X-RAY BINARIES AND THE CURRENT DYNAMICAL STATES OF GALACTIC GLOBULAR CLUSTERS

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ABSTRACT

It has been known for over 30 years that Galactic globular clusters (GCs) are overabundant by orders of magnitude in bright X-ray sources per unit mass relative to the disk population. Recently a quantitative understanding of this phenomenon has developed, with a clear correlation between the number of X-ray sources in a cluster, $N_{\rm X}$, and the cluster's encounter frequency, Γ , becoming apparent. We derive a refined version of Γ that incorporates the finite lifetime of X-ray sources and the dynamical evolution of clusters. With it we find we are able to explain the few clusters that lie off the $N_{\rm X}$ - Γ correlation, and resolve the discrepancy between observed GC core radii and the values predicted by theory. Our results suggest that most GCs are still in the process of core contraction and have not yet reached the thermal equilibrium phase driven by binary scattering interactions.

Subject headings: globular clusters: general — methods: numerical — stellar dynamics

1. X-RAY SOURCES AND CLUSTER DYNAMICS

It was realized more than 30 years ago that Galactic globular clusters (GCs) are overabundant by orders of magnitude in bright X-ray sources per unit mass relative to the disk population (Clark 1975; Katz 1975). It was quickly understood that strong dynamical scattering interactions of binaries in the dense cluster cores should be responsible for this overabundance (Verbunt & Hut 1987). With the advances in X-ray astronomy made possible by observatories such as *Chandra*, the relationship between X-ray sources and core cluster dynamics has recently been quantitatively studied. Pooley et al. (2003) performed Chandra observations of many Galactic GCs down to a limiting luminosity of 4×10^{30} ergs s⁻¹ in the 0.5–6 keV range (which includes low-mass X-ray binaries [LMXBs] in outburst and quiescence, cataclysmic variables [CVs], millisecond pulsars [MSPs], and magnetically active main sequence binaries [ABs]), and looked for correlations between the number of Xray sources in each cluster and properties of the cluster itself. They found the strongest correlation with the "encounter frequency" Γ , a rough estimate of the current dynamical encounter rate in the cluster. More recently, Heinke et al. (2003) and Pooley & Hut (2006) have isolated the quiescent LMXBs (qLMXBs) and CVs, respectively, from the X-ray source populations, and have shown that their numbers are indeed consistent with dynamical formation.

These results represent quantitative, empirical evidence that dynamical encounters are responsible for the formation of Xray sources in clusters. However, they suffer from at least a few drawbacks. First, the correlation between the number of X-ray sources, N_x , and the encounter frequency appears to be sublinear, with $N_{\rm X} \propto \Gamma^{0.74 \pm 0.36}$, although for LMXBs the exponent is 0.97 ± 0.5 (Pooley et al. 2003). Second, there are three clusters for which N_x is significantly larger than predicted by Γ . In the original work of Pooley et al. (2003) it was already clear that NGC 6397 has an N_x that is ~5 times larger than predicted by the N_x - Γ correlation. Recent observations show that N_x is a factor of ~ 2 times that predicted by Γ for NGC 7099 (Lugger et al. 2007), and a factor of ~20 for Ter 1 (Cackett et al. 2006). The common thread among these three clusters is that they are observationally "core-collapsed," while all others in the Pooley et al. (2003) sample are not (Grindlay 2006). (A

possible exception is NGC 6752, whose collapsed core status is debated [Ferraro et al. 2003; Lugger et al. 1995].) A cluster is observationally termed core-collapsed if its surface brightness profile is consistent with a cusp at the limit of resolution, making it more difficult to measure the core radius. As described below, the collapsed core status of a cluster is linked to its dynamical state, implying that cluster evolution complicates the $N_{\rm x}$ - Γ correlation.

2. UNDERSTANDING CLUSTER CORE RADII

The evolution of a GC, being a bound self-gravitating system, is very similar to the evolution of a star, and comprises three main phases. In the "core-contraction phase," the first phase of evolution, the cluster's core contracts on a relaxation timescale, much like a pre-main-sequence star. Once the core density becomes large enough for binary stars to begin strongly interacting dynamically, and thus generating energy via superelastic encounters, the cluster settles into the "binary-burning phase," analogous to the main sequence in stars. In this phase the dynamical properties of the cluster core remain roughly constant (e.g., Gao et al. 1991; Heggie & Hut 2003; Fregeau & Rasio 2007). Once the binary population is exhausted in the cluster core, it will collapse² via the gravothermal instability, leading to extremely high central densities. Deep in collapse, an energy producing event, such as an interaction of a dynamically formed binary, will reverse the collapse, causing the core to rebound and enter the "gravothermal oscillation phase," in which the core continues to collapse and rebound (Heggie & Hut 2003). For a graphical representation of the three main phases of cluster evolution, see Figure 1 of Gao et al. (1991), Figure 5 of Fregeau et al. (2003), or Figure 29.1 of Heggie & Hut (2003).

Since there is strong observational evidence that Galactic GCs were born with significant binary fractions (Hut et al. 1992), and since the binary-burning phase is the longest lived phase of cluster evolution (perhaps tens of Hubble times; Gao et al. 1991; Heggie & Hut 2003; Fregeau & Rasio 2007; Trenti

² Note that due to clashing naming conventions between observation and theory, this core collapse does not necessarily correspond to the observational definition, as discussed in the text. We therefore suggest that some nondynamical term be used in place of the observational "core-collapsed" designation, or at least that the distinction between the observational and theoretical uses of the term be noted when it is used.

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et al. 2007), it is widely believed that most clusters observed today should be in this phase. Early approximate calculations suggested that the ratio of core to half-mass radius, r_c/r_h , in the binary-burning phase, is broadly consistent with observations of the ~80% of Galactic GCs that are not observationally corecollapsed (Gao et al. 1991; Fregeau et al. 2003). Recently, however, more accurate simulations have shown that the early calculations overestimate r_c/r_h by a factor of 10 or more (Heggie et al. 2006; Fregeau & Rasio 2007). These latest results are quite difficult to ignore, since they represent the concordance of two completely independent cluster evolution codes—one direct N-body, with minimal approximations and a natural inclusion of binary interactions; the other the approximate Hénon Monte Carlo method, with direct few-body integration of binary interactions. The values of r_c/r_h in the binary-burning phase agree quite well between the two codes, and furthermore agree quite well with semianalytical theory (Vesperini & Chernoff 1994). The result is that now only the core-collapsed clusters agree with the predicted values of r_c/r_h in the binary-burning phase, implying that if most clusters are in this phase some other energy generation mechanism is responsible for the measured core sizes. Several suggestions have been put forth for the energy source, including, most notably, central intermediate-mass black holes (IMBHs) (Trenti 2006).

One key feature of the evolution of GCs that has escaped careful attention, though, is the timescale of the initial phase of core contraction. Many numerical simulations have shown that it is of order ~10 *initial* half-mass relaxation times (Gao et al. 1991; Giersz & Spurzem 2003; Heggie et al. 2006; Fregeau & Rasio 2007). Since the current half-mass relaxation time for most clusters is ~1 Gyr and was likely much longer in the past, the core contraction phase may easily last longer than a Hubble time (e.g., Hurley 2007). In fact, recent N-body simulations have shown that for a range of initial binary fractions, the evolution of r_c/r_h over a Hubble time is a simple core contraction with no evidence of a binary-burning phase being reached (Hurley 2007). Thus the solution to the discrepancy between theory and observations in r_c/r_h could be the most mundane one, namely that core-collapsed clusters are in the binary-burning phase while the rest are still undergoing core contraction.

3. A REFINED Γ

First introduced by Verbunt & Hut (1987) the encounter frequency Γ is an estimate of the *current* dynamical interaction rate in the cluster, which is assumed to be proportional to the current number of observable X-ray sources. We refine this predictive quantity by including, among other factors, the recent history of the evolution of the core properties. We start with the general form of the interaction rate, then specialize to the Verbunt & Hut (1987) Γ , and our new version.

The interaction rate between two species of objects can be written generally as

$$\Gamma \equiv \frac{dN_{\text{int}}}{dt} = \int \int n_1 n_2 \sigma_{12} \left| \boldsymbol{v}_{12} \right| f(\boldsymbol{v}_{12}) d^3 \boldsymbol{v}_{12} d^3 \boldsymbol{r}, \qquad (1)$$

where n_i is the number density of species i, σ_{12} is the interaction cross section between the two species, v_{12} is their relative speed with f its distribution function, and the integral is carried out over relative velocity space and volume. For the dynamical creation of LMXBs, species 1 represents stellar binaries, while species 2 represents neutron stars. The dynamical formation of

CVs and other low-luminosity X-ray sources is rather more complicated, so species 1 and 2 generally represent single and binary star systems (see, e.g., Fig. 7 of Ivanova et al. 2006). Typically, the integral in equation (1) is approximated as

$$\frac{dN_{\rm int}}{dt} \propto \rho_c^2 r_c^3 / v_\sigma \,, \tag{2}$$

where ρ_c is the core mass density, v_σ is the core velocity dispersion, the integral has been approximated by the core value, the gravitational-focusing dominated cross section has been used, and $\rho_{c,1}/\rho_{c,2}$ is assumed to be constant for all clusters. In addition, it should be pointed out that only the proportionality in equation (1) has been preserved since several factors (some of which are not constant among clusters) have been dropped. More accurate approximations of equation (1) have been used, including numerical integrals over cluster models (Pooley et al. 2003), but all are estimates of the *current* interaction rate.

Dynamically formed X-ray binaries (XRBs) are known to have finite detectable lifetimes. For LMXBs, this lifetime varies from $\sim 10^5 - 10^7$ yr for red giant donors, to ~ 1 Gyr for mainsequence companions, to a few Gyr for ultracompacts (Ivanova et al. 2007). For CVs, this lifetime is ~ 1 Gyr (N. Ivanova 2007, private communication). An additional complication is that dynamically formed XRBs do not necessarily turn on as X-ray sources immediately following a strong interaction. In fact, the interaction that places a binary on the path to becoming an observable X-ray source typically occurs several Gyr before it becomes detectable (Ivanova et al. 2006, 2007).

Since the XRBs we see now formed a few Gyr or more ago, and since the recent dynamical history of clusters may have been quite variable, it is clear that the current number of observable sources should be proportional to the interaction rate integrated over time. We thus write

$$\Gamma \equiv N_{\text{int}} = \iiint n_1 n_2 \sigma_{12} |\mathbf{v}_{12}| f(\mathbf{v}_{12}) d^3 \mathbf{v}_{12} d^3 \mathbf{r} dt.$$
 (3)

We perform the time integration over an interval t_x , the typical detectable lifetime of an XRB, to a time t_i , the typical timescale between strong interaction and observability, in the past. We leave t_x and t_i as parameters, but take $t_x = 1$ Gyr and $t_i = 3$ Gyr as canonical values. We further simplify the integral by writing

$$N_{\rm int} = \int_{t_0 - t_l - t_x}^{t_0 - t_l} f_b f_{\rm co} n_c^2 \sigma_{\rm strong} \sqrt{2} v_\sigma \frac{4\pi}{3} r_c^3 dt, \tag{4}$$

with f_b the core binary fraction, $f_{\rm co}$ the core compact object fraction, n_c the core number density, r_c the core radius, v_σ the 1D core velocity dispersion, and $\sigma_{\rm strong}$ the cross section for a strong interaction between a binary and a single star. The factor of $\sqrt{2}$ is from taking the difference of two Maxwellian velocity distributions. The cross section is $\sigma_{\rm strong} \approx \pi a 2 G M / (\sqrt{2} v_\sigma)^2$, where a is the typical semimajor axis of a binary, M=3m is the total mass of the binary–single system, and m is the typical stellar mass. Taking f_b and $f_{\rm co}$ to be constant over the time

integral and substituting the definition of the core radius, $r_c^2 = 9v_d^2/4\pi Gmn_c$ (Heggie & Hut 2003), yields

$$N_{\rm int} = f_b f_{\rm co} \frac{81\sqrt{2}a}{4Gm} \int_{t_0 - t_1 - t_s}^{t_0 - t_1} v_{\sigma}^3 r_c^{-1} dt.$$
 (5)

This expression is similar to equation (2), but with the core properties integrated over time. We have kept all numerical factors for the sake of completeness—only the proportionality represented by this equation is needed for what follows. By substituting in for the time evolution of the core quantities in the integrand, one can use this expression to differentiate among the three different phases of cluster evolution. We exclude the gravothermal oscillation phase, since in this phase a cluster should have no more than a few binaries in its core (Heggie & Hut 2003), and all clusters considered here with measured core binary fractions show evidence for many more binaries than this (note that a recent estimate puts the core binary fraction of NGC 6397 at $15\% \pm 1\%$; Davis & Richer 2008).

In the binary-burning phase the core radius and velocity dispersion are constant, so the integral is easy to evaluate. For the core contraction phase, we adopt the time evolution of the core radius shown in the *N*-body models of Hurley (2007) approximated as $r_c(t) = r_{c,0}(10 - 9t/t_0)$, with $r_{c,0}$ the current core radius and t_0 the current cluster age. Since there is essentially no core energy support in the core contraction phase, for the relationship between core radius and central velocity dispersion we adopt the self-similar collapse model, with $v_\sigma^2 \propto r_c^{-0.21}$ (Binney & Tremaine 1987). The ratio of the number of interactions for a cluster in the binary-burning phase to the same cluster in core contraction is then

$$\frac{N_{\text{int,bb}}}{N_{\text{int,cc}}} = \left(\frac{t_x}{t_0}\right) \frac{2.835}{\left[t_0 I(t_0 + 9t_l)\right]^{0.315} - \left[t_0 I(t_0 + 9t_l + 9t_x)\right]^{0.315}}, \quad (6)$$

which has a minimum of 2.0 and a maximum of 17.8 in the range $t_x = 10^{-4}$ to 3 Gyr, $t_l = 1$ to 10 Gyr, for $t_0 = 13$ Gyr. For the canonical values of $t_x = 1$ Gyr and $t_l = 3$ Gyr with $t_0 = 13$ Gyr, the value is 5.0. Since the number of X-ray sources should scale roughly linearly with the number of interactions, this suggests that if a cluster is in the binary-burning phase (and has been for a time $t_1 + t_2$ to the present), it should have ~5 times as many X-ray sources than it would if it were in the core contraction phase. (If the Pooley et al. [2003] exponent of 0.74 is adopted this factor is 3.3.) Interestingly, the three clusters in Figure 1 that are observed to be core-collapsed are the three that have a significantly larger N_x than predicted by the standard N_x - Γ correlation, by a factor of ~2 to ~20. NGC 6397 has an overabundance factor of ~2, NGC 7099 a factor of ~5, and Ter 1 a factor of ~20. However, we note that the estimate of Γ for Ter 1 is more uncertain than the rest since it comes directly from equation (2) and not an integral over the cluster profile, and since the velocity dispersion is only estimated as it has not been measured observationally. The similarity between the observed overabundances and those predicted by our simple revision of Γ suggests that the observationally core-collapsed clusters are indeed in the binary-burning phase, while the rest are still in the process of core contraction.

A natural objection to this conclusion is that cluster metallicity may explain the X-ray source overabundance. Studies of bright LMXBs associated with star clusters in external galaxies have shown a strong correlation between cluster metallicity and LMXB incidence, with metal-rich clusters being much

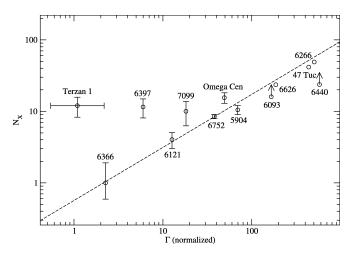


Fig. 1.—Number of observed cluster X-ray sources with $L_{\rm X} \gtrsim 4 \times 10^{30}$ ergs s⁻¹ for several Galactic GCs vs. the encounter rate Γ . The power-law fit and data points are from Pooley et al. (2003) with the exception of NGC 7099 (Lugger et al. 2007) and Ter 1 (Cackett et al. 2006). The $N_{\rm X}$ error bars for Ter 1, NGC 6397, and NGC 7099 represent source counting noise and background source uncertainty, but for the remaining clusters represent only background uncertainty.

more likely to harbor an LMXB (by a factor of 3 or more; Jordán et al. 2004; Sivakoff et al. 2007). However, this result appears to hold only for bright LMXBs. When looking at the low-luminosity LMXB and CV populations in our Galaxy (which comprise the majority of sources in Fig. 1), a correlation between source incidence and cluster metallicity is not clearly apparent (Heinke et al. 2006). In any case, the metallicities of the three overabundant clusters are not significantly larger than those of the other clusters in Figure 1, with [Fe/H] values of -1.30, -1.95, and -2.12 for Ter 1, NGC 6397, and NGC 7099, respectively, with the rest of the clusters ranging in value from -0.34 (NGC 6440) to -1.75 (NGC 6093) (Harris 1996).

Another possibility is that the XRBs in the low- Γ clusters may be mainly primordial, in which case their number should scale with cluster mass. However, were this the case, a look at cluster absolute magnitudes shows that NGC 6121 would have roughly as many sources as NGC 7099 and more than NGC 6397 and Ter 1, and NGC 6366 would have more than Ter 1 (Harris 1996). Note that the 2–5 sources detected in the low- Γ cluster NGC 288 (not included in this study), with $\Gamma \approx 0.53$ in the Pooley et al. (2003) normalization, are likely primordial (Kong et al. 2006).

4. DISCUSSION

This Letter presents the confluence of three suggestive observational and theoretical results into a self-consistent picture. The first is that of the clusters that have been observed sufficiently to determine their XRB population, the three that are core-collapsed are the same three that have a significant X-ray source overabundance (of a factor of ~2 to ~20). The second is the semianalytical result derived in this Letter that a cluster in the binary burning phase for the last few Gyr should have ~5 times more dynamically formed X-ray sources than if it were in the core contraction phase for the same time. The third is the recently confirmed discrepancy between observations and theory for the core radii of Galactic GCs, which suggests that only the observationally core-collapsed clusters are in the binary-burning phase. In light of these facts, the conclusion that seems strongly suggested is that most Galactic GCs are cur-

rently still in the core contraction phase, while only the $\sim 20\%$ of clusters that are core-collapsed are in the binary-burning phase. This goes counter to the widely held belief that most clusters are currently in the binary-burning phase, and complicates the many existing studies that have assumed cluster core properties that are constant with time.

The implications of this result are manifold. There are many studies of the dynamical production of interesting source populations in clusters which assume core properties that are constant with time. These include predictions of the formation of blue stragglers (e.g., Mapelli et al. 2004), the evolution of the core binary fraction (Ivanova et al. 2005), and tidal-capture binaries (e.g., Di Stefano & Rappaport 1994), among others. Revising the results may be as simple as scaling predicted source numbers, but may not be so simple for other quantities.

Studies of GC evolution have shown that clusters starting from very different initial conditions evolve toward a common range of values in many observable structural parameters in the binary-burning phase, including the concentration and ratio of core to half-mass radius (Fregeau et al. 2003; Heggie et al. 2006; Fregeau & Rasio 2007). Since most clusters may not be in the binary-burning phase after all, their observed properties are likely to be more strongly correlated with their initial conditions. This makes modeling of clusters a bit more complicated, but on the other hand allows one to more readily deduce something about the initial properties of clusters.

Perhaps anticlimactically, our results suggest that the alternative energy sources recently proposed for supporting GC cores are not required. These include the suggestion of IMBHs in *many* Galactic GCs (Trenti 2006), enhanced stellar mass loss

from stellar evolution of physical collision products (Chatterjee et al. 2008), mass segregation of compact remnants in young clusters (Merritt et al. 2004), or evaporation of the stellar-mass black hole subsystem in young clusters (Mackey et al. 2007).

Although the picture painted in this Letter is a suggestive one, there are still several caveats and limitations to our analysis. In the derivation of our refined Γ we assume that the core binary fraction and abundance of compact objects are constant over the time of integration. Neither is strictly true, although we expect they will not vary enough to significantly change the overabundance value we derive. We have used only one possible expression for the evolution of the core radius in core contraction. While we expect the general behavior to be very similar to what we have assumed here, more work needs to be done to determine if it is universal. In addition, our analysis ignores the effect of Galactic tidal stripping on cluster mass, which would make a cluster appear overabundant in X-ray sources, and may be relevant for NGC 6397 (Pooley et al. 2003). On the observational side, there are some uncertainties in evaluating Γ , which is dependent on quantities that are somewhat difficult to measure for core-collapsed clusters.

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