COMPARISON OF DISTANCES FROM RR LYRAE STARS, THE TIP OF THE RED GIANT BRANCH, AND CLASSICAL CEPHEIDS

G. A. TAMMANN

Astronomisches Institut der Universität Basel, CH-4102 Binningen, Switzerland; g-a.tammann@unibas.ch

Allan Sandage

Observatories of the Carnegie Institution of Washington, Pasadena, CA 91101

AND

B. REINDL

Astronomisches Institut der Universität Basel, CH-4102 Binningen, Switzerland; reindl@astro.unibas.ch

*Received 2007 August 11; accepted 2008 January 7

ABSTRACT

The extragalactic distance scale relies heavily on Cepheids. However, it has become clear from observations and pulsation models that the slope and zero point of their P-L relations differ from galaxy to galaxy. This makes the determination of Cepheid distances complex and calls for an independent test of their differences. The test is provided by RR Lyrae star distances of 24 galaxies that calibrate the tip of the red giant branch (TRGB; $M_I^{TRGB} = -4.05$), which in turn confirms the adopted Cepheid distances on our 2006 distance scale in 18 cases to within 0.1 mag on average. Relative SN Ia and velocity distances deny a remaining significant metallicity effect of the adopted distances. The new support for these Cepheid distances increases the weight of our previous calibration of the SN Ia luminosity and of the 21 cm line width–luminosity (TF) relation. The value of $H_0 = 62.3$ (± 5) is confirmed on all scales.

Subject headings: distance scale — galaxies: distances and redshifts

1. INTRODUCTION: BAADE'S EARLY ATTEMPT TO COMPARE RR LYRAE VARIABLES AND CLASSICAL CEPHEIDS IN M31

In the late 1940s and early 1950s Baade (1948) and Hubble (1951) had formulated their research plans for observational cosmology using the new Palomar 200 inch (5.08 m) telescope. At various times in the 1930s Hubble had described his early Cepheid distance scale to NGC 6822 (Hubble 1925), M33 (Hubble 1926), and M31 (Hubble 1929) in the Local Group as only "reconnaissance" studies. He had put the distance moduli of these galaxies all at about $\mu = (m - M) = 22.2$ (D = 0.27 Mpc). Hubble then took the scale outward using "brightest stars" (later shown to be H II regions) to the M81/NGC 2403 and M101 groups, both with $\mu = 24.0$ (D = 0.63 Mpc) on his scale, and ultimately to the Virgo Cluster where he adopted $\mu = 26.8$ (D = 2.3 Mpc). Although he was fully aware of the timescale difficulties given by this distance scale that gave a redshift-distance ratio (the Hubble constant) of about 530 km $\rm s^{-1}~Mpc^{-1}$ (the units are assumed in the rest of this paper) with its small expansion age of $1/H_0 = 1.8$ Gyr, he nevertheless believed in this scale as late as 1952. This can be seen in the use by Holmberg (1950) of Hubble's distances in his survey of the groups centered on M81, M101, and the Virgo Cluster, distances that had been recommended by Hubble to him during his Mount Wilson stay to obtain observations of galaxian magnitudes with the 60 inch (1.5 m) telescope. It is also seen in Hubble's reluctance in 1952 to fully accept Baade's revision by about 1.5 mag for the distance moduli of M31 and M33, shown by the use of his scale in the study (Hubble & Sandage 1953) of the bright blue supergiant variables in each, with only a footnote to the new scale by Baade. (That footnote was inserted into the manuscript by one of us, who, at the time, was Baade's student and Hubble's assistant. Hubble only gave his hesitant approval to the footnote in the late drafts of that paper).

Baade had been working in the 1930s on extending the magnitude scales of Seares et al. (1930) in the Mount Wilson Catalogue of Selected Areas, particularly for SA 68, to the faintness required by Hubble's Cepheids in the three Local Group galaxies of NGC 6822, M31, and M33. By the time Baade had resolved the bulge of M31, the face of NGC 205, and the two dwarf companions of M31, NGC 147, and NGC 185 into stars (Baade 1944a, 1944b) and had identified these stars as the tip of the red giant branch in globular clusters (often named the Baade sheet in external galaxies and now called the TRGB), he had corrected Hubble's magnitude scales (which he often called Hubble's "enthusiastic" magnitudes) and changed Hubble's M31 distance modulus (Baade 1944a) to be $\mu=22.4$, which, however, was only 0.2 mag larger than Hubble's value of 22.2.

By 1948 Baade had anticipated that he should have detected RR Lyrae stars in M31 with the 200 inch telescope starting near V = 22.2 because he believed that the mean absolute magnitude of these variables was $M_{\rm pv} = -0.23$. This value had been determined by Shapley (1918) using the method of comparing the apparent magnitudes of two distance indicators (long-period Cepheids and RR Lyrae stars in this case) in aggregates of stars (here the globular clusters) where they appear together, the absolute magnitude calibration of one of which was taken to be known. Shapley had used the method by assuming that the Cepheids in globular clusters were the same as the field Galactic Cepheids whose absolute magnitudes had been calibrated by Hertzsprung (1914), Russell (1913), and Shapley (1918) by the method of statistical parallaxes. Although the theory of the comparison method is correct, the result turned out to be wrong for the Galactic Cepheids but, remarkably, almost correct for the RR Lyrae stars. It was 30 years later that Baade made the distinction between the globular cluster Cepheids (of his Population II) and the Population I Galactic Cepheids. It has turned out that the statistical parallax calibration of the Galactic Cepheids by

Hertzsprung, Russell, and Shapley was too faint by about 2 mag, but Shapley's calibration of the globular cluster Cepheids was close to what we know now to be the correct calibration of the Population II Cepheids, leaving Shapley's value of $\langle M_V \rangle = -0.23$ for the RR Lyrae stars reasonably correct for the purposes of the argument made by Baade (1954) at the Rome 1952 meeting (we adopt $\langle M_V \rangle$ (RR evolved) = +0.52 at [Fe/H] = -1.5 in § 2).

When Baade did not find the M31 RR Lyrae variables at V=22.2, he had two choices. Either the absolute magnitude calibration of the classical (Population I) Cepheids was wrong, and M31 was more distant than Hubble's modulus of 22.2, or Shapley's calibration of the RR Lyrae variables near $M_V=0$ was much too bright. Behr (1951) had suggested that the error was in the Cepheid calibration by nearly 2 mag. Behr's paper was not cited by Baade (although surely he knew of it because he was a voracious devourer of the literature), probably because Baade believed his own arguments to be decisive as he gave them at the 1952 Rome IAU meeting rather than relying on the tricky method of statistical parallaxes of Cepheids, sensitive as the result is to Galactic absorption.

Hence, the method of comparing the apparent magnitudes of various distance indicators with each other where they appear together began with Shapley (1918) in the globular clusters and continued with Baade's spectacular failure to find the M31 RR Lyrae stars at Hubble's distance. The method was successful in the LMC and SMC with the discovery by Thackeray (1954, 1958) of RR Lyrae stars near V=19, which he had also reported in the summary report of Commission 28 at the 1952 Rome IAU meeting. It was Thackeray's discovery of RR Lyrae stars in the LMC and SMC at this faint magnitude, about 2 mag fainter than predicted by the then adopted zero point of the Cepheid period-luminosity (P-L) relation, that cemented the truth of Baade's assertion that a change in the calibration of the P-L relation was needed, and therefore that Hubble's distance to M31 must be too short.

The method of comparisons of Cepheids and RR Lyrae stars in individual galaxies lay dormant until it was taken up again by Walker & Mack (1988) and was also greatly stimulated by the arrival of powerful telescopes including the *Hubble Space Telescope* (*HST*). But the comparison of Cepheids and RR Lyrae stars remained confined to LMC, SMC, and one or two additional galaxies (Smith et al. 1992; van den Bergh 1995; Fusi Pecci et al. 1996; Sandage et al. 1999). Lee et al. (1993), Udalski (2000), and Dolphin et al. (2001) included also the TRGB for comparison (see also Sandage 1971) and the magnitude of the red clump. Sakai et al. (2004) extended the comparison of TRGB and Cepheid distances to 17 galaxies (see also Rizzi et al. 2007).

The importance of comparing different distance indicators, i.e., mainly Cepheids, RR Lyrae stars, and TRGB, to the level of \sim 0.05 mag lies in the new developments that show that the P-L relation of classical Cepheids differs from galaxy to galaxy. It has been argued (Tammann et al. 2003, hereafter TSR03; Sandage et al. 2004, hereafter STR04) that the slope and zero point of the Cepheid P-L relation differ between the Galaxy and LMC and that the difference is likely to be a metallicity effect and may depend on helium as well according to pulsational models (Marconi et al. 2005).

The inequality of Cepheids in different galaxies should not come as a surprise. It was known since Gascoigne & Kron (1965) that SMC Cepheids are bluer than others, which alone precludes identical *P-L* relations. The color difference is not only caused by Fraunhofer blanketing of the metal lines, but it is also due to a real temperature difference of Cepheids at given period as was shown already by Laney & Stobie (1986). Galaxy-specific differences of Cepheids were also demonstrated by differences in their light

curves at a given period (Tanvir et al. 2005; Koen & Siluyele 2007). But it took a wealth of good data to study galaxy-to-galaxy differences of the P-L relations themselves, for instance, photometry of large numbers of Cepheids in the Galaxy, LMC, and SMC (Berdnikov et al. 2000; Udalski et al. 1999a, 1999b); independently determined reddenings (Fernie et al. 1995¹; Udalski et al. 1999a, 1999b); and distances (Feast [1999] for Cepheids in Galactic clusters, Fouqué et al. [2003] and Barnes et al. [2003] for moving-atmosphere Baade-Becker-Wesselink [BBW] distances and distances of LMC and SMC [as compiled from many authors in Tables 6 and 7 below]). These data show that the P-L relation of Cepheids cannot be universal. The reason is that the metal-poor LMC Cepheids are shifted in the luminosity- T_e diagram (i.e., the instability strip in the H-R diagram) to higher temperatures at constant L as compared to metal-rich Galactic Cepheids (STR04). The different slopes, which the Cepheids define in the instability strip, cause also the *slopes* of the *P-L* relations to be different.

The slope difference of the P-L relations of the Galaxy and LMC (and of short- and long-period Cepheids in LMC; see STR04) is particularly troublesome because the difference of the absolute magnitude of the Cepheids in the two galaxies becomes a function of period. While the blue LMC Cepheids are *brighter* than their relatively red Galactic counterparts by as much as $\Delta M_V = 0.36$ mag at $\log P = 0.5$, the difference diminishes with increasing period and changes sign at $\log P = 1.38$. From this follows that if the luminosity difference is interpreted as a metallicity effect, *any metallicity correction must depend on period*.

It is a coincidence that the new *P-L* relations and the period-dependent metallicity corrections lead to distances, which *on average* agree reasonably well with earlier Cepheid distances, although the latter were based on the unjustified assumption that the LMC *P-L* relation of Madore & Freedman (1991) was universal. As an example, the early Cepheid distances of eight or more galaxies by Ferrarese et al. (2000a), Freedman et al. (2001, Table 3, col. [2]), and Tammann et al. (2002) agree with those adopted here and by Sandage et al. (2006, hereafter S06b) to within less than 0.1 mag, regardless of whether some kind of bulk metallicity correction, irrespective of period, is applied and independent of the adopted LMC zero point (for details see S06b).

However, the near agreement of the old and new Cepheid distances collapses if Cepheids with nonaverage properties are considered. For instance, Freedman et al. (2001) and Riess et al. (2005), using the LMC P-L relation of Udalski et al. (1999c) or Thim et al. (2003), have based their luminosity calibration of Type Ia supernovae (SNe Ia) on only six and four galaxies, respectively, whose Cepheids happen to be particularly metal-rich and to have quite long periods. Correspondingly, the new period-dependent metallicity corrections become important and are the main reason why our present distances of these galaxies are longer by \sim 0.3 mag on average than adopted by these authors (Saha et al. 2006, hereafter S06a). The ensuing discrepancy in H_0 as derived from Cepheid-calibrated SNe Ia is on the order of 15%, our scale being longer.

The purpose of this paper is to construct an independent distance scale based on Population II stars in order to test our Cepheid distances. RR Lyrae magnitudes of 24 galaxies are compiled from the literature and uniformly reduced (§ 2.1). They are used to calibrate the absolute magnitude $M_I^{\rm TRGB}$ of the TRGB and to test its dependence on metallicity (§ 2.2). The different P-L relations and their calibration are discussed in § 3. The (satisfactory) comparison of the Cepheid and TRGB distances of 18 galaxies is in § 4. The Hubble diagrams with increasing outreach from TRGB, Cepheid, Cepheid-calibrated 21 cm line width (TF), and SN Ia

¹ See http://ddo.astro.utoronto.ca/cepheids.html.

distances and the resulting value of H_0 are discussed in § 5. The conclusions are in § 6.

2. POPULATION II DISTANCE INDICATORS

2.1. RR Lyrae Stars

2.1.1. The Calibration of RR Lyrae Stars

A summary of many calibration studies of the absolute magnitude of RR Lyrae stars as a function of metallicity has been given elsewhere (Sandage & Tammann 2006, hereafter ST06), the details of which are not repeated here. However, the results are as follows:

- 1. It is almost certain that the relation between M_V^{RR} and [Fe/H] is nonlinear. Most theoretical models of the zero-age horizontal branch (ZAHB) made after about 1990 predict the nonlinearity. Although the M_V luminosity of the level part of the zero-age HB is a function of [Fe/H] (higher metallicity models have fainter ZAHB), the variation of M_V with [Fe/H] becomes progressively smaller as [Fe/H] is decreased (see Fig. 3 of VandenBerg et al. 2000, reproduced as Fig. 9 of ST06). This nonlinearity, of course, applies also to that part of the HB that contains the RR Lyrae variables. Representative theoretical zero-age models are by Lee et al. (1990), Castellani et al. (1991), Bencivenni et al. (1991), Dorman (1992), Caputo et al. (1993), Caloi et al. (1997), Salaris et al. (1997), Cassisi et al. (1999), Ferraro et al. (1999), Demarque et al. (2000), VandenBerg et al. (2000), and Catelan et al. (2004). The nonlinearity also carries over to the HB that has evolved away from the ZAHB.
- 2. There are many observational data that also suggest that the calibration of M_{V}^{RR} with [Fe/H] is nonlinear, many of which are summarized by ST06. Important among these are the analyses of RR Lyrae data in many globular clusters by Caputo et al. (2000). These authors combine a pulsation equation that relates period, luminosity, temperature, and mass with observational data for globular cluster RR Lyrae stars at the blue edge of the instability strip for overtone pulsators and at the red edge for fundamental mode pulsators. Their obvious nonlinear M_V calibration, shown in their Figure 3, is reproduced as Figures 11 and 12 of ST06. Their study using observational data follows earlier nonlinear analyses of M_V^{RR} as a function of [Fe/H] by Caputo (1997), Gratton et al. (1997), De Santis & Cassisi (1999, their Fig. 15), McNamara et al. (2004), and undoubtedly others. The most recent is the study by Bono et al. (2007), where they show the nonlinearity over the entire range of [Fe/H] from 0 to -2.5 (their Fig. 16).
- 3. The zero point of the resulting nonlinear $M_V^{\rm RR}$ -[Fe/H] relation can be found by several methods, some leading to the so-called long RR Lyrae scale that gives $\langle M_V \rangle$ near +0.52 at [Fe/H] = -1.5 and the short scale that gives $\langle M_V \rangle$ near +0.72 at [Fe/H] = -1.5.

There are three calibration methods of high weight that lead to the long scale. In the order that we assign their reliability, they are as follows:

1. The discovery of δ Scuti stars in globular clusters, which are ultra–short-period dwarf Cepheids whose Population II prototype is the low-metallicity pulsator SX Phoenicis (for reviews see Nemec 1989; Nemec & Mateo 1990a, 1990b; McNamara 1997), opened the way for a potentially definitive calibration of RR Lyrae luminosities in globular clusters. There are also a number of such Population II stars in the nearby field with high-weight Hipparcos trigonometric parallaxes. Using these as absolute magnitude calibrators for the globular cluster SX Phoenicis stars gives the distances to globular clusters that contain them, and hence also the

- absolute magnitude of the RR Lyrae stars in these clusters. In this way McNamara (1997) has derived an RR Lyrae calibration that gives $\langle M_V(\text{RR evolved}) \rangle = +0.52$ at [Fe/H] = -1.5.
- 2. Main-sequence fitting of a globular cluster color-magnitude diagram (CMD) to the Hipparcos trigonometric parallax data for field subdwarfs of the appropriate metallicity gives the distance to the cluster. An extensive literature of the complications of the method exists (correction for reddening of the main sequences of the globular clusters, the requirement for precision measurements of [Fe/H] both for the globular clusters and for the appropriate Hipparcos subdwarfs, whether the Lutz-Kelker bias correction should be applied to the *Hipparcos* parallaxes, etc.) and includes papers by Gratton et al. (1997), Reid (1997, 1999), Carretta et al. (2000), and VandenBerg et al. (2000). Representative studies leading to the long distance scale are by Gratton et al. (1997), Carretta et al. (2000), and McNamara et al. (2004), among others cited therein. These calibrations are all consistent with $M_V(RR \text{ evolved}) = +0.52$ at [Fe/H] = -1.5 to generally better than 0.05 mag. A summary by Gratton (1998) to 1998 is important.
- 3. Several recent models of the ZAHB give $M_V = 0.65$ (VandenBerg et al. 2000) and 0.60 (Catelan et al. 2004) at [Fe/H] = -1.5. These must be made 0.09 mag brighter to account for the average evolution away from the ZAHB, which gives a mean $M_V = 0.53$ at [Fe/H] = -1.5 for the average RR Lyrae state. Both of these studies give nonlinear $M_V(Fe/H)$ relations.

 $M_V=0.52$ at [Fe/H] = -1.5 is adopted here. This is in excellent agreement with the RR Lyrae stars in LMC observed by Clementini et al. (2003a) at $\langle m_V^0 \rangle = 19.06$ after correction for absorption and an LMC distance modulus of 18.54 (TSR03, Table 6 with the RR Lyrae entry removed; see also Table 6 below). But it is emphasized that the RR Lyrae calibration does not depend primarily on the adopted LMC distance, which is used in § 3.2.2 to calibrate the P-L relation of the Cepheids in LMC. It is our aim to keep the Population I and Population II distance scales as independent of each other as possible.

Combining the parabolic form of the $M_V^{\rm RR}$ -[Fe/H] relation of Sandage (2006), using the pulsation equation together with the observed log P-[Fe/H] relation for cluster RR Lyrae stars, with the adopted value of $M_V = 0.52$ at [Fe/H] = -1.5 gives the calibration of

$$M_V(\text{RR evolved}) = 1.109 + 0.600([\text{Fe/H}]) + 0.140([\text{Fe/H}])^2$$
(1)

over the metallicity range 0 > [Fe/H] > -2.5. From the way it is derived using the mean evolved level of the RR Lyrae variables in the LMC, equation (1) refers to the mean evolved absolute magnitude of the variables, not the level of the ZAHB at the RR Lyrae position on the HB.

Other methods and analyses lead to a fainter RR Lyrae calibration. A comprehensive review to 1999 is done by Popowski & Gould (1999). In addition to the methods discussed above, these authors analyze two methods: (1) globular cluster kinematics where proper motions are compared with observed radial velocities of individual cluster stars, and (2) where an observed cluster white dwarf sequence is fitted to a calibrated H-R diagram. Altogether they discuss seven methods for an RR Lyrae calibration, keeping the three that they consider the most robust to be statistical parallaxes of field RR Lyrae, trigonometric parallaxes of field RR Lyrae, and internal cluster kinematics. From these they conclude that $\langle M_V(\text{RR evolved}) \rangle = +0.71$ at [Fe/H] = -1.6. This is 0.20 mag fainter than equation (1), which gives $\langle M_V(\text{RR evolved}) \rangle = +0.51$

 $TABLE \ 1 \\ RR \ Lyrae \ Star \ Distances \ of \ 24 \ Galaxies \ (in \ Order \ of \ R.A.)$

Name (1)	N _{RR} (2)	[Fe/H] (3)	⟨ <i>V</i> ⟩ (4)	E(B-V) (5)	A _V (6)	V ⁰ (7)	M_{San}^{V} (8)	M ^V _{Lit} (9)	μ_{new}^{0} (10)	μ_{Lit}^{0} (11)	Telescope (12)	References (13)
NGC 147	36	-1.37	25.29	0.173	0.54	24.75	0.55	0.77	24.20	23.92	200"	1
And III	39	-1.88	25.01	0.057	0.18	24.83	0.47	0.50	24.36	24.33	HST	2
NGC 185	151	-1.37	25.24	0.182	0.56	24.68	0.55	0.77	24.13	23.79	200"	3
NGC 205	30	-0.85	25.54	0.062	0.19	25.35	0.70	0.77	24.65	24.65	200"	4
NGC 224	54	-1.60	25.30	0.062	0.19	25.11	0.51	0.55	24.60	24.55	HST	5
And I	72	-1.46	25.14	0.054	0.17	24.97	0.53	0.57	24.44	24.40	HST	6
SMC	514	-1.70	19.74	0.087	0.27	19.47	0.49		18.98		1.3 m	7
Sculptor	226	-1.70	20.14	0.018	0.06	20.08	0.49	0.43	19.59	19.65	40"	8
IC 1613	13	-1.30	25.00	0.025	0.08	24.92	0.57	0.60	24.35	24.32	HST	9
And II	72	-1.49	24.87	0.062	0.19	24.68	0.53	0.57	24.15	24.06	HST	10
NGC 598	43	-1.30	25.12	Var	Var	25.34	0.57	0.67	24.77	24.67	HST	11
Phoenix	4	-1.40	23.64:	0.016	0.05	23.59	0.54		23.05:		4 m	12
Fornax	197	-1.95	21.27	0.042	0.13	21.14	0.47	0.48	20.67	20.66	HST	13a
Fornax	72	-1.78	21.28	0.042	0.13	21.15	0.48	0.44	20.67	20.72	6.5 m	13b
Fornax	500	-1.81	21.38	0.042	0.13	21.25	0.48		20.77	20.75	1.3 m	13c
LMC	108	-1.46	19.37	0.101	0.31	19.06	0.53	0.61	18.53	18.45	1.54 m	14
Carina	58	-1.90	20.76	0.063	0.20	20.56	0.47	0.58	20.09	20.10	4 m	15a
Carina	33	-2.2:	20.69	0.063	0.20	20.49	0.40	0.57	20.09	19.93	1.3 m	15b
Leo A	8	-1.70	25.10	0.021	0.07	25.03	0.49	0.53	24.54	24.51	3.8 m	16
Leo I	74	-1.82	22.60	0.036	0.11	22.49	0.48	0.44	22.01	22.04	2.2 m	17
Sextans	36	-1.60	20.36	0.050	0.16	20.20	0.51	0.57	19.69	19.67	1 m	18
Leo II	80	-1.90	22.10	0.017	0.05	22.05	0.47	0.44	21.58	21.66	3.6 m	19
UMi	82	-1.90	19.86	0.032	0.10	19.76	0.47	0.60	19.29	19.35	3.52 m	20
Draco	94	-1.60	20.18	0.027	0.08	20.10	0.51	0.69	19.59	19.61	1.2 m	21
Sag dSph	63	-1.79	18.17	0.153	0.47	17.70	0.48	0.52	17.22	17.19	0.9 m	22
NGC 6822	15	-1.92	24.63	0.236	0.73	23.90	0.47	0.50	23.43	23.41	VLT	23
And VI	91	-1.58	25.30	0.064	0.20	25.10	0.51	0.55	24.59	24.56	2.5 m	24

REFERENCES.—(1) Saha et al. 1990; (2) Pritzl et al. 2005; (3) Saha & Hoessel 1990; (4) Saha et al. 1992; (5) Brown et al. 2004; (6) Pritzl et al. 2005; (7) Soszynski et al. 2002; (8) Kaluzny et al. 1995; (9) Dolphin et al. 2001; (10) Pritzl et al. 2004; (11) Sarajedini et al. 2006; (12) Gallart et al. 2004; (13a) Mackey & Gilmore 2003; (13b) Greco et al. 2005; (13c) Bersier & Wood 2002; (14) Clementini et al. 2003a; Soszynski et al. 2003; Alcock et al. 2004; Borissova et al. 2004; (15a) Saha et al. 1986; (15b) Udalski 2000; (16) Dolphin et al. 2002; (17) Held et al. 2000; Held et al. 2001; (18) Mateo et al. 1995a; (19) Demers & Irwin 1993; Siegel & Majewski 2000; (20) Nemec et al. 1988; Bellazzini et al. 2002; Carrera et al. 2002; (21) Bonanos et al. 2004; Grillmair et al. 1998; Nemec 1985; Aparicio et al. 2001; (22) Layden & Sarajedini 2000; Mateo et al. 1995b; (23) Clementini et al. 2003b; McAlary et al. 1983; (24) Pritzl et al. 2002.

at [Fe/H] = -1.6. Among the consequences of this faint calibration is that, using the observations of the mean level of the RR Lyrae in the LMC similar to but earlier than those of Clementini et al. (2003a), Popowski & Gould (1999) derive a distance modulus of the LMC as $\mu^0 = 18.33 \pm 0.08$. This is 0.21 mag smaller than 18.54, which we take to be correct as shown by Table 6 of TSR03 and Table 6 here, supporting the calibration of equation (1), which we adopt as our scale in the remainder of this paper.

The intermediate RR Lyrae calibration by Bono et al. (2007) confirms the nonlinear dependence on [Fe/H]. It is based on convective mixing-length models that give absolute magnitudes averaging 0.1 mag fainter than equation (1) here. Their equation (10) gives $M_V(\text{RR evolved}) = +0.64$ at [Fe/H] = -1.5, which, with $\langle V^{0\,\text{RR}} \rangle = 19.06$ for LMC from sources in Table 1, gives $\mu^0(\text{LMC}) = 18.44$, which is only 0.1 mag less than the adopted value in Table 6 below.

We have not discussed the many calibrations of $\langle M_V^{\rm RR} \rangle$ for individual stars using the moving-atmosphere method (BBW) for which there is a large literature. In the hands of a dozen investigators, these RR Lyrae calibrations cover the range of the shortand long-scale values from $\langle M_V^{\rm RR} \rangle$ of +0.7 to +0.5 mag at [Fe/H] = -1.5 and therefore are of no help here to decide between them at the 0.2 mag level.

2.1.2. Twenty-four RR Lyrae Distances to Nearby Galaxies

The purpose of this section is to summarize the recent data on detection and measurement of the RR Lyrae variables in nearby

galaxies and to use equation (1) to determine RR Lyrae star distances to them.

The literature has been surveyed up to the end of 2006. The results are given in Table 1 ordered by right ascension. Column (2) shows the number of RR Lyrae stars in the particular study. The metallicity given by the original authors is in column (3). Columns (4) and (5) list the measured $\langle V \rangle$ and the E(B-V) reddenings from Schlegel et al. (1998). A few of the papers used the g photometric band (Thuan & Gunn 1976) rather than V. These were transformed to V by $\langle V \rangle - \langle g \rangle = +0.04$ (Saha et al. 1990). Column (6) is the assumed A_V absorption calculated from 3.1E(B-V). The absorption-free V^0 values, found by combining columns (4) and (6), are in column (7). The calculated absolute $\langle M_V(RR \text{ evolved}) \rangle$ magnitude of the RR Lyrae stars using equation (1) is in column (8). The value used by the original authors based on their various adopted RR Lyrae calibrations is in column (9). Column (8) combined with column (7) gives the adopted distance modulus in column (10). The distance from the original authors in column (11) is not necessarily the difference of columns (7) and (9) mainly because of differences of the adopted absorption. The telescope used for the literature study is in column (12). The literature reference is in column (13).

Our adopted $\mu_{\rm new}^0$ values in column (10) are the basis for the distance scale to which all other scales are compared in the remainder of this paper. These adopted RR Lyrae distances agree to 0.04 ± 0.08 with those published by the original authors, or 0.02 ± 0.01 if the early determinations for

 $TABLE\ 2$ Calibration of the TRGB by Means of RR Lyrae Stars

Name	$(V-I)^{\text{TRGB}}$	$\mu_{ m RR}^0$	m_I^{TRGB}	$M_I^{ m TRGB}$	References
(1)	(2)	(3)	(4)	(5)	(6)
Leo A	1.33	24.54	20.53	-4.01	1
Sex dSph	1.35	19.69	15.78	-3.91	2, 3
And I	1.40	24.44	20.49	-3.95	4
UMi	1.40	19.29	15.20	-4.09	5
SMC	1.45	18.98	14.95	-4.03	6
Sculptor	1.47	19.59	15.57	-4.02	7
Draco	1.48	19.59	15.62	-3.97	5
And II	1.51	24.15	20.11	-4.04	4
Carina	1.54	20.09	16.03	-4.06	8
Leo I	1.55	22.01	17.95	-4.06	9
IC 1613	1.56	24.35	20.24	-4.11	7
Leo II	1.60	21.58	17.56	-4.02	2
Phoenix	1.60	23.05:	19.17	(-3.88)	2
Fornax	1.61	20.67	16.68	-3.99	10
NGC 598	1.65	24.77	20.65	-4.12	4, 7, 11
NGC 6822	1.65	23.43	19.35	-4.08	12
And III	1.69	24.36	20.35	-4.01	13
LMC	1.70	18.53	14.54	-3.99	7
NGC 147	1.70	24.20	20.20	-4.00	13, 14
And VI	1.71	24.59	20.45	-4.14	13
NGC 205	1.71	24.65	20.53	-4.12	13
NGC 185	1.76	24.13	19.98	-4.15	7, 13
NGC 224	1.89	24.60	20.46	-4.14	7, 13
Sag dSph		17.22	12.46 ^a	(-4.76)	2
Mean			•••	$-4.05 \pm 0.02 \; (\sigma = 0.08, N = 22)$	• • •

a Poorly defined

REFERENCES.—References to $m_j^{\rm TRGB}$: (1) Dolphin et al. 2003; (2) Karachentsev et al. 2004; (3) Lee et al. 2003; (4) McConnachie et al. 2004; (5) Bellazzini et al. 2002; (6) Udalski 2000; Cioni et al. 2000; (7) Rizzi et al. 2007; (8) Smecker-Hane et al. 1994; Udalski 1998; (9) Bellazzini et al. 2004b; (10) Bersier 2000; (11) Galleti et al. 2004; (12) Gallart et al. 1996; (13) McConnachie et al. 2005; (14) Han et al. 1997.

NGC 147 and NGC 185 are neglected. We take this good agreement as a broad consensus with our equation (1) and our adopted RR Lyrae distance scale.

2.2. The Tip of the Red Giant Branch (TRGB)

The potential of the infrared TRGB magnitude as a distance indicator has been pioneered by Da Costa & Armandroff (1990). The basis of their work is that the cores of red giants with initial masses $\lesssim 2\,M_\odot$ are fully degenerate at the moment when the helium flash occurs, and with nearly constant core masses their luminosity increases only mildly with increasing Z (Rood 1972; Sweigart & Gross 1978). This increase of $M_{\rm bol}$ is compensated in the I band by the increasing effect of line blanketing such that $M_I^{\rm TRGB}$ becomes a useful standard candle for old, metal-poor populations. The importance of this is that $M_I^{\rm TRGB}$ extends the Population II distance scale by a factor of $\sim \! 10$ beyond the reach of RR Lyrae stars. Moreover, a great wealth of apparent $m_I^{\rm TRGB}$ magnitudes has since been accumulated by many authors.

The method of the TRGB is generally employed only in the range -2.3 < [Fe/H] < -0.7 because more metal-rich red giants have an increasingly fainter tip magnitude (see below). If it is restricted to populations older than 7 Gyr, the effect of age is negligible (Lee et al. 1993; Rejkuba et al. 2005).

2.2.1. The Calibration of M_t^{TRGB}

The 24 galaxies with RR Lyrae distances in Table 1 are repeated in Table 2, where also the corresponding apparent m_I^{TRGB} magnitudes (col. [4]) and their references (col. [6]) are given.

The resulting absolute values 2 of $M_I^{\rm TRGB}=m_I^{\rm TRGB}-\mu_{\rm RR}^0$ are given in column (5). The mean of the absolute magnitudes $M_I^{\rm TRGB}$, omitting Sag dSph, whose TRGB is not well defined, and Phoenix, whose RR Lyrae distance is uncertain, is

$$M_I^{\text{TRGB}} = -4.05 \pm 0.02,$$
 (2)

which we adopt. The value holds for a mean TRGB color of $(V-I)^{\text{TRGB}}=1.6$ (see Table 2, col. [2]), which translates into [Fe/H] ~ -1.5 (see below). The standard deviation of the individual determinations of M_I^{TRGB} is $\sigma=0.08$ mag. This is smaller than expected from observational errors alone. It follows from this that the random error of a single RR Lyrae star σ TRGB distance is in any case smaller than 0.1 mag even if metallicity corrections of the TRGB are neglected (see § 2.2.3).

The six late-type galaxies in Figure 1 deviate from the zero line by 0.04 ± 0.03 mag, being brighter. The near agreement of $M_I^{\rm TRGB}$ for late-type galaxies and for dwarf spheroidals indicates that internal absorption in the parent galaxy is negligible for all practical purposes.

2.2.2. Other Calibrations of M_I^{TRGB}

Da Costa & Armandroff (1990) have based their TRGB calibration on globular clusters with RR Lyrae distances; they have obtained $M_I^{\rm TRGB}=-3.98$. From the same method Sakai et al. (2004) have adopted -4.05. The value of -4.06 of Ferrarese et al.

 $^{^2}$ The values of $M_I^{\rm TRGB}$ and $m_I^{\rm TRGB}$ are corrected for Galactic absorption throughout.

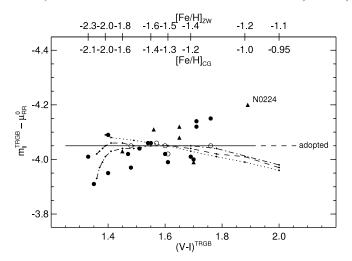


Fig. 1.—Absolute TRGB magnitudes $M_I^{\rm TRGB}$, as determined from the difference of the apparent TRGB magnitude and the RR Lyrae star distance, as a function of the color (V-I) and metallicities of the TRGB. The corresponding metallicities are given at the upper edge of the figure (see text). Note that blueward of (V-I)=1.4 the color becomes insensitive to metallicity. The six late-type galaxies are shown as triangles. The five independent calibrators of Rizzi et al. (2007) are shown as open circles. Semitheoretical predictions of the dependence of $M_I^{\rm TRGB}$ on metallicity of three different groups are drawn; they are normalized to $M_I^{\rm TRGB}=-4.05$ at (V-I)=1.6 (dashed line: Salaris & Cassisi 1998, eq. (5); dot-dashed line: Bellazzini et al. 2004a; dotted line: Rizzi et al. 2007).

(2000b) was calibrated by Cepheids. The luminosity of -4.07 by Bellazzini et al. (2004a) rests on the distances of ω Cen and 47 Tuc. Salaris & Cassisi (1997) have determined the TRGB magnitude from theoretical stellar evolution models and found -4.16, which, however, includes also very short-lived stars. They have revised (1998) their result to -4.27 and -4.24, respectively, depending on the adopted bolometric correction. The result was closely confirmed by Rejkuba et al. (2005) based on the stellar evolution database of Pietrinferni et al. (2004). Bergbusch & VandenBerg (2001, Fig. 15) imply a value close to -4.05 based on the models of VandenBerg et al. (2000). Rizzi et al. (2007) have fitted the HB of five galaxies to the metal-dependent HB of Carretta et al. (2000), whose zero point rests on trigonometric parallaxes; their result is -4.05 ± 0.02 . All values of M_I^{TRGB} quoted in the section refer to a metallicity of [Fe/H] = -1.5. The calibration with RR Lyrae stars in § 2.2.1, which refers to the same metallicity, is in good to excellent agreement with the values quoted here.

2.2.3. The Dependence of M_I^{TRGB} on Metallicity

The absolute magnitudes $M_I^{\rm TRGB}$ of Table 2 are plotted against the color (V-I) of the TRGB in Figure 1. The colors are taken from the original literature. They are converted into metallicities [Fe/H]_{ZW} on the scale of Zinn & West (1984) following Bellazzini et al. (2001) and shown at the upper edge of Figure 1. The metallicities [Fe/H]_{CG} in the system of Carretta & Gratton (1997) are also shown.

The calibrators in Figure 1 suggest an increase of the TRGB luminosity with increasing metallicity, the reality of which we doubt, however. The five calibrators of Rizzi et al. (2007) give a flat calibration, although for a narrower metallicity range of $-1.8 < [{\rm Fe/H}]_{\rm ZW} < -1.3$. Also the models of VandenBerg et al. (2000) do not show a systematic change of the TRGB with metallicity (Bergbusch & VandenBerg 2001; Rejkuba et al. 2005, Fig. 13); the tip becomes *fainter* only for the most metal-rich red giants with (V-I) > 2.0. The strong decline of the tip magnitude redward of this limit has been directly observed in rich populations with a wide metallicity spread as in the Galactic bulge

(Zoccali et al. 2003) and in the halo of NGC 5128 (Rejkuba et al. 2005).

Model-dependent variations of the TRGB magnitude with metallicity have also been determined by Salaris & Cassisi (1998), Bellazzini et al. (2004a), and Rizzi et al. (2007). Their results are displayed in Figure 1, after they are normalized to $M_I^{\rm TRGB} = -4.05$ at (V-I)=1.6. The authors agree that the TRGB magnitude does not change by more than 0.05 mag over the interval $1.4 < (V-I) \lesssim 1.9$ or $-2.0 < [{\rm Fe/H}]_{\rm ZW} < -1.2$. For the most metal-poor red giants the results diverge. For the metal-rich red giants with $[{\rm Fe/H}]_{\rm ZW} > -1.2$ the results agree on a progressive dimming of $M_I^{\rm TRGB}$.

The near constancy of M_I^{TRGB} over a wide metallicity interval is fortunate because no metallicities or colors $(V - I)^{\text{TRGB}}$ are available for most galaxies with known TRGB magnitudes.

In the following a constant value of $M_I^{\rm TRGB} = -4.05$ will be adopted irrespective of metallicity, as Ferrarese et al. (2000b) and Karachentsev et al. (2004, 2006, 2007) in their extensive work on the TRGB have done. Since metal-rich giant branches are unfrequent in old populations, the mean distance error incurred will hardly be larger than 0.05 mag. If one compares the distances of 22 galaxies for which Rizzi et al. (2007) give metal-corrected TRGB distances with those one obtains from a fixed calibration of $M_I^{\rm TRGB} = -4.05$, they differ by only 0.03 mag with a small standard deviation of 0.06 mag. A similar conclusion is reached below, where TRGB distances with and without metallicity corrections are compared with (metallicity-corrected) Cepheid distances (see Table 9 below).

2.2.4. TRGB Distances of Field Galaxies

Karachentsev et al. (2004) have compiled many $m_I^{\rm TRGB}$ magnitudes and have provided additional ones in Karachentsev et al. (2006, 2007). Other authors have observed the TRGB in many additional galaxies. Altogether, $m_I^{\rm TRGB}$ magnitudes are available for 218 (mostly dwarf) galaxies. Since Karachentsev et al. (2007) have used the same TRGB calibration as adopted here, their listed distances remain unchanged. For consistency all distances have been slightly adjusted, where necessary, to the present calibration of $M_I^{\rm TRGB} = -4.05$.

2.2.5. TRGB, the Virgo Cluster, and the Cosmic Distance Scale

Caldwell (2006) has observed the TRGB in the Virgo galaxy NGC 4407 and in five small dwarf galaxies in its vicinity and obtains, with our calibration, a mean distance of $\mu^0=31.08\pm0.05$. An anonymous Virgo dwarf away from other galaxies yields 31.22 with a more realistic error of ±0.17 (Durrell et al. 2007). In view of the depth effect of the Virgo Cluster, which amounts to 2–3 Mpc (e.g., Mei et al. 2007) even on the assumption of sphericity, these first distances to individual cluster members cannot be taken as giving the mean cluster distance. TRGB distances of a statistically fair sample of Virgo Cluster members will be most valuable as a test for the entire distance scale.

TRGB stars have also been detected in the intracluster medium of the Virgo Cluster. Durrell et al. (2002) and Caldwell (2006) quote distances of $31.36^{+0.27}_{-0.17}$ and 31.2 ± 0.09 , respectively. However, these are only *lower* limits to the distance of the cluster core because only the nearest TRGB stars can be detected in the cluster field, while the more distant ones are drowned among the red giant stars on the *near* side.

 $^{^3}$ Durrell et al. (2002) do not actually quote a distance to their Virgo Cluster fields. The value $\mu^0=31.36$ follows from their $I^{\rm TRGB}=27.31$ and the calibration of $M_I=-4.05$ used here.

TABLE 3
A TENTATIVE TRGB CALIBRATION OF THE SN Ia LUMINOSITY

SN (1)	m_V^0 (2)	Galaxy (3)	$\mu_{\text{TRGB}}^{0} $ (4)	Group (5)	$\langle \mu_{\mathrm{TRGB}}^{0} angle \ (6)$	n (7)	References (8)	M_V^0 (SN Ia) (9)
1937C	8.99	IC 4182	28.21	CnV II	28.26	16	1	-19.22
1972E	8.49	NGC 5253	27.89	Cen A	27.89	24	2	-19.40
1989B	10.95	NGC 3627		Leo I	30.43	2	3	-19.48
1998bu	11.04	NGC 3368		Leo I	30.43	2	3	-19.39
Mean								-19.37 ± 0.06

REFERENCES.—(1) Sakai et al. 2004; Rizzi et al. 2007; (2) Sakai et al. 2004; (3) Sakai et al. 2004 for NGC 3351; Sakai et al. 1997 for NGC 3379.

Information on the TRGB distances is available for four SNe Ia at present. Their relevant data are set out in Table 3. The corrected $m_V(\text{max})$ magnitudes of the SNe Ia in column (2) are from Reindl et al. (2005, hereafter R05, Table 2, col. [9]). The TRGB distances of the individual parent galaxies (col. [4]) and of the mean TRGB distances of their respective groups (col. [6]), together with the number of group members involved (col. [7]), are from the sources identified in column (8). All distances are based on $M_I^{\text{TRGB}} =$ -4.05. The resulting absolute magnitude of the four SNe Ia, based on the group distances, is given in column (9). The mean absolute magnitude of $M_V(\text{SNe Ia}) = -19.37 \pm 0.06$ is statistically the same as -19.46 ± 0.04 , which is based on 10 SNe Ia with Cepheid distances (S06b) and has much higher weight. The latter value is also close to various values derived by others, as summarized by Gibson et al. (2000) in the B band (note that $B_{\rm max}^0 - V_{\rm max}^0 = -0.03$; S06b). When the TRGB method can be pushed to yield reliable distances out to \sim 31.5, seven presently known SNe Ia will come into its reach and will yield an independent calibration of H_0 through the TRGB-calibrated Hubble diagram of SNe Ia.

3. POPULATION I DISTANCES

The foundation of the Population I distance scale is classical Cepheids. Their metallicity-dependent P-L relations in B, V, and I have been derived in TSR03 and STR04. It was found that the relatively metal-rich Cepheids in the solar neighborhood ($[O/H]_{T_e} = 8.50$ in the T_e -based scale of Kennicutt et al. [2003] and Sakai et al. [2004]) define a P-L relation that differs in slope and shape from the P-L relation of LMC ($[O/H]_{T_e} = 8.34$). It is therefore not possible to determine an LMC distance from a P-L relation that is based on Galactic Cepheids. The P-L relations of the two galaxies must be independently zero pointed. A more general discussion of the problem follows.

3.1. The Forms of the P-L Relation

The only rationale to assume that the P-L relation is universal is convenience since the time in which it was known that Cepheids in different galaxies have different colors, temperatures, light curves, and slopes of their P-L relations (see § 1). Also the break of the period-color (P-C) and P-L relations of LMC at P=10 days, not yet seen in other galaxies, is alarming (see below). However, the investigation of the shape of the P-L relation of individual galaxies is difficult because the intrinsic width of the instability strip requires very large Cepheid samples distributed over a wide period interval. Such samples are available only for LMC and SMC; they will never become attainable in dwarf galaxies. One has therefore to assume, in first approximation, that the P-L relations are linear.

Even on the assumption of linearity the determination of the *slope* is demanding for several reasons: (1) The Cepheids in many galaxies, particularly the distant ones, are often restricted to $P \gtrsim 10$ days. (2) Selection bias in favor of Cepheids with short periods near to the detection limit (Sandage 1988) tends to flatten the slope. (3) The slope is independent of the reddening only as long as it does not depend on the period, which is not warranted a priori.

The reddening values E(B-V) of the Cepheids in all galaxies considered are derived from (V-I) and in some cases (B-V) colors and an *adopted* template P-C relation with the exception of only three galaxies, viz., the Galaxy, LMC, and SMC. The individual reddenings of the Galactic Cepheids have been derived ab initio by Fernie (1990), Fernie et al. (1995), and others. They have been homogenized and slightly revised by TSR03. The reddenings of the Cepheids in LMC and SMC have been determined from adjacent red clump stars by Udalski et al. (1999a, 1999b).

3.1.1. The Shape of the Galactic P-L Relation

The shape of the P-L relation of the metal-rich Galactic Cepheids ($[O/H]_{T_e} = 8.50$) is determined from two independent methods, both covering a wide period interval:

- 1. Thirty-three Cepheids in clusters are taken from the revised list of Feast (1999; see STR04). Their distances are known from main-sequence fitting relative to the Pleiades, whose distance modulus of $\mu^0 = 5.61 \pm 0.02$ is well determined from different methods, including trigonometric parallaxes (STR04).
- 2. BBW distances are available for 33 partially overlapping Galactic Cepheids from Fouqué et al. (2003) and Barnes et al. (2003). Also included are three additional Cepheids with distances from interferometric diameter measurements (Benedict et al. 2002; Nordgren et al. 2002; Lane et al. 2002; Kervella et al. 2004).

The Cepheids under methods 1 and 2 are corrected for Galactic absorption. The absorption-free magnitudes of the two sets of Cepheids define P-L relations in B, V, and I with very similar slopes. While they agree at $\log P = 0.5$ to within 0.01 mag, they diverge in all three colors by not more than 0.2 mag at $\log P = 1.5$. The adopted mean linear P-L relations in equations (16)–(18) of TSR03 should therefore be good to within ± 0.1 mag over a wide period interval.

The Galactic P-L relation with slope -3.087 ± 0.085 is as linear in all three colors as can be determined from a sample of only 69 Cepheids. Provided that the values of E(B-V) from Fernie et al. (1995) do not systematically overestimate the reddening of long-period Cepheids (a possibility that has been discarded in TSR03), the Galactic P-L relation does not have the break at $\log P = 1.0$ as is observed in LMC (see below). The Galactic P-L relation in B, V, and I is steeper than observed in most other galaxies, but the slope is about equally steep in the metalrich galaxies NGC 3351 and NGC 4321 (Fig. 2), as well as in NGC 224 (M31; §§ 3.1.5 and 3.3).

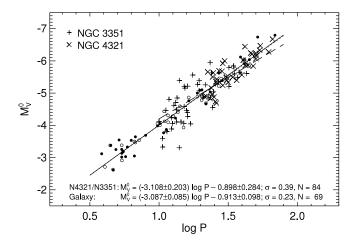


Fig. 2.—P-L relation in V of metal-rich Cepheids in the Galaxy (*circles*), NGC 4321 (*crosses*), and NGC 3351 (*plus signs*). The latter two galaxies define a slope in good agreement with the Galaxy (*dotted line*). For comparison the LMC P-L relation for $\log P > 1.0$ is shown as a dashed line.

From trigonometric parallaxes with the fine guidance sensor on HST Benedict et al. (2007) have derived a very flat slope of the Galactic P-L relation of -2.46 in V, but of their 10 Cepheids only 1 Car has a period significantly larger than 10 days. The flat relation raises several questions: (1) The authors discuss the possibility of a break of the Galactic P-L relation, but this creates more problems than it solves. (2) The flat P-L relation predicts 18 Cepheids in Galactic clusters with periods $0.6 < \log P < 1.0$ to be brighter by 0.16 ± 0.05 than listed in Table 1 of TSR03. These Cepheids lie all in well-defined clusters (not in less reliable associations!), and an *upward* revision of the cluster distance scale by this amount is difficult to accept, particularly since An et al. (2007) have concluded from refined photometry of 7 of the 18 clusters that their distances, if anything, should be shifted downward by 0.12 ± 0.06 . (3) The luminosity difference between the 10 Cepheids by Benedict et al. (2007) and the P-L relation adopted here is not only a function of period but also a function of apparent magnitude. This opens the possibility of astrometric errors as a function of magnitude. Finally, the 10 parallax Cepheids define a P-L relation in V with a random scatter of only 0.11 mag as compared to 0.22 mag in LMC. This confirms the prediction that the intrinsic half-width of the Galactic P-L relation is only 0.08 on the basis of the flat constant-period lines in the Galaxy (STR04). Alone the slope difference of the constantperiod lines between the Galaxy and LMC constitutes an important difference between the Cepheids of these two galaxies.

The HST parallaxes by Benedict et al. (2007) have been augmented by Hipparcos parallaxes including four additional Cepheids by van Leeuwen et al. (2007). These authors have derived a P-L relation from a combination of V and I magnitudes. Since this may conceal differences of the separate P-L relations, the relation in V was derived from their data after correcting for absorption. Not surprisingly, the resulting relation is essentially the same as by Benedict et al. (2007) because most of the weight lies on the HST parallaxes.

Fouqué et al. (2007) rely for the slope of the Galactic P-L relation on 49 BBW infrared surface brightness distances, augmented by the 10 trigonometric parallaxes of Benedict et al. (2007); they exclude Cepheid distances from open clusters. In this way they derive a slope in V of -2.678, significantly flatter than our value of -3.087 and close to the slope one obtains if the LMC Cepheids are (unjustifiedly) fitted with a single slope. The authors admit that the crux of the BBW method is the correct

choice of the p factor that converts observed radial velocities into pulsational velocities. They have taken p from the model of Nardetto et al. (2004), where p depends on the period P. This is unfortunate because any error of the dependence of p(P) translates into an error of the slope of the P-L relation. A weaker p(P) dependence yields steeper P-L relations.

The slope difference between Fouqué et al. (2007) and us (STR04) is caused by their almost exclusive reliance on the BBW method, while we rely on cluster distances that agreed impressively well with the BBW distances available at the time from Fouqué et al. (2003) and Barnes et al. (2003). Our steep slope finds support in metal-rich Cepheids of other galaxies that define, as discussed in § 3.1.5, an equally steep slope.

If Benedict et al. (2007) and Fouqué et al. (2007) claim that the Galactic and LMC P-L relations are indistinguishable, they fail to acknowledge the break of the LMC relation at $\log P = 1.0$ (see § 3.1.2) and its absence in the Galactic Cepheids. In addition, the inequality of the Galactic and LMC P-L relations will be shown in § 3.1.4, independent of any adopted distances, only on the basis of the Cepheid colors.

The pulsation models of Marconi et al. (2005) for highmetallicity Cepheids (Z = 0.02) do not give as steep a slope as we observe in the Galaxy. They obtain the steepest slope for Y = 0.26 with flatter slopes for higher Y(0.28, 0.31) and lower Y(0.25), but even in the first case the slope is significantly flatter than observed. Surprisingly, lower metallicity models with Z = 0.01, Y = 0.26, a composition actually favored for δ Cep by Natale et al. (2008), come close to the observed slope for the Galaxy. Yet the model slopes are not yet definitive because they depend on the position of the red edge of the instability strip, where the treatment of convection is necessary. Also the uneven population of the strip due to temperature-dependent crossing times should be accounted for. Furthermore, the pulsation models show that the *P-L* relation depends not only on *Z* but on *Y* as well. The point is that the models of Marconi et al. (2005) do show that the *P-L* slopes should vary from galaxy to galaxy.

3.1.2. The Shape of the LMC P-L Relation

The shape of the P-L relation in B, V, and I of the low-metallicity Cepheids of LMC ($[{\rm O/H}]_{T_e}=8.36$) is unusually well determined by about 680 Cepheids from the OGLE program (Udalski et al. 1999a) and several other sources (STR04). A linear fit over the entire period interval with slope -2.702 in V is not the optimum fit. A significantly better fit is achieved by two linear lines breaking at P=10 days (Tammann & Reindl 2002; Tammann et al. 2002; Ngeow et al. 2005). The break, also clearly seen in the P-C relations for (B-V) and (V-I) (STR04, Figs. 1a and 1b), withstands several statistical tests (Ngeow et al. 2005; Kanbur et al. 2007; Koen et al. 2007). The break becomes particularly striking if the Cepheids are reduced to the P-L ridgeline by shifting them along constant period lines. The shift is determined by the difference between the observed color (V-I) $_{\rm obs}^0$ of a Cepheid with fixed period and the color (V-I) $_{P}$ -C required by the appropriate P-C relation for this period, i.e.,

$$M_V(\text{Ridge}) = M_V^0 - \beta_{V,V-I} \left[(V-I)_{\text{obs}}^0 - (V-I)_{P-C}^0 \right].$$
 (3)

The coefficient $\beta_{V,V-I}$ is the slope of the constant-period lines in the CMD for M_V versus (V-I). For LMC it was found that $\beta_{V,V-I}=2.43$ (STR04, eq. [29]). The resulting P-L relation with its clear break is shown in Figure 3.

The pulsational models for Z = 0.004, Y = 0.25 of Marconi et al. (2005) fit the observed P-L relation of LMC well, *including*

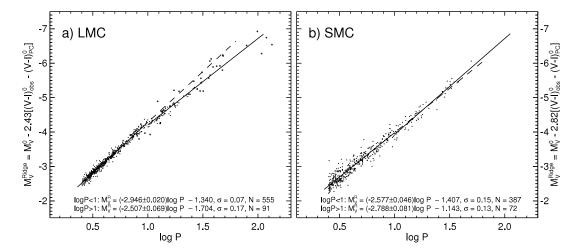


Fig. 3.—(a) Ridgeline P-L relation in V for LMC. The break at $\log P = 1.0$ is highly significant. (b) Ridgeline P-L relation in V for SMC omitting Cepheids with $\log P < 0.4$. The dashed lines are the extrapolations of the P-L relation of the Cepheids with $\log P < 1.0$.

the break at 10 days. The theoretical break is even more pronounced than observed.

3.1.3. The Shape of the SMC P-L Relation

Linear regressions to 459 SMC Cepheids of Udalski et al. (1999b) in the range $0.4 < \log P < 1.7$ give

$$M_B^0 = -(2.222 \pm 0.054) \log P - (1.182 \pm 0.041), \quad (4)$$

$$M_V^0 = -(2.588 \pm 0.045) \log P - (1.400 \pm 0.035),$$
 (5

$$M_I^0 = -(2.862 \pm 0.035) \log P - (1.847 \pm 0.027).$$
 (6)

The constant terms in equations (4)–(6) are based on the distance of $\mu_{\rm SMC}^0 = 18.93$ from Table 7 below. The determination of the exact shape of the SMC P-L relations, however, is subtle. It is known that they turn downward for the many SMC Cepheids with very short periods ($\log P < 0.4$; Bauer et al. 1999). It is difficult to decide whether this should be interpreted as a break at $\log P \sim 0.4$ or as curvature of the P-L relations. The P-C relations in (B - V) and (V - I) clearly suggest an additional break at $\log P = 1.0$ like in LMC. The ridgeline P-L relation in V, constructed with $\beta_{V,V-I} = 2.82$ appropriate for SMC (STR04, Table 4), also suggests the break at $\log P = 1.0$ (at only a 2 σ level), but, contrary to LMC, with the slope increasing above the break point (Fig. 3b). The single-slope SMC P-L relations of equations (4)–(6), however, were deemed to be adequate for the following application to very metal-poor Cepheids (see § 3.3).

3.1.4. The Interplay of the P-L and P-C Relations

The ongoing discussion on the slope of the Galactic P-L relation could still nourish the hope that the P-L relations of classical Cepheids were universal. This hope is unfounded in view of the P-C relations. Because if the B, V, IP-L relations are to be invariable, so must be the P-C relations in (B - V) and (V - I), which are simply the differences of the corresponding P-L relations.

Yet the metal-poor LMC and even more metal-poor SMC Cepheids are on average *significantly* bluer in $(B-V)^0$ than the Galactic Cepheids by 0.07 and 0.14 mag, respectively. The corresponding number for $(V-I)^0$ is 0.05 mag for both galaxies. Thus, the zero points of the B, V, IP-L relations must differ by at least this amount, but the shift could be larger by any additional constant amount. The color behavior of the Cepheids in a two-color diagram $(B-V)^0$ versus $(V-I)^0$ can be explained, ne-

glecting their periods, by atmospheric models (Sandage et al. 1999) as the blanketing effect of the metal lines (see TSR03, Fig. 7a).

But, in addition, the same models show LMC Cepheids to be hotter than Galactic Cepheids *at a given period* by roughly 200 K (see also Laney & Stobie 1986) and to be also hotter at constant luminosity (STR04, Fig. 20). This is, as has been shown, an even stronger luminosity effect than the line blanketing.

If the sizes of the blanketing effect and of the temperature difference were independent of period, the *slopes* of the *P-L* relations could still be the same everywhere, and only their zero points were shifted. However, it is clear that the blanketing effect depends on color and hence on period. Moreover, it was shown in STR04 that also the temperature difference at constant luminosity increases with period. These effects *must* reflect on the slopes of the *P-C* relations. In Table 4 the observed slopes of the *P-C* relations in $(B-V)^0$ and $(V-I)^0$ of the Galaxy, LMC, and SMC are compiled. To avoid further complications with the break at 10 days of at least the LMC relations, the slopes in the interval $0.4 \le \log P \le 1.0$ are only considered. The slope differences in Table 4 between the three galaxies are highly significant at the 2-5 σ level.

The conclusion is that since the *P-C* relations have different slopes in galaxies with different metallicity, the slopes of the *B*, *V*, *I P-L* relations must also vary. It is therefore not anymore the question of whether the *P-L* relations are universal, but only by how much they vary.

A word of warning may here be in place. There are indications that the *P-L* relations in the near-infrared (*JHK*) have nearly the same slope independent of metallicity. If this is the case, this still does not mean, as discussed above, that they have the same zero point. It will therefore be necessary to independently zero point the near-infrared *P-L* relations for Cepheids with different chemical composition.

3.1.5. The Slope of the P-L Relation as a Function of Metallicity

The metallicities (from Sakai et al. 2004) and slopes of the *B*, *V*, and *I P-L* relations of nine galaxies are compiled in Table 5. Only galaxies with well-determined or reasonably well determined slopes are considered. The original sources of the Cepheid data are listed in the last column.

The Cepheids in Sextans A and B are combined to a single *P-L* relation because they have nearly the same (very low) metallicity

TABLE 4 Slopes of P-C relations in the Galaxy, LMC, and SMC (Fits for $0.4 \leq \log P \leq 1.0$)

Color (1)	Galaxy (2)	LMC (3)	Δ (4)	SMC (5)	Δ (6)
$(B-V)^0 \dots \dots$	0.366 ± 0.15 0.256 ± 0.15	0.273 ± 0.024 0.160 ± 0.022	0.093 ± 0.028 0.096 ± 0.027	0.198 ± 0.024 0.199 ± 0.024	$0.168 \pm 0.028 \\ 0.057 \pm 0.027$

and almost identical TRGB distances ($\mu^0 = 25.78$ and 25.79; Rizzi et al. 2007).

The decrease of the P-L slope with decreasing metallicity in Figure 4 is striking. The extreme case of Sextans A and B deserves special emphasis. Confirmatory work would be interesting, although the two small galaxies may not have many more Cepheids than already known (17 over a wide period interval). It is likely that some of the scatter in Figure 4 is intrinsic. The available data for several metal-rich galaxies admittedly suggest that their P-L relations are flatter than in the Galaxy (see § 3.4.4). But Figure 4 leaves no doubt that the P-L slope is correlated with [O/H]. Hence, the P-L relation cannot be universal but must vary from galaxy to galaxy, primarily as a function of [O/H].

3.2. The Zero-Point Calibration of the P-L Relation of Cepheids

3.2.1. The Zero Point of the Galactic P-L Relation

The zero point of the Galactic P-L relation for an adopted metallicity of $[{\rm O/H}]_{T_e}=8.62$ rests on 33 cluster distances (STR04) and 36 BBW distances from Fouqué et al. (2003) and Barnes et al. (2003). The two calibrations agree to 0.07 mag in V at an intermediate period of P=10 days (STR04). New distances of seven clusters by An et al. (2007) suggest smaller distances by \sim 0.1, which brings the two systems to even better agreement. The BBW distances have been revised twice since 2003 (Gieren et al. 2005b; Fouqué et al. 2007), but the effect on the zero point at P=10 days is negligible. The adopted zero point is $M_V=-4.00$. The independent zero point from HST parallaxes by Benedict et al. (2007) is brighter by 0.05, the one of van Leeuwen et al. (2007) by only 0.01.

3.2.2. The Zero Point of the P-L Relation of LMC

The zero point of the LMC *P-L* relations, which holds for a metallicity of $[O/H]_{T_e} = 8.34$ (Sakai et al. 2004), is given by an *adopted* distance of LMC. Thirteen determinations from 1997 to

2002 gave a mean modulus of $\mu^0=18.54\pm0.02$ (STR04). Sixteen newer determinations are compiled in Table 6. The listed errors of the individual distance determinations are from the literature, but since they are incommensurable, a straight mean of 18.53 ± 0.01 has been derived. The value of 18.54 is maintained here. Note that none of the listed distances involve any assumption on the P-L relation, which would make the calibration circular.

If the model P-L relations in V for Z=0.004, Y=0.25 of Marconi et al. (2005) are taken at face value and if they are combined with the observed Cepheids in LMC, one obtains a distance modulus of $\mu_{\rm LMC}^0=18.51\pm0.01$. The result is lower in B and higher in I because the model colors are still redder than observed. The distance becomes smaller by $\sim\!0.1$ mag if the more realistic model with Z=0.008, Y=0.25 is used for LMC.

An LMC Cepheid at P = 10 days is brighter than its Galactic counterpart by 0.25 mag in V. The difference cannot be removed by simply changing the LMC distance because it is wavelength dependent (0.35 in B, 0.15 in I). The erroneous assumption of equal zero points has notoriously led to too small an LMC distance if based on Cepheids.

3.2.3. The Zero Point of the P-L Relation of SMC

The constant terms in equations (4)–(6) are calibrated with an adopted SMC modulus of 18.93 (Table 7) as mentioned before.

In \S 4 the Cepheid distances are compared with the Population II distance indicators. The calibration of the Cepheids should therefore be as free of Population II data as possible. In spite of this, an RR Lyrae star and a TRGB distance are included for the zero-point calibration of each of the P-L relations of LMC and SMC (see Tables 6 and 7). However, their omission would change the calibration by only 0.02 mag. In the case of LMC such a change is negligible because the LMC P-L relation is always used in combination with the independently calibrated Galactic P-L relation. The SMC P-L relation is used for only three galaxies, which follow below.

Galaxy	$[\mathrm{O/H}]_{T_e}$	Slope B	Slope V	Slope I	Error V	Original Source
NGC 3351	8.85		-3.12	-3.38	0.39	Graham et al. (1997)
NGC 4321	8.74		-3.17	-3.43	0.34	Ferrarese et al. (1996)
M31	8.66	-2.55	-2.92		0.21	Vilardell et al. (2007)
Galaxy	8.60	-2.69	-3.09	-3.35	0.09	STR04
LMC ^a	8.34	-2.34	-2.70	-2.94	0.03	Udalski et al. (1999a)
NGC 6822 ^b	8.14		-2.49	-2.81	0.10	Pietrzynski et al. (2004)
NGC 3109	8.06		-2.13	-2.40	0.18	Pietrzynski et al. (2006b)
SMC ^c	7.98	-2.22	-2.59	-2.86	0.05	Udalski et al. (1999b)
IC 1613	7.86	-2.36	-2.67	-2.80	0.12	Antonello et al. (2006)
WLM	7.74		-2.52	-2.74	0.15	Pietrzynski et al. (2007)
Sextans A+B	7.52	-1.43	-1.59	-1.47	0.39	Piotto et al. (1994)

^a Single-fit slope, neglecting the break at P = 10 days.

^b Because of large scatter, the slope depends somewhat on the period cutoff; here $P \ge 5.5$ days.

^c Omitting Cepheids below P = 2.5 days.

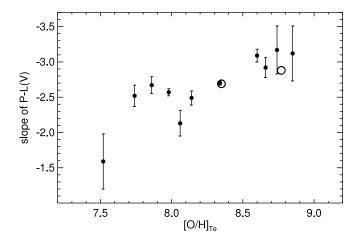


Fig. 4.—Slope of various P-L relations in V as a function of the metallicity $[O/H]_{T_c}$. Open circles are the means of seven galaxies each from S06a (their Fig. 10).

3.2.4. Metallicity Corrections

There is a large literature on metallicity corrections to Cepheid distances (i.e., due to differences in the Cepheid *P-L* relation for different *Y*- and *Z*-values), and we do not review that literature here. A fine review is by Romaniello et al. (2005). For the present paper we use the formulations by S06a.

Cepheid distances derived from V and I magnitudes and the corresponding P-L relations of the Galaxy differ from those using the P-L relations of LMC. Up to periods of $P \lesssim 10$ days the LMC relations yield larger distances, and above this period limit smaller distances. This was ascribed in S06a to the metallicity difference of the Cepheids in the Galaxy ($[\text{O/H}]_{T_e} = 8.60$) and in LMC ($[\text{O/H}]_{T_e} = 8.34$). Correspondingly, Cepheids with Galactic metallicity are reduced with the P-L relations of the Galaxy, and those with the LMC metallicity with the LMC relations. For the distances of Cepheids with intermediate and slightly extrapolated metallicities an interpolation formula was derived in S06a (eq. [10]).

The interpolation formula is given here in tabular form for every increment of $\Delta [{\rm O/H}]_{T_e} = +0.1$ from $[{\rm O/H}] = 8.34$ (Table 8). The entries give the distance modulus change $\Delta \mu$ as a function of period that must be applied to a distance derived from V, I pho-

tometry and based on the LMC *P-L* relations. The table can be read with opposite sign if distances from the Galactic *P-L* relations are to be corrected to lower metallicities.

For a few galaxies outside the range $8.2 < [{\rm O/H}]_{T_e} < 8.7$, the limiting values of 8.2 and 8.7, respectively, have been adopted by S06a.

It may seem paradoxical that metal-rich Cepheids with $\log P > 1.0$ (actually $\log P > 0.933$) yield larger distances than LMC Cepheids although the latter are brighter in V up to $\log P = 1.38$. The reason is that the V and I magnitudes are used to derive not only a true distance but also the reddening. The metal-poor Cepheids being blue yield large reddenings, leading to large absorption corrections and hence to small distances. The effect of metallicity changes on the distance of Cepheids is therefore a combination of their effect on the luminosities and on the inferred absorption corrections.

3.3. A Summary of Available Cepheid Distances

Metallicity-corrected Cepheid distances of 37 galaxies have been derived in S06a from the Galactic and LMC *P-L* relations as given in STR04. Six additional Cepheid distances have since become available.

NGC 55.—Pietrzynski et al. (2006a) have observed 143 Cepheids in V and I in NGC 55 with a metallicity of $[{\rm O/H}]_{T_e}=8.35$, i.e., close to LMC. Using the Udalski et al. (1999c) LMC P-L relation, they have obtained a modulus of $\mu^0=26.45\pm0.05$ if $\mu^0_{\rm LMC}$ is at 18.54. From the best 110 Cepheids and the LMC P-L relation of STR04 we obtain $\mu^0=26.42$ using $\langle E(V-I)\rangle=0.12$ and a small metallicity correction of 0.01 mag. If one applies the Galactic P-L relation instead, which stands for a metallicity of $[{\rm O/H}]_{T_e}=8.60$, one finds $\mu^0({\rm Gal})=26.56$ and a metallicity correction of -0.16 mag from equation (10) in S06a, resulting in a corrected modulus of $\mu^0=26.40$. We adopt 26.41 ± 0.05 for NGC 55.

M31 (*NGC 224*).—Vilardell et al. (2007) have observed hundreds of badly needed Cepheids in this galaxy, 281 of which the authors identify as fundamental pulsators. Unfortunately, the *B*, *V* photometry of these variables with the 2.5 m Isaac Newton Telescope is affected by blends. Vilardell et al. (2007) have found that Cepheids with large amplitudes, i.e., $A_V > 0.8$ mag, are least blemished by blends, and they have kindly provided to us the

TABLE 6
DISTANCE OF LMC FROM LITERATURE SINCE 2002

Author(s)	$(m-M)^0$	Method
Fitzpatrick et al. (2002)	18.50 ± 0.05	Eclipsing binary HV 982
Fouqué et al. (2003)	18.55 ± 0.04	BBW
Clausen et al. (2003)	18.63 ± 0.08	Eclipsing binaries
Clementini et al. (2003a)	18.52 ± 0.09	Review
Groenewegen & Salaris (2003)	18.58 ± 0.08	Main sequence of NGC 1866
Salaris et al. (2003)	18.47 ± 0.01	Red clump stars
Storm et al. (2004)	18.48 ± 0.07	BBW
Feast (2004)	18.48 ± 0.08	Mira variables
Dall'Ora et al. (2004)	18.52 ± 0.03	Semitheoretical
Sakai et al. (2004)	18.59 ± 0.09	TRGB
Alves (2004)	18.50 ± 0.02	Review
Panagia (2005)	18.56 ± 0.05	SN 1987A light echo
Gieren et al. (2005b)	18.53 ± 0.06	BBW
ST06	18.55 ± 0.10	RR Lyrae
Keller & Wood (2006)	18.54 ± 0.02	Bump Cepheids
Sollima et al. (2006)	18.54 ± 0.15	RR Lyrae in K band
Mean	18.53 ± 0.01	

63

TABLE 7 The Distance of SMC from Literature since 2004

Author(s)	$(m-M)^0$	Method
Sakai et al. (2004)	18.96 ± 0.10	TRGB
Storm et al. (2004)	18.88 ± 0.14	BBW
Hilditch et al. (2005)	18.91 ± 0.03	Eclipsing binary
ST06	18.96 ± 0.10	RR Lyrae
Keller & Wood (2006)	18.93 ± 0.02	Bump Cepheids
Mean	18.93 ± 0.02	

subset of the 64 such fundamental pulsators with $0.4 < \log P <$ 1.6. Their mean metallicity is $[O/H]_{T_o} = 8.66$ from their galactocentric distances and the metallicity gradients of Zaritsky et al. (1994). Since this is only slightly more than the adopted value of Galactic values (8.6), it is assumed that the M31 Cepheids follow the Galactic P-C relation. With this assumption individual reddenings E(B-V) were determined, which turn out to increase with period, the mean value being $\langle E(B-V) \rangle = 0.21$. The ensuing absorption-corrected P-L relations are virtually as steep $(-2.916 \pm 0.144 \text{ in } V)$ as in the Galaxy. Comparing these relations with the adopted Galactic P-L relations yields in B and V $\mu^{0} = 24.32 \pm 0.06$. Had we compared with LMC at $\mu^{0} = 18.54$, the modulus would have become 24.18, which is still to be increased by 0.11 mag for the metallicity difference to give $\mu^0 =$ 24.29 (see S06a, eq. [10]). However, the distances are still to be corrected for the amplitude restriction. The largest amplitudes occur in general on the blue side of the instability strip (STR04, Fig. 11). In the Galaxy the 123 Cepheids with $A_V > 0.8$ mag from Berdnikov et al. (2000) are bluer in (B - V) than the total of 321 Cepheids by 0.02 mag. If the same value holds for M31, the above reddenings were underestimated by the same amount and the absorption by 0.06 mag. The distance becomes then 24.26. On the other hand, blue Cepheids are intrinsically brighter than average because of the slope β of the constant-period lines. Yet, since β is quite flat ($\beta_{V,B-V} = 0.6$; STR04) in the Galaxy and presumably in M31, this effect increases the distance by only 0.01, which becomes then $\mu^0 = 24.27$ for M31. The Cepheid distance of M31 is significantly smaller than from RR Lyrae stars (24.53) and the TRGB (24.47). This may be due to remaining blend effects or to an overestimate of the reddening, if the metal-rich M31 Cepheids are intrinsically redder than Galactic Cepheids.

NGC 4258.—Macri et al. (2006) have observed Cepheids in B, V, and I in an outer field of NGC 4258. They have the same metallicity as LMC ($[O/H]_T = 8.36$) according to the metallicity gradient of Zaritsky et al. (1994). The 36 best Cepheids in the field yield $\mu^0 = 29.50 \pm 0.03$ using the LMC P-L relation of STR04 and $\langle E(B-V)\rangle = 0.042$. The Galactic P-L relation yields, after a proper metallicity correction, the same value. The P-L relation in B shows possibly a break at P = 10 days, but even if real this has no effect on the distance determination. For the Cepheids in the inner, metal-rich field of NGC 4258 see § 3.4.4.

NGC 5128 (Cen A).—Forty-five heavily absorbed Cepheids with V and I magnitudes from Ferrarese et al. (2007) in the highly peculiar galaxy NGC 5128 yield $\langle E(V-I) \rangle = 0.50, \mu^0 =$ 27.62 ± 0.04 and $\langle E(V-I) \rangle = 0.42$, $\mu^0 = 27.71 \pm 0.04$, respectively, using the P-L relations of LMC and the Galaxy. Since the metallicity of the Cepheids is unknown, a mean of μ^0 27.67 ± 0.04 is adopted. Following Ferrarese et al. (2007), an absorption-to-reddening factor of $R_V = 2.4$ has been used as measured for NGC 5128 by Hough et al. (1987). Had we assumed the standard absorption factor of $R_V = 3.23$, the mean distance would have become $\mu^0 = 27.54$, which is hardly compatible with

TABLE 8 DISTANCE MODULUS CORRECTIONS TO BE APPLIED TO DISTANCES DERIVED FROM THE LMC P-L Relations in V and I for Any Increase of the Metallicity by $\Delta {\rm [O/H]}=0.1$ from ${\rm [O/H]}_{T_e}^{\rm LMC}=8.34$

$\log P$	$\Delta \mu$
0.50	-0.07
0.75	-0.03
1.00	+0.01
1.25	+0.05
1.50	+0.09

the TRGB distance 27.82 (Karachentsev et al. 2004) or 27.72 (Rizzi et al. 2007).

Two more galaxies with known Cepheids have quite low metallicities, i.e., NGC 3109 and IC 1613 with $[O/H]_{T_0} = 8.06$ and 7.86, respectively, from Sakai et al. (2004), which are close to SMC ($[O/H]_T = 7.98$). In order not to overextrapolate the metallicity corrections of S06a, the two galaxies are tied to the P-L relation of SMC without further metallicity corrections.

NGC 3109.—One hundred two Cepheids from Pietrzynski et al. (2006b) define, after 2 σ clipping, P-L relations with a slope that is even flatter than observed in SMC (Table 5), but with the large scatter of $\sigma = 0.39$. They indicate, if compared with SMC, an internal reddening of $E(V-I) = -0.01 \pm 0.01$, which we take as zero, and a distance modulus of $\mu^0 = 25.41 \pm 0.04$. If the sample is cut at log P = 0.75 to guard against the shortest period Cepheids being possibly overluminous in the mean (Sandage 1988), one obtains $\mu^0 = 25.45 \pm 0.04$, which we adopt. The small reddening may suggest that the Cepheids are even bluer than those of SMC. If the restricted sample of Cepheids had been reduced with the P-L relation of LMC, one would have obtained 25.57. Soszynski et al. (2006) have derived $\mu^0 = 25.61$ (if $\mu_{LMC}^0 = 25.61$) 18.54) from additional magnitudes in J and K and by comparing with LMC. Earlier work on the Cepheids in NGC 3109 is cited by Pietrzynski et al. (2006b).

IC 1613.—Forty-two Cepheids from Antonello et al. (2006) fill exceedingly well the strip in the two-color diagram (B - V)versus (V - I) defined by SMC Cepheids. Six additional Cepheids lie clearly outside that strip and are omitted. The 42 Cepheids are bluer on average by only -0.01 ± 0.01 mag than SMC Cepheids, which we interpret as zero reddening. They define *P-L* relations in B, V, and I with no indication of a break and with slopes that are the same within the errors as the overall slopes in SMC. A comparison of the two sets of Cepheids yields a distance modulus of $\mu^0 = 24.32 \pm 0.02$, somewhat less than the value of 24.50 ± 0.12 from Antonello et al. (2006), who compared with

WLM may tentatively be compared with the *P-L* relation of SMC, although it is very metal deficient ($[O/H]_{T_a} = 7.74$; Sakai et al. 2004). Pietrzynski et al. (2007) have observed 60 Cepheids, of which 3 can be excluded as bright outliers and 1 for lack of complete data. The remaining 56 Cepheids are quite blue and give, if compared with SMC, $E(V-I) = -0.03 \pm 05$, which we take as zero. They define P-L relations ($\sigma = 0.38$ mag) that are even flatter (by 1 σ) than in SMC and significantly flatter than the single-fit P-L relations of LMC (STR04, eqs. [8] and [9]). No bias toward bright Cepheids (Sandage 1988) is seen at short periods. Tied to the V and IP-L relations of SMC, the Cepheids give $\mu^0 = 24.80$ and 24.83, respectively. These values are noticeably smaller than the Pietrzynski et al. (2007) value of 25.18 (if LMC is at 18.54), but the adopted value of $\mu^0 = 24.82$ here

TABLE 9 Comparison of the S06a Cepheid Distances^a with TRGB Distances

		μ_{TRGB}^{0}			Metallicity-corrected	1	
Name	μ_{Cep}^0	$M_I = -4.05$	Source ^b	Δ	μ_{TRGB}^{0}	Δ	Source ^c
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
NGC 55	26.41	26.64	1, 2	-0.23			
NGC 224	24.54	24.46	3, 4	+0.08	24.37	+0.17	12
NGC 300	26.48	26.56	4	-0.08	26.48	+0.00	12
NGC 598	24.64	24.66	3, 4	-0.02	24.71	-0.07	12
NGC 2403	27.43	(27.62)	5	(-0.19)			
NGC 3031	27.80	27.80	4	+0.00	27.70	+0.10	12
NGC 3109	25.45	25.54	4	-0.09	25.57	-0.12	12
NGC 3351	30.10	30.23	4, 6	-0.13	29.92	+0.18	12
NGC 3621	29.30	29.27	4	+0.03	29.26	+0.04	12
NGC 4258	29.50	29.32	7, 8	+0.18	29.37	+0.13	13, 14
NGC 5128	27.67	27.89	9, 10	-0.22	27.90	-0.23	15
NGC 5236	28.32	28.56	11	-0.24			
NGC 5253	28.05	27.89	6	+0.16		•••	
NGC 5457	29.17	29.39	4, 6	-0.22	29.34	-0.17	12
NGC 6822	23.31	23.37	6	-0.09	23.37	-0.06	16
IC 1613	24.32	24.33	4	-0.01	24.38	-0.06	12
IC 4182	28.21	28.19	4	+0.02	28.23	-0.02	12
WLM	24.82	24.87	3, 4	-0.05	24.93	-0.11	12
Mean				$-0.05 \pm 0.03 \; (\sigma = 0.13, N = 17)$		$-0.02 \pm 0.03 \; (\sigma = 0.13, N = 14)$	

Notes.—Some additional distances: NGC 224: $\mu^0 = 24.44 \pm 0.12$ from an eclipsing binary (Ribas et al. 2005); NGC 300: $\mu^0 = 26.41$ from VIJK photometry of Cepheids and assuming $\mu_{\rm LMC}^0 = 18.54$ (Gieren et al. 2005a); NGC 598: $\mu^0 = 24.92 \pm 0.12$ from an eclipsing binary (Bonanos et al. 2006); NGC 4258: $\mu^0 = 29.29 \pm 0.08 \pm 0.07$ from water maser (Herrnstein et al. 1999); NGC 6822: $\mu^0 = 23.35$ from *VIJK* photometry of Cepheids and assuming $\mu_{\rm LMC}^0 = 18.54$ (Gieren et al. 2006).

c References to metallicity-corrected μ_{TRGB}^0 . References.—(1) Seth et al. 2005; (2) Tully et al. 2006; (3) McConnachie et al. 2005; (4) Rizzi et al. 2007; (5) mean distance of six group members; (6) Sakai et al. 2007; (5) mean distance of six group members; (6) Sakai et al. 2007; (6) Rizzi et al. 2007; (7) McConnachie et al. 2008; (8) Rizzi et al. 2007; (9) mean distance of six group members; (10) Sakai et al. 2008; (10) McConnachie et al. 2008; (10) McConnachie et al. 2009; 2004; (7) Mouheine et al. 2005; (8) Macri et al. 2006; (9) Rejkuba et al. 2005; (10) Karataeva et al. 2006; (11) Karachentsev et al. 2007; (12) Rizzi et al. 2007; (13) Macri et al. 2006; (14) Mouhcine et al. 2005; (15) Ferrarese et al. 2007; (16) Sakai et al. 2004.

compares well with the TRGB distance of the galaxy (24.90; Table 9), the fit of the entire CMD (24.88 \pm 0.09; Dolphin 2000), and the position of the HB (24.95 \pm 0.15; Rejkuba et al. 2000).

3.4. Are the Metallicity Corrections to Cepheid Distances Reliable?

Any systematic errors of the adopted metallicity corrections must show by comparing the Cepheid distances with independent distance indicators. The test is independent of zeropoint differences because we seek only the *slope* of the function $\Delta \mu^0 = f([O/H]).$

3.4.1. Comparison of Cepheid Distances with TRGB Distances

Cepheid distances, as well as TRGB distances, are available for 18 galaxies. The low- and high-metallicity Cepheids in the outer and inner field of NGC 5457 are counted twice (Table 9). In the case of NGC 4258 only the Cepheids in the outer field are plotted for reasons given in § 3.3. The differences of the respective distances are plotted against the metallicity $[O/H]_{T_0}$ of the Cepheids in Figure 5. The absence of any significant metallicity dependence on $[O/H]_{T_e}$ is striking.

3.4.2. Comparison with SN Ia Magnitudes

Metallicity-corrected Cepheid distances of 10 galaxies were used in S06b to calibrate the maximum absolute magnitude of their SNe Ia. Any remaining errors of the metallicity correction will become apparent by an unrealistic dependence of the SN Ia luminosities on the Cepheid metallicity $[O/H]_{T_o}$. The test is performed in Figure 6. The formal dependence is statistically insignificant.

3.4.3. Comparison of Cepheid Distances with Velocity Distances

The differences between the Cepheid distances and velocity distances of their parent galaxies are not supposed to be a function of the Cepheid metallicity $[O/H]_{T_o}$. The test is difficult because the galaxies with Cepheid distances are nearby and have small recession velocities that are substantially influenced by peculiar velocities. Galaxies with $\mu^0 < 28.2$ are therefore omitted, as are cluster galaxies and galaxies within 25° from the Virgo Cluster center. The distance differences of the remaining 17 galaxies from Table 8 in S06a and from § 3.3 are plotted against $[O/H]_{T_a}$ in

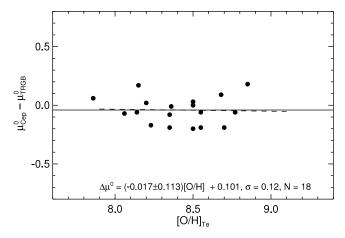


Fig. 5.—Difference $\Delta \mu^0 = \mu_{\text{Cep}}^0 - \mu_{\text{TRGB}}^0$ as a function of $[\text{O/H}]_{T_e}$ for the Cepheids.

Added here are NGC 55, IC 1613, and WLM from § 3.3.

References to μ_{TRGB}^{0}

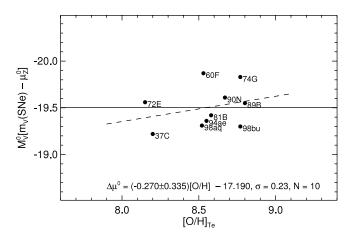


Fig. 6.—Luminosity of SNe Ia as a function of the metallicity of the Cepheids that led to the distance of the parent galaxy.

Figure 7. As in \S 3.4.1, the outer and inner field Cepheids of NGC 5457 are plotted separately, and only those of the outer field of NGC 4258 are considered. A least-squares fit to the data results in some dependence on $[O/H]_{T_e}$ (dashed line), but the statistical error is even larger.

If the evidence of Figures 5–7 is combined, the remaining metal-dependent error of the Cepheid distances amounts to only $\Delta\mu=(0.05\pm0.10)\Delta [{\rm O/H}]_{T_e}.$ Since $\Delta [{\rm O/H}]$ is on the order of unity, the relative distance error between the most metal-poor and most metal-rich galaxies may in fact be zero and, in any case, is likely to be <0.1 mag. This speaks in favor of the present metallicity corrections.

If all Cepheid distances entering Figures 5–7 had been based on the P-L relation of LMC and if no metallicity corrections had been applied, one would have found $\Delta\mu=(0.53\pm0.17)\Delta[\text{O/H}]_{T_e}$. This demonstrates the necessity of metallicity corrections. (It may be noted that the $[\text{O/H}]_{T_e}$ scale of Kennicutt et al. [2003] and Sakai et al. [2004] is compressed by a factor of \sim 1.5 as compared to the old [O/H] scale, which has been widely used in the present context. Therefore, the above relation translates into $\Delta\mu\sim0.35\Delta[\text{O/H}]_{\text{old}}$.)

3.4.4. Comparison with the Cepheids in the Inner Field of NGC 4258

From the water masers moving on Keplerian orbits about the center of NGC 4258 Herrnstein et al. (1999, 2005) have derived a modulus of $29.29 \pm 0.08 \pm 0.07$. Both the Cepheid distance (29.50; see § 3.3) and TRGB distance (29.44; see § 3.3) are larger, but a mean of $\mu^0 = 29.38$ agrees within ≤ 0.12 mag with all three determinations. Cepheids have also been observed in an inner field of NGC 4258 by Macri et al. (2006). Their position in the galaxy with its chemical gradient (Zaritsky et al. 1994) suggests that these Cepheids are as metal-rich as Galactic Cepheids on average. The Cepheids are therefore expected to share the *P-L* relation of the Galaxy, not the LMC. The 81 Cepheids, complying with the typical position in the (B - V) versus (V - I) diagram of classical Cepheids, cover a wide period interval of 0.6 < $\log P < 1.7$. Their absorption-corrected P-L relation can be derived by adopting the above distance of NGC 4258 using an appropriate P-C relation. Macri et al. (2006) have adopted the blue P-C relation of LMC, which, however, is incorrect for the metalrich and necessarily redder Cepheids. If one assumes that these Cepheids follow the same P-C relation as in the Galaxy, one obtains color excesses E(B-V) that are nearly independent of period. This lends support to the assumption that the P-C relations of the inner field and of the Galaxy have the same slope.

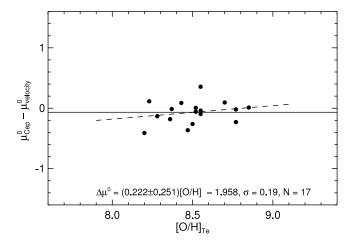


Fig. 7.—Difference of $\Delta \mu = \mu_{\text{Cep}}^0 - \mu_{\text{velocity}}^0$.

The resulting P-L relations in B, V, and I are very flat; in fact, in spite of the high metallicity they are as flat as in LMC (not considering the break at P=10 days) and as flat as in the outer field. The observation that the inner field Cepheids agree with the LMC Cepheids to within 0.1 mag at all periods depends on the additional assumption that the P-C relations of the inner field and of the Galaxy have not only the same slope but also the same zero point.

This is not to suggest that the above combination of a Galactic *P-C* relation (for high-metallicity Cepheids) and an LMC *P-L* relation (for low-metallicity Cepheids) could give a consistent solution. Rather, it is likely that the flat slope of the metal-rich Cepheids in the inner field of NGC 4258, in contrast to Figure 4, is caused by a second parameter other than [O/H], possibly by *Y* as mentioned before.

It is fortunate that the P-L relation of the inner field, as derived here, crosses the Galactic P-L relation at $\log P \sim 1.5$, which happens to be the median period of the known Cepheids in most galaxies outside the Local Group. It makes therefore little difference for the derived distances which of the two P-L relations applies to a given set of high-metallicity Cepheids.

4. COMPARISON OF THE ZERO POINTS OF THE POPULATION I AND POPULATION II DISTANCES

The Cepheid distances of 18 galaxies introduced in § 3.3 can be compared with their corresponding TRGB distances (Table 9). The comparison is equivalent to a comparison of Cepheid with RR Lyrae star distances because the TRGB distances are so tightly linked with the RR Lyrae stars through Table 2. On average the Cepheid distances are smaller than the TRGB distances by only 0.04 ± 0.03 . If instead the 13 galaxies are compared for which metal-corrected TRGB distances are given in the literature, the difference $\mu_{\rm Ceph}^0 - \mu_{\rm TRGB}^0$ becomes -0.02 ± 0.03 . This agreement of the Population I and Population II distances of nearby galaxies is as good as can possibly be expected.

Rizzi et al. (2007) have compared the TRGB distances of 15 galaxies with their Cepheid distances, but the latter are derived from the old P-L relation of Madore & Freedman (1991) without corrections for metallicity. Rizzi et al. (2007) find $\langle \mu_{\text{Ceph}}^0 - \mu_{\text{TRGB}}^0 \rangle = 0.01 \pm 0.03$. The good agreement is no surprise because it was stated already in \S 1 that several old Cepheid distance scales, prior to the one adopted by Freedman et al. (2001), agree on average well with those adopted here.

Only 10 galaxies used for the comparison by Rizzi et al. (2007) are also contained in Table 9. LMC and SMC are omitted because

they are used here to calibrate the *P-L* relation of their specific metallicity. We do not have a reliable template *P-L* relation for the most metal-poor galaxies of Rizzi et al. (2007) (Sextans A/B).

5. THE LOCAL AND NOT SO LOCAL HUBBLE DIAGRAMS

The TRGB and Cepheid distances, as well as the Cepheidcalibrated TF and SN Ia distances, can be used to construct distance-calibrated Hubble diagrams that reach progressively deeper into the cosmic expansion field.

The various distances are transformed to the barycenter of the Local Group, which is assumed to lie at the distance of 0.53 Mpc in the direction of M31, i.e., at two-thirds of the way to this galaxy, because the galaxies outside the Local Group expand presumably away from the barycenter and not away from the observer. The barycentric distances are designated with r^{00} and μ^{00} , respectively.

The heliocentric velocities are corrected to the barycenter of the Local Group following Yahil et al. (1977) and for a self-consistent Virgocentric infall model assuming a local infall vector of 220 km s^{-1} and a density profile of the Virgo complex of r^{-2} (Yahil et al. 1980; Dressler 1984; Kraan-Korteweg 1986; de Freitas Pacheco 1986; Giraud 1990; Jerjen & Tammann 1993). The choice of these particular corrections among others proposed in the literature is justified because they give the smallest scatter in the Hubble diagrams (see S06b). The velocities are not corrected for the projection angle between the observer and the Local Group barycenter as seen from the galaxy because it affects the velocities by less than 2% for all galaxies beyond 3 Mpc.

5.1. The Hubble Diagram from TRGB

The galaxies outside the Local Group, for which TRGB distances are available (§ 2.2.4), are plotted in a Hubble diagram (Fig. 9a). The nearest galaxies reflect clearly the effect of the gravitational pull of the Local Group, suggesting that the zero-velocity surface lies at a distance of ≤ 1.6 Mpc from the barycenter of the Local Group. The 59 galaxies with $\mu_{\rm TRGB}^0 > 28.2$ (4.4 Mpc) define a value of $H_0 = 61.7 \pm 1.5$ km s⁻¹ Mpc⁻¹. This is a very local value extending to only $\mu^0 = 30.4$ (12 Mpc). The scatter about the Hubble line of $\sigma_m = 0.39$ mag cannot be caused by distance errors, but is explained by (one-dimensional) peculiar motions of ~ 90 km s⁻¹ on average (S06b).

5.2. The Hubble Diagram from Cepheids

There are 34 galaxies in S06b outside the Local Group whose Cepheid distances have been derived following the precepts given in §§ 3.1 and 3.2. Three additional Cepheid distances are given in § 3.3. The velocities of the total of 37 galaxies are plotted against v_{220} in Figure 9b. The 30 galaxies with $\mu_{\text{Ceph}}^0 > 28.2$, and excluding the deviating case of NGC 3627, define a Hubble line with $H_0 = 63.1 \pm 1.8$ out to $\mu_{\text{Ceph}}^0 \sim 32.0$ (25 Mpc). As in the case of the TRGB distances, the scatter of $\sigma_m = 0.33$ must be caused mainly by peculiar velocities on the order of 150 km s⁻¹ at a median velocity of about 1000 km s⁻¹ (assuming a random error of the Cepheid distances of 0.15 mag).

The agreement of the local value of H_0 from TRGB magnitudes and from Cepheids to within 2% suggests that the zero-point errors of the two independent methods do not accumulate to more than 0.04 mag.

5.3. The Hubble Diagram from TF

A complete sample of 104 inclined spiral galaxies with v_{220} < 1000 km s⁻¹ with known 21 cm line widths was discussed in

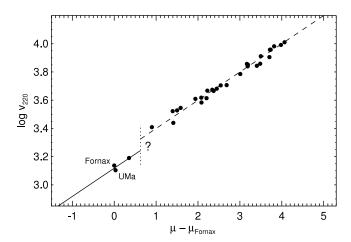


Fig. 8.—Hubble diagram of 31 clusters with relative 21 cm line width distances from Masters et al. (2006). The zero point is arbitrarily set at the Fornax Cluster. Note the spurious break of the Hubble line (see text).

S06b. This small distance limit was chosen to define as complete a distance-limited sample as possible. The zero point of the TF distances was calibrated with 31 galaxies for which also Cepheid distances are available from S06a. The Hubble diagram of the sample galaxies is repeated in Figure 9c from S06b, but added is the mean TF distance of $\mu^{00}=31.61$ of a complete sample of 49 Virgo Cluster spirals plotted at the mean cluster velocity of $\langle v_{220}\rangle=1152~{\rm km~s^{-1}}$. Also added is the UMa cluster with $\mu^{00}=31.45~{\rm and}~\langle v_{220}\rangle=1270~{\rm km~s^{-1}}$. The TF distance of UMa is taken from Tully & Pierce (2000), who obtained $\mu^0=31.35\pm0.06$ from 38 cluster members with B, R, I, and K' photometry. After recalibrating their 24 calibrators with the present Cepheid distances (S06a), one obtains $\mu^0=\mu^{00}=31.45$. The value is adopted here, although the UMa sample may not be strictly complete as to the faintest cluster spirals.

The TF distances in Figure 9c give a Hubble constant of $H_0 = 59.0 \pm 1.9$ out to ~ 16 Mpc. This agrees well with H_0 from TRGB distances (Fig. 9a), but it is 1.6σ less than determined in Figure 9b from Cepheids. This difference, however, cannot be real because the TF distances depend entirely on the calibration through Cepheids. Rather, it reflects on the reliability of the TF method. In any case the scatter in Figure 9c is very large (0.69 mag). This cannot be attributed to peculiar motions, which contribute only $\sigma_m \sim 0.3$ mag in Figures 9a and 9b. Even if some of the scatter is caused by observational errors of the input parameters, the intrinsic dispersion is large. This makes the TF method vulnerable to Malmquist bias if magnitude-limited samples are used instead of complete distance-limited samples. This is the reason why the more distant clusters of Tully & Pierce (2000), which are expected to suffer at least some magnitude bias, are not considered here.

Masters et al. (2006) have measured I-band TF distances for an average of 25 galaxies in 31 clusters between 1100 km s⁻¹ < $v_{\rm CMB} \lesssim 10,000$ km s⁻¹. The clusters define an impressively tight Hubble diagram with a scatter of $\sigma_m \approx 0.15$ mag, comparable only to distant SNe Ia (R05). However, the diagram has no zero-point calibration and does not per se define a value of H_0 . The authors propose to calibrate their TF relation by local galaxies with Cepheid distances. Yet, however fair their selection criteria may be for the galaxies in the different clusters, the same criteria cannot be applied to a distance-limited yet highly incomplete sample of field galaxies with Cepheid distances. This would be decisive in view of the large intrinsic scatter of the TF relation. Therefore, any value of H_0 derived from the two sets of differently selected galaxies remains unreliable. A safer way would be to calibrate

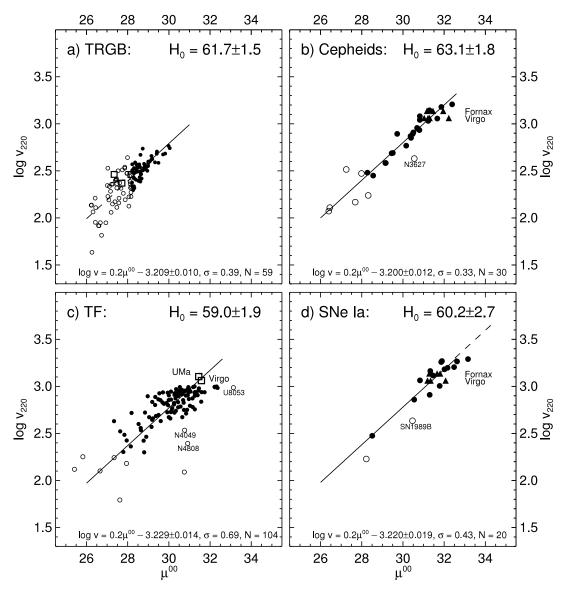


Fig. 9.—Distance-calibrated Hubble diagrams for (a) TRGB distances (the M81, Cen A, and IC 342 groups are shown as squares at their mean position), (b) Cepheid distances (the Virgo Cluster and Fornax Cluster members are plotted at $v_{220}=1152$ and $1371~\rm km~s^{-1}$, respectively), (c) 21 cm line width distances of a complete sample of field galaxies with $v_{220}<1000~\rm km~s^{-1}$ (the Virgo Cluster and the UMa Cluster [at $v_{220}=1236~\rm km~s^{-1}$] are also shown), and (d) SN Ia distances with $v_{220}<2000~\rm km~s^{-1}$ (the dashed line is the downward extension of the Hubble line defined by 62 SNe Ia with 3000 km s⁻¹ < $v_{\rm CMB}<20,000~\rm km~s^{-1}$ and reflecting the large-scale value of H_0 [from S06b]). Triangles denote cluster members. Open symbols are objects with $\mu^0<28.2~\rm cm~s^{-1}$ and a few deviating objects (identified); open symbols are not considered for the solution.

the Hubble diagram with the nearest two clusters of their sample with independently known distances, i.e., the Fornax Cluster and UMa Cluster. However, as seen in Figure 8, their relative distances do not put the nearest clusters with $v_{220} < 2000 \text{ km s}^{-1}$ on the same Hubble line as the more distant clusters. The latter are shifted by $\Delta \log v = 0.08$ or $\Delta \mu = 0.40$, as compared to the nearest clusters. The corresponding increase of H_0 by $\sim 20\%$ at ~ 2000 km s⁻¹ is denied by the Hubble diagram of SNe Ia (see, e.g., Fig. 9d; R05, Fig. 15). The break of the Hubble line suggests that the selection criteria for the individual galaxies in the near and distant clusters resulted in two incompatible cluster samples. The unrealistic break of the Hubble line is not caused by corrections of the velocities for the cosmic microwave background (CMB) motion. It persists whether the nearest clusters are corrected for CMB motion or not. In Figure 8 the nearest clusters are *not* corrected for this motion because the comoving volume extends to at least 3000 km s⁻¹ (Federspiel et al. 1994, Figs. 17-19; Dale & Giovanelli 2000).

5.4. The Hubble Diagram from SN Ia Distances

Figure 9d shows the Hubble diagram of local SNe Ia with $v_{220} < 2000 \text{ km s}^{-1}$. The SNe Ia are drawn from the homogeneously reduced list of SN Ia magnitudes (R05). Their mean Cepheid-calibrated absolute magnitude is adopted as $M_V^0(\text{max}) = -19.46$ (S06b). Of the 22 SNe Ia, 1 has $\mu^0 < 28.2$, and SN 1989B in NGC 3627 is an outlier. Three SNe Ia each in the Virgo Cluster and Fornax Cluster are plotted at their mean cluster velocity. The 20 adopted SNe Ia give $H_0 = 60.2 \pm 2.7$ with a scatter of $\sigma_m = 0.43$. Both values are statistically the same as those derived from Cepheids in Figure 9b. The statistical agreement in H_0 must be expected because the zero point of the SNe Ia depends entirely on the Cepheids, but the SNe Ia extend the Hubble diagram to 30 Mpc and beyond (see below). The similar scatter of the SNe Ia and Cepheids in their respective Hubble diagrams suggests that they are equally good distance indicators.

The weighted mean of H_0 from TRGB distances, Cepheids, TF distances, and SNe Ia is 61.3 ± 1.0 . There is no hint that the mean value of H_0 varies significantly from about 4 to 30 Mpc. Clear deviations from a steady Hubble flow are detected only from the pull of the Local Group and from the Virgocentric flow. Other deviations near local mass concentrations are expected to exist (e.g., Klypin et al. 2003, their Figs. 5–7), but the present method considering relatively few galaxies is not suitable to detect them. The distance independence of the *mean* value of H_0 , however, is the more significant as the distant SNe Ia with 3000 km s⁻¹ < $v_{\rm CMB} < 20,000$ km s⁻¹ yield the same value of H_0 does not depend on distance.

6. CONCLUSIONS AND OUTLOOK

The local agreement of the Population I and Population II distance scales is encouraging. Population I Cepheids, as well as SNe Ia and the TF relation, both calibrated through Cepheids, on the one hand and Population II RR Lyrae stars and the magnitude of the TRGB, based on these stars, on the other yield highly consistent distances for many individual galaxies. Moreover, they all agree with a local value of the Hubble constant of $H_0 = 62.3$. Finally, the SNe Ia carry the distance scale into the cosmic expansion field out to $\sim 20,000 \, \mathrm{km \, s^{-1}}$ and prove that H_0 is virtually unchanged in the free field.

The Population II distance scale alone leads through RR Lyrae stars and the TRGB to a minimum distance of the Virgo Cluster of $\mu^0 \geq 31.3$. If one wants to drive the Population II distances to cosmic scales, one may note that the four SNe Ia discussed in § 2.2.5 give a preliminary mean TRGB-calibrated luminosity of $M_V = -19.37 \pm 0.06$. Yet the value of $M_V = -19.46 \pm 0.04$

from 10 Cepheid-calibrated SNe Ia (S06b) has still much higher weight. Nevertheless, the agreement is encouraging at this stage. Other TRGB-calibrated SNe Ia will become available in the future.

Even if the agreement of H_0 from independent distances of Population I and II objects is accidental, it is now unlikely that the systematic error of H_0 is as large as 10%. If the systematic error is in fact as high as 0.2 mag, or 10% in distance, this translates to ± 5 units in H_0 , as stated in the Abstract.

If the value of $H_0=62$ is taken at face value and combined with WMAP data of the CMB fluctuation spectrum, it poses constraints on the equation of state $w=p/\rho$ of the dark energy. According to Spergel et al. (2007), the WMAP3 data give $\Omega_m h^2=0.128\pm0.008$ ($h\equiv H_0/100$), from which follows then a rather high matter density parameter of $\Omega_m=0.33$. This value disfavors a universe with w=-1 at the 2 σ level (see Figs. 15 and 16 of Spergel et al. 2007) and suggests a quintessence model with w>-1. The high matter density Ω_m is not favored, however, by the large-scale distribution of the luminous red galaxies in the Sloan Digital Sky Survey if it is combined with the WMAP3 data. In that case a closed universe with $\Omega_{\rm total}\sim 1.02$ is compatible with $H_0=62$ (Tegmark et al. 2006, Fig. 13). This illustrates that a reliable value of H_0 imposes stringent constraints on any cosmological models.

We thank Abhijit Saha and Norbert Straumann for helpful discussions. Francesc Vilardell has kindly made available the selected sample of Cepheids in M31. A. S. thanks the Observatories of the Carnegie Institution for postretirement facilities. We thankfully acknowledge the helpful comments of an anonymous referee.

REFERENCES

— 1954, in Trans. IAU VIII (Rome 1952, meeting), Report of Commission 28 (Cambridge: Cambridge Univ. Press), 397

Barnes, T., Jeffreys, W., Berger, J., Mueller, P., Orr, K., & Rodriguez, R. 2003, ApJ, 592, 539

Bauer, F., et al. 1999, A&A, 348, 175

Behr, A. 1951, Astron. Nachr., 279, 97

Bellazzini, M., Ferraro, F. R., Origlia, L., Pancino, E., Monaco, L., & Oliva, E. 2002, AJ, 124, 3222

Bellazzini, M., Ferraro, F. R., & Pancino, E. 2001, ApJ, 556, 635

Bellazzini, M., Ferraro, F. R., Sollima, A., Pancino, E., & Origlia, L. 2004a, A&A, 424, 199

Bellazzini, M., Gennari, N., Ferraro, F. R., & Sollima, A. 2004b, MNRAS, 354, 708

Bencivenni, D., Caputo, F., Manteign, M., & Quarta, M. L. 1991, ApJ, 380, 484 Benedict, G. F., et al. 2002, AJ, 124, 1695

——. 2007, AJ, 133, 1810

Berdnikov, L. N., Dambis, A. K., & Voziakova, O. V. 2000, A&AS, 143, 211 Bergbusch, P. A., & VandenBerg, D. A. 2001, ApJ, 556, 322

Bersier, D. 2000, ApJ, 543, L23

Bersier, D., & Wood, P. R. 2002, AJ, 123, 840

Bonanos, A. Z., Stanek, K. Z., Szentgyorgyi, A. H., Sasselov, D. D., & Bakos, G. A. 2004, AJ, 127, 861

Bonanos, A. Z., et al. 2006, ApJ, 652, 313

Bono, G., Caputo, F., & di Criscienzo, M. 2007, A&A, 476, 779

Borissova, J., Minniti, D., Rejkuba, M., Alves, D., Cook, K. H., & Freeman, K. C. 2004, A&A, 423, 97

Brown, T. M., Ferguson, H. C., Smith, E., Kimble, R. A., Sweigart, A. V., Renzini, A., & Rich, R. M. 2004, AJ, 127, 2738

Caldwell, N. 2006, ApJ, 651, 822

Caloi, V., D'Antona, F., & Mazzitelli, I. 1997, A&A, 320, 823

Caputo, F. 1997, MNRAS, 284, 994

Caputo, F., Castellani, V., Marconi, M., & Ripepi, V. 2000, MNRAS, 316, 819Caputo, F., de Rinaldis, A., Manteiga, M., Pulone, L., & Quarta, M. L. 1993, A&A, 276, 41

Carrera, R., Aparicio, A., Martínez-Delgado, D., & Alonso-García, J. 2002, AJ, 123, 3199

Carretta, E., & Gratton, R. G. 1997, A&AS, 121, 95

Carretta, E., Gratton, R. G., Clementini, G., & Fusi Peci, F. 2000, ApJ, 533, 215Cassisi, S., Castellani, V., degl'Innocenti, S., Salaris, M., & Weiss, A. 1999, A&AS, 134, 103

Castellani, V., Chieffi, S., & Pulone, L. 1991, ApJS, 76, 911

Catelan, M., Pritzl, B. J., & Smith, H. A. 2004, ApJS, 154, 633

Cioni, M.-R. L., van der Marel, R. P., Loup, C., & Habing, H. J. 2000, A&A, 359, 601

Clausen, J. V., Storm, J., Larsen, S. S., & Giménez, A. 2003, A&A, 402, 509Clementini, G., Gratton, R., Bragaglia, A., Carretta, E., Di Fabrizio, L., & Maio, M. 2003a, AJ, 125, 1309

Clementini, G., Held, E. V., Baldacci, L., & Rizzi, L. 2003b, ApJ, 588, L85

Da Costa, G. S., & Armandroff, T. E. 1990, AJ, 100, 162

Dale, D. A., & Giovanelli, R. 2000, in ASP Conf. Ser. 201, Cosmic Flows Workshop, ed. S. Courteau & J. Willick (San Francisco: ASP), 25

Dall'Ora, M., et al. 2004, ApJ, 610, 269

de Freitas Pacheco, J. A. 1986, Rev. Mex. AA, 12, 74

Demarque, P., Zinn, R., Lee, Y. W., & Yi, S. 2000, AJ, 119, 1398

Demers, S., & Irwin, M. J. 1993, MNRAS, 261, 657

De Santis, R., & Cassisi, S. 1999, MNRAS, 308, 97

Dolphin, A. E. 2000, ApJ, 531, 804

Dolphin, A. E., et al. 2001, ApJ, 550, 554

_____. 2002, AJ, 123, 3154

——. 2003, AJ, 125, 1261

Dorman, B. 1992, ApJS, 81, 221 Dressler, A. 1984, ApJ, 281, 512

Dressier, A. 1964, ApJ, 261, 512

Durrell, P. R., Ciardullo, R., Feldmeier, J. J., Jacoby, G. H., & Sigurdsson, S. 2002, ApJ, 570, 119

Durrell, P. R., et al. 2007, ApJ, 656, 746

```
Feast, M. W. 1999, PASP, 111, 775
        2004, in IAU Colloq. 193, Variable Stars in the Local Group, ed. D. W.
  Kurtz & K. R. Pollard (ASP Conf. Ser. 310; San Francisco: ASP), 304
Federspiel, M., Sandage, A., & Tammann, G. A. 1994, ApJ, 430, 29
Fernie, J. D. 1990, ApJS, 72, 153
Fernie, J. D., Beattie, B., Evans, N. R., & Seager, S. 1995, Inf. Bull. Variable Stars,
  4148, 1, http://www.astro.utoronto.ca/DDO/research/cepheids/cepheids.html
Ferrarese, L., Mould, J. R., Stetson, P. B., Tonry, J. L., Blakeslee, J. P., & Ajhar,
  E. A. 2007, ApJ, 654, 186
Ferrarese, L., et al. 1996, ApJ, 464, 568
        2000a, ApJS, 128, 431
        2000b, ApJ, 529, 745
```

Ferraro, F. R., Messineo, M., Fusi Pecci, F., de Palo, M. A., Stranieero, O., Chieffi, A., & Limongi, M. 1999, AJ, 118, 1738

Fitzpatrick, E. L., Ribas, I., Guinan, E. F., De Warf, L. E., Maloney, F. P., & Massa, D. 2002, ApJ, 564, 260

Fouqué, P., Storm, J., & Gieren, W. 2003, in Stellar Candles for the Extragalactic Distance Scale, ed. D. Alloin & W. Gieren (Berlin: Springer), 21

Fouqué, P., et al. 2007, A&A, 476, 73

Freedman, W. L., et al. 2001, ApJ, 553, 47

Fusi Pecci, F., et al. 1996, AJ, 112, 1461

Gallart, C., Aparicio, A., Freedman, W. L., Madore, B. F., Martínez-Delgado, D., & Stetson, P. B. 2004, AJ, 127, 1486

Gallart, C., Aparicio, A., & Vilchez, J. M. 1996, AJ, 112, 1928

Galleti, S., Bellazzini, M., & Ferraro, F. R. 2004, A&A, 423, 925

Gascoigne, S. C. B., & Kron, G. E. 1965, MNRAS, 130, 333

Gibson, B. K., et al. 2000, ApJ, 529, 723

Gieren, W., Pietrzynski, G., Nalewajko, K., Soszynski, I., Bresolin, F., Kudritzki, R.-P., Minniti, D., & Romanowsky, A. 2006, ApJ, 647, 1056

Gieren, W., Pietrzynski, G., Soszynski, I., Bresolin, F., Kudritzki, R.-P., Minniti, D., & Storm, J. 2005a, ApJ, 628, 695

Gieren, W., Storm, J., Barnes, T. G., Fouqué, P., Pietrzynski, G., & Kienzle, F. 2005b, ApJ, 627, 224

Giraud, E. 1990, A&A, 231, 1

Graham, J. A., et al. 1997, ApJ, 477, 535

Gratton, R. G. 1998, MNRAS, 296, 739

Gratton, R. G., Fusi Pecci, F., Carretta, E., Clementini, G., Corsi, C. E., & Lattanzi, M. 1997, ApJ, 491, 749

Greco, C., et al. 2005, preprint (astro-ph/0507244)

Grillmair, C. J., et al. 1998, AJ, 115, 144

Groenewegen, M. A. T., & Salaris, M. 2003, A&A, 410, 887

Han, H., Hoessel, J. G., Gallagher, J. S., Holtsman, J., & Stetson, P. B. 1997, AJ, 113, 1001

Held, E. V., Clementini, G., Rizzi, L., Momany, Y., Saviane, I., & Di Fabrizio, L. 2001, ApJ, 562, L39

Held, E. V., Saviane, I., Momany, Y., & Carraro, G. 2000, ApJ, 530, L85 Herrnstein, J. R., Moran, J. M., Greenhill, L. J., & Trotter, A. S. 2005, ApJ,

Herrnstein, J. R., et al. 1999, Nature, 400, 539

Hertzsprung, E. 1914, Astron. Nachr., 196, 201

Hilditch, R. W., Howarth, I. D., & Harries, T. J. 2005, MNRAS, 357, 304

Holmberg, E. 1950, Medd. Lunds Astron. Obs. II, 128, 1

Hough, J. H., Bailey, J. A., Rouse, M. F., & Whittet, D. C. B. 1987, MNRAS, 227, 1P

Hubble, E. 1925, ApJ, 62, 409

. 1926, ApJ, 63, 236

-. 1929, ApJ, 69, 103

-. 1951, Trans. Am. Philos. Soc., 51, 461

Hubble, E., & Sandage, A. 1953, ApJ, 118, 353

Jerjen, H., & Tammann, G. A. 1993, A&A, 276, 1

Kaluzny, J., Kubiak, M., Szymanski, M., Udalski, A., Krzeminski, W., & Mateo, M. 1995, A&AS, 112, 407

Kanbur, S. M., Ngeow, C., Nanthakumar, A., & Stevens, R. 2007, PASP, 119,

Karachentsev, I. D., Karachentseva, V. E., Huchtmeier, W. K., & Makarov, D. I. 2004, AJ, 127, 2031

Karachentsev, I. D., et al. 2006, AJ, 131, 1361

. 2007, AJ, 133, 504

Karataeva, G. M., Tikhonov, N. A., Galazutdinova, O. A., & Hagen-Thorn, V. A. 2006, Astron. Lett., 32, 236

Keller, S. C., & Wood, P. R. 2006, ApJ, 642, 834

Kennicutt, R. C., Jr., Bresolin, F., & Garnett, D. R. 2003, ApJ, 591, 801

Kervella, P., Nardetto, N., Bersier, D., Mourard, D., & Coudé du Foresto, V. 2004, A&A, 416, 941

Klypin, A., Hoffman, Y., Kravtsov, A. V., & Gottlöber, S. 2003, ApJ, 596, 19

Koen, C., Kanbur, S., & Ngeow, C. 2007, MNRAS, 380, 1440

Koen, C., & Siluyele, I. 2007, MNRAS, 377, 1281

Kraan-Korteweg, R. C. 1986, A&AS, 66, 255

Lane, B., Creech-Eakman, M., & Nordgren, T. 2002, ApJ, 573, 330

Laney, C. D., & Stobie, R. S. 1986, MNRAS, 222, 449

Layden, A. C., & Sarajedini, A. 2000, AJ, 119, 1760

Lee, M. G., Freedman, W. L., & Madore, B. F. 1993, ApJ, 417, 553

Lee, M. G., et al. 2003, AJ, 126, 2840

Lee, Y. W., Demarque, P., & Zinn, R. 1990, ApJ, 350, 155

Mackey, A. D., & Gilmore, G. F. 2003, MNRAS, 345, 747

Macri, L. M., Stanek, K. Z., Bersier, D., Greenhill, L. J., & Reid, M. J. 2006, ApJ, 652, 1133

Madore, B. F., & Freedman, W. L. 1991, PASP, 103, 933

Marconi, M., Musella, I., & Fiorentino, G. 2005, ApJ, 632, 590

Masters, K. L., Springob, C. M., Haynes, M. P., & Giovanelli, R. 2006, ApJ,

Mateo, M., Fischer, P., & Krzeminski, W. 1995a, AJ, 110, 2166

Mateo, M., Kubiak, M., Szymanski, M., Kaluzny, J., Krzeminski, W., & Udalski, A. 1995b, AJ, 110, 1141

McAlary, C. W., Madore, B. F., McGonegal, R., McLaren, R. A., & Welch, D. L. 1983, ApJ, 273, 539

McConnachie, A. W., Irwin, M. J., Ferguson, A. M. N., Ibata, R. A., Lewis, G. F., & Tanvir, N. 2004, MNRAS, 350, 243

. 2005, MNRAS, 356, 979

McNamara, D. H. 1997, PASP, 109, 1221

McNamara, D. H., Rose, M. B., Brown, P. J., Ketcheson, D. I., Maxwell, J. E., Smith, K. M., & Wooley, R. C. 2004, in Variable Stars in the Local Group, ed. D. W. Kurtz & K. R. Pollard (San Francisco: ASP), 525

Mei, S., et al. 2007, ApJ, 655, 144

Mouhcine, M., Ferguson, H. C., Rich, R. M., Brown, T. M., & Smith, T. E. 2005, ApJ, 633, 810

Nardetto, N., Fokin, A., Mourard, D., Mathias, P., Kervella, P., & Bersier, D. 2004, A&A, 428, 131

Natale, G., Marconi, M., & Bono, G. 2008, ApJ, 674, L93

Nemec, J. M. 1985, AJ, 90, 204

. 1989, in The Use of Variable Stars in Fundamental Problems of Astronomy, ed. E. P. Schmidt (Cambridge: Cambridge Univ. Press), 215

Nemec, J. M., & Mateo, M. 1990a, in ASP Conf. Ser. 10, The Evolution of the Universe of Galaxies, ed. R. G. Kron (San Francisco: ASP), 134

. 1990b, in ASP Conf. Ser. 11, Confrontation between Stellar Pulsation and Evolution (San Francisco: ASP), 64

Nemec, J. M., Wehlau, A., & Mendes de Oliveira, C. 1988, AJ, 96, 528

Ngeow, C.-C., Kanbur, S. M., Nokolaev, S., Buonacorsi, J., Cook, K. H., & Welch, D. L. 2005, MNRAS, 363, 831

Nordgren, T. E., Lane, B. F., Hindsley, R. B., & Kervella, P. 2002, AJ, 123,

Panagia, N. 2005, in Cosmic Explosions, ed. J. M. Marcaide & K. W. Weiler (Berlin: Springer), 585

Pietrinferni, A., Cassisi, S., Salaris, M., & Castelli, F. 2004, ApJ, 612, 168 Pietrzynski, G., Gieren, W., Udalski, A., Bresolin, F., Kudritzki, R.-P., Soszynski, I., Szymanski, M., & Kubiak, M. 2004, AJ, 128, 2815

Pietrzynski, G., et al. 2006a, AJ, 132, 2556

2006b, ApJ, 648, 366

2007, AJ, 134, 594

Piotto, G., Capaccioli, M., & Pellegrini, C. 1994, A&A, 287, 371

Popowski, P., & Gould, A. 1999, in Post Hipparcos Standard Candles, ed. A. Heck & F. Caputo (Dordrecht: Kluwer), 53

Pritzl, B. J., Armandroff, T. E., Jacoby, G. H., & Da Costa, G. S. 2002, AJ, 124, 1464

-. 2004, AJ, 127, 318

. 2005, AJ, 129, 2232

Reid, I. N. 1997, AJ, 114, 161

1999, ARA&A, 37, 191

Reindl, B., Tammann, G. A., Sandage, A., & Saha, A. 2005, ApJ, 624, 532 (R05) Rejkuba, M., Greggio, L., Harris, W. E., Harris, G. L. H., & Peng, E. W. 2005, ApJ, 631, 262

Rejkuba, M., Minniti, D., Gregg, M. D., Zijlstra, A. A., Alonso, M. V., & Goudfrooij, P. 2000, AJ, 120, 801

Ribas, I., Jordi, C., Vilardell, F., Fitzpatrick, E. L., Hilditch, R. W., & Guinan, E. F. 2005, ApJ, 635, L37

Riess, A. G., et al. 2005, ApJ, 627, 579

Rizzi, L., Tully, R. B., Makarov, D., Makarova, L., Dolphin, A. E., Sakai, S., & Shaya, E. J. 2007, ApJ, 661, 815

Romaniello, M., Primas, F., Mottini, M., Groenewegen, M., Bono, G., & Francois, P. 2005, A&A, 429, L37

Rood, R. T. 1972, ApJ, 177, 681

Russell, H. N. 1913, Science, 37, 651

Saha, A., & Hoessel, J. G. 1990, AJ, 99, 97

Saha, A., Hoessel, J. G., & Krist, J. 1992, AJ, 103, 84

Saha, A., Hoessel, J. G., & Mossman, A. E. 1990, AJ, 100, 108

Saha, A., Monet, D. G., & Seitzer, P. 1986, AJ, 92, 302

Saha, A., Thim, F., Tammann, G. A., Reindl, B., & Sandage, A. 2006, ApJS, 165, 108 (S06a)

Sakai, S., Ferrarese, L., Kennicutt, R. C., Jr., & Saha, A. 2004, ApJ, 608, 42 Sakai, S., Madore, B. F., Freedman, W. L., Lauer, T. R., Ajhar, E. A., & Baum, W. A. 1997, ApJ, 478, 49

Salaris, M., & Cassisi, S. 1997, MNRAS, 289, 406

1998, MNRAS, 298, 166

Salaris, M., degl'Innocenti, S., & Weiss, A. 1997, ApJ, 479, 665

Salaris, M., Percival, S., & Girardi, L. 2003, MNRAS, 345, 1030

Sandage, A. 1971, in Pontificiae Academiae Scientiarum Scripta Varia, Nuclei of Galaxies, ed. D. J. K. O'Connell (Amsterdam: North Holland), 601

-. 1988, PASP, 100, 935 -. 2006, AJ, 131, 1750

Sandage, A., Bell, R. A., & Tripicco, M. J. 1999, ApJ, 522, 250

Sandage, A., & Tammann, G. A. 2006, ARA&A, 44, 93 (ST06)

Sandage, A., Tammann, G. A., & Reindl, B. 2004, A&A, 424, 43 (STR04)

Sandage, A., Tammann, G. A., Saha, A., Reindl, B., Macchetto, F. D., & Panagia, N. 2006, ApJ, 653, 843 (S06b)

Sarajedini, A., Barker, M. K., Geisler, D., Harding, P., & Schommer, R. 2006, AJ, 132, 1361

Schlegel, D., Finkbeiner, D., & Davis, M. 1998, ApJ, 500, 525

Seares, F. H., Kapteyn, J. C., & van Rhijn, P. J. 1930, Mount Wilson Catalogue of Photographic Magnitudes in Selected Areas 1-139 (Washington, DC: Carnegie Institution of Washington), Publication No. 402

Seth, A. C., Dalcanton, J. J., & de Jong, R. S. 2005, AJ, 129, 1331

Shapley, H. 1918, ApJ, 48, 89

Siegel, M. H., & Majewski, S. R. 2000, AJ, 120, 284

Smecker-Hane, T. A., Stetson, P. B., Hesser, J. E., & Lehnert, M. D. 1994, AJ, 108, 507

Smith, H. A., Silbermann, N. A., Baird, S. R., & Graham, J. A. 1992, AJ, 104,

Sollima, A., Cacciari, C., & Valenti, E. 2006, MNRAS, 372, 1675

Soszynski, I., Gieren, W., Pietrzynski, G., Bresolin, F., Kudritzki, R.-P., & Storm, J. 2006, ApJ, 648, 375

Soszynski, I., et al. 2002, Acta Astron., 52, 369

2003, Acta Astron., 53, 93

Spergel, D. N., et al. 2007, ApJS, 170, 377

Storm, J., Carney, B. W., Gieren, W. P., Fouqué, P., Latham, D. W., & Fry, A. M. 2004, A&A, 415, 531

Sweigart, A. V., & Gross, R. G. 1978, ApJS, 36, 405

Tammann, G. A., & Reindl, B. 2002, Ap&SS, 280, 165

Tammann, G. A., Reindl, B., Thim, F., Saha, A., & Sandage, A. 2002, in ASP Conf. Ser. 283, A New Era in Cosmology, ed. T. Shanks & N. Metcalfe (San Francisco: ASP), 258

Tammann, G. A., Sandage, A., & Reindl, B. 2003, A&A, 404, 423 (TSR03) Tanvir, N. R., Hendry, M. A., Watkins, A., Kanbur, S. M., Berdnikov, L. N., &

Ngeow, C. C. 2005, MNRAS, 363, 749

Tegmark, M., et al. 2006, Phys. Rev. D, 74, 123507

Thackeray, A. D. 1954, in Trans. IAU VIII (Rome 1952, meeting), Report of Commission 28 (Cambridge: Cambridge Univ. Press), 397

1958, in Report to the 1957 Vatican Conference on Stellar Populations, Specola Vaticana, Vol. 5, ed. D. J. K. O'Connell, 195

Thim, F., Tammann, G. A., Saha, A., Dolphin, A., Sandage, A., Tolstoy, E., & Labhardt, L. 2003, ApJ, 590, 256

Thuan, T. X., & Gunn, J. E. 1976, PASP, 88, 543

Tully, R. B., & Pierce, M. J. 2000, ApJ, 533, 744

Tully, R. B., et al. 2006, AJ, 132, 729

Udalski, A. 1998, Acta Astron., 48, 113

2000, Acta Astron., 50, 279

Udalski, A., Soszynski, I., Szymanski, M., Kubiak, M., Pietrzynski, G., Wozniak, P., & Zebrun, K. 1999a, Acta Astron., 49, 223

1999b, Acta Astron., 49, 437

Udalski, A., Szymanski, M., Kubiak, M., Pietrzynski, G., Szymanski, M., Wozniak, P., & Zebrun, K. 1999c, Acta Astron., 49, 201

VandenBerg, D. A., Swenson, E. J., Rogers, F. J., Iglesias, C. A., & Alxander, D. R. 2000, ApJ, 532, 430

van den Bergh, S. 1995, ApJ, 446, 39

van Leeuwen, F., Feast, M. W., Whitelock, P. A., & Laney, C. D. 2007, MNRAS, 379, 723

Vilardell, F., Jordi, C., & Ribas, I. 2007, A&A, 473, 847

Walker, A., & Mack, A. R. 1988, AJ, 96, 872

Yahil, A., Sandage, A., & Tammann, G. A. 1980, in Physical Cosmology, ed. E. Balian, J. Audouze, & D. N. Schramm (Amsterdam: North Holland), 127

Yahil, A., Tammann, G. A., & Sandage, A. 1977, ApJ, 217, 903

Zaritsky, D., Kennicutt, R. C., Jr., & Huchra, J. P. 1994, ApJ, 420, 87

Zinn, R., & West, M. J. 1984, ApJS, 55, 45

Zoccali, M., et al. 2003, A&A, 399, 931