COINCIDENCES OF DARK ENERGY WITH DARK MATTER: CLUES FOR A SIMPLE ALTERNATIVE?

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ABSTRACT

A rare coincidence of scales in standard particle physics is needed to explain why Λ or the negative pressure of cosmological dark energy (DE) coincides with the positive pressure P_0 of random motion of dark matter (DM) in bright galaxies. Recently Zlosnik and coworkers proposed to modify the Einstein curvature by adding nonlinear pressure from a medium flowing with a four-velocity vector field U^{μ} . We propose to check whether a smooth extension of general relativity with a simple kinetic Lagrangian of U^{μ} can be constructed, and whether the pressure can bend spacetime sufficiently to replace the roles of DE, cold DM, and heavy neutrinos in explaining anomalous accelerations at all scales. As a specific proof of concept we find a vector-for- Λ model (V Λ model) and its variants. With essentially *no free parameters*, these appear broadly consistent with the solar system, gravitational potentials in dwarf spiral galaxies and the Bullet Cluster of galaxies, the early universe with inflation, structure formation, and big bang nucleosynthesis, and late acceleration with a 1 : 3 ratio of DM : DE.

Subject headings: cosmology: theory - dark matter - gravitation

The incompleteness of standard physics and Einstein's general relativity (GR) is evident from the smallness of the cosmological constant Λ or the vacuum energy density $\Lambda c^2/8\pi G \sim (0.001 \text{ eV})^4$, compared to the expected quantum pressure $c^5 m_{\rm P}^4 / \hbar^3 \sim (10^{28} \text{ eV})^4$ at scales of the Planck mass $m_{\rm P} = (\hbar c/G)^{1/2}$. Current speculations about the new physics of Λ are as free as analogous speculations about the *Pioneer* anomaly (Turyshev et al. 2006); both represent acceleration discrepancies of order $\sim 7a_0$, driven by unidentified (likely unrelated) pressures ~72 P_0 , where $a_0 \equiv 1.2$ Å s⁻² and $P_0 \equiv$ $a_0^2/8\pi G$ are scales of acceleration and pressure. On intermediate scales, galaxy clusters and spiral galaxies often reveal a discrepant acceleration of order $(0.1-2)a_0$. GR, if sourced primarily by baryons and photons with negligible mass density of neutrinos and other particles in the standard model or variations, appears to be an adequate and beautiful theory in the inner solar system, but it appears increasingly inadequate in accounting for astronomical observations as we move up in scale from 100 AU to 1 kpc to 1 Gpc. The universe made of known material of positive pressure should show a decelerating expansion as an open universe, but instead it is turning into an accelerating one now, evidenced by much dimmer supernovae detected at redshift unity. A standard remedy to restore harmony with GR and fit successfully larger scale observations (Spergel et al. 2007 and references therein) is to introduce a "dark sector," in which two exotic components dominate the matter-energy budget of the universe at redshift z with a split of Ω_{DE} : $\Omega_{\text{DM}} = 3 : (1 + z)^3$ approximately: dark energy (DE) as a negative-pressure and nearly homogeneous field described by unknown physics, and cold dark matter (DM) as a collisionless and pressureless fluid motivated by perhaps MSSM (minimal supersymmetric extension of the standard model) physics. However, anticipating several new particles from the Large Hadron Collider, the success of this concordance model still gives little clue to the physics governing the present 1:3 ratio of its constituents. This ratio is widely considered improbable, because standard particle physics expects a ratio 1 : 10^{120} . Here we speculate whether the 3 : $(1 + z)^3$ ratio could come from a coincidence of scales of $a_0 \equiv 1.2$ Å s⁻² with a cosmological baryon energy density $\rho_b c^2 \sim 3.5 \times (1+z)^3 P_0$.

A deeper link of DM and DE.—It is curious that the distribution of DM in dwarf galaxies is extremely ordered, something that the cuspy Λ CDM halos are still struggling to explain even with maximum baryonic feedback (Gnedin & Zhao 2002). For example, on galactic scales the Newtonian gravity of DM $g_{\text{DM}} = V_c^2/R - g_B$ and the Newtonian gravity of baryons $g_B = GM_B/R^2$ have a tight correlation:

$$(g^2/g_B)^n - g^n \approx a_0^n, \quad g \equiv g_{\rm DM} + g_B,$$
 (1)

where $n \ge 1$ (Zhao & Famaey 2006). This rule holds approximately at all radii R of all spiral galaxies of baryonic mass $M_B(R)$ and circular velocity $V_c(R)$ within the uncertainty of the stellar mass-to-light ratio and object distance. For low surface brightness galaxies or at the very outer edge of bright spirals, the gravity g is weaker than a_0 , and our empirical formula predicts $g^2/g_B = (V_c^2/R)^2/(GM_B/R^2) = V_c^4/(GM_B) \sim a_0$, which is essentially the normalization of the (baryonic) Tully-Fisher relation (McGaugh 2005). Bulges and the central parts of elliptical galaxies are dominated by baryons inside a transition radius where the baryons and DM contribute about equally to the rotation curve; equation (1) predicts $g_{DM} = g_B = a_0/2$. We can define a DM pressure $P_0 \equiv a_0(a_0/8\pi G)$ at the transition by multiplying the local gravity $(g_{DM} + g_B) = a_0$ with the DM column density above this radius $g_{\rm DM}/4\pi G = a_0/8\pi G$. This scale P_0 appears on larger scales too. All X-ray clusters have gas pressure and DM random energy density comparable to P_0 . The amplitude of the scale a_0 appears in the r^{-1} cusp of CDM halos too (Xu et al. 2007; Kaplinghat & Turner 2002). These can be understood since the last scattering shell at z = 1000 has a thickness $2L \sim 10$ Mpc and contains typical potential wells of depth $c^2/N \sim (1000 \text{ km s}^{-1})^2$ due to inflation, where $N \equiv 10^5$; hence the typical internal acceleration is $c^2/N/L \sim 0.2a_0$. Also, a DM sphere of radius 5 Mpc becoming nonlinear now would fall in with an acceleration ~200 \times $H_0^2 \times 5$ Mpc ~ a_0 . While correlations of baryons and DM can generally be understood in a galaxy formation theory where DM and baryons interact, the unlimited freedom of dark particles means a good spread in DM concentration, and hence

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the correlation would have substantial history-dependent variance from galaxies to galaxies and radii to radii. For example, DM is unexpected in tidal dwarf galaxies, but is observed because of its a_0 acceleration (Gentile et al. 2007). The tightness of such hidden regulations on DM at all radii for all galaxies is anomalous, at least challenging in the standard framework.

It is even more curious that DM in various systems and DE are tuned to a common scale P_0 , hence requiring a coincidence in two dark sectors. These empirical facts are unlikely random coincidences of the fundamental parameters of the dark sectors. Since all these anomalies are based on the gravitational acceleration of ordinary matter in GR, one wonders whether the dark sectors are not just a sign of an overlooked possible field in the gravitational sector.

Continuing along the lines of Zhao (2007), here we propose to investigate whether the roles of both DM and DE could be replaced by a vector field in a modified metric theory. This follows from two long lines of investigations pursued by Kostelecky & Samuel (1989), Jacobson & Mattingly (2001), Carroll & Lim (2004), and others on the consequences of symmetry-breaking in string theory, and by Milgrom (1983), Bekenstein (2004), Sanders (2005), Skordis et al. (2006), Zhao & Famaey (2006), and others driven by astronomical needs. These two independent lines were first merged by the pioneering work of Zlosnik et al. (2007). *The existence of an explicit Lagrangian* satisfying the main constraints for the solar system, galaxy rotation curves, and cosmological concordance ratio remains to be demonstrated.

Leading up to the vector field.—In Einstein's theory of gravity, the slightly bent metrics for a galaxy in a uniformly expanding background set by the flat FRW cosmology is given by

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -\left(1 + \frac{2\Phi}{c^2}\right)d(ct)^2 + \left(1 - \frac{2\Psi}{c^2}\right)a(t)^2dl^2, \quad (2)$$

where $dl^2 = dx^2 + dy^2 + dz^2$ is the Euclidian distance in Cartesian coordinates. In the collapsed region of galaxies, the metric is quasi-static with the potential $\Phi(t, x, y, z) = \Psi(t, x, y, z)$ due to DM plus baryons, which all follow the geodesics of $g_{\mu\nu}$.

Modified gravity theories are often inspired to preserve the weak equivalence principle; i.e., particles or small objects continue on the geodesics of the above physical metric independently of their chemical composition. Unlike in Einstein's theory, the strong equivalence principle and CPT can be violated by, e.g., creating a preferred frame using a vector field. The Einstein-Aether theory of Jacobson & Mattingly (2001) is such a simple construction, where a unit vector field U^{μ} is designed to couple only to the metric but not to matter directly. It has a kinetic Lagrangian with linear superposition of quadratic covariant derivatives $\nabla(c^2 U)\nabla(c^2 U)$, where $c^2 U^{\mu}$ is constrained to be a timelike four-momentum vector per unit mass by $-g_{\mu\nu}U^{\mu}U^{\nu} = 1$. The norm condition means the vector field introduces up to 3 new degrees of freedom; e.g., a perturbation in the FRW metric (eq. [2]) has $c^2 U_{\mu} \equiv g_{\mu\nu} c^2 U^{\nu} \approx (c^2 + \Phi)$, A_x/c , A_y/c , A_z/c), containing a four-vector made of an electriclike potential Φ and three new magnetic-like potentials. But for spin-0 mode perturbations with a wavenumber vector \boldsymbol{k} , we can approximate $U_{\mu} - (1, 0) \approx (\Phi/c^2, kV/c)$, which contains just one degree of freedom, i.e., the flow potential V(t, x, y, z). We expect that an initial fluctuation of $c|\mathbf{k}|V \sim |\Phi| \sim c^2 N^{-1} \equiv$ $10^{-5}c^2$ can be sourced by a standard inflaton; the vector field tracks the spectrum of metric perturbation (Lim 2005).

Most recently, Zlosnik et al. (2007) suggested replacing the

linear $\lambda \nabla U \nabla U$ with a nonlinear kinetic Lagrangian $F(\lambda \nabla U \nabla U)$ to extend Jacobson's framework. They showed that this class of nonlinear models is promising for producing the DE effect in cosmology and the DM-like effect in the weak-field limit. Here we continue along the lines of the pioneering authors, but aim for a single Lagrangian with parameters in a good match with basic observations of a range of scales.

A simple Lagrangian for Λ .—The difficulty of writing down a specific Lagrangian is that there are infinite ways to form pressure-like terms quadratic to covariant derivatives of the vector field. Simplicity is the guide when choosing gravity since GR plus Λ CDM largely works. Let us start with forming two pressure terms for any four-momentum-like field A^{μ} with a positive norm $mc^2 \equiv (-g_{\alpha\beta}A^{\alpha}A^{\beta})^{1/2}$ by

$$8\pi G\mathcal{J}(A) \equiv \frac{1}{3} \left(\frac{\nabla_{\alpha} A^{\alpha}}{m}\right)^2, \quad 8\pi G\mathcal{K}(A) \equiv \frac{\nabla_{\parallel} A^{\alpha}}{m} \frac{\nabla_{\parallel} A_{\alpha}}{m}, \quad (3)$$

where the right-hand sides are covariant with dimension of acceleration squared, and $\nabla_{\parallel} = A^{\alpha} \nabla_{\alpha}$ or ∇_{α} stands for the covariant derivative with spacetime coordinates along the direction of the vector *A* or the dummy index α , respectively. From these we can generate two simpler pressure terms *K* and *J* of the unit vector field U^{α} by

$$J \equiv \mathcal{J}(U) \sim 0, \ K \equiv \mathcal{K}(U) \sim \frac{|\nabla \Phi|^2}{8\pi G} \quad \text{in galaxies,}$$
$$J \equiv \mathcal{J}(U) \sim \frac{3c^2 H^2}{8\pi G}, \ K \equiv \mathcal{K}(U) \sim 0 \quad \text{in a flat universe,}$$
(4)

where the approximations hold for U^{α} with negligible spatial components and a nearly flat metric (eq. [2]). Note the J and K are constructed so that we can control timelike Hubble expansion and spacelike galaxy dynamics *separately*.² The Kterm, with a characteristic pressure scale $a_0^2/8\pi G = P_0$ in galaxies, is the key for our model. The J-term, meaning critical density, has a characteristic scale $N^2 P_0 \sim 10^{10} P_0$: at the epoch of recombination z = 1000 when baryons, neutrinos, and photons contribute \sim (8, 3, 5) $\times 10^9 P_0$, respectively, to the term $J = 3c^2 H^2/8\pi G$, the epochs of equality and recombination nearly coincide.

Now we are ready to construct our total action $S = \int d^4x |-g|^{1/2} \mathcal{L}$ in physical coordinates, where the Langrangian density

$$\mathcal{L} = \frac{R}{16\pi G} + L_m + L_J + L_K + (U_{\nu}U^{\nu} + 1)L^m, \qquad (5)$$

where *R* is the Ricci scalar, and L_m is the ordinary matter Lagrangian. For the vector field part, L^m is the Lagrangian multiplier for the unit norm and we propose the new Lagrangian

$$L_{J} = \int_{0}^{J} \lambda_{N} \left(\sqrt{\frac{|J|}{P_{0}}} \right) dJ, \quad L_{K} = \int_{\infty}^{K} \lambda_{n} \left(\sqrt{\frac{|K|}{P_{0}}} \right) dK, \quad (6)$$

² A full study should include spacelike terms $8\pi GK_{12} \equiv 2g^{\alpha\beta}(c^2\nabla_{\alpha}U^{\gamma})(c^2\nabla_{\beta}U_{\gamma}) - \frac{2}{3}(c^2\nabla_{\alpha}U^{\alpha})^2$ and $8\pi GK_{13} \equiv 2g^{\alpha\beta}(c^2\nabla_{\alpha}U^{\gamma})(c^2\nabla_{\beta}U_{\gamma}) - 2(c^2\nabla_{\alpha}U^{\beta})(c^2\nabla_{\beta}U^{\alpha})$ that change the details of structure formation, PPN parameters, and gravitational waves, which are beyond our goal here.

where the nonnegative continuous functions $\lambda_i(x) = [0, \lambda(x) - \lambda(N)]_{\max}$, $\lambda(x) = (1 + x/i)^{-n}$, where the subscript i = either n or N. Incidentally, n = 0 gives GR. The cutoffs (e.g., with $n = \pm 1$) guarantee a bounded Hamiltonian with kinetic terms L_K and L_J always bounded between $\pm N^2 P_0$ [e.g., in a lab near Earth $K \sim (10^{13} - 10^{22})P_0 > N^2 P_0$, so $L_K = 0$]. The condition at the tidal boundary K = J = 0 is well behaved too (see eqs. [44]–[48] of Famaey et al. 2007 on the Cauchy problem). Note that $1 - dL_K/dK > \mu_{\min} \equiv (1 + N/n)^{-n} \sim 10^{-15}$ and $1 - dL_J/dJ > \mu_B \equiv (1 + N/N)^{-n} \sim 2^{-3}$.

Taking variations of the action with respect to the metric and the vector field, we can derive the modified Einstein's equation (EE) and the dynamical equation for the vector field. The expressions are generally tedious (A. Halle 2007, in preparation), but the result simplifies in the perturbation- and matter-dominated regime that is of interest to us. As anticipated in Lim (2005) the *ij*-cross term of the EE yields $\Psi - \Phi = 0$ for all our models, which means incidentally twice as much deflection for light rays as in the Newtonian regime. As anticipated in Dodelson & Liguori (2006), the *ti*-term of the EE can be cast into that of an unstable harmonic oscillator equation with a negative string constant $\ddot{V} + b_1H\dot{V} - (1 - \mu_B)b_2H^2V = S(\Phi, \Psi)$ if $(1 - \mu_B) > 0$, so we expect that HV tracks Φ . The *tt*-term of the EE takes the form

$$8\pi G\rho = 3\mu_B H^2 + 2\nabla \cdot \left[(1 - \lambda_n)\nabla\Phi\right] - \Lambda_0 - Q(\dot{\Phi}, \dot{V}, V),$$
(7)

where we approximated $1 - \lambda_N(x) \sim 2^{-n} = \mu_B$ as a constant in the matter-dominated regime where $J < N^2 P_0$ and the *Q*-term is zero for static galaxies and a uniform FRW flat cosmology. So the *tt*-term of the EE reduces to the simple form

$$4\pi G\rho = \nabla^2 \Phi - \nabla \cdot \left[\lambda_n \left(\frac{|\nabla \Phi|}{a_0}\right) \nabla \Phi\right] \quad \text{in galaxies,} \quad (8)$$

$$\frac{8\pi G\bar{\rho}}{3\mu_B} = H^2 - \frac{\Lambda_0}{3\mu_B} \quad \text{in matter-dominated FRW.} \quad (9)$$

Here the pressure from the vector field creates new sources for the curvature. The term $\nabla[\lambda_n(x)\nabla\Phi]/4\pi G$ in the Poisson equation acts as if one were adding DM for quasi-static galaxies. A cosmological constant in the Hubble equation is created by

$$\frac{\Lambda_0 c^2}{8\pi G} = -\int_{\infty}^0 \lambda_n(x) d(P_0 x^2) \approx \frac{2(nP_0)^2}{(n-1)(n-2)}.$$
 (10)

For binary stars and the solar system, $4\pi G\rho - \nabla^2 \Phi \approx 0$ is true because the gravity at distances 0.3–30 AU from a Sunlike star is much greater than the maximum vector field gradient strength Na_0 , so $dL_K/dK = 0$; in fact, $|\nabla \Phi| \approx GM_{\odot}/r^2 \sim$ $(10^9-10^5)a_0$, and the typical anomalous acceleration is $Na_0\mu_{\min} \sim 10^{-10}a_0$, well below the current detection limit of $10^{-4}a_0$ (Soreno & Jezter 2006). This might explain why most tests of non-GR effects around binary pulsars and black holes and in the solar system yield negative results; Pluto at 40 AU and the *Pioneer* satellites at 100 AU might show interesting effects. Extrapolating the analysis of Foster & Jacobson (2006), we expect GR-like PPN (parameterized post-Newtonian) parameters and gravitational wave speeds in the inner solar system. *Near the edges of galaxies*, we recover the nonrelativistic theory of Bekenstein & Milgrom (1984) with a function

$$\mu(x) \equiv 1 - \lambda_n(x) \sim \mu_{\min} + x, \quad \text{if } x = \frac{|\nabla \Phi|}{a_0} \ll 1. \quad (11)$$

Note that $\mu(x) \rightarrow x$; hence rotation curves are asymptotically flat except for a negligible correction $\mu_{\min} \sim 10^{-15}$. In the intermediate regime x = 1 our function with $1 - \lambda_n(x) \sim 0.55-0.6$ for n = 2-5. Equation (1) argues that galaxy rotation curves prefer a relatively sharper transition than $\mu(x) = x/(1 + x) = 0.5$ at x = 1 (Famaey et al. 2007), where we can identify $g_B/(g_{\text{DM}} + g_B) = \mu(x)$. So our model should fit observed rotation curves.

For the Hubble expansion, the vector field creates a cosmological-constant-like term $\Lambda_0 c^2 / 8\pi G \approx 9P_0$ below the zero point of the energy density in the solar system because the zero point of our Lagrangian (eq. [6]) is chosen at $N^2 P_0 \leq K < +$ ∞ . During matter domination, the contribution of matter $8\pi G\rho$ and Λ_0 to the Hubble expansion H^2 (eq. [9]) is further scaled up because the effective gravitational constant $G_{\rm eff}$ = $G/\mu_B = 2^n G \ge G$, with GR being the n = 0 special case.³ Coming back to the original issue of the 3:1 ratio of matter density to our cosmological constant, equation (9) predicts that $(\Lambda_0 c^2 / 8\pi G\mu_B) : (\bar{\rho}_b c^2 / \mu_B) \sim (9P_0 / \mu_B) : [4(1+z)^3 P_0 / \mu_B], \text{ which}$ is close to the desired $3 : (1 + z)^3$ ratio. Adding neutrinos makes the explanation slightly poorer. So the DE scale is traced back to a separate coincidence of scale, i.e., the present baryon energy density $\bar{\rho}_b c^2 \sim 4P_0$, where P_0 contains a scale a_0 for the anomalous accelerations on galactic scales. Our model predicts that DE is due to a constant of vacuum, preset by the modification parameter *n* of the gravity; n = 0 gives GR.

In our model, the effective DM (the dog) follows the baryons (the tail) throughout the universal $(1 + z)^3$ expansion with a ratio set by *n*. To fit the Λ CDM-like expansion exactly, we note the Hubble equation for a flat FRW cosmology with the vector field and standard mix of baryons, neutrinos, and photons $\Omega_b h^2/0.02 \approx (\Omega_p h^2/0.002)(0.07 \text{ eV}/m_p) \approx \Omega_{\text{ph}} h^2/0.00025 \sim 1$ yields at the present epoch

$$\frac{\Omega_b + \Omega_{\nu} + \Omega_{\rm ph}}{\mu_B} = 1 - \frac{\Lambda_0}{3\mu_B H_0^2} = \Omega_m^{\Lambda \rm CDM}.$$
 (12)

The second equality fixes $\mu_B^{-1} = 2^n = 8-8.4$ if we adopt $a_0/c \approx H_0/6 \approx 12$ km s⁻¹ Mpc⁻¹ and $\Omega_m^{\Lambda \text{CDM}} = 0.25-0.3$. The first equality would predict an uncertain but very small neutrino mass $m_{\nu} \sim \pm 0.3$ eV.

Big bang nucleosynthesis (BBN) also anchors any modification to GR. In the radiation-dominated era $|J| = 3c^2H^2/8\pi G \gg N^2P_0$, the dynamics are driven by

$$8\pi G\rho \approx 3H^2 - \Lambda_0 - \Lambda_N$$
 in radiation-dominated FRW,

where $\Lambda_N c^2/8\pi G = -\int_0^\infty \lambda_N(x)d(P_0N^2x^2) = -N^2P_0/8$ for n = 3 is a finite negative number, much smaller than the radiation pressure $\sim (z/1000)^3 N^2 P_0$. So the early universe is GR-

(13)

³ While Carroll & Lim (2004) found a scaling down of *G* because they were interested in stable spin-0 modes with $(1 - \mu_B) < 0$ for a restricted class $(c_4 = 0 \neq c_1)$ of Jacobson's models, Dodelson & Liguori (2006) argue that an unstable growth of the vector field is helpful to structure growth in many gravity theories.

like, especially the Hubble parameter at BBN, insensitive to the precise value of $N^2 P_0$.

Note a more general version of our vector-for- Λ model has a Lagrangian

$$L_{K} + L_{J} = \lambda_{K} \mathcal{K}(U\lambda_{K}^{1/\mathcal{N}}) + \lambda_{J} \mathcal{J}(U\lambda_{J}^{1/\mathcal{N}}) - P_{0} \mathcal{V}(\lambda_{K}, \lambda_{J}), \quad (14)$$

with 4 vector degrees in $U\lambda_J^{1/N}$ and 1 scalar degree of freedom in $\lambda_{\kappa}/\lambda_{\mu}$. Our simple model is equivalent to the special case of two nondynamical scalar fields λ_K and λ_J with $1/N \sim 1/N \rightarrow$ 0, hence $\mathcal{K} = \mathcal{K}(U) = K$ and $\mathcal{J} = \mathcal{J}(U) = J$ (eq. [3]). The potential is smooth with $P_0 \mathcal{V}(\lambda_K, \lambda_J) = \int_{\mu_{\min}}^{1} [\mathcal{H}(\mu_{\min} + \lambda_K - \lambda_J) - (N^2/n^2)\mathcal{H}(\lambda - \lambda_J - 2^{-n})]P_n d\lambda$, where $P_n \equiv (\lambda^{-1/n} - \lambda_J) = (\lambda^{-1/n} - \lambda_J) = (\lambda^{-1/n} - \lambda_J)$ $(1)^2 n^2 P_0$ and $\mathcal{H}(y)$ is the Heaviside function of y. A vector field $A_{\mu} \approx (mc^2 + m\Phi, mcA)$ with a mass scale *m* has a quantum degeneracy pressure limit $\sim (c^5/\hbar^3)m^4$. It is intriguing that our model suggests the existence of a zero-point vacuum energy $\Lambda_0 c^2 / 8\pi G \sim P_0 \mathcal{V}(1, 1) \sim 9P_0 \sim (0.001 \text{ eV})^4$. And the (positive) radiation pressure at the epoch of baryon-radiation equality coincides with the cutoff energy density $P_0 \mathcal{V}(0, 0) \sim -N^2 P_0 \sim$ $-(0.3 \text{ eV})^4$, and the vacuum-to-cutoff energy density ratio ~ $9/N^2 \sim 10^{-9}$ coincides with the cosmic baryon-to-photon or baryon-to-neutrino number ratio $\eta \sim 3 \times 10^{-10}$, due to a tiny asymmetry with antibaryons. Can theories such as quantum gravity and inflation explain these coincidences? Understanding these might give clues to how the four-vector potential of photons decouples from the baryon current vector, and decouples from our E&M-like vector field A^{μ} in spontaneous symmetry breaking in string theory (Kostelecky & Samuel 1989; Carroll & Shu 2006; Ferreira et al. 2007).

Massive neutrinos are optional for our model because the L_j term creates a massive-neutrino-like effect in cosmology without affecting galaxy rotation curves. There are a few ways to create the impression of a fluid of 2 eV neutrinos in clusters of galaxies as well (Angus et al. 2007; Sanders 2005; Zlosnik et al. 2007). For example, a general Lagrangian with $\mathcal{N} \sim n$ would have new dynamical freedoms $\mu \equiv 1 - \lambda_k$ and $1 - \lambda_j$, which satisfy second-order differential equations in time in

galaxies, reminiscent of fluid equations for DM. Then the Bekenstein-Milgrom μ -function would acquire a history-dependent nonlocal relativistic correction of order $c/Na_0\tau \sim 1$ if the temporal variation (relaxation) timescale τ of the scalar field λ_{κ} is comparable to the Hubble time. This dynamical correction is hard to simulate, but is most important at the tidal boundary of (merging) systems where a condensate of the dynamical freedoms λ_{κ} and λ_{J} oscillates rapidly and could in principle act as an extra DM source for explaining some outliers to the Bekenstein-Milgrom theory, e.g., the merging Bullet Cluster with its efficient lensing and high speed (Angus & McGaugh 2007). A dynamical field λ_{J} is desirable as an inflaton to seed perturbations (Kanno & Soda 2006).

In summary, we demonstrate as a proof of concept that *at least one alternative* Lagrangian for gravity (eqs. [5] and [14]) can be sketched out to resemble GR plus Λ CDM on large scales and in the Hubble expansion, and to make an excellent fit to the rotation curve data of dwarf galaxies. The keys are a zeropoint pressure scale P_0 at the edge of galaxies, and a universal convergence source term $[(1 - \mu_B)/8\pi G](c^2\nabla_{\alpha}U^{\alpha})^2$ below the cutoff pressure N^2P_0 , which is near the epoch of equality and the last scattering. However, the CMB should be sensitive to the $\mu_B \equiv 2^{-n}$ modification parameter.⁴ It should be feasible to falsify the present model and variations by simultaneous fits to supernova distances and the CMB.

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⁴ In the radiation-dominated era, the perturbed Poisson equation for radiation is approximately $16\pi G\delta\rho \approx 2 |\mathbf{k}|^2 \Phi$, at very short wavelength (hence large $x \sim |\mathbf{k}| \Phi/a_0 \gg 1$), where $\Phi = \Psi$. But after recombination, $Q \sim 2q |\mathbf{k}|^2 \Phi \propto 2(1 - \mu_B) |\mathbf{k}|^2 \Phi$, resembling a dissipationless DM term $4\pi G\delta\rho_{\rm DM}$ to make the matter perturbation grow as $4\pi G\delta\rho \approx (1 - q) |\mathbf{k}|^2 \Phi$.

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