

CLOSURE RELATIONS FOR $e^\pm$ PAIR SIGNATURES IN GAMMA-RAY BURSTS

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ABSTRACT

We present recipes to diagnose the fireball of gamma-ray bursts (GRBs) by combining observations of $e^\pm$ pair signatures (the pair annihilation line and the cutoff energy due to the pair creation process). Our recipes are largely model-independent and extract information even from the nondetection of either pair signature. We evaluate physical quantities such as the Lorentz factor, optical depth, and pair-to-baryon ratio only from the observable quantities. In particular, we can test whether the prompt emission of GRBs comes from the pair/baryonic photosphere or not. The future *Gamma-Ray Large Area Space Telescope (GLAST)* satellite will provide us with good chances to use our recipes via either detection or nondetection of pair signatures.

Subject headings: gamma rays: bursts — gamma rays: theory — plasmas

1. INTRODUCTION

Gamma-ray bursts (GRBs) are among the most mysterious objects in the universe. Various models have been suggested, but no conclusive picture of them has yet been obtained (see reviews in, e.g., Mészáros 2006; Zhang 2007). One of the leading models is the optically thin internal shock model, in which the prompt emission is explained by electromagnetic radiation from relativistic electrons accelerated in internal shocks (see, e.g., Rees & Mészáros 1994). Another leading model is the photospheric emission model, in which the prompt emission comes from the photospheric radius r_{ph} at which the Thomson optical depth is unity, i.e., $\tau = 1$ (see, e.g., Rees & Mészáros 2005). The possibility that a fireball contains copious $e^\pm$ pairs (a pair-dominated fireball) is also discussed by many authors. In particular, we recently proposed that the pair photosphere is unstable and that it is capable of generating the observed nonthermal spectrum with high radiative efficiency (Ioka et al. 2007). The existence of copious pairs can extend the photosphere compared to the baryonic photosphere, which is determined by baryon-related electrons. Such pairs could be produced via dissipation processes such as internal shocks and magnetic reconnection.

Prompt gamma rays are typically radiated at ~ 100 keV. Observationally, even more high-energy photons have been detected by the EGRET detector. Such high-energy emissions are theoretically expected due to radiation processes such as synchrotron and/or inverse Compton emission. Sufficiently high-energy photons cannot avoid the pair-production process, which leads to a cutoff energy due to pair creation. On the other hand, there may be many pairs that can be seen as pair-annihilation lines via the pair-annihilation process (Ioka et al. 2007; Pe’er & Waxman 2004; Pe’er et al. 2006). The future *GLAST* satellite will be a suitable detector to observe such pair signatures, as a pair-annihilation line and/or cutoff energy.

Obviously, such pair signatures convey important information about GRB fireballs. For example, the cutoff energy due to pair creation carries information about the bulk Lorentz factor of a fireball. This possibility has already been investigated by several authors (Baring & Harding 1997; Lithwick & Sari 2001; Razzaque et al. 2004). However, there are few studies focusing on both the pair-annihilation line and the cutoff energy due to pair creation.

In this paper, we propose that by combining these two pair signatures, we can get more information about the GRB fireball (§ 2). Even if we cannot detect either pair signature, the non-detection itself provides information (§ 3). We show that observations of pair signatures can allow us to evaluate the Lorentz factor, the optical depth of a fireball, and the pair-to-baryon ratio, among other quantities. In particular, we derive these relations only from observable quantities, and make analyses as model-independent as possible. Our recipes are especially productive in testing the pair photospheric emission model (§ 4).

Throughout the paper, we assume that we know the gamma-ray spectrum in the wide energy range (e.g., the high-energy spectral index β , etc.), the source redshift z from other observations, and hence the luminosity $\varepsilon L_\varepsilon$ at given observed energy ε from the observed flux (see Fig. 1).

2. DIAGNOSING THE FIREBALL BY $e^\pm$ PAIR SIGNATURES

Let us assume that we can find a pair-annihilation line in the spectrum of the prompt emission (Fig. 1), which typically peaks at

$$\varepsilon_{\text{ann}} \simeq \frac{\Gamma}{1+z} m_e c^2. \quad (1)$$

This expression is valid as long as pairs forming a pair-annihilation line are nonrelativistic. This is a reasonable assumption, since the cooling time of sufficiently relativistic pairs t_{cool} due to magnetic and/or photon fields is usually much shorter than the pair-annihilation time t_{ann} . However, we note that the line would be broadened by a dispersion of the Doppler factor. Therefore, gamma rays due to pair annihilation will be observed as a “bump” rather than a “line.” There are several possible causes for line broadening. First, the order unity distribution of the Lorentz factor in the emission region can broaden the line by order unity even when pairs are nonrelativistic in the comoving frame. Second, the order unity line broadening may also be caused by observing a section of the emission region with an opening angle $\sim 1/\Gamma$ rather than a small spot, so that the Doppler factor toward the observer is different by order unity between the center and the edge of the observed emission region. Third, the order unity variation of the Lorentz factor may also occur within the dynamical time. Recent observations may suggest that the emission

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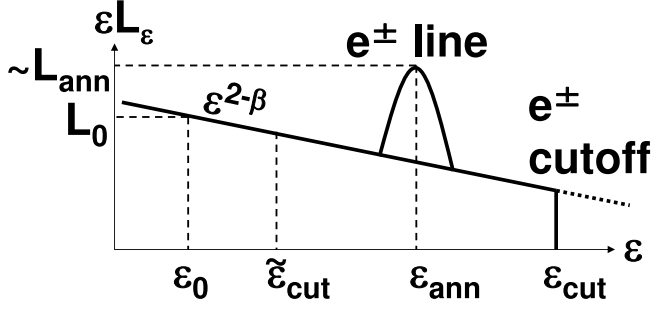


FIG. 1.—Schematic picture of the GRB spectrum, showing the observable quantities of pair signatures [L_{ann} , L_0 , ε_0 , ε_{cut} , $\varepsilon_{\text{ann}} \simeq \Gamma m_e c^2 / (1+z)$, and β]. The recipes to constrain the physical quantities of a GRB fireball using only observable quantities are as follows. (I) The case where we can observe both the pair-annihilation line and the cutoff energy due to pair creation, i.e., ε_{ann} , L_{ann} , and ε_{cut} ; if we also know the kinetic luminosity of baryons L_p , we can measure τ , $\tau_{\pm}$, and $n_{\pm}/n_p$ from eqs. (14), (16), and (17). Without L_p , we obtain the inequalities (18) from $L_p > 0$, while we have an upper limit on τ and lower limits on $\tau_{\pm}$ and $n_{\pm}/n_p$ from the assumption $L_p \lesssim L_{\gamma}$ by replacing L_p with L_{γ} in eqs. (14), (16), and (17). If the inequality (13) is satisfied, the fireball is pair-dominated, $\tau \approx \tau_{\pm}$, and we can use eqs. (11) and (12) instead of eqs. (14) and (15). (II) The case where we only observe ε_{cut} , not L_{ann} and ε_{ann} ; with L_p , we can set upper limits on τ , $n_{\pm}/n_p$, and $\tau_{\pm}$ by replacing L_{ann} with $L_0 [\Gamma m_e c^2 / (1+z) \varepsilon_0]^{2-\beta}$ in eqs. (14), (16), and (17). Without L_p , we obtain the inequality (20) from $L_p > 0$, while we set an upper limit on τ from $L_p \lesssim L_{\gamma}$ by replacing L_p and L_{ann} with L_{γ} and $L_0 [\Gamma m_e c^2 / (1+z) \varepsilon_0]^{2-\beta}$, respectively, in eq. (14). $\Gamma < [(1+z)\varepsilon_{\text{cut}}/m_e c^2]$ should be acquired by other means. (III) The case where we only observe L_{ann} and ε_{ann} , not ε_{cut} ; we regard the observed maximum energy ε_{max} as the lower limit on the true cutoff energy ε_{cut} . With L_p , we can set upper limits on τ and $\tau_{\pm}$ as well as a lower limit on $n_{\pm}/n_p$ by replacing ε_{cut} with $f_{\text{max}} \equiv f(\varepsilon_{\text{max}}, \Gamma)$ in eqs. (14), (16), and (17). Without L_p , we obtain the inequality (22) from $L_p > 0$, while we obtain an upper limit on τ as well as a lower limit on $n_{\pm}/n_p$ from $L_p \lesssim L_{\gamma}$ by replacing L_p and ε_{cut} with L_{γ} and f_{max} , respectively, in eqs. (14) and (16). Such arguments can be also applied to the completely thin fireballs. (IV) The recipes (I)–(III) are especially valuable to test the pair photospheric emission model. The inequalities (18), (20), and (22) are useful to constrain $\tau_{\pm}$. This model gives $\tau_{\pm} \sim 1$ in case I, and eq. (24) if L_{ann} is comparable to the underlying continuum emission. The photospheric radius can also be estimated.

is radiatively very efficient (Ioka et al. 2006; Zhang et al. 2007), and efficient internal dissipation may make the fireball radiation-dominated. If it is, the Lorentz factor will increase as $\Gamma \propto r$, and the Lorentz factor will vary by order unity within the dynamical time. Therefore, we can expect that all three effects may broaden the line by order unity.

The total luminosity of the pair-annihilation line L_{ann} (Coppi & Blandford 1990; Svensson 1982), the kinetic luminosity of pairs $L_{\pm}$, and the kinetic luminosity of baryons L_p are given by

$$L_{\text{ann}} \simeq \frac{3}{8} n_+ n_- \sigma_T c (2m_e c^2) (4\pi r^2 \Delta') \Gamma^2, \quad (2)$$

$$L_{\pm} = n_{\pm} c (2m_e c^2) (4\pi r^2) \Gamma^2, \quad (3)$$

$$L_p = n_p c (m_p c^2) (4\pi r^2) \Gamma^2, \quad (4)$$

respectively. Here r is the emission radius, Δ' is the comoving width of the emission region, and $n_+ = n_{\pm}$, $n_- = n_{\pm} + n_p$, $n_{\pm}$, and n_p are the comoving density of positrons, electrons, $e^{\pm}$ pairs, and baryon-related electrons, respectively. We have assumed that most of the sufficiently relativistic pairs cool down in the dynamical time. Combining expressions of L_{ann} and $L_{\pm}$ leads to

$$L_{\text{ann}} \simeq \frac{3}{16} L_{\pm} \tau_{\pm} \left(1 + \frac{n_p}{n_{\pm}} \right), \quad (5)$$

where $\tau_{\pm} \simeq 2n_{\pm} \sigma_T \Delta'$ denotes the optical depth against pairs.

Pair-creation processes such as $\gamma\gamma \rightarrow e^+e^-$ and $e\gamma \rightarrow ee^+e^-$ prevent sufficiently high-energy photons from escaping the source.

Usually, the most important pair-creation process is $\gamma\gamma \rightarrow e^+e^-$ (Razzaque et al. 2004). The optical depth for this process, $\tau_{\gamma\gamma}$, at some energy ε can be evaluated for a given photon spectrum. The elaborate evaluation of $\tau_{\gamma\gamma}$ is possible if we know the spectrum in detail (see, e.g., Coppi & Blandford 1990; Baring 2006; Baring & Harding 1997; Gupta & Zhang 2008). Here we assume a power-law photon spectrum for simplicity, i.e., with the luminosity $\varepsilon L_{\varepsilon}(\varepsilon) = L_0 (\varepsilon/\varepsilon_0)^{2-\beta}$ for $\beta > 2$. Then, we have (Gould & Schröder 1967; Lightman & Zdziarski 1987; Svensson 1987; Lithwick & Sari 2001; Baring 2006)

$$\tau_{\gamma\gamma}(\varepsilon) \simeq \xi(\beta) n_{\gamma}(\varepsilon_{\gamma} > \tilde{\varepsilon}) \sigma_T \Delta', \quad (6)$$

$$\left[\tilde{\varepsilon} = \frac{(\Gamma m_e c^2)^2}{(1+z)^2 \varepsilon} \right],$$

where the comoving density of photons whose energies are larger than $\tilde{\varepsilon}$ is given by

$$n_{\gamma}(\varepsilon_{\gamma} > \tilde{\varepsilon}) = \frac{L_0}{4\pi r^2 \Gamma c \varepsilon_0 (1+z)} \int_{\tilde{\varepsilon}}^{\infty} \frac{d\varepsilon_{\gamma}}{\varepsilon_0} \left(\frac{\varepsilon_{\gamma}}{\varepsilon_0} \right)^{-\beta}, \quad (7)$$

and $\tilde{\varepsilon}$ is the energy of a photon that interacts with the photon of energy ε at the pair-creation threshold. Here $\xi(\beta)$ is a numerical factor that depends on the photon index (Gould & Schröder 1967; Lightman & Zdziarski 1987; Svensson 1987; Coppi & Blandford 1990; Lithwick & Sari 2001; Baring 2006; Gupta & Zhang 2008), and $\xi(\beta)/(\beta-1)$ decreases with β ; its values are $\xi(\beta) = 11/90 \simeq 0.12$ and $\xi(\beta) = 7/75 \simeq 0.093$ for $\beta = 2$ and 3, respectively.³ For the isotropic photon distribution with an infinite power law, we can use $\xi(\beta) \simeq 7(\beta-1)/[6\beta^{5/3}(\beta+1)]$ for $1 < \beta < 7$ (Svensson 1987; Baring 2006). Note that L_0 is related to the observed (time-resolved) flux $\varepsilon F_{\varepsilon}(\varepsilon)$ by

$$\varepsilon F_{\varepsilon}(\varepsilon = \varepsilon_0) = \frac{L_0}{4\pi d_L^2}, \quad (8)$$

where d_L is the luminosity distance to the source. Unless a fireball is completely thin, where all the photons can escape without attenuation, the cutoff energy ε_{cut} exists due to the pair-creation process $\gamma\gamma \rightarrow e^+e^-$, where $\tau_{\gamma\gamma}(\varepsilon_{\text{cut}}) = 1$ (Fig. 1). With equation (3), $\tau_{\gamma\gamma}(\varepsilon_{\text{cut}}) = 1$ is rewritten as

$$1 = \tau_{\gamma\gamma}(\varepsilon_{\text{cut}}) \simeq \frac{L_0}{L_{\pm}} \tau_{\pm} f(\varepsilon_{\text{cut}}, \Gamma), \quad (9)$$

where

$$f(\varepsilon_{\text{cut}}, \Gamma) \simeq \xi(\beta) \frac{\Gamma m_e c^2}{(1+z) \varepsilon_0} \int_{\varepsilon_{\text{cut}}}^{\infty} \frac{d\varepsilon_{\gamma}}{\varepsilon_0} \left(\frac{\varepsilon_{\gamma}}{\varepsilon_0} \right)^{-\beta}. \quad (10)$$

Note that we may arbitrarily take ε_0 by adjusting L_0 . We also note that ε_{cut} is larger than ε_{ann} , as long as ε_{cut} is determined by the pair-creation process $\gamma\gamma \rightarrow e^+e^-$ (and we have also assumed that electrons and positrons are accelerated enough to emit high-energy photons with $\varepsilon > \varepsilon_{\text{cut}}$ via, e.g., synchrotron or inverse

³ When we assume an isotropic photon spectrum with an infinite power law, we have $\xi(\beta = 2) = 11/90 \simeq 0.12$. This value is obtained by various authors, e.g., Gould & Schröder (1967), Svensson (1987), Baring (2006), and Gupta & Zhang (2008). Lithwick & Sari (2001) adopt a factor of 2 smaller value, as discussed in Zhang & Mészáros (2001).

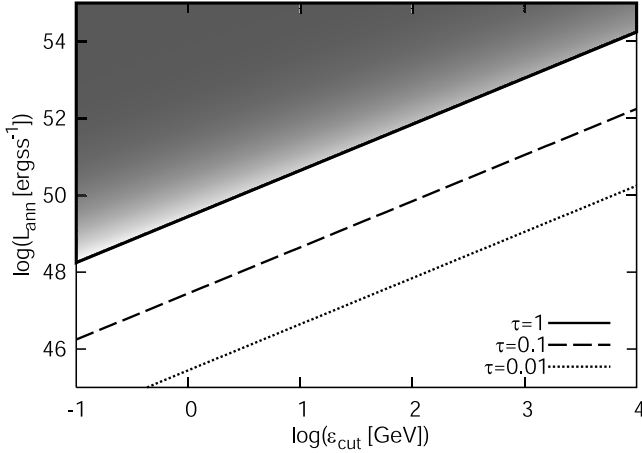


FIG. 2.—Relation between the cutoff energy ε_{cut} and total luminosity of a pair annihilation line L_{ann} for a given optical depth τ . Lines are calculated by exploiting eqs. (10) and (14). Parameters used are $L_p = 10^{50}$ ergs s $^{-1}$, $L_0 = 10^{51}$ ergs s $^{-1}$, $\varepsilon_0 = 10^{2.5}$ keV, $\Gamma = 10^{2.5}$, $\beta = 2.2$, and $z = 0.1$. In this case, the fireball is pair-dominated in this figure, and we can use eq. (11) instead of eq. (14). The shaded region shows $\tau \gtrsim 1$, where photons suffer from Compton scattering. If we can obtain necessary quantities such as L_{ann} in the scheme of Fig. 1, we can estimate τ by plotting observational quantities in this figure. Note that ε_{cut} should be larger than $\varepsilon_{\text{ann}} \simeq \Gamma m_e c^2 / (1 + z)$ for typical photon spectra, as long as ε_{cut} is determined by the pair-creation process.

Compton radiation processes). This is because an assumed photon spectrum has $\beta > 1$ (which is typically expected for prompt emission), and hence the photon number density decreases with photon energies. Therefore, photons with $\varepsilon \lesssim \varepsilon_{\text{ann}}$ do not have enough target photons with $\tilde{\varepsilon} \gtrsim \varepsilon_{\text{ann}}$ to be attenuated at $\varepsilon_{\text{cut}} \lesssim \varepsilon_{\text{ann}}$. Otherwise, the created pairs would make the optical depth τ larger than unity. In this case, the cutoff energy is determined by the Compton down-scattering process rather than the pair-creation process for the assumed spectrum (see, e.g., Lithwick & Sari 2001). Although we hereafter focus on cases where ε_{cut} is determined by the pair-creation cutoff, there are possibilities of $\varepsilon_{\text{cut}} \lesssim \varepsilon_{\text{ann}}$ for $\tau \gtrsim 1$. We may be able to check $\varepsilon_{\text{cut}} \lesssim \varepsilon_{\text{ann}}$ and $\tau \gtrsim 1$, if we can observe the Lorentz factor Γ by other means as well as the cutoff energy ε_{cut} . We would expect that high-energy gamma rays come from the region where $\tau \sim 1$, as long as the dissipation continues until $r \sim r_{\text{ph}}$ and the emission from $r \sim r_{\text{ph}}$ is not negligible. This is because high-energy gamma rays from the region where $\tau \gg 1$ are significantly down-scattered. We would also expect that the GRB radiative efficiency is small (contrary to the observations) if the prompt emission comes only from $\tau \gg 1$, since almost all the energy goes into the afterglow. We also note that in some models, such as the slow dissipation scenario (Ghisellini & Celotti 1999), high-energy photons with $\varepsilon > \varepsilon_{\text{cut}}$ may not be produced, because electrons and positrons are not accelerated enough (Pe’er et al. 2006).

2.1. Closure Relations for the Pair-dominated Fireball

Now let us assume a pair-dominated fireball, $n_p < 2n_\pm$. Then we can solve equations (5) and (9) for the two unknown quantities $\tau_\pm$ and $L_\pm$ as

$$\tau \approx \tau_\pm \simeq \left[\frac{16}{3} \frac{L_{\text{ann}}}{L_0 f(\varepsilon_{\text{cut}}, \Gamma)} \right]^{1/2} \quad (11)$$

$$L_\pm \simeq \left[\frac{16}{3} L_0 L_{\text{ann}} f(\varepsilon_{\text{cut}}, \Gamma) \right]^{1/2}. \quad (12)$$

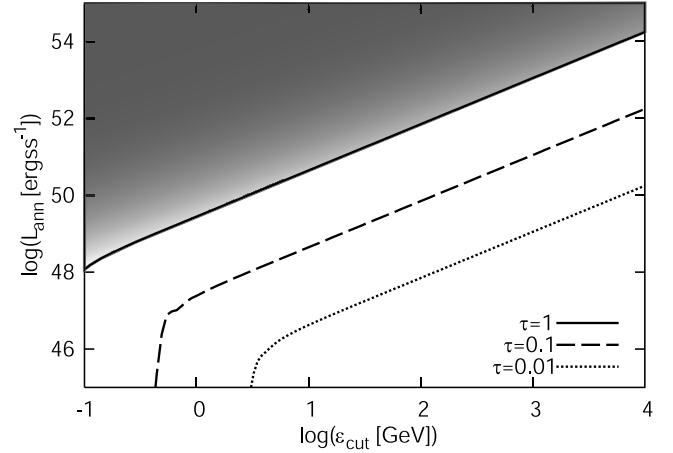


FIG. 3.—As in Fig. 2, but for $L_p = 10^{52}$ ergs s $^{-1}$. In this case, the fireball is not pair-dominated for sufficiently small ε_{cut} , given fixed τ . When the fireball is baryon-dominated, we cannot expect a pair-annihilation line. As pairs become dominant, L_{ann} increases sharply.

Remarkably, the above two quantities are expressed only in terms of observable quantities [L_{ann} , L_0 , ε_0 , ε_{cut} , $\varepsilon_{\text{ann}} \simeq \Gamma m_e c^2 / (1 + z)$, and β], so we can evaluate $\tau_\pm \approx \tau$ and $L_\pm$. Note that we have not assumed the frequently used relation $r \approx 2\Gamma\Delta'$, which is expected in the internal shock model. Because we have not specified the model, our recipes are largely model-independent in that sense.

The absence of σ_T in equations (11) and (12) just comes from the fact that the pair-annihilation, pair-creation, and Compton scattering are all basic two-body interaction processes with cross section $\sim \sigma_T$. Ambiguities arising from the transformation between the comoving frame and observer frame are canceled, because the transformation between the two frames is the same for L_0 , $L_\pm$, and L_{ann} .

Equations (11) and (12) are useful because they enable us to estimate $\tau \approx \tau_\pm$ and $L_\pm$ from observational quantities only, although there will be possible uncertainties due to, e.g., observational difficulties in evaluating ε_{cut} , ε_{ann} , and L_{ann} . In Figures 2 and 3, we demonstrate that we can obtain information on τ for a given burst (especially a given pulse). Observations of pair signatures will enable us to plot the point in such a figure and to compare it with lines expressing optical depths. Of course, a line for a given τ is different among bursts with different parameter sets. However, we could see the tendency of the distribution of the optical depth for some bursts (or pulses) with a similar parameter set. In this case, lines for a given optical depth can be expressed as “a band” with a finite width. We think that the plot without lines for optical depths may also be useful. More and more observations of pair signatures will allow us to plot points with optical depths in the $\varepsilon_{\text{cut}}-L_{\text{ann}}$ plane.

The assumption $n_p < 2n_\pm$ can be checked a posteriori by the observations. From equations (3) and (4), we have the condition for the fireball to be pair-dominated,

$$\frac{m_p L_\pm}{m_e L_p} \simeq \frac{2n_\pm}{n_p} > 1, \quad (13)$$

which can be checked if we can measure L_p from other observations. For example, we could obtain $L_p \sim L_p^{\text{AG}}$, where L_p^{AG} is the kinetic luminosity of baryons estimated from the afterglow observations. Note that the inequality (13) just means that the

pair photospheric radius should be larger than the baryonic photospheric radius, i.e., $r_{\text{ph},\pm} > r_{\text{ph},p}$. In particular, we have a closure relation $\tau_{\pm} \simeq 1$ for prompt emission arising from a pair photosphere.

The kinetic luminosity of baryons may be usually less than the observed gamma-ray luminosity, $L_p \lesssim L_\gamma$, as inferred by recent observations that the prompt emission is radiatively very efficient (Ioka et al. 2006; Zhang et al. 2007). It is not very convincing yet, since we cannot measure the precise GRB energy at present. But once it is observationally established, we will obtain the useful sufficient condition. If the sufficient condition, $m_p L_{\pm}/m_e L_\gamma > 1$, is satisfied, we can justify pair-dominance in the inequality (13) by observations. This sufficient condition will be useful, as we do not need to evaluate L_p .

2.2. More General Relations

As shown in previous subsections, the signatures of pair annihilation and creation are useful as a diagnostic tool of pair-dominated fireballs in GRBs. However, the fireball cannot be pair dominated where the inequality (13) is not satisfied. Taking into account the term $n_p/n_{\pm} \simeq 2m_e L_p/m_p L_{\pm}$ in equation (5), we can derive the quadratic equation for $L_{\pm}$ from equations (5) and (9), and generalize equations (11) and (12) as

$$\tau \simeq \left(\frac{16}{3} \frac{L_{\text{ann}}}{L_0 f_{\text{cut}}} + \frac{m_e^2 L_p^2}{m_p^2 L_0^2 f_{\text{cut}}^2} \right)^{1/2}, \quad (14)$$

$$L_{\pm} \simeq \left(\frac{16}{3} L_0 L_{\text{ann}} f_{\text{cut}} + \frac{m_e^2 L_p^2}{m_p^2} \right)^{1/2} - \frac{m_e}{m_p} L_p, \quad (15)$$

where we have defined $f_{\text{cut}} \equiv f(\varepsilon_{\text{cut}}, \Gamma)$, and $\tau \simeq (2n_{\pm} + n_p)\sigma_T \Delta' = \tau_{\pm}(1 + n_p/2n_{\pm})$ is the optical depth of the emission region. We can also evaluate the pair-to-baryon ratio and the optical depth against pairs as

$$\frac{2n_{\pm}}{n_p} \simeq \left(1 + \frac{16m_p^2 L_{\text{ann}} L_0 f_{\text{cut}}}{3m_e^2 L_p^2} \right)^{1/2} - 1, \quad (16)$$

$$\tau_{\pm} \simeq \frac{m_e L_p}{m_p L_0 f_{\text{cut}}} \left[\left(1 + \frac{16m_p^2 L_{\text{ann}} L_0 f_{\text{cut}}}{3m_e^2 L_p^2} \right)^{1/2} - 1 \right]. \quad (17)$$

Compared to equations (11) and (12), we need additional information on the amount of baryons, L_p , to obtain τ , $L_{\pm}$, and $\tau_{\pm}$. If we take the no-pair limit $2n_{\pm} \ll n_p$ in equations (14), (15), and (17), we find that τ does not depend on L_{ann} , and $L_{\pm}, \tau_{\pm} \rightarrow 0$, as expected.

Even if we cannot estimate L_p , we have useful constraints only from pair signatures. First, we can show

$$\tau_{\pm} < \left(\frac{16}{3} \frac{L_{\text{ann}}}{L_0 f_{\text{cut}}} \right)^{1/2} < \tau. \quad (18)$$

The above inequalities can be derived by exploiting $L_p > 0$ for equations (14) and (17), respectively. Therefore, observations of pair signatures give us an upper limit on the optical depth for Compton (or Thomson) scattering from pairs. In particular, we can exclude the pair photospheric emission model when we have $\tau_{\pm} \ll 1$. Second, with $L_p \lesssim L_\gamma$, we can observationally set an upper limit on τ as well as lower limits on $\tau_{\pm}$ and $2n_{\pm}/n_p$ by replacing L_p with L_γ in equations (14), (16), and (17).

3. CASES FOR LIMITED OBSERVATIONS

3.1. The Case of Nondetected Pair-Annihilation Lines

We can gain some information about the fireball even if a pair-annihilation line is not observed. The nondetection of the pair-annihilation lines means that

$$L_{\text{ann}} \lesssim \varepsilon L_\varepsilon(\varepsilon_{\text{ann}}) = L_0 \left[\frac{\Gamma m_e c^2}{(1+z)\varepsilon_0} \right]^{2-\beta}. \quad (19)$$

If we can measure L_p , we can set upper limits on τ , $2n_{\pm}/n_p$, and $\tau_{\pm}$ by replacing L_{ann} with $L_0[\Gamma m_e c^2/(1+z)\varepsilon_0]^{2-\beta}$ in equations (14), (16), and (17).

Even when we cannot estimate L_p , the inequalities (18) where $L_p > 0$ is used yield a looser constraint on the optical depth from pairs as

$$\tau_{\pm} \lesssim \left\{ \frac{16}{3f_{\text{cut}}} \left[\frac{\Gamma m_e c^2}{(1+z)\varepsilon_0} \right]^{2-\beta} \right\}^{1/2}. \quad (20)$$

If the right-hand side of the above inequality is smaller than unity, i.e.,

$$\varepsilon_{\text{cut}} \gg \left[\frac{16(\beta-1)}{3\xi(\beta)} \right]^{1/(\beta-1)} \left(\frac{\Gamma}{1+z} \right) m_e c^2,$$

we can exclude the pair photospheric emission model. If we use $L_p \lesssim L_\gamma$ instead of $L_p > 0$, we obtain an upper limit on τ by replacing L_p and L_{ann} with L_γ and $L_0[\Gamma m_e c^2/(1+z)\varepsilon_0]^{2-\beta}$, respectively, in equation (14).

Note that we have implicitly assumed that Γ is already determined by another means. At least we have $1 \leq \Gamma < (1+z)\varepsilon_{\text{cut}}/m_e c^2$. We can estimate Γ from $\tau_{\gamma\gamma}(\varepsilon_{\text{cut}}) = 1$ in equation (6) if we give the emission radius r . For example, r may be estimated from the frequently used relation $r \approx 2\Gamma^2 c \delta t_{\text{decay}}/(1+z)$, where the decay time of a pulse δt_{decay} is basically determined by the angular spreading timescale (Baring & Harding 1997; Lithwick & Sari 2001). The possible thermal emission component may be also useful in estimating Γ (Pe'er et al. 2007).

3.2. The Case of Nondetected Cutoff Energy

Because of the limited sensitivity of the detector, the observed maximum energy ε_{max} may be smaller than the true cutoff energy ε_{cut} . As seen in equation (10), $f(\varepsilon, \Gamma)$ increases with ε for $\varepsilon \lesssim \varepsilon_{\text{cut}}$, as long as the cutoff energy is determined by the pair-creation process. [More precisely, $\tau_{\gamma\gamma}(\varepsilon)$; hence $f(\varepsilon, \Gamma)$ typically reaches almost the maximum value around $\varepsilon \sim \tilde{\varepsilon}_{\text{peak}}$ for the low-energy spectral index $\alpha \lesssim 1$, where $\varepsilon_{\text{peak}}$ is the peak energy. On the other hand, $\tau_{\gamma\gamma}(\varepsilon)$ always increases with ε for $\alpha \gtrsim 1$.] Then, we have

$$f(\varepsilon_{\text{cut}}, \Gamma) \gtrsim f(\varepsilon_{\text{max}}, \Gamma). \quad (21)$$

If we can measure L_p , we can set upper limits on τ and $\tau_{\pm}$, as well as a lower limit on $2n_{\pm}/n_p$, by replacing f_{cut} with $f_{\text{max}} \equiv f(\varepsilon_{\text{max}}, \Gamma)$ in equations (14), (16), and (17).

Without knowing L_p , the inequalities (18), where $L_p > 0$ is used, yield the looser upper limit on $\tau_{\pm}$ as

$$\tau_{\pm} \lesssim \left(\frac{16}{3} \frac{L_{\text{ann}}}{L_0 f_{\text{max}}} \right)^{1/2}. \quad (22)$$

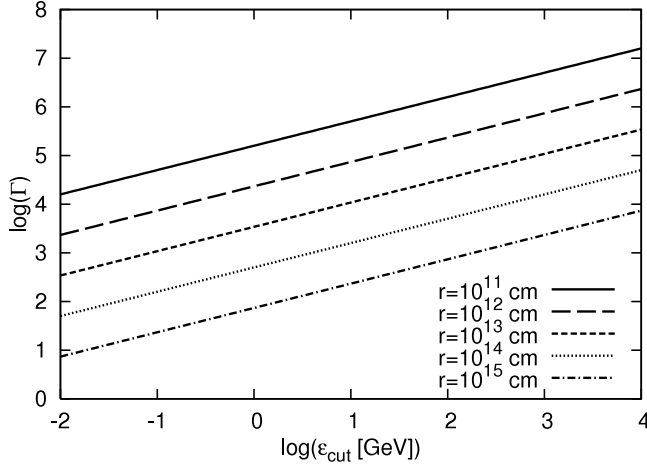


FIG. 4.— Relation between the cutoff energy ε_{cut} and bulk Lorentz factor Γ for given collision radii r . A simple power-law photon spectrum is assumed. Parameters used are $E_0 = 10^{51}$ ergs, $\varepsilon_0 = 10^{2.5}$ keV, and $z = 0.1$. Note that $\varepsilon_{\text{cut}} \lesssim \varepsilon_{\text{peak}}$ is also implicitly assumed.

If the right-hand side of the above inequality is less than unity, i.e.,

$$\varepsilon_{\text{max}} \gg \left[\frac{16(\beta - 1)}{3\xi(\beta)} \right]^{1/(\beta-1)} \frac{\Gamma m_e c^2}{1+z} L_{\text{ann}} L_0^{-1} \left[\frac{\Gamma m_e c^2}{(1+z)\varepsilon_0} \right]^{2-\beta},$$

the pair photospheric emission model is ruled out. If we use $L_p \lesssim L_\gamma$ instead of $L_p > 0$, we obtain an upper limit on τ as well as a lower limit on $2n_\pm/n_p$ by replacing L_p and f_{cut} with L_γ and f_{max} , respectively, in equations (14) and (16).

The above arguments in this subsection can be applied even when the fireball is completely thin, i.e., the cutoff energy due to the pair-creation process in the source does not exist. If we know that this is the case from other means, we can replace $f_{\text{max}} \equiv f(\varepsilon_{\text{max}}, \Gamma)$ with $f(\varepsilon_{\text{peak}}, \Gamma)$ in the inequality (21) for $\alpha \lesssim 1$.

4. IMPLICATIONS

In this paper, we have shown that pair signatures can provide useful information about the fireballs in GRBs using only observable quantities. The strategy for acquiring physical quantities is summarized in the caption of Figure 1.

4.1. Examination of r and Γ

The determination of emission radii r is important not only for specifying the prompt emission model, but also for various other model predictions (e.g., neutrino production in the internal shock model is sensitive to emission radius r ; Murase & Nagataki 2006; Murase et al. 2006). After our work on pair signatures, Gupta & Zhang (2008) recently focused on this issue of the unknown emission radius. They re-expressed the cutoff energy as a function of r and Γ . By using equations (6) and (7), we can see that the emission radius r is obtained from observationally determined ε_{cut} , ε_0 , and the radiation energy of a subshell at ε_0 , $E_0 \sim L_0 \delta t_{\text{rise}} / (1+z)$, if we know Γ by other means (see Fig. 4). Here, δt_{rise} is the rise time of a pulse, which is basically determined by the comoving width of the subshell, $\Delta' \approx \Gamma c \delta t_{\text{rise}} / (1+z)$. Equation (1) is one way to determine Γ . Other means (e.g., by using the photospheric emission component; Pe'er et al. 2007) are also useful.

On the other hand, the emission radius can be also estimated via the relation $r \approx 2\Gamma^2 c \delta t_{\text{decay}} / (1+z)$, as noted in § 3.1. Once this relation is validated, we can compare the emission radius estimated from it with that determined from ε_{cut} . In other words, we can test whether Γ determined by equation (9) and $r \approx 2\Gamma^2 c \delta t_{\text{decay}} / (1+z)$ is consistent with Γ estimated from equation (1) and other means, or not. Because the derived Γ should be consistent if the emission radius is the same, they will be useful as another closure relation (see § 5). Note that we have not so far assumed the relation $r \approx 2\Gamma \Delta' \approx 2\Gamma^2 c \delta t_{\text{rise}} / (1+z)$, which is expected in the internal shock model, but may not be true. In fact, models other than the internal shock model do not always predict $r \approx 2\Gamma \Delta'$, but can lead to $r \gg 2\Gamma \Delta'$.

4.2. Test of the Pair Photospheric Emission Model

As already noted, pair signatures are especially useful to test the pair photospheric emission model, in which the prompt emission comes from $r_{\text{ph}} \approx r_{\text{ph},\pm}$. We can measure $\tau_\pm$ by equation (17) with L_p , and an upper limit on $\tau_\pm$ by the inequalities (18) without L_p . If we can observe either the pair-annihilation line or the cutoff energy due to pair creation, an upper limit on $\tau_\pm$ is obtained by equation (17) with the inequalities (19) or (21) for known L_p , and by the inequality (20) or (22) for unknown L_p . When the fireball is pair-dominated, i.e., the inequality (13) is satisfied, we have $\tau \approx \tau_\pm$. In addition, under the photospheric emission model, we expect that high-energy gamma rays are produced by dissipation around the photosphere (which may occur at the subphotosphere) and emerge from the emission region at $r \sim r_{\text{ph}}$. Therefore, the pair photospheric emission model predicts $\tau \approx \tau_\pm \sim 1$ in equations (11) or (14).

When the fireball is pair-dominated, the photospheric radius where $\tau \approx \tau_\pm \simeq 1$ can be expressed as

$$r_{\text{ph}} \approx r_{\text{ph},\pm} \simeq \frac{f_{\text{cut}} L_0 \sigma_T}{4\pi m_e c^3 \Gamma^3 q}, \quad (23)$$

where $q \equiv r_{\text{ph}} / \Gamma \Delta'$, which is expected to be an order unity factor in the internal shock model. Equation (23) is essentially the same equation as that shown in Rees & Mészáros (2005). Note that the relation $r \approx 2\Gamma \Delta'$ expected in the internal shock model leads to $q = 2$.

When $L_{\text{ann}} \sim \varepsilon L_\gamma(\varepsilon_{\text{ann}}) = L_0 [\Gamma m_e c^2 / (1+z)\varepsilon_0]^{2-\beta}$, the pair photospheric emission model, under which we expect $\tau \approx \tau_\pm \sim 1$, predicts a unique relation between ε_{cut} and ε_{ann} . Equation (11) yields (after the integration over ε_γ in eq. [10], which is the expression of f_{cut}),

$$\frac{\varepsilon_{\text{cut}}}{\varepsilon_{\text{ann}}} \sim \left[\frac{(\beta-1)}{\tau_\pm^2} \frac{16}{3\xi(\beta)} \right]^{1/(\beta-1)} \sim \left[(\beta-1) \frac{16}{3\xi(\beta)} \right]^{1/(\beta-1)}. \quad (24)$$

If pairs are created by the underlying continuum photons, the pair-annihilation line cannot exceed the continuum emission much prominently, i.e., $L_{\text{ann}} \sim L_0 [\Gamma m_e c^2 / (1+z)\varepsilon_0]^{2-\beta}$ (Ioka et al. 2007; Pe'er & Waxman 2004; Pe'er et al. 2006). Therefore, the relation (24) could be satisfied for many bursts under the pair photospheric emission model. Superposing low-quality spectra of many events by adjusting either ε_{ann} or ε_{cut} could help to find the other feature in this model.

5. DISCUSSION

Although pair signatures give us useful information, caution is warranted, because there are some uncertainties in obtained

quantities, and we have made several assumptions in deriving the equations and inequalities shown in this paper.

First, we have assumed that all the photons come from the same emission region. However, this might not be true. Although we assume the same emission radius for pair signatures as a first consideration, actual emissions may not come from the same emission radius. For example, let us consider cases where high-energy gamma rays come from two different emission radii r_1 and r_2 ($r_1 < r_2$). There will be three possibilities: case A, where the observed pair-annihilation line comes from r_1 , while the pair-creation cutoff coming from r_2 , $\varepsilon_{\text{cut},2}$ is higher than that from r_1 , $\varepsilon_{\text{cut},1}$; case B, where the observed pair-annihilation line comes from r_2 , while the pair-creation cutoff coming from r_1 , $\varepsilon_{\text{cut},1}$ is higher than that from r_2 , $\varepsilon_{\text{cut},2}$; and case C, where both the observed pair-annihilation line and (higher) pair-creation cutoff come from r_1 or r_2 .

We can further refine these into various subcases. In case A1, where the underlying continuum comes predominantly from r_1 at $\varepsilon \lesssim \varepsilon_{\text{cut},1}$, we will ideally see $\varepsilon_{\text{cut},1}$ coming from r_1 , as well as $\varepsilon_{\text{cut},2}$, which is the higher cutoff. Because we have higher $\tau_{\gamma\gamma}$ at smaller r (and/or for larger E_0), with a given Γ , the former cutoff could be naturally lower than the latter. If we see two ε_{cut} values in the photon spectrum (i.e., one due to $\varepsilon_{\text{cut},1}$, the other to $\varepsilon_{\text{cut},2}$, which is higher), our recipes would be applied to the line and the lower cutoff $\varepsilon_{\text{cut},1}$. In case A2, the underlying continuum predominantly comes from r_2 at $\varepsilon \lesssim \varepsilon_{\text{cut},2}$; if the outflow has similar Γ at the two emission radii, we would expect that time-resolved detailed observations could separate different emission radii; if we can use $r \approx 2\Gamma^2 c \delta t_{\text{decay}} / (1+z)$, the larger emission radius r leads to the longer δt_{decay} . On the other hand, if the outflow has different Γ values at the two emission radii, it would be useful to determine Γ independently in various ways. Although it may be observationally difficult, other means of estimation [e.g., by using the photospheric emission component and/or the relation $r \approx 2\Gamma^2 c \delta t_{\text{decay}} / (1+z)$] in addition to equation (1) would be useful in enabling us to evaluate Γ . If emissions come from the same emission radius, we expect that all of the Γ we obtain should be consistent. In case B1, where the underlying continuum dominantly comes from r_2 at $\varepsilon \lesssim \varepsilon_{\text{cut},2}$, since the pair-creation cutoff from r_1 is higher, and not completely masked by the underlying continuum from r_2 , we can see $\varepsilon_{\text{cut},2}$ below $\varepsilon_{\text{cut},1}$ as in case A1. We may apply our recipes to the line and the lower cutoff $\varepsilon_{\text{cut},2}$. In case B2, where the underlying continuum dominantly comes from r_1 at $\varepsilon \lesssim \varepsilon_{\text{cut},1}$, the higher cutoff $\varepsilon_{\text{cut},1}$ comes from the inner radius r_1 , while $\varepsilon_{\text{cut},2}$ is masked, and the observed pair-annihilation line is generated at the larger radius r_2 . This would typically require that the Lorentz factor at the outer emission radius r_2 is smaller than that at the inner emission radius r_1 , because the prominent pair-annihilation line and the lower $\varepsilon_{\text{cut},2}$ would mean copious pairs and photons at r_2 . Therefore, evaluation of Lorentz factors by several means would be important. In case C1, where both pair signatures come from r_1 while the underlying continuum dominantly comes from r_2 at $\varepsilon \lesssim \varepsilon_{\text{cut},2}$, we can see $\varepsilon_{\text{cut},1}$ above $\varepsilon_{\text{cut},2}$ in principle. Hence, our recipes can be applied to the line and higher cutoff, while if we use the lower cutoff, we could obtain a Lorentz factor that is inconsistent with other estimations. In case C2, where both pair signatures come from r_2 while the underlying continuum predominantly comes from r_1 at $\varepsilon \lesssim \varepsilon_{\text{cut},1}$, similar to case C1, we would ideally see two ε_{cut} . We can apply our recipes to the higher cutoff, and then obtain the Lorentz factor that is consistent with the estimation from equation (1).

Therefore, we should apply our recipes to time-resolved spectra, if possible, and then compare the Lorentz factors obtained by

several means in order to check the consistency. When we have two or more cutoffs, we can select the cutoff that provides the consistent Lorentz factor. Once we see that emissions come from the same radius, the recipes described in this paper can be used to obtain information about the fireballs of GRBs.

Second, we have assumed that sufficiently relativistic electrons cool down rapidly, $t_{\text{cool}} \ll t_{\text{ann}}$, which is expected in many models (Ioka et al. 2007; Pe'er & Waxman 2004; Pe'er et al. 2005). However, the pair-annihilation line might come from relativistic pairs. For example, in the slow dissipation scenario (Ghisellini & Celotti 1999), e.g., as might be expected from magnetic reconnection, the typical electron Lorentz factor at the end of the dynamical time γ_{cool} could be larger than unity (Pe'er et al. 2006). If $\gamma_{\text{cool}} > 1$, we should use $\varepsilon_{\text{ann}} \sim \Gamma \gamma_{\text{cool}} m_e c^2 / (1+z)$ instead of equation (1), and the expression of L_{ann} should also be modified [where L_{ann} is suppressed for $\gamma_{\text{cool}} \beta_{\text{cool}}^2 / (1 + \beta_{\text{cool}}^2) \gtrsim 1$; Svensson 1982]. In such a case, it becomes more difficult to observe the pair-annihilation line, since the width of the pair-annihilation line is broadened by more than an order of unity in energy due to the broad distributions of relativistic pairs, although we can check this observationally (Svensson 1982). If we can specify the distributions of electrons and positrons properly (e.g., thermal distributions), we could evaluate L_{ann} , Γ , and the shape of the pair-annihilation line with elaborate observational results in the future. But where the distributions of electrons and positrons are unknown, they are model-dependent, as demonstrated in Pe'er et al. (2005, 2006), which would cause possible ambiguities for our recipes.

Third, we have also assumed that the cutoff energy ε_{cut} is determined by attenuation via $\gamma\gamma \rightarrow e^+e^-$ in the source. However, the attenuation due to interaction with cosmic infrared background photons should be also taken into account when ε_{cut} is sufficiently high. This cosmic attenuation effect can make it difficult to determine the cutoff energy at the source, ε_{cut} . The observed maximum energy might also represent the maximum energy of accelerated electrons. In order to evaluate ε_{cut} properly, careful analyses will be needed. The secondary delayed emission may also be useful (Murase et al. 2007). Note that we can apply the recipe in § 3.2 even without the true ε_{cut} .

Pair signatures may be detected by the future *GLAST* satellite. However, the detection of pair-annihilation lines may be difficult due to line broadening, as discussed in § 2. Lines are observed as bumps, so that evaluated τ and $L_{\pm}$ will have uncertainties by some factor, due to observational difficulties in the precise determination of L_{ann} and Γ . We also anticipate applying our recipes to single pulses. Some GRBs can be regarded as single pulse events. For example, some bright bursts such as BATSE trigger numbers 647 and 999 exhibited relatively smooth, long, single pulses, which are well separated from other pulses. For such single pulses, we may expect emissions from the approximately same emission radius, although the spectrum also shows time-dependent evolution. Our recipes could also be applied to flares, where wider and smoother pulses are seen (Burrows et al. 2005; Ioka et al. 2005); a flare may be expected to come from approximately the same emission radius. However, the detection of pair-annihilation lines will be more difficult observationally, because pair-annihilation lines from flares are typically expected at ~ 10 MeV if the Lorentz factor of flare outflows is ~ 10 . Furthermore, emissions from flares will be contaminated by afterglow components.

The height of the pair-annihilation line may be comparable to the underlying continuum emission. Therefore, we have to collect sufficiently many photons to identify the pair-annihilation line. For example, if the height of the pair-annihilation line is larger

than the underlying continuum by a factor of ~ 2 , we need to collect ~ 20 photons for the 3σ detection at $\sim \varepsilon_{\text{ann}}$. When the spectrum of the prompt emission is expressed by a power law extending to sufficiently high energies, *GLAST*/LAT is expected to find ~ 70 GRBs per year under the criterion that >10 photons per bursts are collected for the energy threshold 30 MeV (Omodei et al. 2006). This suggests that, if a significant fraction of GRBs are accompanied by pair-annihilation lines, we expect good opportunities to see them.

We also note that there may be some uncertainties in determining ε_{cut} . Opacity skin effects can sometimes render the exponential attenuation $\exp(-\tau_{\gamma\gamma})$ a poor descriptor of attenuation with $1/(1 + \tau_{\gamma\gamma})$, which leads to broken power laws rather than

exponential turnovers (Baring 2006; Baring & Harding 1997). We expect that such ambiguities could be solved by observing the maximum energy for many events.

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