The Astrophysical Journal, 665:1277–1287, 2007 August 20

GROWTH OF STRUCTURE SEEDED BY PRIMORDIAL BLACK HOLES

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Received 2006 August 29; accepted 2007 April 9

ABSTRACT

We discuss the possibilities for the growth of primordial black holes (PBHs) via the accretion of dark matter. In agreement with previous works, we find that accretion during the radiation-dominated era does not lead to a significant mass increase. However, during matter domination, PBHs may grow by up to 2 orders of magnitude in mass through the acquisition of large dark matter halos. We discuss the possibility of PBHs being an important component in dark matter halos of galaxies, as well as their potential to explain the ultraluminous X-ray sources (ULXs) observed in nearby galactic disks. We point out that although PBHs are ruled out as the dominant component of dark matter, there is still a great deal of parameter space that is open to their playing a role in the modern-day universe. For example, a primordial halo population of PBHs each at $10^{2.5}\,M_{\odot}$, making up 0.1% of the dark matter, grows to $10^{4.5}\,M_{\odot}$ via the accumulation of dark matter halos and accounts for $\sim$10% of the dark matter mass by a redshift of $z \approx 30$. These intermediate-mass black holes may then "light up" when passing through molecular clouds, becoming visible as ULXs at the present day, or they may form the seeds for supermassive black holes at the centers of galaxies.

Subject headings: accretion, accretion disks — black hole physics

Online material: color figures

1. INTRODUCTION

It is well established that supermassive black holes (SMBHs) with masses in the range $m_{\rm BH} \sim 10^6$–$10^{9.5}\,M_{\odot}$ reside in the centers of spheroidal systems (Bernardi et al. 2003). One can make a convincing case that these have grown largely through accretion, with the consequent energy emission observed in electromagnetic output and jets at an efficiency of $\epsilon \sim 0.1$ (Sołtan 1982; Yu & Tremaine 2002). Observations of distant quasars have shown us that these SMBHs are already in place by redshifts of 6 and greater, but the mechanism of their formation remains a mystery.

Motivated by this question, we examine the potential for primordial black holes (PBHs) to grow through accretion to become seed masses for SMBHs. Primordial black holes, defined as black holes forming in the early universe without stellar progenitors, were first proposed by Zel'dovich & Novikov (1967) and Hawking (1971) as a possible consequence of the extremely high densities achieved in the big bang model. If they do indeed form in the early universe and can avoid evaporation (Hawking 1975) up to the present day, they must still exist and they may be important. We use a combination of analytical and numerical methods to follow PBH growth through the radiation- and matter-dominated eras and show that a PBH can multiply its mass by up to 2 orders of magnitude through the accretion of a dark matter halo.

If PBHs can grow sufficiently by accretion (or if they are very large at birth), they may account for intermediate-mass black holes (IMBHs) in the mass range $10^2\,M_{\odot} \lesssim M \lesssim 10^4\,M_{\odot}$, which have been suggested as the engines behind ultraluminous X-ray sources (ULXs) recently discovered in nearby galactic disks (Dewangan et al. 2006; Madhusudhan et al. 2006; Miller 2005; Mushotzky 2004). On the basis of the observed ULX luminosities of $\sim 10^{39}$ erg s^{-1}, stellar-mass black holes are ruled out unless the emission is highly beamed; there is at present no evidence for beaming (Mushotzky 2004; Freeland et al. 2006; Poutanen et al. 2007; Miniutti et al. 2006). IMBHs have the appropriate Eddington luminosity to explain ULXs, but since there is currently no easy way to produce black holes of this mass from stellar collapse at the abundances observed (Fryer & Kalogera 2001), their origins are highly debated. Some have suggested that ULXs could be powered by black hole remnants of an early population of very massive, metal-poor stars (Heger et al. 2003), but there is no consensus on the possible abundance of such remnants (see, e.g., Kuranov et al. 2007; Micic et al. 2007).

We suggest that PBHs may be able to grow to sufficient masses through the accretion of dark matter halos to account for a pervasive population of IMBHs. These IMBHs may also be important in the buildup of SMBHs, as suggested by recent numerical studies (Micic et al. 2005), a scenario that may be testable with the Laser Interferometer Space Antenna (LISA) in coming years (Micic et al. 2006; Will 2004; Fregeau et al. 2006).

Aside from observational indications, there have been physics-based inquiries indicating the plausibility of the production of PBHs through a variety of mechanisms in the early universe (Carr 2005), as briefly reviewed in § 2. One may wish to ask at this point what constraints on PBH production exist, given current observational limits. This important question is addressed in § 8.1 and Figure 4. We will show that there is ample room for a population of PBHs that is both permitted and interesting.

Our study of PBH accretion is organized as follows. In § 2 we discuss theories of PBH formation. In § 3 we outline our accretion model. Sections 4 and 5 describe the accretion calculations in the radiation and matter eras, respectively, while in § 6 we do a combined calculation for both eras. In § 7 we give our results for the total accretion possible for a PBH, and in § 8 we discuss the implications of our findings for the possible importance of PBHs in the present-day universe. Appendices A and B present details of our accretion calculations.

2. PRIMORDIAL BLACK HOLE PRODUCTION

There has been a great deal of interest in primordial black hole production in the early universe, resulting in the proposal of a variety of formation mechanisms. We refer the reader to a review of PBHs (Carr 2005) for an overview of the possibilities, briefly summarized here. In one mechanism of interest (Jedamzik 1997), PBHs form at a QCD phase transition at $\sim 1\ M_{\odot}$, a scale of interest for microlensing studies. PBHs formed at higher mass through other mechanisms may be natural candidates to solve other problems, such as that of the nature of ULX engines. In general, most mechanisms create PBHs at about the horizon mass, given by (Carr 2005)

$$M_H(t) \approx \frac{c^3 t}{G}, \tag{1}$$

or, in terms of cosmic temperature T,

$$M_H(T) \approx (1\ M_{\odot}) \left(\frac{T}{100\ \mathrm{MeV}}\right)^{-2} \left(\frac{g_{\mathrm{eff}}}{10.75}\right)^{-1/2}, \tag{2}$$

where g_{eff} is the number of effective relativistic degrees of freedom.

Briefly, some common mechanisms for creation of PBHs in the early universe are as follows. (1) PBHs formed at the QCD phase transition (mentioned above), when conditions temporarily allow regions of modest overdensity to collapse into black holes when they enter the horizon (Jedamzik 1997). The PBHs formed in this way would have a mass spectrum strongly peaked at the QCD epoch horizon mass ($\sim 1\ M_{\odot}$). (2) The collapse of rare peaks in the density field of the early universe. In this case, the probability of PBH formation at a given epoch is determined by the nature and evolution of the perturbations. (3) The collapse of cosmic string loops (Caldwell & Casper 1996; Garriga & Sakellariadou 1993; Hawking 1989; Polnarev & Zembowicz 1991; MacGibbon et al. 1998). Due to frequent collisions and reconnections, cosmic strings may occasionally form loops compact enough that the loop is within its Schwarzschild radius in every dimension. (4) A soft equation of state (Khlopov & Polnarev 1980). If the equation of state becomes soft (e.g., during a phase transition), PBHs may form at peaks in density as pressure support weakens. (5) Bubble collisions (Crawford & Schramm 1982; Hawking et al. 1982; La & Steinhardt 1989). During spontaneous symmetry breaking, bubbles of broken symmetry may collide, in some cases focusing energy at a point and producing a black hole. In this mechanism, the PBHs would form at the horizon mass of the phase transition. (6) Collapse of domain walls (Berezin et al. 1983; Ipser & Sikivie 1984). Closed domain walls forming at a second-order phase transition may collapse to form PBHs. In the case of thermal equilibrium, this would result in very small masses, but see Rubin et al. (2000) for a discussion of how nonequilibrium conditions may result in significant PBH masses.

For cases in which the mass of the PBH is low at creation, the PBH may evaporate before the present day through Hawking radiation (Hawking 1975). The limiting mass for evaporation by the present day is 10^{15} g; in general the evaporation time is given by (Carr 2003)

$$\tau(M) \approx \frac{\hbar c^4}{G^2 M^3} \approx 10^{64} \left(\frac{M}{M_{\odot}}\right)^3 \ \mathrm{yr}. \tag{3}$$

If PBHs are to arise directly from primordial density perturbations, it is required that the scale of fluctuations set down by inflation be "blue"; i.e., the spectrum must have more power on small scales. In terms of inflationary parameters, this implies that the scalar spectral index $n > 1$, which is disfavored in the latest *WMAP* results (Spergel et al. 2007).

Some mechanisms, such as the collapse of density peaks, may result in PBHs forming in clusters. For a discussion of the consequences of clustering, see Chisholm (2006). Formation via domain wall collapse, as discussed in Dokuchaev et al. (2004), may also lead to clustering, without relying on initial dark matter perturbations. In that scenario, primordial black holes can grow through mergers to form galaxies without the help of initial perturbations in the dark matter.

In this work, we assume that the PBHs are rare and isolated rather than appearing in clusters, but we expect that clustered PBHs would increase accretion power, so in that respect our treatment is a conservative one. We may refer to a specific PBH seed mass when convenient for illustrative purposes, but it should be noted that our results are independent of the PBH formation mechanism.

3. ACCRETION MODEL

3.1. *Setup*

We model accretion of matter onto primordial black holes in both the radiation era and the matter era. In both cases we follow the calculations for radial infall, following previous work on the growth of clusters (Gunn & Gott 1972; Bertschinger 1985; Fillmore & Goldreich 1984). Acknowledging that in a realistic accretion model the infall is unlikely to be perfectly radial, we make the simplifying assumption that the angular momentum of the infalling matter causes it to accrete in a halo around the PBH rather than incorporating itself into the PBH itself. This assumption is conservative from the standpoint of an estimation of the PBH's mass increase.

A PBH clothed by a dark matter halo will have the accreting power of an object having the total mass of the PBH plus the halo, to the extent that the accretion radius (e.g., Bondi radius) is larger than the radius of the PBH dark matter halo. However, constraints on the PBH's effect on the power spectrum apply only to the seed mass, not to the total mass of the clothed PBH, since the mass accreted by the PBH is drawn from the surrounding matter, and the additional mass is therefore "compensated." In other words, a region may be defined around the PBH for which the overdensity is due only to the original PBH, with no contribution from the accreted mass.

The increased mass due to the accreted halo will aid in the accretion of baryons by increasing the overall mass in the system. It will also assist in mass gain via mergers, as the dark matter halo will contribute to dynamical friction.

3.2. *Assumptions*

In all cases, we use the cosmological parameters derived from the third-year *WMAP* data release (*WMAP* III; Spergel et al. 2007). Specifically, we use the parameter set derived from the assumption of a flat, ΛCDM universe, with the combination of *WMAP* III and all other data sets ($\Omega_{\Lambda} = 0.738$, $\Omega_m = 0.262$, $h = 0.708$, and $\sigma_8 = 0.751$). In both the radiation- and matter-era calculations, we consider the accretion of dark matter only. In the radiation era, we assume that the radiation is too stiff to accrete appreciably, as suggested in many past analyses (Carr & Hawking 1974; Custódio & Horvath 1998; Niemeyer & Jedamzik 1999). In the matter era, we ignore the small contribution to the PBH mass that is due to the accretion of baryons. We make the further assumptions that each PBH is stationary and isolated, and that the surrounding matter is initially in the Hubble flow.

In all our accretion models, we end the calculation at a sufficiently high redshift that the effect of the cosmological constant is negligible. For completeness, however, we include in Appendix B the outline of the calculation with the cosmological constant included.

For a more detailed analysis of the consequences of gas accretion onto PBHs, we refer the reader to a companion paper (M. Ricotti et al. 2007, in preparation; hereafter Paper II). Also, Ricotti (2007) discusses in detail "Bondi-type solutions" including cosmological effects and the growth of the "clothing" dark matter halo on which we focus in the present work.

3.3. *PBH Velocities*

Our accretion estimate would decrease if the PBHs were moving quickly relative to the dark matter surrounding them; here we assume that the PBHs are initially stationary. We justify this assumption by considering the likely effect of nearby density perturbations in the dark matter. At any epoch, we can estimate the mass scale at which structures are becoming nonlinear by calculating the variance of density perturbations from an estimate of the matter power spectrum. The following fitting formula approximates the 2 σ mass perturbations:

$$M_{2\sigma} = (1 \times 10^{17}\ M_\odot) \exp\left[-5.57(1+z)^{0.57}\right]. \tag{4}$$

From this, we may calculate the characteristic circular velocity and thus the typical proper velocity of PBHs as a function of redshift:

$$v_p \sim v_c = (17\ \mathrm{km\ s^{-1}}) \left(\frac{M_{2\sigma}}{10^8\ M_\odot}\right)^{1/3} \left(\frac{1+z}{10}\right)^{1/2}. \tag{5}$$

For redshifts down to $z \sim 30$, we consider the PBHs to be stationary.

Peculiar velocities become important when PBHs get captured by the dark halos of the first galaxies with mass $M \sim 10^5$–$10^6\ M_\odot$ forming at $z < 30$. For $100 < z < 1000$, PBHs move at about the sound speed with respect to the gas. This is due to Silk damping of gas perturbations, while PBHs are not affected by it. In linear theory, the velocity of the gas and dark matter are simply related to the power of the respective density perturbations. The peculiar velocity affects the gas accretion rate and the radiative efficiency, but it does not affect the calculations of the growth of the dark halo in this work. On the other hand, through a similar calculation one can find the angular momentum of the accreted dark matter. In Paper II, we find that the angular momentum is sufficiently large to forbid direct accretion of dark matter onto the central PBH, which leads to a halo profile that is a power law with slope -2.25.

We stop our calculation at redshift $z = 30$ and do not extrapolate all the way to $z = 0$ because the assumption of accretion from a uniform density medium with no external tidal forces fails at late times. However, in principle, in rare cases where the PBHs were isolated and had not come into contact with nonlinear systems, an additional mass gain of at most a factor of 31 would be possible, bringing the total growth since formation to be on the order of 10^3 times the original seed mass. Such uninterrupted growth is unlikely, as it could only occur if the PBHs were neither incorporated into larger galactic halos nor subjected to strong tidal forces.

4. RADIATION ERA

The details of an analytical estimate of accretion in the radiation era can be found in Appendix A. Here we outline the basic idea and quote the result of a numerical calculation.

We begin the calculation at a redshift of $z \approx 10^7$. The exact starting redshift we choose in the radiation era does not affect the final result. In the radiation era, the motion of a dark matter shell at a distance r from the black hole is governed by the differential equation

$$\frac{d^2r}{dt^2} = -\frac{Gm_{\mathrm{BH}}}{r^2} - \frac{1}{4}\frac{r}{t^2}, \tag{6}$$

where m_{BH} is the black hole mass and t is time. With the initial conditions

$$r = r_i, \quad \frac{dr}{dt} = H_i r_i = \frac{1}{2}\frac{r_i}{t_i} \tag{7}$$

at $t = t_i$, we evolve these equations forward in time until matter-radiation equality at $z_{\mathrm{eq}} \approx 3 \times 10^3$. When a shell turns around ($\dot{r} = 0$), we assign the matter in that shell to the PBH's dark matter halo. We find that the PBH can accrete a dark matter halo that is on the order of its original mass:

$$\frac{m_{h,\,\mathrm{rad}}}{m_{\mathrm{BH}}} \approx 1. \tag{8}$$

5. MATTER ERA

The evolution of a spherically symmetric overdensity in the matter-dominated era has been treated in the case of the growth of clusters (Gunn & Gott 1972; Bertschinger 1985; Fillmore & Goldreich 1984). These analyses neglect the effect of the cosmological constant, assuming that $\Omega_\Lambda = 0$ and $\Omega_m = 1$. In the general case in which $\Lambda \neq 0$, the equation of motion of a shell of dark matter at a radius r from the PBH becomes (Lahav et al. 1991)

$$\frac{d^2r}{dt^2} = -\frac{Gm_{\mathrm{BH}}}{r^2} + \frac{\Lambda r}{3}. \tag{9}$$

The cosmological constant term affects the accretion at redshifts that are of order 1, but since we halt our accretions at higher redshift, we find growth that is consistent with the Gunn & Gott (1972) and Bertschinger (1985) result:

$$m_h \sim t^{2/3}, \tag{10}$$

with the turnaround radius of the dark matter halo (which we identify as the effective radius of the halo) growing with time as

$$r_{\mathrm{ta}} \sim t^{8/9}. \tag{11}$$

We choose the turnaround radius as the halo radius to be able to better compare with calculations such as those in Bertschinger (1985), although this choice does not significantly affect the final result. The details of the $\Lambda \neq 0$ calculation are discussed in Appendix B.

For a PBH that begins growing at matter-radiation equality and stops at $z = 30$, we find that the halo increases its total mass as

$$\frac{m_{h,\,\mathrm{matt}}}{m_{\mathrm{BH}}} \approx 100. \tag{12}$$

In the general case of a PBH growing in the matter era, the mass increase from $z_{\rm eq}$ to z is

$$\frac{m_{h,\,\rm matt}}{m_{\rm BH}} \approx 100\left(\frac{31}{1+z}\right). \tag{13}$$

6. GENERAL CASE

In addition to approximate calculations specific to the matter and radiation eras, we also present the general result, which spans both eras and includes (for completeness) consideration of the cosmological constant. We start with the radial infall equation for a shell of matter:

$$\frac{d^2r}{dt^2} = \frac{-4\pi G r}{3}(\rho_m + 2\rho_r) + \frac{\Lambda c^2 r}{3}. \tag{14}$$

For computational convenience, we recast this equation in terms of derivatives with respect to redshift, and we switch to comoving coordinates. After some algebra, we are left with two differential equations, one for the comoving radial coordinate, $x(z)$, and one for the peculiar velocity of a shell, $v(x, z)$, defined by

$$v = \frac{dr}{dt} - Hr \tag{15}$$

$$= \frac{d(ax)}{dt} - Hax, \tag{16}$$

where $a = 1/(1+z)$. The integration equations take on a simple form in the new coordinates:

$$\frac{dx}{dz} = \frac{-v}{H}, \tag{17}$$

$$\frac{dv}{dz} = av + \frac{G[M_{\rm acc}(x) + m_{\rm BH}]}{Hax^2}, \tag{18}$$

where $M_{\rm acc}(x)$ is defined as the excess matter over the background-density matter within the comoving radius x (i.e., the matter previously accreted into the halo region around the black hole).

7. RESULTS

7.1. *Mass Accretion*

Our results from the combined calculation are consistent with those we obtained treating the matter and radiation eras separately. A PBH can grow by 2 orders of magnitude through the accumulation of a dark matter halo from early in the radiation era to $z \sim 30$, with the halo mass increasing in proportion to the cosmic scale parameter $a = 1/(1+z)$:

$$m_h(z) = \phi_i\left(\frac{1000}{1+z}\right)m_{\rm BH}, \tag{19}$$

where the proportionality constant $\phi_i \approx 3$. Figure 1 summarizes our mass accretion result.

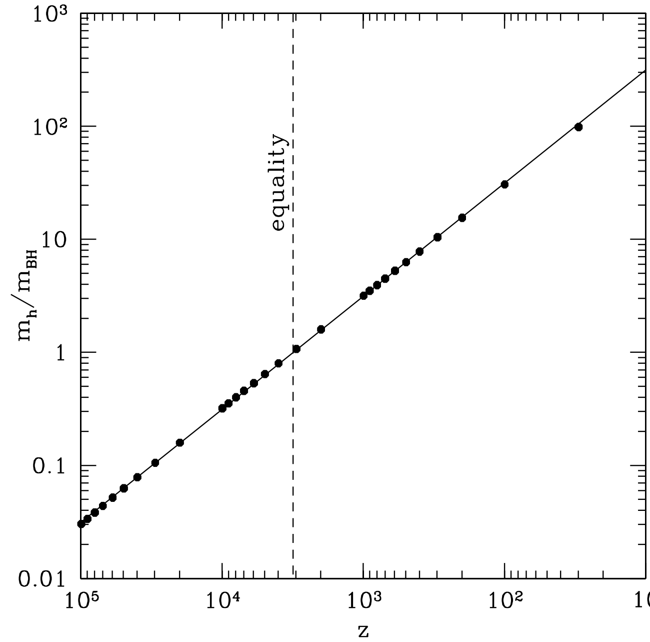

Fig. 1.—Accreted halo mass vs. redshift. The halo radius is defined at an overdensity of $\delta = 2$. We include a dashed line to indicate the redshift of matter-radiation equality.

7.2. *Halo Profile*

In a previous study, Bertschinger (1985) performed analytical calculations of radial infall onto a central overdensity and onto a black hole; the difference in the two calculations was that in the former case, the particles could oscillate through the center, whereas in the latter case they were absorbed by the black hole (as is the case in our simulation). Bertschinger obtained a $\rho(r) \sim r^{-2.25}$ profile for the extended overdensity and a $\rho(r) \sim r^{-1.5}$ profile for the black hole case.

Our simulation resulted in a profile of $\rho(r) \sim r^{-3}$ for the outer parts of the halo; we did not resolve the inner parts, which we expect to have a profile of $\rho(r) \sim r^{-1.5}$. We obtain this profile because for the sake of the accretion calculation we have assumed perfectly radial accretion; if we were to explicitly include the nonradial nature of the halo formation, we would obtain $\rho(r) \sim r^{-2.25}$, as in Bertschinger (1985). See Ricotti (2007) for further discussion on the profile shape and its effect on the gas accretion rate. Our profile is illustrated in Figure 2.

Since a power-law profile has no sharp cutoff in radius, we must choose a criterion by which to define the matter within the halo. We may choose either the turnaround radius (the distance out to which shells have broken free of the Hubble flow) or a cut on the overdensity versus radius; both criteria give similar results. Using the turnaround radius definition, the comoving radius of the dark matter halo at redshift z from accretion beginning at matter-radiation equality is

$$x_{\rm ta} = 1.30\left(\frac{1+z}{31}\right)^{-1/3}\left(\frac{m_{\rm BH}}{100\ M_\odot}\right)\ {\rm kpc} \tag{20}$$

for $z \lesssim 100$. The radius defined by a cut on overdensity can be read off Figure 2.

7.3. *Density Parameter in PBHs*

As the masses of the clothed PBHs increase, so does their overall density parameter. Given an initial matter fraction of

$$\omega_{{\rm BH},i} \equiv \frac{\Omega_{{\rm BH},i}}{\Omega_{m,i}}, \tag{21}$$

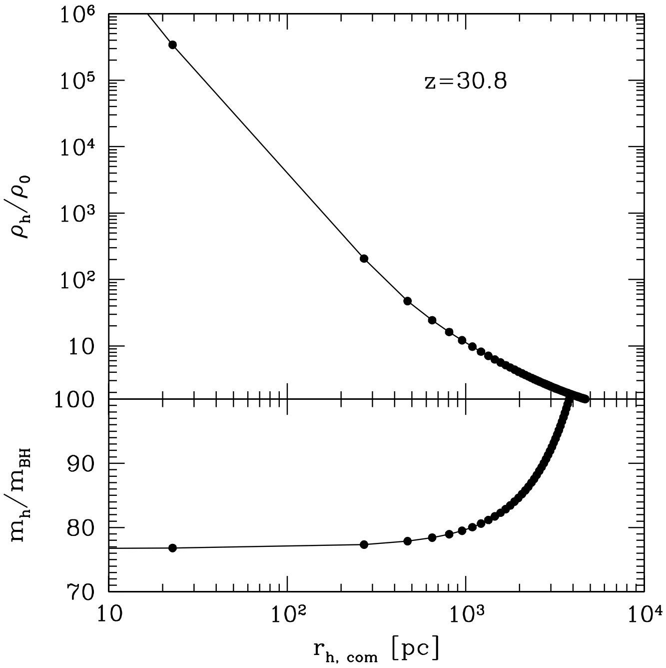

FIG. 2.—Dark matter halo profile. *Top*, halo overdensity vs. comoving radius from PBHs; *bottom*, halo mass vs. comoving radius from PBHs. In the inner parts of the halo, the density falls off as r^{-3}, and the profile flattens in the outer regions.

the final matter fraction increases in proportion to the clothed PBH mass:

$$\frac{\omega_{\mathrm{BH},f}}{\omega_{\mathrm{BH},i}} = \frac{m_{\mathrm{BH},f}}{m_{\mathrm{BH},i}}, \tag{22}$$

where $m_{\mathrm{BH},f} = m_{\mathrm{BH}} + m_h(z)$ includes the PBH and the accreted halo.

In Figure 3, we illustrate that the proportional mass increase, while not dependent on the mass of the PBH, does depend on the proportion of the dark matter made up of PBHs. If the PBHs

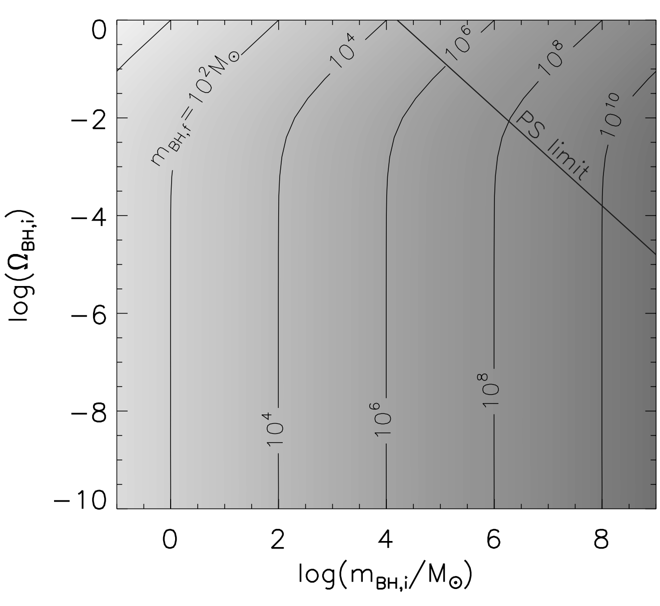

FIG. 3.—Final mass per PBH plotted as a function of initial mass and initial PBH density parameter, with the Afshordi et al. (2003) limit (marked PS) included for reference (see § 8.1). [*See the electronic edition of the Journal for a color version of this figure.*]

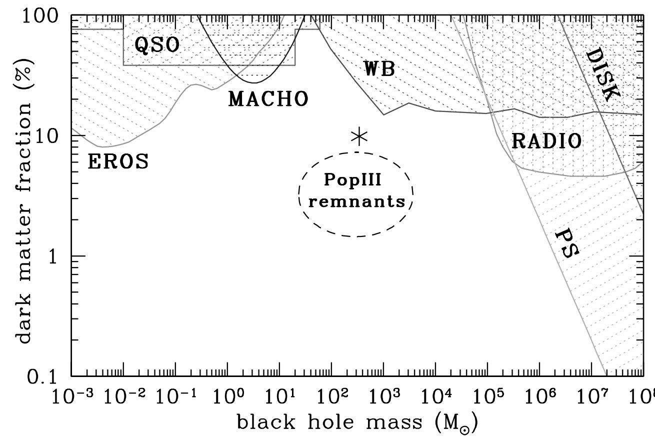

FIG. 4.—Observational constraints on black holes in the Galactic halo (see § 8.1) from microlensing experiments (EROS and MACHO), quasar variability studies (QSO), compact radio source lensing (RADIO), the stability of wide binaries (WB), the high-wavenumber matter power spectrum (PS), and the heating of the Galactic disk (DISK). The region labeled "PopIII remnants" represents a rough estimate of the region of parameter space relevant to a scenario in which Population III star remnant IMBHs are responsible for ULX observations. The asterisk gives the position of the example of a PBH ULX used in the Abstract (see § 8.2). [*See the electronic edition of the Journal for a color version of this figure.*]

were to begin to dominate the dark matter, they would grow less because of the decrease in the density of dark matter.

8. DISCUSSION AND CONCLUSIONS

Our results suggest that PBHs could grow significantly after their formation by acquiring a dark matter halo and that the resulting clothed black holes could make up an interesting fraction of the dark matter. In the following discussion, we show that current observations are not in conflict with this conclusion, and in fact there is ample room both observationally and theoretically for PBHs to play a role in the universe today.

8.1. *Observational Limits on PBHs*

We include in Figure 4 a plot of the current observational limits on PBHs over a wide range of masses and dark matter fractions. Here we describe the limits illustrated in the plot.

Most limits on PBHs in the present-day universe are derived from considerations of PBHs as dark matter candidates. PBHs massive enough to escape evaporation would certainly qualify as "cold" and "dark" matter; however, their existence could have noticeable effects on processes from nucleosynthesis to galaxy formation. If PBHs form at very early times, and thus with very low masses ($\lesssim 10^{10}$ g), they could interfere with nucleosynthesis by emitting particles during their evaporation (Kohri & Yokoyama 2000). The abundance of PBHs forming after nucleosynthesis is constrained by measurements of the baryon fraction of the universe (Novikov et al. 1979). Massive PBHs can also be constrained by dynamical considerations in the low-redshift universe. Some of the strongest current constraints are derived from the wealth of data from microlensing searches in the Galaxy (Alcock et al. 2000, 2001; Afonso et al. 2003). The frequency and character of observed microlensing events constrains black holes in the mass range 0.1–1 $M_\odot$ to make up less than ~20% of the dark matter in the Galactic halo (Alcock et al. 2000; Gould 2005). The limits from microlensing are shown in Figure 4, labeled "MACHO" and "EROS."

For larger masses, constraints on PBH dark matter can be found by examining the effect of PBHs on the matter power spectrum. Calculating the excess in the power spectrum that Poisson-distributed

PBHs would contribute, Afshordi et al. (2003) find an upper limit on present-day PBH masses of a few times $10^4\ M_{\odot}$, which is consistent with but tighter than the previous constraint at $10^6\ M_{\odot}$ based on the heating of the Galactic disk (Lacey & Ostriker 1985). However, we note that this limit assumes that all the dark matter is in PBHs: $\Omega_{\rm BH} = \Omega_{\rm DM}$. If this assumption is relaxed, we find that a wide range of masses can be accommodated at lower density parameters. Specifically, we find that the product of the initial density parameter and the initial mass are constrained by

$$m_{\rm BH}\Omega_{\rm BH} < \frac{m_{\rm PS}}{x}, \tag{23}$$

where x is the factor by which the PBHs increase in mass via accretion and $m_{\rm PS}$ is $\sim 10^4\ M_{\odot}$, the limit derived from Afshordi et al. (2003). We include this power spectrum limit in the constraint plot (Fig. 4) with the label "PS."

Other limits can be placed by considering the effects of compact objects along the line of sight that are lensing more distant sources (Wambsganss 2007). Dalcanton et al. (1994) search for the slight amplification in the continuum emission of quasars that would be expected if black holes were to cross the line of sight during an observation. With a large sample of observations, they are able to place limits on black holes in the range $\sim 10^{-3}\ M_{\odot}$ to $\sim 300\ M_{\odot}$. This limit is included in the constraint plot (Fig. 4) and labeled "QSO." Wilkinson et al. (2001) place limits on the abundance of massive black holes in the universe that are based on their predicted effect of creating multiple images of compact radio sources. Studying a sample of 300 sources, they find a null result and from that can place a constraint on the density of black holes along the line of sight. Their constraint is included in Figure 4 with the label "RADIO." Coincident with the compact radio source study, another group found a similar constraint by searching for the same lensing effect in gamma-ray burst light curves (Nemiroff et al. 2001). The results of the two studies are consistent with each other, so for simplicity we include only the Wilkinson et al. (2001) result in Figure 4.

Finally, a limit on black holes with masses of $\sim 10\ M_{\odot}$ and up can be placed by observing widely orbiting binary systems in the Galaxy (Yoo et al. 2004). If a compact object passes between the two companion stars in a binary, the orbits of the stars will be perturbed. Yoo et al. (2004) use this to estimate how many compact objects with the ability to disturb a binary system exist in the halo. This constraint is included in Figure 4 and is labeled "WB." We point out that none of the above observations significantly constrain black holes making up less than $\sim 10\%$ of the dark matter for a wide range of masses.

8.2. *PBHs as ULXs*

This part of parameter space is consistent with an interpretation of the recent ULX observations as accreting, intermediate-mass black holes in nearby galaxies. In this scenario, ULXs occur when IMBHs residing in the galaxy's halo pass through molecular clouds in the disk (Mapelli et al. 2006; Miller 2005; Winter et al. 2006). The enhanced density in a molecular cloud is sufficient to trigger gas accretion, which causes the IMBH to emit X-ray radiation. These sources would be transient, and in any given galaxy the number of sources detected at any given time would depend on the number and distribution of the IMBH population and the fraction of the disk made up of molecular clouds.

Several recent papers explore the possibilities for making ULXs with IMBHs. Mii & Totani (2005) estimate the number of ULXs expected if they are the result of IMBHs passing through molecular clouds. Using the mass and dark matter fraction estimated for IMBHs if they are the compact remnants of Population III stars (Madau & Silk 2005), $M_{\rm IMBH} \sim 10^2$–10^3 and $\Omega_{\rm IMBH}/\Omega_b \sim 0.1$, they find that the estimated number of ULXs is consistent with observations. These results do not depend on the nature of the IMBHs; if the IMBHs were primordial in nature rather than Population III star remnants, they would also be capable of producing the observed sources.

Although these results are encouraging, the present lack of understanding of the nature of ULXs and the many uncertainties that go into predictions of the consequences of a halo population of IMBHs mean that the issue is far from resolved. A recent paper drawing on the Mii & Totani (2005) result uses an ensemble of N-body simulations to place constraints on the number of IMBHs in the Milky Way halo by drawing on the fact that we have not observed any ULXs in our galaxy (Mapelli et al. 2006). These authors find in one simulation that for a halo population of $\sim 10^5$ IMBHs incorporating a fraction of 0.1% of the baryons and distributed in an NFW profile (Navarro et al. 1997), the predicted number of ULXs per galaxy is on the order of 1; however, the number of lower luminosity X-ray sources is overproduced. The constraint found by this method may be applicable to our own Galaxy, but it has not as yet been extended to other galaxies, where ULXs are observed. Since the predictions depend strongly on not only the distribution and number of IMBHs, but also on the properties of the gas in the galactic disk and the efficiency of the black hole accretion (which in turn depends on whether or not an accretion disk is formed), it is difficult to generalize them to other systems.

Other authors have suggested that ULXs may be due to IMBHs accreting from captured stellar companions rather than molecular clouds (Pooley & Rappaport 2005; Patruno et al. 2006; Madhusudhan et al. 2006). In this case, the ULXs would "turn on" when residing in dense star clusters.

More detailed observations and simulations are required to answer the ULX question. Here we merely point out that the case for IMBH ULXs is an interesting one, easily consistent with current constraints on the dark matter fraction in black holes, and as we show, PBHs may account for or grow into IMBHs by the present era. Thus, PBHs should be considered viable candidates that would explain these mysterious sources.

In Figure 4, we include as a region of interest the area in parameter space explored in the Mii & Totani (2005) paper (labeled "PopIII remnants"). We also mark with an asterisk the position of the scenario discussed in the Abstract: a population of $10^{2.5}\ M_{\odot}$ PBHs making up 0.1% of the dark matter and growing through accretion to incorporate 10% of the dark matter by the present day (we mark the mass of the seed PBH only, going on the conservative assumption that the PBH's dark matter halo is not directly accreted). Both these points lie in a region of acceptable parameter space for the explanation of ULXs with IMBHs, but the exact extent of the region is difficult to define, given the uncertainties mentioned above.

8.3. *PBHs as SMBH Seeds*

The question of the origin of supermassive black holes at high redshifts has attracted a great deal of attention since quasars have been discovered at redshifts of $z > 6$, implying that black holes as massive as $\sim 10^9\ M_{\odot}$ (Fan et al. 2003; Barth et al. 2003; Willott et al. 2003) exist when the universe is less than 1 Gyr old. It has proven difficult to find a mechanism that can create such massive black holes so quickly. Most proposals require smaller black holes to act as seeds for the buildup of SMBHs. In some cases, these seeds form directly from the collapse of halos (Lodato & Natarajan 2006; Spaans & Silk 2006). In others, the seeds are the remnants of Population III stars (Shapiro 2005; Volonteri & Rees 2005), or they might be primordial. The question of whether or not SMBHs can be grown from the merging of stellar-mass black holes has also been discussed in recent work (see Li et al. [2007] and references therein). While in each scenario a case may be made for the ability of these seeds to result in the SMBHs we observe as quasars, sometimes requiring the invocation of self-interacting dark matter (Ostriker 2000; Hu et al. 2006) or the accretion of scalar fields (Bean & Magueijo 2002), there is as yet no consensus on the matter. We suggest that PBHs may be viable SMBH seed candidates because of their ability to build up large dark matter halos that may assist in further growth through baryon accretion later on. Furthermore, since they form earlier than Population III stars, they have more time to grow in the epoch before quasars are observed.

8.4. *Conclusions*

We have shown that primordial black holes can grow significantly after formation through the accretion of a dark matter halo. In the radiation era this can lead to an increase of total mass that is of order unity, while during matter domination, the mass can grow by roughly 2 orders of magnitude. Although a dark matter halo may not significantly increase the mass of the seed black hole itself due to the lack of a mechanism to dissipate angular momentum, the accumulation of a halo cannot be ignored when considering the ability of a PBH to accrete gas in later eras (in the case of collisional dark matter [Ostriker 2000], the PBH can directly accrete significant amounts of dark matter). We have also shown that the parameter space available to PBHs as components of dark matter components is sufficient for them to play an interesting role in galaxies. PBHs may be viable candidates for the seeds of intermediate-mass black holes, which are possibly responsible for ultraluminous X-ray sources, or they may play a role in seeding the supermassive black holes currently found in the centers of galaxies.

This material is based on work supported under a National Science Foundation Graduate Research Fellowship. The authors thank Niyaesh Afshordi, Bernard Carr, Andy Gould, Joshua Green, Martin Rees, Ed Sirko, David Spergel, Paul Steinhardt, and everyone at Underground Coffee for helpful feedback and suggestions.

APPENDIX A

RADIATION ERA ANALYTICAL APPROXIMATION

The equation describing the dynamics of a dark matter shell at a distance r from a black hole of mass $m_{\rm BH}$ at a time t is

$$\frac{d^2r}{dt^2} = -\frac{Gm_{\rm BH}}{r^2} - \frac{1}{4}\frac{r}{t^2}. \tag{A1}$$

In the radiation era, we have $H = 1/(2t)$. We define $r_i = r(t = t_i)$, which leads to

$$\frac{dr}{dt}(t = t_i) = H_i r_i = \frac{1}{2}\frac{r_i}{t_i}, \tag{A2}$$

where H_i is the initial Hubble parameter.

We now consider the unperturbed solution, where $m_{\rm BH} = 0$. This reduces the above second-order equation to

$$\frac{d^2r}{dt^2} = -\frac{1}{4}\frac{r}{t^2}. \tag{A3}$$

We take the unperturbed behavior to be a power law:

$$r_0(t) = r_i\left(\frac{t}{t_i}\right)^{\alpha}. \tag{A4}$$

Then

$$\frac{d^2r_0}{dt^2} = \frac{r_i}{t_i^{\alpha}}\alpha(\alpha - 1)t^{\alpha-2} = -\frac{1}{4}\frac{r_0}{t^2}, \tag{A5}$$

which we can solve to get

$$\alpha = \frac{1}{2}. \tag{A6}$$

This gives us the time evolution of the unperturbed solution:

$$r_0 = \frac{r_i}{t_i^{1/2}}. \tag{A7}$$

We can now take $r = r_0 + \delta r$. Differentiating twice, we get

$$\ddot{r} + \delta\ddot{r} = -\frac{Gm_{\rm BH}}{r_0^2(1 + \delta r/r_0)^2} - \frac{1}{4}\frac{r_0}{t^2} - \frac{1}{4}\frac{\delta r}{t^2}. \quad (A8)$$

Substituting in our solutions for r_0 and $\ddot{r}_0$, we obtain

$$\delta\ddot{r} = -\frac{Gm_{\rm BH}}{r_0^2(1 + \delta r/r_0)^2} - \frac{1}{4}\frac{\delta r}{t^2}. \quad (A9)$$

The above equation is still exact, but we can take $\delta r/r$ to be small to get the lowest order solution,

$$\delta\ddot{r} + \frac{1}{4}\frac{\delta r}{t^2} = -\frac{Gm_{\rm BH}}{r_i^2}\frac{t_i}{t}. \quad (A10)$$

For simplicity, we now define $x \equiv \delta r/r_i$ and $\tau \equiv t/t_i$. This gives us

$$\ddot{x} + \frac{1}{4}\frac{x}{\tau^2} = -\frac{Gm_{\rm BH}t_i^2}{r_i^3\tau}. \quad (A11)$$

Defining

$$\epsilon = \frac{Gm_{\rm BH}t_i^2}{r_i^3} \approx \left(\frac{\delta\rho}{\rho}\right)_i, \quad (A12)$$

we have

$$\ddot{x} + \frac{1}{4}\frac{x}{\tau^2} = -\frac{\epsilon}{\tau}. \quad (A13)$$

The solution of the homogeneous equation, $\ddot{x} + x/(4\tau^2) = 0$, is $x \propto \tau^\alpha$. Finding the particular integral yields $x = -4\epsilon\tau$, which gives a general solution of

$$x = A\tau^{1/2} - 4\epsilon\tau. \quad (A14)$$

We solve for A by considering that at $\tau = 1$, $x = 0$, which makes $A = 4\epsilon$. Thus, we have

$$x(\tau) = 4\epsilon(\tau^{1/2} - \tau) = \frac{\delta r}{r_i}(\tau). \quad (A15)$$

For those shells of matter bound to the central black hole, each shell will have a turnaround time (the time at which the shell ceases to expand in the Hubble flow and begins to fall back) and a collapse time (the time when the radius of the shell goes to zero). We define a shell's collapse time as the time when $r = 0$:

$$r = r_0 + \delta r = r_0 + \frac{\delta r}{r_i}r_i = 0. \quad (A16)$$

Rewriting this with x and τ, we have

$$r_i\tau_{\rm coll}^{1/2} + x_{\rm coll}r_i = 0, \quad (A17)$$

and with the solution for x, this gives

$$\tau_{\rm coll} = \left(\frac{1 + 4\epsilon}{4\epsilon}\right)^2, \quad (A18)$$

$$t_{\rm coll} = t_i\left(\frac{1 + 4\epsilon}{4\epsilon}\right)^2. \quad (A19)$$

To find the amount of matter accreted by the black hole in the radiation era, we choose an initial time and find the amount of matter that is accreted between that time and the time of matter-radiation equality. This will likely be an overestimate, however, as any interactions or other effects are more likely to slow accretion by pulling matter away from the black hole. This calculation will find the amount of matter that had sufficient time to accrete, assuming that the accretion is steady and undisturbed. Here we perform a rough estimate of the amount of matter accreted.

We choose the initial time to be $z_i \approx 10^7$. The final time is the time of matter-radiation equality, $t_f = t_{\rm eq}$, corresponding to $z_{\rm eq} \approx 3 \times 10^3$. During the radiation epoch, $t \propto (1+z)^{-2}$, so we have $t_i/t_f \approx 10^{-7}$. Setting $t_f = t_{\rm coll}$, we have

$$\frac{t_{\rm coll}}{t_i} = \left(\frac{1+4\epsilon_{\rm min}}{4\epsilon_{\rm min}}\right)^2 = 10^7, \tag{A20}$$

where $\epsilon_{\rm min}$ reflects the fact that this is a maximal estimate of the accretion. Since we defined $\epsilon \equiv Gm_{\rm BH}/(r_i^3 t_i^{-2})$, we can now write

$$r_{i,\max}^3 = \frac{Gm_{\rm BH}}{t_i^{-2}\epsilon_{\rm min}}. \tag{A21}$$

Then

$$m_{\rm acc,max} = \frac{4}{3}\pi r_{i,\max}^3 \rho_{m,i} = \frac{4}{3}\pi \frac{Gm_{\rm BH}}{t_i^{-2}\epsilon_{\rm min}}\rho_{m,0}(1+z_i)^3, \tag{A22}$$

which gives us

$$\frac{m_{\rm acc,max}}{m_{\rm BH}} = \frac{4\pi G\rho_{m,0}}{3t_i^{-2}\epsilon_{\rm min}}(1+z_i)^3. \tag{A23}$$

With some manipulation, this becomes

$$\frac{m_{\rm acc,max}}{m_{\rm BH}} = \frac{2}{9}\left(\frac{t_i}{t_f}\right)^2\left(\frac{t_f}{t_0}\right)^2\frac{\Omega_{m,0}}{\epsilon_{\rm min}}(1+z_i)^3, \tag{A24}$$

where the subscript 0 refers to the present era. This is not exact, since we are implicitly assuming that the present era is completely matter-dominated; i.e., $\Omega_{m,0} = 1$. However, this can be neglected if we instead use $z \approx 30$ as the final time; in this case, the fraction t_i/t_0 is not significantly changed and the factor $\Omega_{m,0}$ becomes unity. For $\epsilon_{\rm min}$, we use $\delta\rho/\rho \approx 2.5 \times 10^{-5}$. Here $t_f/t_0 = 6 \times 10^{-6}$, $t_i/t_f = 9 \times 10^{-8}$, and $z_i = 10^7$, which gives us

$$\frac{m_{\rm acc,max}}{m_{\rm BH}} \approx 2.6. \tag{A25}$$

Thus, we see that the fractional increase in mass of the black hole in the radiation era is on the order of 1.

APPENDIX B

MATTER-ERA CALCULATION WITH COSMOLOGICAL CONSTANT

To calculate the radial infall of dark matter with the effect of the cosmological constant included, we solve (Lahav et al. 1991)

$$\frac{d^2r}{dt^2} = -\frac{GM_i}{r^2} + \frac{\Lambda r}{3}. \tag{B1}$$

The mass internal to a dark matter shell initially at r_i is given by

$$M_i = \frac{4}{3}\pi\rho_{m,i}r_i^3 + m_{\rm BH}, \tag{B2}$$

where r_i is the initial physical radius, with the initial density contrast defined as

$$\Delta_{c,i} = \frac{M_i}{(4/3)\pi r_i^3\rho_{c,i}} - 1. \tag{B3}$$

From this, Lahav et al. (1991) derive the equation of motion,

$$\frac{d^2A}{d\tau^2} = -\frac{1}{2}(\Delta_{c,i}+1)A^{-2} + \lambda_i A, \tag{B4}$$

where $\lambda_i = \Omega_{\Lambda,i}$, $\tau = H_i t$, and A is the scale factor of the shell, $R(t) = A(t, r_i)r_i$. The initial conditions for the Hubble flow are $A_i = 1$ and $dA/d\tau = 1$. This must be solved numerically for the initial density contrast $\Delta_{c,i}$ corresponding to the shell collapsing at the final time τ_f. The mass accreted from τ_i to τ_f is given by

$$m_{\rm acc} = m_{\rm BH}\left(\frac{1 - \Omega_{\Lambda,i} - \Omega_{{\rm BH},i}}{\Delta_{c,i} + \Omega_{\Lambda,i} + \Omega_{{\rm BH},i}}\right). \tag{B5}$$

As a result of the presence of the cosmological constant, there will be a last bound shell, which is the last shell of matter that can in principle be accreted by the black hole. All shells internal to this are bound and will turn around and fall back, but those beyond it will continue expanding in the Hubble flow. On the basis of the calculation in Subramanian et al. (2000), we find the last bound shell to be at an initial radius of

$$r_\lambda^3 = \frac{1}{2\pi}\frac{m_{\rm BH}}{\rho_{m,i}}\left(2\frac{\Omega_{\Lambda,i}}{\Omega_{m,i}}\right)^{-1/3}, \tag{B6}$$

where $\Omega_{m,i}$ is the initial density parameter in matter. The mass within the last bound shell is then

$$m_\lambda = (4\pi/3)\rho_{m,i} r_\lambda^3, \tag{B7}$$

$$m_\lambda = \frac{2}{3}\left(\frac{1 - \Omega_{\Lambda,i}}{2\Omega_{\Lambda,i}}\right)^{1/3} m_{\rm BH}. \tag{B8}$$

For the case of accretion beginning at matter-radiation equality, the ratio of the mass within the last bound shell to the initial mass of the PBH will be $m_\lambda/m_{\rm BH} \approx 1500$. This is the mass increase that is possible, in principle, for each PBH. However, as we derive this by assuming each PBH to be isolated, it may occur that the PBH will run out of matter to accrete before this limit is reached, as it has all been accreted by neighboring black holes, or that infall of much of this mass would occur after the present epoch. More realistically, as the PBHs grow, they will begin to interact with one another, and their cluster dynamics and mergers will have to be considered. The last bound shell mass can therefore be considered a strict upper limit on the accretion.

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