that corresponds to a lobe-filling ZAMS donor star in Figure 1. Several features are noted for the bifurcation periods in Figure 1. First, the bifurcation periods for all the four models decrease with increasing initial secondary mass from $0.5 M_\odot$ to $1.3 M_\odot$. Second, in models with saturated MB, there exists an upper limit of the initial secondary mass, beyond which no bifurcation period exists. This upper limit is in the range ~ 1.2 – $1.3 M_\odot$ for model 3, and ~ 1.3 – $1.4 M_\odot$ for models 2 and 4. Third,

Table 1
Calculated Results of the Bifurcation Periods for Different Binary Models

| $M_{2,i}$ | P_{bif} (days) | | | | P_{rlof} (hr) | | | |
|---------------|------------------|---------|---------|---------|-----------------|---------|---------|---------|
| | Model 1 | Model 2 | Model 3 | Model 4 | Model 1 | Model 2 | Model 3 | Model 4 |
| $0.5 M_\odot$ | 3.20 | 4.08 | 4.12 | 4.01 | 31.8 | 16.5 | 18.2 | 13.6 |
| $0.6 M_\odot$ | 3.02 | 3.41 | 3.40 | 3.35 | 25.6 | 11.3 | 11.1 | 10.0 |
| $0.7 M_\odot$ | 2.86 | 2.81 | 2.79 | 2.77 | 21.5 | 10.0 | 9.8 | 9.6 |
| $0.8 M_\odot$ | 2.77 | 2.41 | 2.38 | 2.37 | 19.0 | 10.1 | 9.9 | 9.8 |
| $0.9 M_\odot$ | 2.73 | 2.14 | 2.09 | 2.10 | 17.9 | 10.7 | 10.3 | 10.4 |
| $1.0 M_\odot$ | 2.28 | 1.29 | 1.25 | 1.27 | 18.3 | 11.4 | 10.8 | 11.1 |
| $1.1 M_\odot$ | 1.51 | 0.63 | 0.59 | 0.61 | 18.9 | 11.7 | 10.8 | 11.2 |
| $1.2 M_\odot$ | 1.01 | 0.55 | 0.48 | 0.51 | 19.1 | 11.8 | 10.5 | 11.0 |
| $1.3 M_\odot$ | 0.86 | 0.52 | | 0.46 | 19.2 | 12.0 | | 11.1 |
| $1.4 M_\odot$ | 0.90 | | | | 20.7 | | | |
| $1.6 M_\odot$ | 0.95 | | | | 22.3 | | | |
| $1.8 M_\odot$ | 0.98 | | | | 22.8 | | | |
| $2.0 M_\odot$ | 0.92 | | | | 21.6 | | | |

comparing the bifurcation periods of model 1 with those of models 2–4 indicates that the MB law plays the most important role in determining the values of the bifurcation periods: different MB laws can change the bifurcation periods by as much as $\sim 60\%$, compared to $\sim 14\%$ (see Table 1) caused by different mass-loss mechanisms.

In Table 1, we also present P_{rlof} following Podsiadlowski et al. (2002). In model 1, we get $P_{rlof} \simeq 18.3$ hr for $M_{2,i} = 1 M_\odot$, which is in close line with the result of Podsiadlowski et al. (2002; 17.7 hr with $Z = 0.001$, $Y = 0.27$), where the difference could be explained as the difference between the metallicities we used. When we use saturated MB, P_{rlof} decreases to ~ 11 hr.

According to the calculated orbital period evolutions, LMXBs can be classified into three categories: the diverging systems with $P_f \gg P_i$, the converging systems with $P_f \ll P_i$, and the parallel systems with $P_f \sim P_i$. As an example, we present the

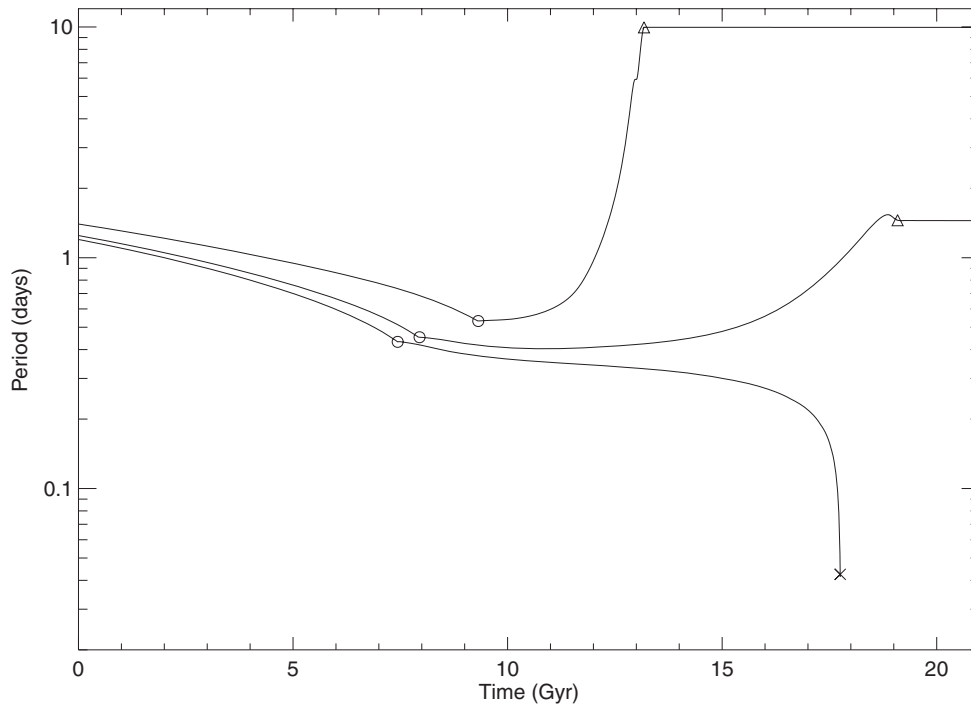


Figure 2. Period evolutions of an LMXB with $M_{2,i} = 1.0 M_{\odot}$ and $P_i = 1.20, 1.25, 1.40$ days in model 3. The circle, the triangle, and the cross mark the onset of the RLOF, the end of the RLOF, and the end of the calculation, respectively.

calculated results for a $1.4 M_{\odot} + 1.0 M_{\odot}$ binary in model 3, to illustrate the three kinds of evolutionary sequences in Figure 2. The corresponding bifurcation period is found to be 1.25 days, and the initial orbital periods are chosen to be $P_i = 1.20, 1.25$, and 1.40 days, which represent the converging, parallel, and diverging systems, respectively.

3.2. Effect of MB and Mass Loss

Pylyser & Savonije (1988) have emphasized the effect of MB on the evolution of LMXBs. Comparing the results of models 1 and 2 presented in Table 1, we find that the bifurcation periods with traditional MB law are smaller (larger) than those with saturated MB law, when the initial secondary star mass $M_{2,i}$ is less (larger) than $0.7 M_{\odot}$.

Our results suggest that mass loss also influences the value of P_{bif} , though in a less important way compared with MB. The bifurcation periods in nonconservative models 3 and 4 are lower than those in model 2, in which conservative mass transfer has been assumed. This result is consistent with van der Sluys et al. (2005b) but contradicts Ergma et al. (1998).

It is also interesting to see whether an UCXB can form with saturated MB. For an LMXB with an initial orbital period below the bifurcation period, mass transfer is mainly driven by the loss of angular momentum. The orbital period will decrease with the donor mass until a minimum period is reached. Paczynski & Sienkiewicz (1981) found a minimum period of about 80 minutes without MB, while Podsiadlowski et al. (2002) showed that minimum orbital periods less than 11 minutes could be reached for binaries with an initial orbital period very close to the bifurcation period if traditional MB is included, but in a time longer than the age of the universe. van der Sluys et al. (2005b) further investigated this “magnetic capture” scenario for the formation of UCXBs. Our calculations show that when the initial orbital period is close to the bifurcation period, ultracompact systems ($P < 1$ hr) can indeed form with saturated MB, but also

in a time longer than the age of the universe. For example, for an LMXB with $M_{2,i} = 1.3 M_{\odot}$ and $P_i = 0.46$ days in model 4, a final period of $P_f = 22$ minutes can be reached after $\gtrsim 15$ Gyr of mass transfer. All the works done by previous authors show that a more efficient angular momentum loss mechanism is required to produce UCXBs within 13.7 Gyr in this scenario.

3.3. Semianalytical Method

In this subsection, we will try to use a semianalytical method to understand our numerical results. First from Equation (2), we have the following equation

$$\frac{3}{2} \frac{\dot{J}}{J} = \frac{3}{2} \left(\frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} \right) - \frac{1}{2} \frac{\dot{M}_1 + \dot{M}_2}{M_1 + M_2} + \frac{1}{2} \frac{\dot{P}}{P}. \quad (10)$$

If we assume a fraction α of the mass lost by the donor is accreted by the NS, i.e., $\dot{M}_1 = -\alpha \dot{M}_2$, we can write the period derivative as

$$\frac{\dot{P}}{P} = 3 \frac{\dot{J}}{J} - A(M_1, M_2, \alpha) \frac{\dot{M}_2}{M_2}, \quad (11)$$

where

$$A(M_1, M_2, \alpha) = \frac{3M_1^2 + 2(1 - \alpha)M_1M_2 - 3\alpha M_2^2}{M_1(M_1 + M_2)}. \quad (12)$$

Our analysis is limited to binary evolution with $M_2 < M_1$. In this case, it is clearly seen that mass transfer increases the orbital period and angular momentum loss decreases the orbital period. The bifurcation period is decided by the balance of these two factors.

Keeping the orbital period unchanged (i.e., $\dot{P} \simeq 0$), we calculate the maximum mass transfer rates for orbital periods from 0.35 days to 0.95 days. This period interval covers the whole range of the bifurcation periods obtained in this work

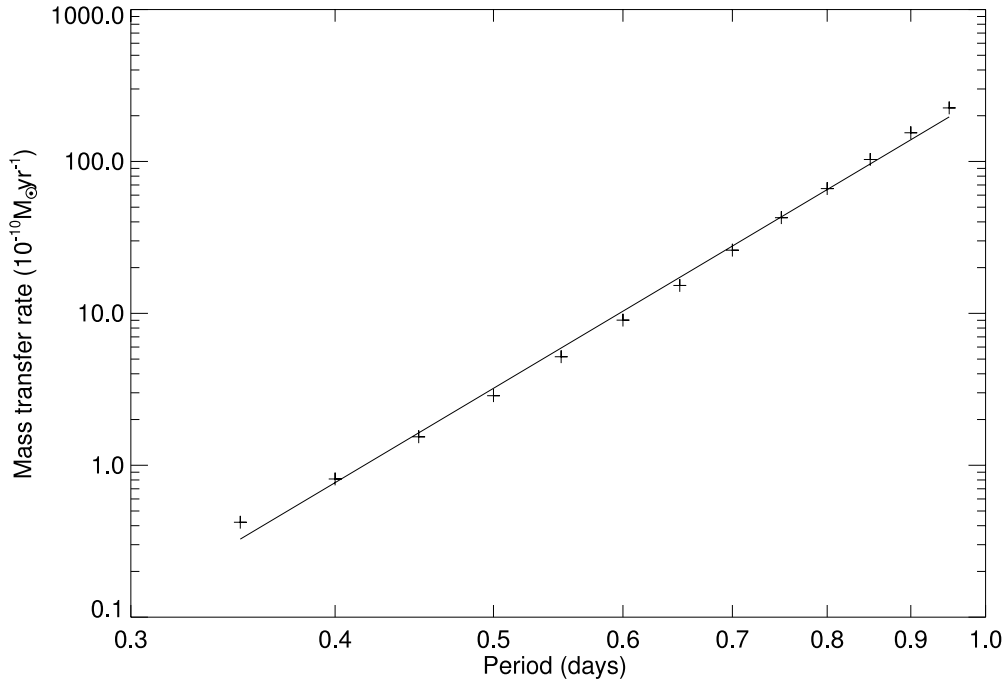


Figure 3. Maximum mass transfer rates in an LMXB consisting of $1.4 M_{\odot}$ neutron star and a $1 M_{\odot}$ secondary at fixed orbital periods from 0.35 days to 0.95 days. The crosses mark the calculated data and the solid line represents a logarithmic fit.

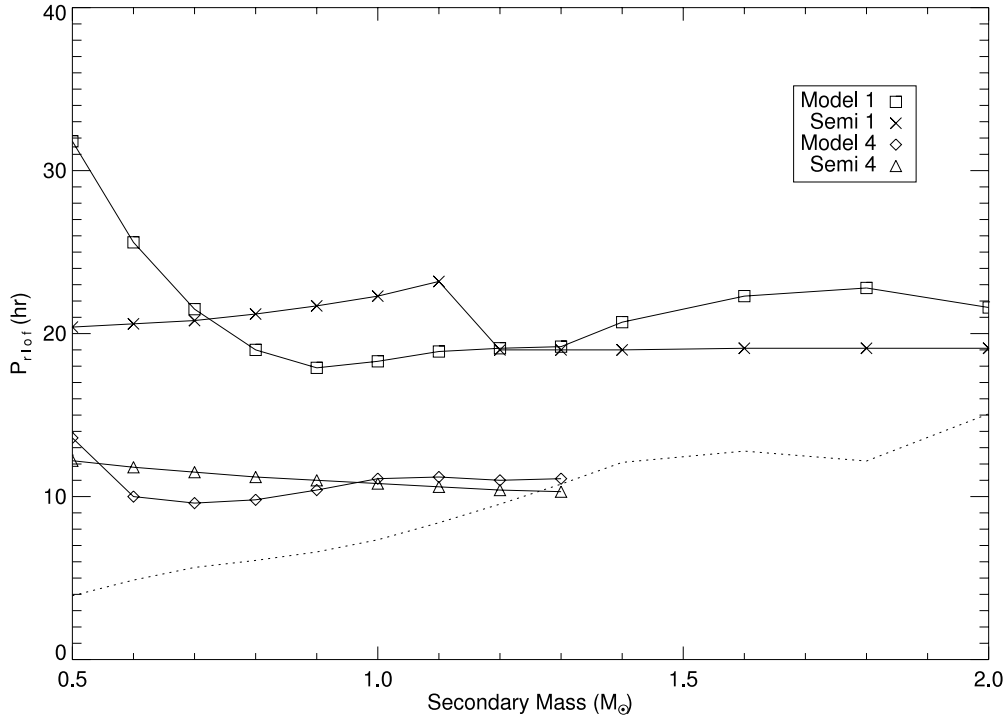


Figure 4. Comparison of the semianalytical results of P_{rlof} with the numerical values for model 1 and model 4. The dotted line shows the minimum initial period P_{ZAMS} that corresponds to a Roche lobe filling ZAMS secondary star. Here, the semianalytical 1 results are calculated with $\alpha = 1$ for $M_{2,i} \leq 1.1 M_{\odot}$, and $\alpha = 0$ for $M_{2,i} \geq 1.2 M_{\odot}$ where $\alpha = 1$ should be used. The reasons why we do this are given in the text.

and in Podsiadlowski et al. (2002). We show the calculated mass transfer rates in Figure 3, and find that they can be fitted by an approximate expression as

$$-\dot{M}_2(P) \simeq 2.73 \times 10^{-8} (P/\text{day})^{6.41(\pm 0.11)} M_{\odot} \text{yr}^{-1}. \quad (13)$$

Then we calculate the mean mass transfer rates with constant P and find that they lie between $\dot{M}_2(P)/2$ and $\dot{M}_2(P)$. This means

that if we use Equation (13) to calculate P_{rlof} , it will deviate no more than $\sim 10\%$ from the true value. The mean mass transfer rates here are calculated as follows. We fix the binary period P in a constant value in our code, and then evolve the donor from its initial mass $M_{2,i}$ to the time when it loses half of its initial mass $0.5M_{2,i}$. This mass transfer process takes a time of $T_{1/2}$. Then we use $0.5M_{2,i}/T_{1/2}$ as the mean \dot{M}_2 for this period P . If we assume that angular momentum loss is dominated by

saturated MB when $P < 10$ days, from Equations (2) and (7) we have

$$\frac{\dot{J}}{J_{\text{orb}}} = -K \omega_{\text{crit}}^2 \left(\frac{R_2}{R_\odot}\right)^{0.5} \left(\frac{M_2}{M_\odot}\right)^{-0.5} \left(\frac{4\pi^2}{G}\right)^{2/3} \frac{(M_1 + M_2)^{1/3}}{M_1 M_2} \times P^{-4/3} \simeq -6.2 \times 10^{-11} (P/\text{day})^{-1} \text{yr}^{-1}. \quad (14)$$

Here, we adopt $K = 2.7 \times 10^{47} \text{ g cm}^2 \text{ s}$ (Andronov et al. 2003), $\omega_{\text{crit}} = 2.9 \times 10^{-5} \text{ Hz}$ (Sills et al. 2000), $M_1 = 1.4 M_\odot$, $M_2 = 1 M_\odot$, and replace the radius of the secondary R_2 with its Roche lobe radius $R_{\text{L},2}$ from Equation (1). Combining Equations (11)–(14) with $\dot{P} = 0$, we obtain $P_{\text{rlof}} \simeq 12.4 \text{ hr}$ for conservative mass transfer ($\alpha = 1$), and $P_{\text{rlof}} \simeq 10.7 \text{ hr}$ for nonconservative mass transfer ($\alpha = 0$). These values agree well with our numerical results (~ 10.8 – 11.1 hr). If the traditional MB law is used, similarly, from Equation (6) we get

$$\frac{\dot{J}}{J_{\text{orb}}} = -3.8 \times 10^{-30} M_2 R_2^4 \omega^3 G^{-2/3} \frac{(M_1 + M_2)^{1/3}}{M_1 M_2} \omega^{1/3} \simeq -4.6 \times 10^{-9} (P/\text{day})^{-2/3} \text{yr}^{-1}, \quad (15)$$

for $M_1 = 1.4 M_\odot$ and $M_2 = 1 M_\odot$. Combining Equations (11)–(13) and (15) with $\dot{P} = 0$, we get $P_{\text{rlof}} \simeq 22.3 \text{ hr}$ for conservative mass transfer ($\alpha = 1$), which is about 20% larger than $\sim 18.3 \text{ hr}$ from our numerical calculations and $\sim 17.7 \text{ hr}$ in Podsiadlowski et al. (2002). The main reason for this difference is that we use the constant value $1.4 M_\odot$, $1 M_\odot$ for M_1 , M_2 in Equation (12), which should change with time to $\sim 2.2 M_\odot$, $\sim 0.2 M_\odot$. This will decrease the coefficient in Equation (12) and increase the value of P_{rlof} by $\sim 10\%$ – 20% . For donor mass $\geq 1.2 M_\odot$, this will increase the value of P_{rlof} by as much as $\sim 30\%$. So it is better to use $\alpha = 0$ instead of $\alpha = 1$ for donors mass $\geq 1.2 M_\odot$, which could yield more accurate results (from our numerical results we find that the deviation of P_{rlof} between conservative and nonconservative mass transfer is smaller than 10%).

Using Equations (11)–(15), we also compute the semianalytical results of P_{rlof} for 0.5 – $2 M_\odot$ donors, and compare them with our numerical results of models 1 and 4 in Figure 4, where the semianalytical 4 results are calculated with $\alpha = 0$. When we calculate the semianalytical 1 results in Figure 4, for the reasons mentioned above and below, we use $\alpha = 1$ for $M_{2,i} \leq 1.1 M_\odot$ and $\alpha = 0$ for $M_{2,i} \geq 1.2 M_\odot$, where $\alpha = 1$ should be used. A few points need to be noted for the semianalytical results in Figure 4. First with the above-mentioned equations it is impossible to compute the P_{rlof} for binaries with $M_{2,i} \geq 1.4 M_\odot$ under conservative mass transfer, since both terms in the right side of Equation (11) are negative when $\alpha = 1$ and $M_{2,i} \geq 1.4 M_\odot$, and there will be no solutions for $\dot{P} = 0$. We instead adopt $\alpha = 0$ when $M_{2,i} > 1.4 M_\odot$ (from our numerical results we find that the deviation of P_{rlof} between conservative and nonconservative mass transfer is smaller than 10%). Second, Equation (13) is derived only for $1 M_\odot$ donor star rather than donors in the whole mass range (0.5 – $2 M_\odot$), because in the latter case it is impossible to find a unified expression of the mass transfer rate like Equation (13). As seen in Figure 4, the difference between the semianalytical and numerical results is generally smaller than 20% except for donors smaller than $0.7 M_\odot$. The reasons for the big discrepancies when $M_{2,i} < 0.7 M_\odot$ are discussed in Section 4.

4. DISCUSSION AND CONCLUSIONS

Motivated by new ideas about MB and mass loss in LMXB evolution, we have made a systematic investigation on the bi-

furcation periods in binary models, taking into account different MB laws and mass-loss mechanisms. We find that the strength of MB is the dominant factor in determining the value of bifurcation periods compared with mass loss. The stronger the MB, the larger the bifurcation periods. This also results in an upper limit for the secondary masses beyond which no converging systems exist.

In our calculations we assume either fully conservative (models 1 and 2) or nonconservative (models 3 and 4) mass transfer to constrain the bifurcation period distribution in different mass transfer modes. From the expression of $A(M_1, M_2, \alpha)$ we always have

$$A(M_1, M_2, 1) = 3 - \frac{3M_2}{M_1} < A(M_1, M_2, 0) = 3 - \frac{M_2}{M_1 + M_2}, \quad (16)$$

which means that nonconservative mass transfer contributes more to the increase of the orbital period than conservative mass transfer. This explains why we generally have a lower bifurcation period in the nonconservative mass transfer models (models 3 and 4) than in the conservative mass transfer model (model 2) under the same MB law. The real situation may lie between these two extreme cases. For binary systems with donors $M_{2,i} \sim 0.5$ – $0.8 M_\odot$, it would take more than 13.7 Gyr before mass transfer begins via RLOF. So the bifurcation period for these systems seems meaningless, unless there exist some unknown mechanisms of loss of orbital angular momentum.

In our semianalytical analysis in Section 3.3 we use the condition $\dot{P} \sim 0$ to derive the values of P_{rlof} . This expression seems different from $P_f \simeq P_{\text{rlof}}$, which is the original definition of bifurcation period. We argue here that these two expressions are roughly the same except for binaries with $M_{2,i} \geq 1.4 M_\odot$ under conservative mass transfer ($\alpha = 1$), the reason of which has been given in Section 3.3. For $M_{2,i} < 1.4 M_\odot$, we find that \dot{P}/P always scales with P from Equations (11) and (13)–(15). This means that if initially $\dot{P} > 0$ (< 0), \dot{P}/P will become larger (smaller) during the evolution, leading to monotonic increase (decrease) of the period, as seen in Figure 2. So for these systems $P_f \sim P_{\text{rlof}}$ is approximately equivalent with $\dot{P} \sim 0$. Several rough assumptions in this semianalytical method contribute to the discrepancies between the semianalytical results and the numerical results in Figure 4, especially for $M_{2,i} < 0.7 M_\odot$. First is the use of $\dot{P} \sim 0$ as the definition of P_{rlof} , which may not work well sometimes. Second is the use of Equation (13), which is most suitable for binaries with $M_{2,i} = 1 M_\odot$ as pointed out in Section 3.3. Third is the assumption we made that MB law is the dominated mechanism for the angular momentum loss, while the true case is that the MB may not work sometimes (for example when the convective envelope is too small). Fourth is that we use a constant initial value of $M_{1,i}$, $M_{2,i}$ for M_1 , M_2 in Equation (12) and Equation (14)–(15), while in the true case M_1 , M_2 should change with time. This will cause a big problem for $\alpha = 1$ when $M_{2,i} > 1 M_\odot$, which has been pointed out in Section 3.3. Fifth reason is the use of $\omega_{\text{crit},\odot}$ in Equation (14) as the value of ω_{crit} for all the donors ranging from $0.5 M_\odot$ to $1.3 M_\odot$. At last we conclude that (1) \dot{P} is a fair definition of bifurcation period, and (2) the period evolution during the mass-transfer phase is in first approximation sufficiently well described by the balance of mass transfer and angular momentum loss caused by MB. For these rough assumptions made in this semianalytical method, its results agree with the numerical results only in the leading order.

Our numerical calculations show that there is an upper limit for the donor mass beyond which no converging systems will form. Pylyser & Savonije (1988) found that, in the case of $M_{1,i} = 4.0 M_{\odot}$, there is no converging system existing if $M_{2,i} > 1.7 M_{\odot}$, and concluded that for any given initial accretor mass there exists a maximum initial secondary mass for the formation of converging systems. From our calculations with $M_{1,i} = 1.4 M_{\odot}$, we find an upper limit for the initial secondary mass $M_{2,i}$ between 1.2 and $1.4 M_{\odot}$ under saturated MB. The reason is that for binaries with a MS donor of initial mass $> 1.4 M_{\odot}$, the bifurcation period is shorter than the minimum ZAMS period, so that these systems will diverge. For traditional MB, this upper limit is $> 2 M_{\odot}$, beyond the range of donor masses we adopt.

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