STOKES TRAPPING AND PLANET FORMATION

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ABSTRACT

It is believed that planets are formed by aggregation of dust particles suspended in the turbulent gas forming accretion disks around developing stars. We describe a mechanism, termed "Stokes trapping," by which turbulence limits the growth of aggregates of dust particles, so that their Stokes number (defined as the ratio of the damping time of the particles to the Kolmogorov dissipation timescale) remains close to unity. We discuss possible mechanisms for avoiding this barrier to further growth. None of these is found to be satisfactory, and we introduce an alternative hypothesis, which does not involve the growth of small clusters of dust grains.

Subject headings: accretion, accretion disks — planets and satellites: formation — turbulence

Online material: color figures

1. BACKGROUND

It is widely believed that planets are formed by aggregation of dust particles in an accretion disk surrounding a growing star; the fact that solar planets have orbits that are roughly circular and coplanar with the Sun's equator is readily explained by this model. According to this picture, planet formation is a two-stage process (Safranov 1969; Goldreich & Ward 1973; Armitage 2007). The final stage must be driven by gravitational forces, but initially the density of dust particles is not sufficiently high for gravitational instability to overcome centrifugal forces. It is believed that the dust particles initially aggregate by random collisions and that when the aggregates have grown to a sufficient size, they sink to the midplane of the accretion disk. Gravitational collapse to form planets can commence if the density at the midplane of the disk becomes sufficiently high. In this paper we are concerned with the first ("kinetic") phase of planet formation in this standard model.

The kinetic aggregation process in protoplanetary disks (reviewed by Dominik et al. 2007; Henning et al. 2006; Armitage 2007) has distinctive features. The dust particles are surmised to be in a highly turbulent environment (at least during periods when the star is accreting material at a substantial rate) because laminar flows could not dissipate energy sufficiently rapidly to account for observed accretion rates. Unless some mechanism enables the particles to fuse together chemically, they must form very weak aggregates (bound by van der Waals and possibly electrostatic forces) until gravitational collapse is well advanced: this implies that the aggregates are very fragile unless they can be heated to high temperatures. Also, the aggregates of dust particles may be fractal structures, with a fractal dimension D.

The principal purpose of this paper is to argue that the standard model for planet formation by aggregation of dust grains is highly problematic, because the dust grains are unable to aggregate in the turbulent environment of the accretion disk. Furthermore, we argue that the turbulence would prevent even very large aggregates from settling to the midplane of the disk. Both of these difficulties are discussed in earlier works (for example, the fragility of aggregates is discussed by Youdin & Shu [2002] and Benz [2000],

and the difficulty of dust settling in a turbulent environment is considered in Weidenschilling & Cuzzi [1993]; other works are cited below). The earlier literature, however, assumes that the difficulties may be removed by refinements of the standard model. Here we give a systematic discussion of the aggregation of small clusters and argue that the difficulties are so severe that it is necessary to develop an alternative theory.

A note about terminology is in place here. We describe the condensed material in the interstellar medium as consisting of dust *grains*, which may form *aggregates*, bound by van der Waals or electrostatic forces. We argue that these aggregates may *cluster* together in space, without coming into contact. We also describe properties of small fragments of solid material in a turbulent gas in a more general context, without needing to specify whether they are dust grains or aggregates; in such cases we refer to *particles* suspended in the gas.

2. SUMMARY OF RESULTS

This paper uses a recently developed theory for the collision rate of turbulent aerosols (Wilkinson et al. 2006; Mehlig et al. 2007) to draw conclusions about the aggregation of dust particles in a standard model for a protostellar accretion system. An important parameter is the Stokes number, St, which is the ratio of the timescale for an aggregate of dust grains to be slowed by drag forces (the "stopping time") to the dissipative correlation time of the turbulent forcing. Note that our definition of the Stokes number differs from that used in some other works on planet formation (for example, Youdin 2004; Bauer et al. 2007; Youdin & Lithwick 2007), which define the Stokes number as the ratio of the stopping time to the orbital period, although our definition is that which is used in the fluid dynamics literature, for example, in Falkovich et al. (2002), Duncan et al. (2005), and Bec et al. (2006).

Our principal conclusions may be summarized as follows. It has been argued (Mizuno et al. 1988) that the relative velocity of the dust aggregates, and therefore their rate of collision, increases significantly when aggregates grow to a size such that their Stokes number is of order unity. Our recent work (Wilkinson et al. 2006; Mehlig et al. 2007) enables this effect to be quantified in terms of the turbulence intensity. We use our results to show that the relative velocity of particles suddenly increases by several orders of magnitude when the Stokes number becomes of order unity. We argue that this effect results in the formation of a "Stokes trap":

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aggregates with Stokes numbers significantly larger than unity will collide with sufficiently high relative velocity that they fragment upon collision, thereby replenishing the population of aggregates with smaller Stokes number. This process would result in a stable population of aggregates with a Stokes number that has a distribution spanning St $\approx$ 1. The production of planets therefore depends on the particle aggregation process being able to escape from this Stokes trap. We consider four scenarios by which escape from the Stokes trap might happen:

1. *Clustering.*—Particles in a turbulent flow with Stokes numbers of order unity have been shown to cluster together. This clustering effect has been ascribed to particles being "centrifuged" away from vortices (Maxey 1987). More recent work has shown that the particles cluster onto a fractal set (Sommerer & Ott 1993; Bec et al. 2006; Duncan et al. 2005) of dimension D_{cl}. Note that there are two fractal dimensions in this problem: the dust particle aggregates may be fractals of dimension D, and these aggregates cluster onto a set of dimension D_{cl}. It is conceivable that this clustering effect might create a means of escaping the Stokes trap, but we have not been able to identify a mechanism that would be sufficiently effective in the model which we have considered.

It has previously been suggested that this clustering effect may play a role in planet formation (Cuzzi et al. 2001), but this work does not consider the possibility that aggregates are fragmented when their Stokes number is close to unity. The clustering effect only occurs when St $= O(1)$, but without fragmentation the aggregation process produces a broad (and time-dependent) distribution of aggregate sizes. The clustering mechanism would then only act on a small proportion of the aggregates. Our Stokes-trapping mechanism avoids this difficulty, by ensuring that most of the aggregates have a Stokes number that is of order unity.

2. *Temperature variations.*—The temperature of the dust aggregates could vary dramatically as they are swept back and forth between the surface and the midplane of the accretion disk by turbulent motion of the gas. High temperatures can facilitate chemical reactions, as can the condensation of water and other volatile materials when the dust aggregate encounters low-temperature regions. Chemical processes could cause the dust aggregates to become sufficiently tightly bound that they are less easily fragmented in collisions. In addition, condensation tends to occur more readily in corners, favoring the formation of compact aggregates. Below we argue that the accretion disk is sufficiently optically thick at the crucial stage of its development that there can be a marked difference between the surface and midplane temperatures. However, unless the star is accreting at a high rate, at distances significantly greater than 1 AU the temperatures are low everywhere across the profile of the disk. We conclude that chemical processes are probably not significant unless accretion is occurring at a very high rate.

3. *Fractal aggregates.*—The Stokes trap can also be avoided if the dust particles form aggregates with a fractal dimension D less than 2; in this case we find that the Stokes number does not increase as the size of the aggregate increases. Some experiments on dust grains (Blum et al. 2000; Krause & Blum 2004) have shown evidence that they form aggregates with a fractal dimension approximately equal to 1.4; others (Wurm & Blum 1998) obtained values close to 1.9. It should be noted that these experiments only observe the dust particle aggregates for a short time after their creation, whereas in protoplanetary processes the aggregates have plenty of time to relax to a more compact state, which would be energetically favored due to increased contact

area. The structure of dust aggregates in real protoplanetary systems is therefore highly uncertain. The hypothesis that the dust aggregates are tenuous structures with $D \leq 2$ does not appear to provide a satisfactory resolution, because these tenuous aggregates will always be advected with the gas and will not sink to the midplane. We also show that tenuous aggregates may be torn apart by shearing forces.

4. *Quiescence.*—It might be expected that the relative velocities of the dust particle aggregates are reduced if the rate of accretion is reduced. We write the rate of accretion onto the star as $\dot{M} = 10^{-7} \Lambda \ M_{\odot} \ \mathrm{yr}^{-1}$, where the coefficient is chosen so that $\Lambda = 1$ corresponds to a typical rate of accretion (Armitage 2007). We investigate the scaling of the relative velocity of suspended particles with the scaling parameter Λ and find that it increases as Λ decreases. We conclude that the Stokes trap is still present in relatively quiescent disks, where Λ is small.

Our calculations are based on estimates using Kolmogorov's theory for turbulence (Frisch 1995) in combination with a standard model for an accretion disk (Shakura & Sunyaev 1973). In our application of the Kolmogorov theory we assume that the size scale of the largest eddies, L is proportional to the scale height of the disk, H, writing $L = \ell H$ (we show that this is equivalent to the "α-prescription"; Shakura & Sunyaev 1973). We also consider how the results depend on the additional scaling parameter ℓ.

Because of the large uncertainties in the parameters of the model, our calculations are to be interpreted as illustrations of the Stokes trapping principle, rather than as quantitative predictions. Order-of-magnitude changes in the uncertain parameters will not alter our principal conclusion about the effectiveness of the Stokes trap in limiting dust aggregation. For simplicity we confine attention to the case where the heating of the disk is active (that is, dominated by dissipation in the disk rather than by direct illumination from the star). Our calculations lead to the estimate that typical particle sizes of Stokes-trapped aggregates at 10 AU from the star are of order 25 μm.

In principle it is possible to gain information about the sizes of particles suspended in accretion disks around young stars from spectroscopic studies, but this is an ill-conditioned problem (Carciofi et al. 2004), with uncertain spectroscopic data. Studies of the "silicate feature" (at approximately 10 μm wavelength) in a survey of spectra of accretion disks around young stars by Kessler-Silacci et al. (2007) are interpreted as indicating typical particle sizes between 3 and 10 μm, between 10^{-3} and 10 AU from the star. Similar estimates were obtained by van Boekel et al. (2003) and Apai et al. (2005). By contrast, spectral analysis in the millimeter range (probing predominantly the outer regions of the disk) has been interpreted as indicating that particles larger than 50 μm may be present (Wood et al. 2002; Bauer et al. 2007).

Even if the particle growth process can circumvent the Stokes trap, the particle aggregates must collapse to the midplane of the disk for gravitational instability to become effective. If we make the assumption that the integral scale of the turbulence is comparable to the thickness of the accretion disk (that is, if ℓ is of order 1), we show that advection of particles by the largest turbulent eddies presents another very significant barrier to planet formation: turbulence will prevent aggregates of centimeter size from settling to the midplane, unless the rate of accretion is very substantially smaller than $10^{-7} \ M_{\odot} \ \mathrm{yr}^{-1}$.

We conclude that there is no satisfactory theory for the formation of planets by aggregation of submicron-sized dust grains.

An alternative theory is therefore required. In § 9 we introduce an alternative hypothesis.

3. COLLISION PROCESSES IN TURBULENT AEROSOLS

Throughout we assume that the drag force on a particle is proportional to the difference in velocity between the particle and the surrounding gas, so that the equation of motion is

$$\ddot{\boldsymbol{r}} = \gamma[\boldsymbol{u}(\boldsymbol{r},t) - \dot{\boldsymbol{r}}], \tag{1}$$

where $\boldsymbol{r}$ is the position of the particle and $\boldsymbol{u}(\boldsymbol{r}, t)$ is the fluid velocity field (until the particles come into contact). This equation is familiar in the context of Stokes's law for the drag on a sphere, where the damping rate γ is proportional to the kinematic viscosity ν.

We assume that the gas has a multiscale turbulent flow that is described by the Kolmogorov theory of turbulence (Frisch 1995). The length scale and timescale associated with the smallest eddies where energy is dissipated are denoted by η and τ, respectively. According to the Kolmogorov theory of turbulence, these quantities are determined by the rate of dissipation per unit mass, $\mathcal{E}$, and the kinematic viscosity ν:

$$\eta = \left(\frac{\nu^3}{\mathcal{E}}\right)^{1/4}, \quad \tau = \left(\frac{\nu}{\mathcal{E}}\right)^{1/2}. \tag{2}$$

If the turbulent motion is driven by forces acting on a length scale $L \gg \eta$, the velocity fluctuations of the fluid have a power-law spectrum for wavenumbers between $1/L$ and $1/\eta$ (Frisch 1995).

We define the Stokes number to be $\mathrm{St} = 1/(\gamma\tau)$. Very small particles are advected with the gas flow, but in addition they have small random velocities due to Brownian diffusion, which can cause them to collide at a rate $\mathcal{R}_d$. They can also collide due to the shearing motion of the flow, an effect discussed by Saffman & Turner (1956) in the context of rainfall from turbulent clouds. These advective collisions occur at a rate $\mathcal{R}_a$, which turns out to be negligible compared to Brownian diffusion in the present context (see § 6).

When the Stokes number approaches unity, there is a dramatic increase in the relative velocity of suspended particles due to the formation of fold caustics in their velocity field, illustrated for the case of one spatial dimension in Figure 1. When faster particles overtake slower ones, the manifold representing the phase-space distribution of the particles develops folds, and the velocity field of the particles goes from being single valued to multivalued (three-valued, in this illustration). Before the caustics form, the relative velocity of the particles is due to their Brownian diffusion. After the folds have formed, the relative velocity may be orders of magnitude higher. The rate of collision $\mathcal{R}$ for a suspended particle in a turbulent flow is approximated by a formula presented by Mehlig et al. (2007):

$$\mathcal{R} = \mathcal{R}_d + \mathcal{R}_a + \exp(-A/\mathrm{St})\mathcal{R}_g. \tag{3}$$

Here $\mathcal{R}_g$ is the collision rate predicted by a model introduced by Abrahamson (1975), often termed the "gas-kinetic model," in which the suspended particles move with velocities that become uncorrelated with each other and with the gas flow. A less precise formula was suggested by Falkovich et al. (2002), and the idea that there is a dramatic change in the relative velocity of particles at $\mathrm{St} \approx 1$ appears to have been originally proposed by Mizuno et al. (1988) and Markiewicz et al. (1991). The exponen-
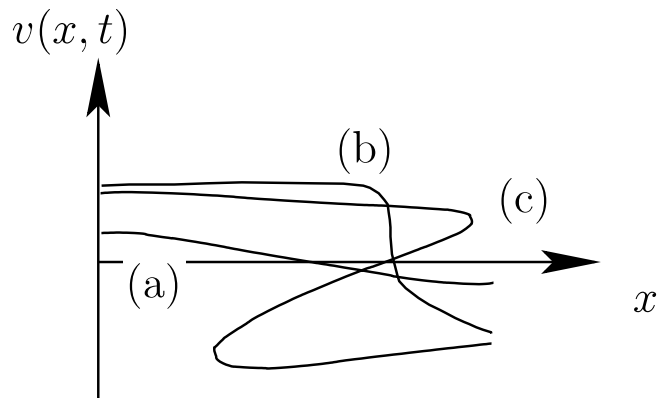

Fig. 1.—Fold caustics. The velocity $v(x, t)$ of the suspended particles is initially single valued (a), but if the motion of the particles is overdamped, fast particles may catch up with (b) and overtake (c) slower ones. The velocity field becomes triple valued in the region between two fold caustics, which are singularities of the projection of the phase-space manifold onto coordinate space. In the region between the caustics the suspended particles have a relative velocity, and their trajectories cross. [*See the electronic edition of the Supplement for a color version of this figure.*]

tial term in equation (3) describes the fraction of the coordinate space for which the velocity field is multivalued; A is a "universal" dimensionless constant. The nonanalytical dependence of the rate of caustic production on St was noted by Wilkinson & Mehlig (2005), and recent simulations of Navier-Stokes turbulence suggest that $A \approx 2$ (Falkovich & Pumir 2007). The rate $\mathcal{R}_g$ greatly exceeds $\mathcal{R}_d$, but the gas kinetic theory is only applicable when the velocity field of the suspended particles is multivalued. The exponential term arises because the formation of caustics is determined by a process involving escape from an attractor by a diffusion process, similar to the Kramers model for a chemical reaction. The exponential term is therefore analogous to the Arrhenius term $\exp(-E/k_{\mathrm{B}}T)$ in the expression for the rate of an activated chemical reaction (Wilkinson & Mehlig 2005).

At large Stokes number the rate of collision of particles of radius a is

$$\mathcal{R}_g = 4\pi a^2 n \langle \Delta v \rangle, \tag{4}$$

where n is the number density of particles and Δv is the relative speed of two suspended particles at the same position in space. (Angular brackets denote averages throughout.) In a multiscale turbulent flow, when $\gamma\tau \ll 1$ the motion of the suspended particles is underdamped relative to the motion on the dissipative scale, but it is overdamped relative to slower long-wavelength motions in the fluid. The relative velocity of two nearby particles is a result of the different histories of the particles. If we follow the particles far back in time to when they had a large separation, their velocities were very different, but these velocity differences are damped out when the particles approach each other. Mehlig et al. (2007) surmised the variance of the relative velocities using the Kolmogorov scaling principle. In the following we consider, for simplicity, only the case of a symmetric collision where both particles have the same damping rate, γ. In the case of collisions between particles of very unequal sizes, we may still use the estimate below, using the smaller of the two values of γ.

When the particles are underdamped relative to the smallest dissipative scale, but overdamped relative to the "integral" (driving) scale, we can apply the Kolmogorov cascade principle (Frisch 1995) that motion in the inertial range is independent of the mechanism of dissipation. It is therefore determined by the

rate of dissipation $\mathcal{E}$, but it does not depend on ν. The moments of the relative velocity therefore depend only on $\mathcal{E}$ and γ. For the second moment, dimensional considerations imply that (Mehlig et al. 2007)

$$\langle \Delta v^2 \rangle = K \frac{\mathcal{E}}{\gamma}, \qquad (5)$$

where Kolmogorov's theory of turbulence suggests that K is a universal constant, but in practice K should have a weak dependence on Reynolds number due to intermittency effects (Frisch 1995). The constant K can be determined by simulation of Navier-Stokes turbulence, but this has not yet been done. We use $K = 1$ in the remainder of this article.

Völk et al. (1980; see also Mizuno et al. 1988; Markiewicz et al. 1991) have discussed relative velocities in the large Stokes number limit, expressing their results in terms of properties of the integral velocity and timescales: their results are equivalent to equation (5) but are harder to apply because the precise form of the integral scale motion is highly uncertain, whereas the rate of dissipation in equation (5) is determined directly from the accretion rate and the mass per unit area in the accretion disk.

4. PROPERTIES OF DUST-PARTICLE AGGREGATES

4.1. *Damping Rate*

During the kinetic phase of planet formation, the size a of the suspended particles is usually smaller than the mean free path of the gas, λ. In this case the drag force is proportional to $\rho_g c_s \bar{A}(\boldsymbol{u} - \dot{\boldsymbol{r}})$, where ρ_g is the gas density, $\bar{A}$ is the angular average of the projected area of the particle, and c_s is the mean molecular speed of the gas (Epstein 1924). In this case the damping rate is

$$\gamma \sim \frac{\rho_g c_s \bar{A}}{m}, \qquad (6)$$

where m is the mass of the particle. In the case of spherical particles, the coefficient in equation (6) is 4/3 (Epstein 1924).

4.2. *Structure of Aggregates*

In this paper the suspended particles are assumed to consist of dust grains, of typical size a_0, which are weakly bound together by electrostatic and van der Waals interactions. These aggregates may have a fractal structure, such that the number of particles in an aggregate of linear dimension a is

$$N \sim \left(\frac{a}{a_0} \right)^D, \qquad (7)$$

where D is a fractal dimension. The damping rate for an aggregate of N dust grains with size a_0 composed of material of density ρ_p is therefore estimated as

$$\gamma \sim \frac{\rho_g c_s}{\rho_p a_0} N^{(2-D)/D} = \gamma_0 \left(\frac{a}{a_0} \right)^{2-D}. \qquad (8)$$

Since St $= 1/(\gamma\tau)$, we have St $\propto (a/a_0)^{D-2}$ and conclude that if $D \leq 2$, the Stokes number does not increase as the mass of the dust aggregate increases.

In the early stages of aggregation, we expect that the mean free path of the suspended particles is very large compared to their size, and the relevant fractal growth mechanism is ballistic aggregation (where particles incident with random linear trajectories adhere to the aggregate on contact) rather than diffusion-limited
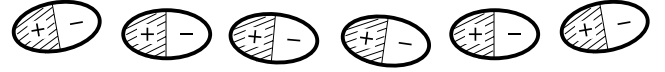

FIG. 2.—Schematic illustration of a chain of dust grains. Dust grains in a low-gravity environment have been found to form aggregates with a low fractal dimension. This may be due to particles having an electric dipole, causing them to form chains. (In this schematic illustration we show a perfect dipolar chain, which would lead to a fractal dimension of unity; in reality there are numerous reasons why the chain can be branched.)

aggregation (where an incoming particle executes a random walk). Numerical experiments (Nakamura et al. 1994) on ballistic-limited aggregation suggest $D \approx 2.98$ for a ballistic particle cluster aggregate (BPCA) model (particles adhering to a stationary aggregate) and $D \approx 1.93$ for a ballistic cluster cluster aggregate (BCCA) model (where aggregates of particles move and collide with each other). (Note that the use of the term "cluster" in the designations BPCA and BCCA differs from that used in this paper, as explained at the end of § 1.)

However, experiments on agitated dust grains (Blum et al. 2000; Krause & Blum 2004) have shown evidence of aggregates with a lower fractal dimension, $D \approx 1.4$. The lower fractal dimension observed in these experiments may be related to electrostatic effects: for example, if agitation of dust particles causes them to acquire dipolar charges, they will tend to form strings (dimension $D = 1$), as illustrated in Figure 2. The timescales on which these aggregates are observed is very short compared to the astrophysical context which we consider (and also the number of particles in the aggregates are not very large). The appropriate value for D is, therefore, highly uncertain. It is probably not universal throughout the accretion disk structure.

Our interest is in aggregates that undergo energetic collisions (which are sufficient to cause fragmentation in some cases) and that have a long time to relax into a compact configuration with the lowest energy. In the following we therefore usually assume that the aggregates are predominantly in compact configurations, with $D \approx 3$, but in many cases below we continue to give formulae for general values of D.

4.3. *Strength of Aggregates*

The other distinctive aspect of the dust-grain aggregates concerns their fragility under collisions. There is a substantial literature on this topic: see Dominik & Tielens (1997), Youdin (2004), and references therein. In the following we introduce a new treatment, valid in the limit where the grains are very small.

We mentioned earlier that the aggregates are bound together by van der Waals forces, which means that they must be very fragile. For a collision between two aggregates of size a_1 and a_2, there will be a velocity v_{cr} above which the average change in the mass of the larger particle is negative. It is extremely hard to estimate this critical velocity for collisions of aggregates. We estimate this critical velocity below for the collision of two grains. We assume that the critical velocity is approximately independent of size, so that the two-grain estimate is sufficient for the general case. In the following we therefore use v_{cr} to refer to the mean value of the relative velocity below which *pairs* of grains remain bound. This assumption is similar to one proposed in a much more detailed discussion (Dominik & Tielens 1997), which also discusses the effects of the objects rolling across each other in glancing collisions.

We motivate this assumption by the following argument. When two aggregates collide, initially the collision only influences the grains on the surface of the aggregates in the vicinity of the point
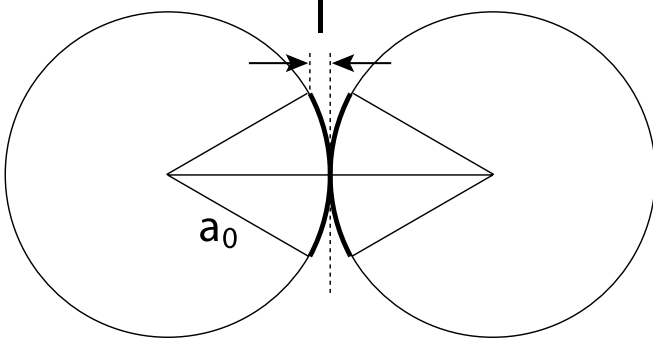
Fig. 3.—Two spheres in contact with a surface area $\pi l a_0$ with separation less than $2l$.

of collision. If the aggregates collide with a relative velocity v, some of the grains will acquire relative velocities that are comparable to v. If v/v_{cr} is significantly greater than unity, grains on the periphery of the colliding aggregates will be able to escape.

In order to estimate the critical relative collision velocity above which fragmentation occurs we therefore estimate the critical velocity v_{cr} above which two dust grains do not remain bound after a binary collision. This is estimated by equating the kinetic energy of the collision process between two dust grains, treated as spheres of radius a_0, to the binding energy.

The binding energy is usually estimated by a theory proposed by Chokshi et al. (1993) (and discussed in simpler terms by Youdin 2004), which assumes that the spheres deform so that they are in contact on a small circular patch. By balancing the force required to deform the spheres against a derivative of the surface energy released by bringing them into contact, the equilibrium deformation and hence the binding energy are estimated in terms of a Young's modulus for the spheres, Y, and their surface energy of attraction, σ. Omitting the dimensionless prefactors (which depend on making more precise definitions of Y and σ) the surface deformation δ and binding energy E_b are estimated to be

$$\delta \sim \left(\frac{\sigma^2 a_0}{Y^2}\right)^{1/3}, \quad E_b \sim \left(\frac{\sigma^5 a_0^4}{Y^2}\right)^{1/3}. \tag{9}$$

The latter expression gives an estimate for the critical velocity v_{cr}, obtained in Chokshi et al. (1993):

$$v_{cr} \sim \left(\frac{\sigma^5}{\rho_p^3 Y^2 a_0^5}\right)^{1/6}. \tag{10}$$

Using values for quartz of $Y = 5.4 \times 10^{10}$ N m^{-2}, $\sigma = 2.5 \times 10^{-2}$ N m^{-1}, and $\rho_p = 2.6 \times 10^3$ kg m^{-3} quoted in Chokshi et al. (1993), we estimate $\delta \approx 3 \times 10^{-11}$ m and $v_{cr} \approx 0.2$ m s^{-1} for grains of size $a_0 = 10^{-7}$ m. The validity of this approach is questionable when the value of the deformation is small compared to the size of an atom, as is found in this example. We therefore propose an alternative estimate of the binding energy.

We assume that there is a van der Waals binding energy δE_1 between atoms, which operates over a range l, which we equate with the typical interatomic distance. We assume that there is a binding energy $\delta E_1/l^2$ per unit area for all parts of two spheres that are within a distance $2l \ll a_0$ of each other, where δE is the van der Waals binding energy between two atoms and l is the range over which it acts. Two spheres in contact have a surface area $\pi l a_0$ with separation less than $2l$ (see Fig. 3), so that the binding energy is

$E_b \sim \pi \delta E_1 a_0/l$. Unless the grains are made of very plastic material, particles are not expected to aggregate if their (center-of-mass frame) kinetic energy is large compared to the binding energy. Writing $E_b \sim m v_{cr}^2 \sim \rho_p a_0^3 v_{cr}^2$, we estimate a critical velocity

$$v_{cr} = \sqrt{\frac{\delta E_1}{\rho_p l a_0^2}}. \tag{11}$$

Taking $\delta E_1 \approx 10^{-21}$ J, $l = 3 \times 10^{-10}$ m, and $a_0 = 10^{-7}$ m, we find that $v_{cr} \approx 3 \times 10^{-1}$ m s^{-1} for binding of microscopic grains. Thus, collisions between aggregates with relative velocities of order 0.3 m s^{-1} have the potential to dislodge particles from an aggregate.

It is instructive to compare the microscopic and the macroscopic models, equations (10) and (11). The surface energy and Young's modulus can be related to microscopic parameters by writing $\sigma \sim \delta E_1/l^2$, where $\delta E_1 \approx 10^{-21}$ J is the van der Waals interaction energy and $Y \sim \delta E_2/l^3$, where $\delta E_2 \approx 10^{-18}$ J is the characteristic energy scale for covalent bonds; these expressions give values that are consistent with the values for σ and Y for quartz quoted above. Expressing equation (10) in terms of microscopic quantities gives

$$v_{cr} \sim \sqrt{\frac{\delta E_1}{\rho_p l a_0^2}} \left(\frac{\delta E_1}{\delta E_2}\right)^{1/3} \left(\frac{a_0}{l}\right)^{1/6}. \tag{12}$$

We see that the two estimates (10) and (11) differ by dimensionless ratios with small exponents, and the values obtained from the two formulae are therefore typically quite similar. Whichever of equations (10) or (11) gives the larger value should be used to estimate the critical velocity. The material parameters can make a significant difference; in particular, ice has a much larger surface energy and is softer than quartz, and the predicted critical velocity is approximately an order of magnitude larger for ice (Chokshi et al. 1993).

There are significant sources of uncertainty in these estimates for the critical velocity for binding single grains, which are very hard to quantify. One is the effect of surface roughness. This will reduce the effective contact area between two grains and therefore reduce v_{cr}. The effect of electrostatic charging of dust grains and aggregates is harder to quantify. It is possible that the microscopic grains will all tend to acquire a positive charge due to ionization by energetic photons. This would reduce the binding energy. Another possibility is that grains acquire random charges through frictional charge transfer in collisions. This may increase the critical velocity for pairs of grains, because of electrostatic attraction of oppositely charged particles. This effect will be reduced by cancellation of charges for larger aggregates.

Experiments to determine the critical velocity for collisions of grains have been performed by projecting small silica spheres at a glass surface (Poppe & Blum 1997). These experiments have usually showed that the spheres adhere to the surface at velocities that are approximately 10 times higher than those predicted by equation (10). The reasons for this discrepancy are not understood. Although the effects of charges on the particles were quantified in these experiments, it is possible that effects of electrostatic charges on the surface might account for the discrepancy.

5. TURBULENCE IN PROTOPLANETARY DISKS

We now consider estimates relating to dust particles suspended in gas surrounding a growing star. Observational evidence suggests that there is usually sufficient angular momentum that the

TABLE 1

ASSUMED VALUES FOR PARAMETERS

Variable	Symbol	Value
Mass of gas molecules	$m_g = \mu m_H$	$\mu = 2.34$
Molecular collision cross section	S_{col}	2×10^{-19} m^2
Mass of the star	M	1.99×10^{30} kg
Typical size of interstellar dust grains	a_0	10^{-7} m
Density of suspended particles	ρ_p	2×10^3 kg m^{-3}
Mass fraction of suspended particles to gas molecules	κ	3/170

gas forms an accretion disk, and we use the steady state disk model described by Shakura & Sunyaev (1973). The kinetic and potential energy of the material in this cloud must be dissipated into heat and radiated away in order to allow the material to fall into the growing star; we consider the case in which the disk is primarily heated by dissipation, rather than by radiation from the star. It is known that stars can form quite rapidly (over a time-scale of $t_{acc} \approx 10^6$ yr). The process is so fast that either shocks or turbulent processes must play a role in the dissipation of energy: the rate of dissipation by laminar flow is too small by many orders of magnitude. It has been argued that magnetohydrodynamic mechanisms provide the necessary large-scale instability (Armitage 2007), but the gas is only weakly ionized, and the small-scale dissipation is expected to be described by conventional hydrodynamics, for which the Kolmogorov theory of turbulence is applicable. Shocks dissipate a finite fraction of the kinetic energy of a gas almost instantaneously, so that their role must be short lived, and throughout most of the accretion process, turbulence is the dissipation mechanism.

In the following we consider a star of mass M (taken to be 1 $M_\odot$) surrounded by a proto–planetary nebula, which is collapsing into the star at a rate $\dot{M}$. We assume

$$\dot{M} = 10^{-7}\Lambda \ M_\odot \ \text{yr}^{-1}. \tag{13}$$

Most discussions of protoplanetary accretion systems assume that $\Lambda \approx 1$. We assume that the accretion zone is predominantly molecular hydrogen (we take the mean molecular mass of the gas molecules $m_g = \mu m_H$, with $\mu = 2.34$ and $m_H = 1.67 \times 10^{-27}$ kg being the mass of atomic hydrogen). For estimating the mean free path of the gas, we assume that the molecular collision cross section is $S_{col} = 2 \times 10^{-19}$ m^2. These and other assumed parameter values are collected in Table 1. We use

$$c_s^2 = k_B T / m_g \tag{14}$$

to relate c_s, a characteristic speed that is of order the speed of sound, to temperature.

The accretion disk is characterized by its mass density per unit area $\Sigma(R)$ at a distance R from the star, its scale height $H(R)$, its rate of dissipation per unit area $Q(R)$, and its surface temperature $T(R)$. Using the continuity equations for flow of mass and angular momentum within a thin accretion disk which is in a quasi-stationary state, the rate of dissipation per unit area Q at radius R can be obtained in terms of the rate of accretion, $\dot{M}$ (Shakura & Sunyaev 1973):

$$Q(R) = \frac{3}{8\pi}\frac{GM\dot{M}}{R^3}\left[1 - \left(\frac{R_c}{R}\right)^{1/2}\right]. \tag{15}$$

Here R_c is the radius of the core of the accretion system. In the following we concentrate on the region $R/R_c \gg 1$ and do not carry forward the final factor of the above expression. Note that this formula is independent of the actual mechanism of dissipation. The rate of dissipation per unit mass is obtained from the mass per unit area $\Sigma(R)$ of the accretion disk at radius R, and the density of gas $\rho_g(R)$ at the midplane of the disk is related to the scale height:

$$\mathcal{E}(R) = \frac{Q(R)}{\Sigma(R)}, \quad \rho_g(R) = \frac{\Sigma(R)}{\sqrt{2\pi}H(R)}, \tag{16}$$

where in the second expression we assume a Gaussian density profile with variance H. Consider next how to estimate the height of the disk, $H(R)$. The gas is in Maxwell-Boltzmann equilibrium in the gravitational potential, and we assume that the contribution to the potential from the disk itself is negligible. The potential energy at a distance z from the midplane is $\Phi(z, R) = GMm_g z^2/2R^3$, so that the gas density at distance z from the midplane of the disk is $\rho(z, R) = \rho_g(R)\exp\left[-z^2/2H^2(R)\right]$, the scale height of the disk being

$$H(R) = \left[\frac{k_B T(R)R^3}{GMm_g}\right]^{1/2} = \frac{c_s(R)}{\Omega(R)}, \tag{17}$$

where $\Omega(R) = \left(GM/R^3\right)^{1/2}$ is the Keplerian orbital angular frequency for a mass M at radius R.

We assume that the surface temperature $T(R)$ is determined by the rate at which the thermal energy created by dissipation of turbulent motion can be radiated away. This assumption is justifiable as long as Λ in equation (13) is not too small, but when $\Lambda \ll 1$ the rate of heating by radiation from the star is expected to be significant (Armitage 2007). Assuming that the surface of the disk behaves as a black body with emissivity ε, one obtains

$$Q(R) = 2\varepsilon\sigma T^4(R) \sim \frac{3}{8\pi}\frac{GM\dot{M}}{R^3}, \tag{18}$$

where σ is the Stefan-Boltzmann constant. Assuming $\varepsilon \approx 1$, one obtains the radial temperature profile

$$T(R) = \left(\frac{3\Omega^2\dot{M}}{16\pi\sigma}\right)^{1/4} = T(R_0)\Lambda^{1/4}\left(\frac{R}{R_0}\right)^{-3/4}, \tag{19}$$

where R_0 is a convenient reference radius, which we take to be $R_0 = 1$ AU $= 1.5 \times 10^{11}$ m, and $T(R_0)$ is the temperature at R_0 with Λ set equal to unity. Given the radial dependence $T(R)$ of the temperature, the radial dependence of the velocity of sound and

Variable	Symbol	Equation	$X(R_0)$	δ_R	δ_Λ	δ_ℓ
Surface temperature	T	eq. (19)	130 K	$-3/4$	$1/4$	0
Speed of sound.....................	c_s	eq. (20)	670 m s^{-1}	$-3/8$	$1/8$	0
Disk height...........................	H	eq. (21)	3.4×10^9 m	$9/8$	$1/8$	0
Surface density.....................	Σ	eq. (25)	280 kg m^{-2}	$-3/4$	$3/4$	-2
Gas density...........................	ρ_g	eq. (16)	3.3×10^{-6} kg m^{-3}	$-15/8$	$5/8$	-2
Dissipation rate	$\mathcal{E}$	eq. (16)	0.11 m^2 s^{-3}	$-9/4$	$1/4$	2
Gas mean free path...............	λ	$\mu m_H/(\sqrt{2}\rho_g S)$	0.42 m	$15/8$	$-5/8$	2
Kinematic viscosity...............	ν	$\nu = \lambda c_s$	280 m^2 s^{-1}	$3/2$	$-1/2$	2
Kolmogorov length...............	η	eq. (2)	120 m	$27/16$	$-7/16$	1
Kolmogorov time...................	τ	eq. (2)	52 s	$15/8$	$-3/8$	0
Kolmogorov velocity	u_K	η/τ	2.3 m s^{-1}	$-3/16$	$-1/16$	1
Integral velocity	u_L	$u_L = (\mathcal{E}L)^{1/3}$	710 m s^{-1}	$-3/8$	$1/8$	1
Integral timescale	t_L	$t_L = L/u_L$	4.6×10^6 s	$3/2$	0	0

of the disk thickness can be determined. Using equations (17) and (18) we find

$$c_s(R) = c_s(R_0)\Lambda^{1/8}\left(\frac{R}{R_0}\right)^{-3/8}, \qquad (20)$$

$$H(R) = H(R_0)\Lambda^{1/8}\left(\frac{R}{R_0}\right)^{9/8}. \qquad (21)$$

(Again, the dependence on Λ is not shown explicitly in the list of arguments.) In order to estimate the radial dependence of other quantities, it is necessary to determine the radial dependence of the density. This is achieved as follows. Applying the continuity equation for angular momentum leads to a relation between Σ and an effective kinematic viscosity (termed the eddy viscosity), ν_{eff}, that is the diffusion coefficient characterizing the transport of angular momentum across the accretion disk (Shakura & Sunyaev 1973):

$$\dot{M} = 3\pi\nu_{\text{eff}}(R)\Sigma(R). \qquad (22)$$

The eddy viscosity in turn is determined by the size L of the largest eddies in the turbulent flow (the integral scale). We estimate $\nu_{\text{eff}} \sim L^2/t_L$, where t_L is the eddy turnover time for an eddy of size L. The Kolmogorov scaling argument implies that $t_L \sim (L^2/\mathcal{E})^{1/3}$, so that we estimate

$$\nu_{\text{eff}} \sim L^{4/3}Q^{1/3}(R)\Sigma^{-1/3}(R). \qquad (23)$$

It is natural to assume that L is of order $H(R)$, and we write

$$L = \ell H(R), \qquad (24)$$

where ℓ is a parameter. Taking equations (17), (18), (23), and (24) together, we find

$$\Sigma(R) = \frac{2\sqrt{2}}{9\pi\ell^2}\frac{\dot{M}}{H^2\Omega}. \qquad (25)$$

Usually it is assumed (Shakura & Sunyaev 1973) that $\nu_{\text{eff}} \sim \alpha c_s H$ (where $\alpha < 1$ is a dimensionless coefficient). Our own approach is equivalent to this α-prescription: using equation (25) in equation (23), we obtain

$$\nu_{\text{eff}} = \frac{3}{2\sqrt{2}}\ell^2 c_s H. \qquad (26)$$

Thus, we see that the α-prescription is equivalent to assuming that the integral scale of the turbulence is smaller than H by a factor $\ell \approx \sqrt{\alpha}$. It is widely accepted that many observations are compatible with $\alpha \approx 10^{-2}$ (Hartmann et al. 1998), corresponding to $L \approx H/10$. Accordingly, wherever we quote numerical values for quantities without specifying how they scale with ℓ, we have set $\ell = 0.1$. An advantage of our "ℓ-prescription" is that it makes the physical nature of the adjustable parameter clearer than for the standard α-prescription. The rather small values of α indicated by observations could be indicative of a fundamental problem with a theory. Our alternative approach is reassuring because it indicates that a more physically transparent parameter, $\ell = L/H$, is not in fact a very small number.

These results allow us to determine the power-law radial dependence of other quantities. The results for those determining the gaseous component of the accretion disk are summarized in Table 2, writing a generic variable in the form

$$X = X(R_0)\left(\frac{R}{R_0}\right)^{\delta_R}\Lambda^{\delta_\Lambda}\ell^{\delta_\ell}, \qquad (27)$$

where $R_0 = 1$ AU $= 1.5 \times 10^{11}$ and $X(R_0)$ is the value of X at radius R_0, with $\Lambda = \ell = 1$.

6. RELATIVE VELOCITIES, COLLISION RATES, AND STOKES TRAPPING

Next we consider the behavior of dust grains suspended in the gas forming the accretion disk. We consider the case where the dust grains are almost all submicron-sized particles: we assume they are composed of material with density $\rho_p = 2 \times 10^3$ kg m^{-3} and that these are initially spherical particles of radius $a_0 = 10^{-7}$ m. Following Hayashi (1981), we assume that the mass ratio κ of suspended particles to gas molecules is $\kappa = 3/170 \approx 0.018$.

The results described above can be used to estimate collision rates. (Many of the formulae below are standard results from kinetic theory and have been used to estimate collision rates between dust particles in earlier works on planet formation: see, for example, Weidenschilling 1980; Beckwith et al. 2000 and works cited therein.) We consider two cases. First, we estimate the collision rate when the Stokes number is small and when the collision mechanism is Brownian diffusion (we shall see that advective collisions [Saffman & Turner 1956] make a negligible contribution). Second, we also require the rate of collision for large Stokes

TABLE 3

Estimates of Quantities Characterizing the Suspended Particles (Quoted to Two Significant Figures)

Variable	Symbol	Equation	$X(R_0, a_0)$	δ_R	δ_Λ	δ_ℓ	δ_a
Damping rate	γ	eq. (8)	0.11 s^{-1}	$-9/4$	$3/4$	-2	$2-D$
Number density	n	eq. (28)	7.0×10^7 m^{-3}	$-15/8$	$5/8$	-2	$-D$
Diffusive velocity	v_d	eq. (29)	1.4×10^{-2} m s^{-1}	$-3/8$	$1/8$	0	$-D/2$
Diffusive collision rate	$\mathcal{R}_d$	eq. (30)	1.8×10^{-7} s^{-1}	$-9/4$	$3/4$	-2	$(4-3D)/2$
Advective collision rate	$\mathcal{R}_a$	eq. (31)	1.4×10^{-15} s^{-1}	$-15/4$	1	-2	$2-D$
Relative velocity	$\langle \Delta v^2 \rangle^{1/2}$	eq. (5), $K=1$	0.98 m s^{-1}	0	$-1/4$	2	$(D-2)/2$
Gas kinetic collision rate	$\mathcal{R}_g$	eq. (32)	3.6×10^{-6} s^{-1}	$-27/16$	$3/16$	1	$(2-D)/2$
Stokes number	St	eq. (33)	0.17	$3/8$	$-3/8$	2	$D-2$

number. Both estimates require the number density of dust grains. We write

$$n \sim \frac{\kappa \rho_g}{m_0} \left(\frac{a}{a_0}\right)^{-D} = n_0 \left(\frac{a}{a_0}\right)^{-D}, \qquad (28)$$

where D is a fractal dimension, $\kappa \approx 0.018$ is the ratio of the mass densities of condensed to gaseous matter, and $m_0 = 4\pi \rho_p a_0^3/3$ is the mass of a microscopic dust grain; the second equality defines n_0.

The mean speed of particles due to Brownian motion is

$$\langle v_d \rangle \sim c_s \sqrt{\frac{\mu m_H}{N m_0}} = v_0 \left(\frac{a}{a_0}\right)^{-D/2}, \qquad (29)$$

where the second equality defines v_0. Our estimate of the collision rate due to Brownian diffusion for a general value of the fractal dimension of the aggregates is

$$\mathcal{R}_d \sim 4\pi\sqrt{2} n a^2 \langle v_d \rangle \sim 4\pi\sqrt{2} n_0 a_0^2 v_0 \left(\frac{a}{a_0}\right)^{(4-3D)/2}$$
$$= \mathcal{R}_{d0} \left(\frac{a}{a_0}\right)^{(4-3D)/2}, \qquad (30)$$

and the factor $\mathcal{R}_{d0}$ is given in Table 3. The collision rate due to advective shearing is approximately

$$\mathcal{R}_a \sim \frac{n a^3}{\tau}. \qquad (31)$$

The gas kinetic collision rate (taking $K=1$ in eq. [5]) is

$$\mathcal{R}_g = 4\pi n a^2 \sqrt{\langle \Delta v^2 \rangle} = 4\pi n_0 a_0^2 \sqrt{\frac{\mathcal{E}}{\gamma_0}} \left(\frac{a}{a_0}\right)^{(2-D)/2}$$
$$= \mathcal{R}_{g0} \left(\frac{a}{a_0}\right)^{(2-D)/2}. \qquad (32)$$

The Stokes number is

$$\mathrm{St} = \frac{1}{\gamma \tau} = \mathrm{St}_0 \left(\frac{a}{a_0}\right)^{D-2}. \qquad (33)$$

All of these quantities have a power-law dependence on radius and particle size: for the generic quantity X we write

$$X = X(R_0, a_0) \left(\frac{R}{R_0}\right)^{\delta_R} \Lambda^{\delta_\Lambda} \ell^{\delta_\ell} \left(\frac{a}{a_0}\right)^{\delta_a}. \qquad (34)$$

[Again, it is to be understood that $X(R_0, a_0)$ is evaluated for $\Lambda = \ell = 1$.] The quantities determining the collision rates are collected in Table 3.

The growth of the aggregates due to collisions could be limited by two factors. First, the rate of collisions usually decreases as the size of the aggregates increases, and the collision rate could become negligible when the particles grow to a sufficiently large size. Second, large aggregates could be more vulnerable to being fragmented upon collision.

To illuminate a discussion of whether these limitations occur in practice, we now consider the properties of particles with size a chosen so that St $= 1$. For compact particles ($D = 3$) we find that their size is

$$a^* = a_0 \left(\frac{1}{\mathrm{St}_0}\right)^{1/(D-2)} \sim a^*(R_0) \Lambda^{3/8} \ell^{-2} \left(\frac{R}{R_0}\right)^{-3/8}, \qquad (35)$$

with $a^*(R_0) \approx 6 \times 10^{-7}$ m; for $\ell = 0.1$ this leads to particles of size $a^* \approx 25$ μm at $R = 10$ AU. It is also of interest to evaluate the collision rate of these particles. Below we describe a mechanism that will lead to the aggregates becoming compact, so we concentrate on the predictions of this calculation when $D = 3$. The approximate size, collision rates, and relative velocity for compact aggregates with Stokes number unity are listed in Table 4. Note that the diffusive collision rate is very small, but it increases abruptly (by approximately 5 orders of magnitude, for the case in which $\ell = 0.1$) when the size of the particles increases such that the Stokes number exceeds unity (see Fig. 4). These calculations also show that the relative velocity for collisions of particles with Stokes number unity is comparable to the critical velocity for microscopic dust aggregates to be fragmented.

If the lifetime of the accretion system is $t_{\rm acc} \approx 10^6$ yr, an aggregation process with rate $\mathcal{R}$ will not be significant if $\mathcal{R}t_{\rm acc} \ll 1$. For compact particles the diffusive collision rate decreases rapidly with particle size. At the size where St $= 1$, the diffusive collision rate is extremely slow (of order one collision per 10^5 yr when $\ell = 0.1$). The slowing down of the collision rate as the particle size increases is therefore quite close to becoming an insurmountable bottleneck.

For compact aggregates ($D \approx 3$), the collision rate for the gas kinetic mechanism is much larger than the diffusive rate when St $\gg 1$, and it also decreases less rapidly as the size of the particles increases. The estimate for $\mathcal{R}_g$ above indicates that the gas kinetic collision mechanism is sufficiently rapid that it does not limit the growth of aggregates, at least until they are so large that the approximations in equation (33), such as assuming that the drag is determined by the Epstein formula, break down. We conclude that kinetic factors do not limit the growth of aggregates, once particles become sufficiently large such that St > 1.

TABLE 4
Estimates for Compact Particles ($D = 3$) with St $= 1$

Variable	Symbol	Equation	$X(R_0)$	δ_R	δ_Λ	δ_ℓ
Stokes-trapped particle size	a^*	eq. (35)	5.7×10^{-7} m	$-3/8$	$3/8$	-2
Diffusive collision rate in Stokes trap	$\mathcal{R}_d^*$	eq. (30)	2.3×10^{-9} s^{-1}	$-21/16$	$3/16$	3
Advective collision rate in Stokes trap	$\mathcal{R}_a^*$	eq. (31)	2.5×10^{-13} s^{-1}	$-39/8$	$17/8$	-8
Gas kinetic collision rate in Stokes trap	$\mathcal{R}_g^*$	eq. (32)	3.6×10^{-6} s^{-1}	$-27/16$	$3/16$	1
Turbulent relative velocity at St $= 1$	$\langle \Delta v \rangle^*$	$(\mathcal{E}\tau)^{1/2}$	2.3 m s^{-1}	$-3/16$	$-1/16$	1

Note.—The results are of the form of eq. (27).

As the size a of the aggregates increases, however, so does their relative velocity $\langle \Delta v \rangle$. For example, compact aggregates of size $a = 0.1$ m collide with a relative velocity of approximately 10 m s^{-1} if we take $\ell = 0.1$. Such a collision would certainly cause the aggregates to fragment upon impact.

We have arrived at the following picture of the initial stages of the growth of dust particle aggregates. Initially, dust particles will aggregate by Brownian diffusion. This process continues until the Stokes number of the larger particles is of order unity. Then the velocity of these particles starts to separate from that of the gas due to the formation of fold caustics. This greatly increases the relative velocity between particles, as illustrated in Figure 1. When these larger particles collide with each other they have sufficient kinetic energy that they cause each other to fragment, so that particles with large Stokes number fragment to replenish the supply of particles with small Stokes number. This results in a steady state distribution of particle sizes, corresponding to Stokes numbers of order unity: we call this the "Stokes trap."

We remark that (Cuzzi et al. 2001) have also made estimates relating to aggregates with Stokes number unity in a discussion of clustering effects. They did not suggest that the aggregates would be subject to the Stokes trapping effect.
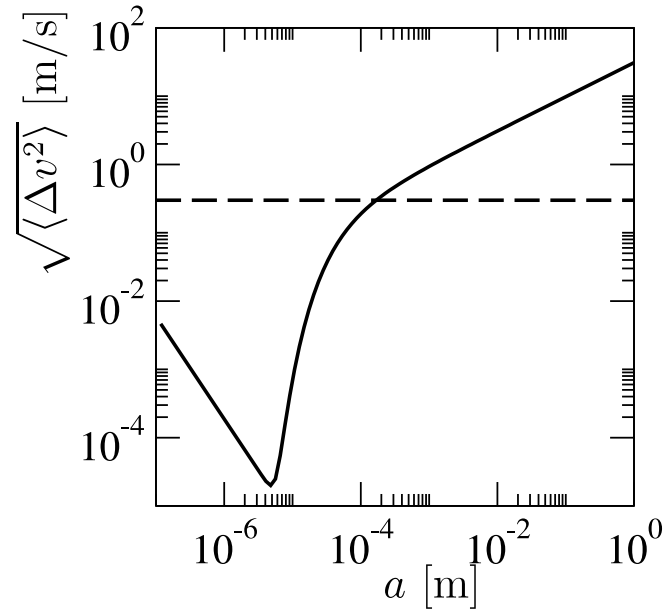

Fig. 4.—Relative collision velocity $\langle \Delta v^2 \rangle^{1/2}$ as a function of the aggregate size, a, illustrating the abrupt increase of the collision velocity at Stokes number unity. The plot, using data from Table 4 in eqs. (3)–(5), applies to compact ($D = 3$), equal-sized aggregates, for the case $R = 10$, $\ell = 0.1$. The critical $v_{\rm cr}$ is shown as a horizontal line. [See the electronic edition of the Supplement for a color version of this figure.]

7. MECHANISMS FOR AVOIDING THE STOKES TRAP

In the following we propose four mechanisms by which the Stokes trap might be avoided.

7.1. Escaping the Stokes Trap by Clustering

It is known that small particles suspended in a turbulent flow can cluster, even if the fluid flow is incompressible, due to effects of the inertia of the particles. This effect was proposed by Maxey (1987), who suggested that heavy particles would be "centrifuged" out of vortices and would tend to cluster in regions of low vorticity: this effect is often referred to as "preferential concentration" because it is thought that the particles cluster in regions of low vorticity.

The argument by Maxey does not give a quantitative description of the nature of the clustering effect at long times. Sommerer & Ott (1993) suggested that particles in random fluid flows cluster onto a fractal set of dimension $D_{\rm cl}$ and proposed using the Lyapunov dimension (first defined by Kaplan & Yorke 1979) to characterize this set. Recent numerical experiments (Bec et al. 2006) have studied how the (Lyapunov) fractal dimension $D_{\rm L}$ of particles in a three-dimensional turbulent flow varies as a function of Stokes number: the minimum value is $D_{\rm L} \approx 2.6$ for St $= 0.55$, $D_{\rm L} = 3$ for St $>$ St$_c \approx 1.7$, and $D_{\rm L} \to 3$ as St $\to 0$. Theoretical work (Duncan et al. 2005) has shown that the fractal clustering may be explained without the centrifuge (or preferential concentration) effect: a model for which it is absent gives a minimum dimension $D_{\rm L} \approx 2.65$, in good agreement with simulations.

The Stokes-trapped dust aggregates will be clustered, but if the clustering is confined to length scales below the Kolmogorov length η, the effect is not sufficiently strong to initiate gravitational collapse: the mass of solid matter within a Kolmogorov length is

$$ M_{\rm K} \sim \kappa \rho_g \eta^3 \approx 1.0 \times 10^{-3} \text{ kg } \Lambda^{-11/16} \ell \left(\frac{R}{R_0}\right)^{51/16}. \quad (36) $$

The size of the clusters created by the preferential concentration effect cannot significantly exceed this value, and this mass is much too small for gravitational effects to become significant.

We thus conclude that although the clustering effect is promoted by the Stokes trap, clustering of dust aggregates onto a fractal set cannot provide inhomogeneities that are sufficient to trigger gravitational instability in protoplanetary systems. However, it is possible that this clustering effect could influence the scattering of electromagnetic radiation by the dust grains.

Some other mechanisms for clustering have been proposed and are discussed in Barge & Sommeria (1995) and Johansen et al. (2007). We argue that these mechanisms can be discounted because they assume the existence of much larger aggregates than the Stokes-trapped size.

7.2. *Escaping the Stokes Trap by Chemical Processes*

We propose that the midplane of the accretion disk can be at a much higher temperature than its surface. The dust aggregates circulate between regions with different temperatures. In the lower temperature regions, water and organic compounds can condense onto the aggregates. When the particles circulate to higher temperature regions, chemical reactions may occur. This is expected both to compactify the aggregate and to make it much more resistant to being fragmented in collisions. It is also possible that the temperature in regions close to the star is sufficiently high that at least some of their component materials are melted. This will also produce much more robust and compact aggregates, able to withstand collisions at much higher relative velocities.

We have assume that the temperature of the accretion disk is determined by dissipative heating, and we have estimated the surface temperatures, which are quite low. However, if the disk is optically thick in the spectral region corresponding to the Wien wavelength, the midplane temperature may be considerably higher than the surface temperature.

The optical thickness W is the ratio of the height of the disk to the photon mean free path. The optical properties are dominated by the dust particles if $W \gg 1$, and when the particles are larger than the wavelength we may assume that their optical cross section S_{opt} is approximately the square of their linear dimension: $S_{opt} \approx a^2$. If the density of particles is n, the optical mean free path is $\lambda_{opt} \sim 1/(nS_{opt})$. Thus, for compact particles ($D = 3$), the optical thickness is

$$W \sim Hna^2. \qquad (37)$$

The ratio of the interior temperature T_{int} of the accretion disk to its surface temperature T is

$$\frac{T_{int}}{T} \sim W^{1/4}. \qquad (38)$$

Very small particles, with dimension small compared to the Wien wavelength corresponding to temperature T, do not absorb or scatter radiation effectively. If the aggregates are very large, their number is correspondingly reduced, so that they make a smaller contribution to the optical thickness. From the data above, we see that the Stokes-trapped particles have sizes that are comparable to, but somewhat larger than, the Wien wavelength corresponding to the surface temperature. The Stokes-trapped particles are therefore of approximately optimal size to increase the optical thickness. The optical thickness corresponding to Stokes-trapped particles is (for compact particles, $D = 3$):

$$W^* \sim \frac{Hn_0a_0^3}{a^*} \sim 410\Lambda^{3/8}\left(\frac{R}{R_0}\right)^{-3/8}. \qquad (39)$$

The temperature at the center of the disk is therefore expected to exceed the surface temperature by a factor of approximately 4 in the range $R = 1$–10. AU

The chondrules, grains of typically submillimeter size that are found in many meteorites, show evidence of having been heated to very high temperatures, causing melting. The temperatures required are considerably higher than those which we predict above (except very close to the star). In addition, the exposure of chondrules to high temperatures may have been brief compared to the circulation time t_L; otherwise the chondrules may have evaporated.

7.3. *Escaping the Stokes Trap by Forming Tenuous Aggregates*

We have already remarked that if the fractal dimension D of the dust aggregates is $D \leq 2$, then the Stokes number does not increase as the size of the aggregate increases. Such aggregates would avoid the Stokes trap, but there are three reasons why these tenuous aggregates do not provide a satisfactory solution to the problem of planet formation.

First, it might be thought that as such aggregates grow their collision rate would become so small that growth would, for all practical purposes, cease. In § 6 it was determined how the collision rate for fractal aggregates with dimension D scales as a function of their characteristic size, a: we found $\mathcal{R}_d \propto a^{(4-3D)/2}$. The collision rate therefore increases with the aggregate size when $D < 4/3$, because such very tenuous aggregates are so extended that they entangle each other. Experiments on growing aggregates in negligible gravity have produced objects with very low fractal dimensions of roughly $D = 1.4$ (Blum et al. 2000; Krause & Blum 2004), rather smaller than the values predicted for kinetic aggregation processes. The dust aggregates observed in low-gravity experiments are only observed for a short period compared to the time between collisions between dust aggregates, so that the structure of the aggregates in protostellar accretion systems is uncertain, but there is probably no kinetic limitation on the growth of very tenuous clusters.

A second possibility is that the tenuous aggregates are torn apart by the effect of the shearing motion of the fluid. We estimate this effect as follows. Consider the force required to split an aggregate of fractal dimension D and size a, composed of particles of size a_0. The number of bonds at an equatorial plane of the aggregate is approximately

$$N_b \sim \left(\frac{a}{a_0}\right)^{D-1}. \qquad (40)$$

Each bond has a breaking strain F_b. Grains in contact have an energy per unit area $\delta E_1/l^2$ per unit area, acting over a range l. The binding energy is $E_b = \pi\delta E_1 a_0/l$ (see § 4.3). Writing $E_b = F_b l$, we estimate

$$F_b \sim \frac{\delta E_1}{l^2}a_0. \qquad (41)$$

The drag force on a single grain due to fluid motion with a speed u relative to the fluid is $F = m_0\gamma_0 u$. When $D \leq 2$, the drag force pulling the aggregate apart is comparable to the sum of the forces that are predicted by applying this formula to each grain. The relative velocity of the gas at opposite sides of the aggregate is a/τ, so that the force acting to break up the aggregate is

$$F_{br} \sim \frac{N\gamma_0 m_0 a}{\tau} \sim \frac{m_0\gamma_0 a_0}{\tau}\left(\frac{a}{a_0}\right)^{D+1}. \qquad (42)$$

The ratio of the applied force to the critical force is $F_{br}/N_b F_b$; note that this ratio is independent of D (provided $D \leq 2$). Setting this ratio equal to unity, we estimate the size a_{max} of aggregates that will be torn apart by shearing forces:

$$a_{max} \approx 78 \text{ m } \Lambda^{-9/16}\ell\left(\frac{R}{R_0}\right)^{33/16}. \qquad (43)$$

The estimates leading to this result are valid provided the aggregate is small compared to the Kolmogorov length, $a \ll \eta$; this condition is only marginally satisfied. It is not clear how the shear forces increase with aggregate size once the aggregate is large compared to the Kolmogorov correlation scale, but it is hard to escape the conclusion that tenuous aggregates are torn apart by shear forces when they become sufficiently large. The mass contained in these tenuous aggregates is clearly comparable to that estimated in equation (36) and is too small to initiate gravitational collapse directly.

A third problem is that aggregates with Stokes number less than unity are always advected with the gas. Even if they could become very massive, they would never collapse to the midplane of the disk, as long as the motion of the gas remains turbulent.

7.4. *Escaping the Stokes Trap by Reducing the Accretion Rate*

If the accretion rate $\dot{M}$ is reduced (by choosing a small value for the parameter Λ in eq. [13]), there is a corresponding reduction in the turbulence intensity $\mathcal{E}$. We might expect that there would be also be a reduction in the relative velocities of suspended particles. This latter expectation is false, because a reduction in the rate of accretion results in a reduced density of the gas, and the suspended particles are more lightly damped by the gas (note that if γ decreases the relative velocity increases; see eq. [5]).

This principle is illustrated by results in Table 4, which show that the Stokes-trapped particle size decreases as the accretion rate parameter Λ decreases (because $\delta_\Lambda > 0$ for a^*). A further demonstration comes from considering the size of particles that are not fragmented on collision. If the relative velocity is less than the critical velocity v_{cr} for disintegration of dust aggregates, then they continue to grow on collision. The size of particles for which the turbulent relative velocity exceeds v_{cr} is

$$a_{cr} = 1.0 \times 10^{-7} \text{ s}^2 \text{ m}^{-1} \; v_{cr}^2 \ell^{-4} \Lambda^{1/2}. \quad (44)$$

We see that, contrary to intuition, the attainable particle size decreases as the rate of accretion decreases.

8. SETTLING AND GRAVITATIONAL COLLAPSE

Even if the Stokes trap is circumvented, the particles must grow to a substantial size in order for them to settle to the midplane of the accretion disk. In the following we illustrate this fact by estimating the size at which particles start to settle to the midplane, according to our model. We find that settling only starts at large particle sizes (approximately 0.1 m, where the use of the Epstein formula for the damping rate becomes questionable). We also estimate the critical height of the dust layer for the onset of gravitational collapse and find that this is a small fraction of the height of the gas layer.

8.1. *Critical Size for Settling*

The integral scale fluctuations with velocity u_L will advect aggregates away from the midplane. This effect is opposed by gravitational attraction to the midplane. The gravitational acceleration at a distance z from the midplane is

$$g(R,z) = \frac{GM}{R^3} z = \Omega^2 z. \quad (45)$$

If the integral timescale of the turbulence, t_L, is large compared to the relaxation time γ^{-1}, then the aggregates only settle to the midplane if their terminal speed $u_t = g(R,z)/\gamma$ exceeds u_L. There is thus no significant settling unless $\gamma < \gamma_s$, where

$$\gamma_s = \frac{\Omega^2 H}{u_L} = 1.8 \times 10^{-7} \text{ s}^{-1} \; \ell^{-1} \left(\frac{R}{R_0} \right)^{-3/2}. \quad (46)$$

In the case of compact aggregates ($D = 3$), we conclude that particles start to settle to the midplane when their size exceeds

$$a_s \approx a_0 \left(\frac{\gamma_0}{\gamma_s} \right) \approx 6.0 \times 10^{-2} \text{ m } \Lambda^{3/4} \ell^{-1} \left(\frac{R}{R_0} \right)^{-3/4}. \quad (47)$$

When considering very large aggregates, the damping rate γ may be sufficiently small such that $\gamma t_L \ll 1$. In addition, in cases where $\ell = L/H \ll 1$, aggregates must be advected by many eddies to traverse the height of the disk. In these cases, the effect of the turbulence must be modeled as a sequence of random impulses on the aggregate (Cuzzi et al. 1993; Youdin & Lithwick 2007). We argue that, because of the Stokes trapping effect, the aggregates never grow to a sufficient size for the condition $\gamma t_L \ll 1$ to become relevant.

8.2. *Critical Height for Gravitational Collapse*

A stationary gas of solid particles with mass density ρ_{sol} is unstable against gravitational collapse on a timescale $t_{coll} \sim (G\rho_{sol})^{-1/2}$. If the gas consists of particles with typical relative speed Δv, then gravitational collapse (the Jeans instability) occurs on length scales greater than $\Delta v t_{coll}$. In the case of an accretion disk, the question of gravitational instability can be complicated by the rotational motion and the finite thickness of the disk.

The gravitational stability of a uniform thin disk of area density Σ may be described by giving a dispersion relation for the frequency ω of a density perturbation with wavenumber k

$$\omega^2 = \Omega^2 - 2\pi G\Sigma|k| + v_s k^2, \quad (48)$$

where v_s is a two-dimensional sound velocity (Binney & Tremaine 1988). The system becomes gravitationally unstable when $\omega^2 < 0$ for some value of k. For a collisionless system, we have $v_s = 0$, and the instability arises when the term containing the gravitational constant is larger in magnitude that the centrifugal term. The instability is favored by choosing a large value for k, but if we consider very large values of k, the approximation of treating the mass distribution as two dimensional fails. We therefore assume that the largest possible value of k is $2\pi/H$, where H is the height of the disk.

The solid material in the disk (with area density $\kappa\Sigma$) therefore becomes gravitationally unstable when its scale height, H_{sol}, is less than a critical value, which is given (approximately) by

$$H_{cr} = \frac{4\pi^2 \kappa G\Sigma}{\Omega^2} \approx 3.3 \times 10^5 \text{ m } \Lambda^{3/4} \ell^{-2} \left(\frac{R}{R_0} \right)^{9/4}. \quad (49)$$

Note that $H_{cr}/H \ll 1$, implying that to trigger a gravitational instability, particles must be very much heavier than particles of the size a_{cr} (given by eq. [47]).

9. CONCLUDING REMARKS

In this paper we have analyzed the consequences of turbulence for a standard model of the formation of planets by aggregation of submicron-sized dust. This led us to introduce the concept of the

Stokes trap, a mechanism that limits the growth of dust aggregates in turbulent protostellar accretion disks and thus constitutes a barrier to planet formation. The particle sizes predicted by the Stokes trap mechanism are so small that turbulent fluctuations would never allow the aggregates to settle to the midplane of the disk and achieve sufficient density to trigger gravitational instability. The Stokes trap must therefore be avoided if the planets are to form.

We have proposed four possible ways of avoiding the Stokes trap and assessed these mechanisms in the light of Kolmogorov's scaling principle for turbulence in combination with a standard model for the accretion disk. Particles clustering due to preferential concentration cannot produce sufficiently large clusters. Chemical processes that consolidate aggregates will strengthen them but not to the extent that large aggregates would be invulnerable to fragmentation by collision. If the aggregates are very tenuous fractals, with dimension $D \leq 2$, their Stokes number remains very small, but the aggregates may be torn apart by shearing forces when they become sufficiently large (and even if this difficulty can be overcome, they will not settle to the midplane of the disk). Finally, we observed that as the disk becomes less active (that is, as $\dot{M}$ decreases), the relative velocity of the suspended aggregates increases, so that the difficulty is not removed as the disk becomes quiescent.

Even if the aggregates escape from the Stokes trap and continue growing, our estimates indicate that they must reach very large sizes before they can overcome the effects of turbulence and start to settle to the midplane. The turbulence intensity would have to be reduced by many orders of magnitude before this could happen.

We conclude that a model for planet growing by aggregation of small dust grains is highly problematic. It is therefore necessary to examine other possibilities. The difficulties with the theory are avoided if the protoplanetary accretion disk already contains large objects with a gravitational escape velocity that is large enough to prevent them from being disrupted by collisions with dust grains and aggregates. A star forms when a cloud of interstellar gas satisfies conditions that allow it to undergo gravitational collapse. It is likely that at the same time as the star forms, other regions of the cloud in the vicinity of the nascent star will condense by gravitational attraction. Condensed objects much smaller than the star itself may be drawn into the accretion disk of the star and eventually become planets lying on roughly circular orbits in the equatorial plane of the star. Thus, we propose an alternative picture for the formation of planets, which we term "concurrent collapse," in which planets are formed as gravitationally bound systems at the same time as the star. During the life of the accretion disk they become coupled by friction and mass transfer to the circular motion of the accretion disk. While in the accretion disk their structure may be radically transformed: their mass could be substantially increased by further accretion of material or reduced by ablation or evaporation of lighter elements.

One possible criticism of this scenario is that the density ρ of the gas cloud at temperature T that collapses to form the star will be associated with a mass scale, the Jeans mass,

$$M_{\rm J} \sim \left[\frac{1}{\rho} \left(\frac{k_{\rm B} T}{G m_{\rm H}} \right)^3 \right]^{1/2}. \qquad (50)$$

It may be argued that the Jeans mass is a lower limit to the mass of objects that could form by gravitational collapse, determined by the density of the interstellar medium. There are two possible routes to producing gravitational collapse with a lower mass. One of these is spontaneous fragmentation: as gravitational collapse proceeds, the density of some regions will increase, thus lowering the Jeans mass estimate that applies for further collapse. The process of gravitational collapse is the same as that which forms stars, and it has been argued (Low & Lynden-Bell 1976) that there is a lower limit to the mass of stars that can form due to fragmentation of a gravitationally collapsing cloud. It is argued that opacity is an increasing function of temperature and that the fragmentation ceases when the collapsing gas cloud becomes opaque, making it unable to lose energy efficiently: this lower limit is estimated to be approximately $10^{-2} M_{\odot}$. Another mechanism for reducing the Jeans mass, termed "turbulent fragmentation," has been proposed more recently. Simulations of gravitational collapse show that shock waves develop, which can produce a highly nonuniform density. The regions of highest density can correspond to greatly decreased Jeans masses (Padoan & Nordlund 2002, 2004). This has been proposed as a mechanism to explain the production of brown dwarfs, but there does not appear to be any reason why objects of significantly lower mass could not be produced.

We emphasize that the initial sizes of the planets when they condense could be very different from their final sizes. In particular, after the planets form, they might initially have noncircular orbits out of the plane of the accretion disk. The planets would therefore be moving rapidly relative to the gas of the accretion disk. It might be expected that in this case a high proportion of the original mass could be removed by ablation. Collisions between the planets would also be possible, resulting in the creation of large numbers of much smaller fragments.

For the time being this concurrent collapse hypothesis must be supported by the implausibility of the standard dust aggregation model. Because the additional hypothesis only concerns conditions at early stages of the life of the stellar accretion system, its implications for the eventual structure of a solar system are uncertain and require considerable additional research.

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