The Astrophysical Journal, 689: L33–L36, 2008 December 10

ALAS, THE DARK MATTER STRUCTURES WERE NOT THAT TRIVIAL

Kasper B. Schmidt,[1,2] Steen H. Hansen,[1] and Andrea V. Macciò[2]
Received 2008 September 25; accepted 2008 October 20; published 2008 November 14

ABSTRACT

The radial density profile of dark matter structures has been observed to have an almost universal behavior in numerical simulations; however, the physical reason for this behavior remains unclear. It has previously been shown that if the pseudo phase-space density, ρ/σ_d^ϵ, is a beautifully simple power law in radius, with the "golden values" $\epsilon = 3$ and $d = r$ (i.e., the phase-space density is only dependent on the radial component of the velocity dispersion), then one can analytically derive the radial variation of the mass profile, dispersion profile, etc. That would imply, if correct, that we just have to explain why $\rho/\sigma_r^3 \sim r^{-\alpha}$, and then we would understand everything about equilibrated DM structures. Here we use a set of simulated galaxies and clusters of galaxies to demonstrate that there are no such golden values, but that each structure instead has its own set of values. Considering the same structure at different redshifts shows no evolution of the phase-space parameters toward fixed points. There is also no clear connection between the halo virialized mass and these parameters. This implies that we still do not understand the origin of the profiles of dark matter structures.

Subject headings: dark matter — galaxies: halos — methods: data analysis — methods: numerical

1. INTRODUCTION

According to numerical simulations of dark matter (DM) structures, the mass density profile, $\rho(r)$, changes from something with a fairly shallow profile in the central region, $\gamma \equiv d \ln \rho / d \ln r \sim -1$ (or maybe 0), to something steeper in the outer region, $\gamma \sim -3$ (or maybe steeper) (Navarro et al. 1996; Moore et al. 1998; Diemand et al. 2007; see also Reed et al. 2005; Stoehr 2006; Navarro et al. 2004; Graham et al. 2006; Merritt et al. 2006; Ascasibar & Gottloeber 2008). For the largest structures, such as galaxy clusters, there appears to be fair agreement between numerical predictions and observations concerning the central steepness (Pointecouteau et al. 2005; Sand et al. 2004; Buote & Lewis 2004; Broadhurst et al. 2005; Vikhlinin et al. 2006); however, for smaller structures, such as galaxies or dwarf galaxies, observations tend to indicate central cores (Salucci et al. 2003; Gilmore et al. 2007; Wilkinson et al. 2004). Few purely theoretical attempts have been made to understand the origin of this density profile (e.g., González-Casado et al. 2007; Henriksen 2007), with varying level of success.

A completely different approach is to search for simple phenomenological relations in the numerical simulations, such as finding straight lines in some parameter space. The idea is then that such phenomenological relations may reduce the complexity of the Jeans equation, which can then be solved analytically.

One of the most successful attempts in this direction was sparked by the discovery that the pseudo phase-space density is approximately a power law in radius, $\rho/\sigma_r^3 \sim r^{-\alpha}$ (Taylor & Navarro 2001). The most simple analytical solutions to this problem showed that the density slopes could vary in the range from -1 to -3 (Hansen 2004a), in excellent agreement with numerical results of Navarro et al. (1996). The analytical investigations were taken to a higher level in Austin et al. (2005), who demonstrated that there is a characteristic value $\alpha = 1.944$ when one considers isotropic structures. Shortly after, Dehnen & McLaughlin (2005) used the results of numerical simulations (Diemand et al. 2004a, 2004b) to show that the "golden values" $\alpha = 1.944$ and $\epsilon = 3$ indeed provides a very good fit, when one is using the *radial* velocity dispersion in the pseudo phase-space density. Dehnen & McLaughlin (2005, hereafter DM05) also solved the Jeans equation under this assumption, and demonstrated explicitly that one can thus derive analytically all relevant profiles for the DM structure. Many other authors have considered similar pseudo phase-space densities, e.g., Hansen et al. (2006), Knebe & Wießner (2006), Stadel et al. (2008), Knollmann et al. (2008), Ascasibar & Gottloeber (2008), Zait et al. (2008), Van Hese et al. (2008), Navarro et al. (2008), and Lapi & Cavaliere (2008).

All this implies that if we can explain the origin of the very simple connection, $\rho/\sigma_r^3 \sim r^{-\alpha}$, then we have a complete understanding of the DM structures. However, this carries the implicit assumption that the three golden values are indeed the same for all structures, namely that $\alpha = 1.944$, and that the relevant quantity to consider is the radial dispersion, σ_d^ϵ with $d = r$ and $\epsilon = 3$.

Here we use the results of recent numerical simulations to demonstrate that this is *not* the case, and that there is *no* simple universal pseudo phase-space density for equilibrated DM structures. Given our findings it therefore appears that few theoretical approaches that successfully explain the origin of the cosmological profiles, such as the Barcelona model (Manrique et al. 2003; González-Casado et al. 2007), remain.

2. GENERALIZED PSEUDO PHASE-SPACE DENSITY

In order to test whether a generalized phase-space density exists, we consider the relation[3]

$$\rho/\sigma_d^\epsilon \propto r^{-\alpha}. \quad (1)$$

[1] Dark Cosmology Centre, Niels Bohr Institute, University of Copenhagen, Juliane Maries Vej 30, 2100 Copenhagen, Denmark.
[2] Max-Planck-Institut für Astronomie, Königstuhl 17, 69117 Heidelberg, Germany.

[3] As pointed out by Jin H. An, it would be very natural to perform an investigation similar to the one presented here on a generalized phase-space density of the form $\rho/(\sigma_r^{k_1}\sigma_t^{k_2})$, where k_1 and k_2 are the parameters to be explored (as done with D and ϵ here).

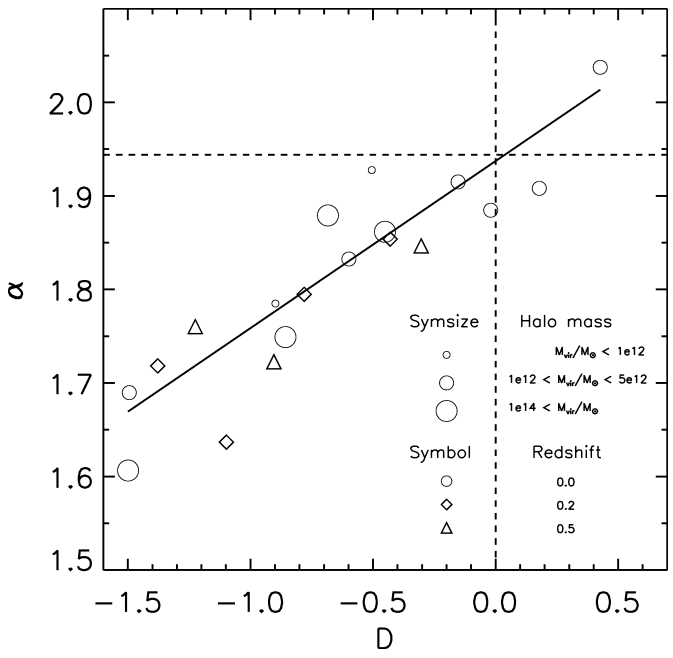

FIG. 1.—Optimized D and α values from the MC code plotted against each other. The vertical and horizontal dashed lines indicate the golden values suggested by Austin et al. (2005) and Dehnen & McLaughlin (2005). The solid line is a linear fit to the results for the $z = 0$ structures (*circles*), and is given by $\alpha = (0.19 \pm 0.02)D + (1.94 \pm 0.02)$. This indicates that the suggested golden values are just a consequence of using σ_r in the phase-space density relation. The results for the galaxy-size WMAP1 structures at $z = 0.2$ (*diamonds*) and $z = 0.5$ (*triangles*) are overplotted for comparison, and shows a trend very similar to the $z = 0$ case. No correlation between halo mass (indicated by different symbol size) and the fitting parameters is detected.

Here we have defined the general velocity dispersion as (Hansen 2007; Schmidt 2008)

$$\sigma_d^\epsilon = \sigma_r^\epsilon (1 + D\beta)^{\epsilon/2} . \quad (2)$$

Here $\beta(r) = 1 - \sigma_{\rm tan}^2/\sigma_{\rm rad}^2$ is the usual velocity anisotropy parameter, where $\sigma_{\rm rad}$ and $\sigma_{\rm tan}$ are the radial and tangential component of the velocity dispersion, respectively. Thus, setting $D = 0$ corresponds to using the radial component of the DM structure velocity dispersion in the phase-space density expression. Allowing $D \neq 0$, the phase-space density depends on a velocity dispersion that can be any combination of $\sigma_{\rm rad}$ and $\sigma_{\rm tan}$, e.g., $D = -2/3$ corresponds to using $\sigma_{\rm tot}$, and $D = 1$ corresponds to $\sigma_{\rm tan}$.

Here we use the rather simple analytical pseudo phase-space density; however, the actual six-dimensional phase-space density is different from this one and is not a power law in radius according to simulations (Stadel et al. 2008).

3. NUMERICAL SIMULATIONS

To test if the values of D, α, and ϵ are the same for all structures, we used a set of intermediate- and high-resolution simulated DM structures. These structures are all created using the PKDGRAV tree code by Joachim Stadel and Thomas Quinn (Stadel 2001). The high-resolution simulation Via Lactea includes one highly equilibrated structure of mass $M_{\rm halo} = 1.77 \times 10^{12}\ M_\odot$, containing about 84 million particles (Diemand et al. 2007). This structure did not experience any major mergers since $z = 1$, and all the quantities are extracted in spherical bins. The rest of the structures are galaxy-size and cluster-size DM halos based on either a *WMAP* 1 year or 3 year cosmology. The initial conditions for these structures are generated with the GRAFIC2 package (Bertschinger 2001). The starting redshifts z_i are set to the time when the standard deviation of the smallest density fluctuations resolved within the simulation box reaches 0.2 (the smallest scale resolved within the initial conditions is defined as twice the intraparticle distance). All the halos were identified using a spherical overdensity algorithm (Macciò et al. 2007). The cluster-like halos have been extracted from a 63.9 Mpc h^{-1} simulation containing 600^3 particles, with a mass resolution of $m_p = 8.98 \times 10^7\ M_\odot\ h^{-1}$. The masses of the clusters used for this study are 2.1, 1.8, and $1.6 \times 10^{14}\ M_\odot\ h^{-1}$. The galaxy-size halos have been obtained by resimulating halos found in the previous simulation at high resolution. The simulated halos are in the mass range 0.9–$2.5 \times 10^{12}\ M_\odot\ h^{-1}$ and have a mass resolution of $m_p = 4.16 \times 10^5\ M_\odot\ h^{-1}$. That gives a minimum number of particles per halo of about 2.5×10^6. The high-resolution cluster $C_{\rm HR}W3$ has 11 million particles within its virial radius and a mass of $M = 1.81 \times 10^{14}\ M_\odot\ h^{-1}$.

From these numerical simulations we directly calculate all the relevant quantities, such as $\rho(r)$, $\sigma_r(r)$, $\sigma_\theta(r)$, and $\sigma_\phi(r)$, where the σ's are combined to obtain $\beta(r)$. These profiles can then be compared to the pseudo phase-space density defined in equation (1).

4. MONTE CARLO CODE

In order to test whether the suggested golden values of DM05 and Austin et al. (2005) do indeed exist, we wrote a Monte Carlo (MC) code to optimize the parameters of the phase-space density in equation (1), for each of the simulated DM structures.

The MC code is based on the temperature annealing principle (Kirkpatrick et al. 1983; Hansen 2004b). We want to optimize the parameter set (D,α,ϵ), so that it makes the left and right-hand sides of equation (1) converge toward the expected power-law relation. In order to do that we search the parameter space and for each jump estimate the χ^2 value of the relation defined by

$$\chi^2 = \sum_i \left[\frac{f_1(x_i) - f_2(x_i)}{df_2(x_i)}\right]^2 . \quad (3)$$

Here x_i corresponds to the data input from the simulations, f_1 and f_2 corresponds to the left and right-hand sides of the relation, and df_2 is the error on f_2. Since we are dealing with simulations, we have no reasonable estimate of the error df_2. Therefore, we use $df_2 = 0.05\ (\rho/\sigma_r^3)$. Choosing different kinds of errors (e.g., ρ/σ_D^ϵ, ρ/σ_D^3, ρ/σ_r^ϵ, and $r^{-\alpha}$) with different magnitudes (0.05, 0.07, and 0.10) has no significant systematic effect on the final result. Since there is the possibility of local minima in the χ^2 landscape, we have implemented the metropolis choice in our code (Metropolis et al. 1953). All technical details of this code can be found in Schmidt (2008).

5. RESULTS

Combining the simulated DM structures with the MC code, we are able to estimate the parameters that optimizes the phase-space density relation for each structure. If there should be a general phase-space density relation, each structure should have the same optimized parameters. We see in Figure 1 that this is not the case. In Figure 1 we have indicated the suggested golden values as horizontal and vertical dashed lines.

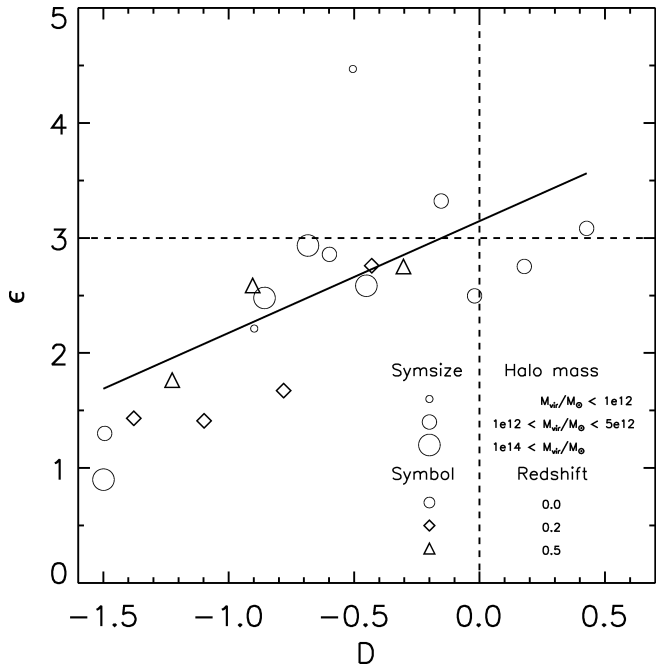

FIG. 2.—Same as Fig. 1, but for D and ϵ with a linear fit given by $\epsilon = (0.97 \pm 0.37)D + (3.15 \pm 0.29)$.

The obtained (roughly) linear relations in Figures 1 and 2 are

$$\alpha = (0.19 \pm 0.02)D + (1.94 \pm 0.02), \quad (4)$$

$$\epsilon = (0.97 \pm 0.37)D + (3.15 \pm 0.29). \quad (5)$$

This shows that a generalized phase-space density relation does not exist. Thus, our results suggest that the hunt for a physical explanation of the (often assumed universal) power-law appearance of ρ/σ^3 is probably a dead end. It seems that this expression is nothing more than a possible fitting function with the nice property of having the same physical units as the phase-space density.

If we force $D = 0$ (like DM05), we get from equations (4) and (5) that $\alpha = 1.94 \pm 0.02$ and $\epsilon = 3.15 \pm 0.29$. These values are in excellent agreement with the results from DM05. If we use other values of D, i.e., phase-space densities with combinations of the different velocity dispersion components, we get the parameter values listed in Table 1.

Knowing that the parameters that optimizes the phase-space density relation in equation (1) for simulated structures at redshift $z = 0$ are related, it would be interesting to see whether such relations are also present at higher redshifts. We use $z > 0$ snapshots for the WMAP1 galaxy-size structures (clusters are not significantly relaxed at high redshifts). Running the MC code with these DM halos gave results very similar those for the $z = 0$ structures. In general, we did not find any indication of a redshift dependence for the optimized values, showing that there is no special attractor for the values of D, α, and ϵ. The results at $z > 0$ (*triangles and diamonds*) are plotted together with the $z = 0$ results in Figures 1–3. The linear relations obtained between the optimized parameters are not significantly affected by redshift, and therefore our calculations suggest that equations (4) and (5) are valid for all redshifts.

Furthermore, we find no significant correlation between the virial mass of the simulated structures and the α, ϵ, and D values (see Figs. 1–3).

TABLE 1

RESULTS FOR DIFFERENT PHASE-SPACE DENSITIES USING THE RELATIONS IN EQUATIONS (4) AND (5)

D	Phase-Space Density	α	ϵ
1	$\rho/\frac{1}{2}(\sigma_\phi^2 + \sigma_\theta^2)^{\epsilon/2}$	2.13 ± 0.03	4.12 ± 0.47
0*	ρ/σ_r^ϵ	1.94 ± 0.02	3.15 ± 0.29
−1	$\rho/[2\sigma_r^2 - \frac{1}{2}(\sigma 2\phi + \sigma 2\theta)]^{\epsilon/2}$	1.75 ± 0.03	2.18 ± 0.47
−2/3	$\rho/[\frac{1}{3}(\sigma 2r + \sigma 2\phi + \sigma 2\theta)]^{\epsilon/2}$	1.81 ± 0.02	2.50 ± 0.38

NOTES.—The phase-space densities are extracted from eq. (2) using the corresponding D values. The case marked with an asterisk (*) is the one used in Hansen (2004a), Austin et al. (2005), and Dehnen & McLaughlin (2005).

6. CONCLUSIONS

Using a set of numerically simulated galaxy and cluster-sized DM structures and analyzing them with a Monte Carlo code, we show that no generalized pseudo phase-space density relation seems to exist in general. We have thus shown that the previously suggested relation $\rho/\sigma^3 \sim r^{-\alpha}$ does not hold universally. The redshift and mass independence of our results show that there is no special attractor for the parameters describing the generalized phase-space density.

Instead, we happen to identify a set of seemingly linear relations between the parameters D, α, and ϵ (describing the generalized pseudo phase-space density from eq. [1]), which we have parameterized in equations (4) and (5).

Thus, given our findings that ρ/σ^3 is nothing but a nice fitting formulae and not a physical attractor, we are still far from truly understanding the density profile of DM structures.

It is a pleasure to thank Juerg Diemand for kindly providing part of the numerically simulated data used in the figures. We are grateful to Jin H. An and Bo V. Hansen for fruitful discussions. Part of the numerical simulations were performed on the PIA cluster of the Max-Planck-Institut für Astronomie at the Rechenzentrum in Garching. The Dark Cosmology Centre is funded by the Danish National Research Foundation.

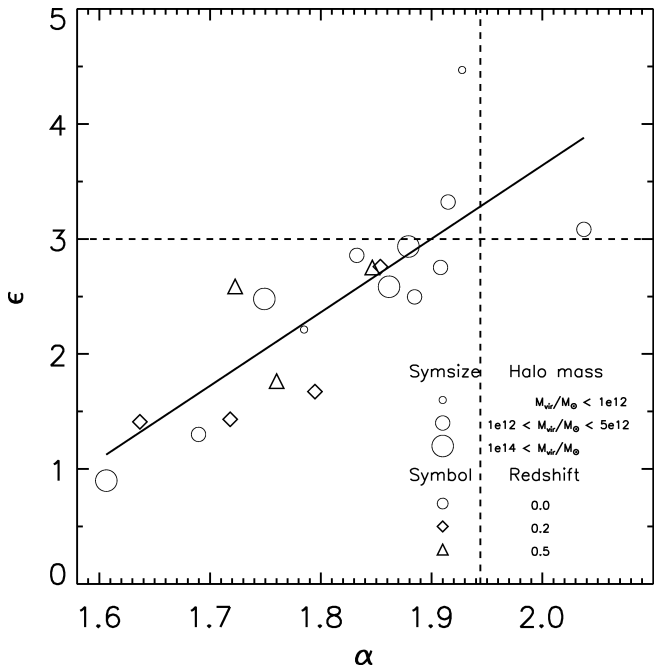

FIG. 3.—Same as Fig. 1, but for α and ϵ with a linear fit given by $\epsilon = (6.39 \pm 1.44)\alpha + (-9.14 \pm 2.65)$.

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