

# RELATIONS BETWEEN THE LUMINOSITY, MASS, AND AGE DISTRIBUTIONS OF YOUNG STAR CLUSTERS

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## ABSTRACT

We derive and interpret some relations between the luminosity, mass, and age distributions of star clusters, denoted here by  $\phi(L)$ ,  $\psi(M)$ , and  $\chi(\tau)$ , respectively. Of these,  $\phi(L)$  is the easiest to determine observationally, whereas  $\psi(M)$  and  $\chi(\tau)$  are more informative about formation and disruption processes. For a population of young clusters, with a relatively wide range of ages,  $\phi(L)$  depends on both  $\psi(M)$  and  $\chi(\tau)$  and thus cannot serve as a proxy for  $\psi(M)$  in general. We demonstrate this explicitly by four illustrative examples with specific forms for either  $\psi(M)$  or  $\chi(\tau)$ . In the special case in which  $\psi(M)$  is a power law and is independent of  $\chi(\tau)$ , however,  $\phi(L)$  is also a power law with the same exponent as  $\psi(M)$ . We conclude that this accounts for the observed similarity between  $\phi(L)$  and  $\psi(M)$  for the young clusters in the Antennae galaxies. This result reinforces our picture in which clusters form with  $\psi(M) \propto M^{-2}$  and are then disrupted rapidly at a rate roughly independent of their masses. The most likely disruptive process in this first stage is the removal of interstellar matter by the energy and momentum input from young stars (by photoionization, winds, jets, and supernovae). The few clusters that avoid this “infant mortality” are eventually disrupted in a second stage by the evaporation of stars driven by two-body relaxation, a process with a strong dependence on mass. We suspect this picture may apply to many, if not all, populations of star clusters, but this needs to be verified observationally by determinations of  $\psi(M)$  and  $\chi(\tau)$  in more galaxies.

*Subject headings:* galaxies: star clusters — stars: formation

## 1. INTRODUCTION

In this paper, we derive and interpret some relations between the luminosity, mass, and age distributions of young star clusters. We define these “distributions” (probability densities, in fact) as follows:  $\phi(L)dL$  is the fraction of clusters with luminosities between  $L$  and  $L + dL$ ,  $\psi(M)dM$  is the fraction of clusters with masses between  $M$  and  $M + dM$ , and  $\chi(\tau)d\tau$  is the fraction of clusters with ages between  $\tau$  and  $\tau + d\tau$ . We focus here on the question: To what extent does  $\phi(L)$  reflect  $\psi(M)$ , and how does this depend on  $\chi(\tau)$ ?

For a population of clusters with the same mass-to-light ratio, resulting from a common age and stellar mass function (and neglecting any mass dependence of stellar escape rates),  $\phi(L)$  always has the same form as  $\psi(M)$ . This assumption is often made in studies of old globular clusters, when their colors or other evidence indicates that the spread in ages is modest or narrow (in a fractional sense). However, for a population of clusters with a wide range of mass-to-light ratios,  $\phi(L)$  can be very different from  $\psi(M)$ . This is the generic case when the spread in ages is broad in fractional terms, as happens when clusters form continually up to the present time. We are concerned in this paper primarily with this situation.

The luminosity function of a population of clusters is relatively easy to determine because it requires observations in only one photometric band. Consequently, there is now an extensive literature on this subject. In studies with deep observations of large samples of young clusters, the luminosity function is often found to have an approximate power-law form,  $\phi(L) \propto L^\alpha$ , with an exponent near  $\alpha \approx -2$ . Early examples of this result include the Milky Way (van den Bergh & LaFontaine 1984), the Large Magellanic Cloud (Elson & Fall 1985), and Messier 33 (Christian & Schommer 1988). A more recent example, and the one that motivates the work presented here, is the interacting Antennae galaxies (Whitmore et al. 1999). In some cases, the luminosity function is a power law in a first approximation, but also appears

to have some secondary convex curvature in log-log plots. However, this curvature often has only marginal statistical significance, and conceivably it could be the result of subtle systematic effects, such as undercorrections for incompleteness or photometric errors at faint magnitudes. We ignore such curvature here in the interest of keeping our analysis as simple and transparent as possible.

Because the luminosity function is affected by the fading of the stellar populations within the clusters, it does not tell us directly about the formation and disruption of the clusters. The mass and age distributions are more fundamental in this regard. In particular, for the youngest clusters,  $\psi(M)$  is likely to reflect fairly directly the physical processes involved in the formation of the clusters and/or their parent molecular clouds. The initial mass function of clusters thus plays a role in the theory of cluster formation similar to that of the initial mass function of stars in the theory of star formation. The age distribution, in principle, reflects a combination of the formation and disruption histories of the clusters. In practice, however,  $\chi(\tau)$  is primarily a diagnostic of disruption processes, because it usually has a stronger dependence on  $\tau$  than is plausible for the formation history.

The mass and age distributions require photometry in several bands, corrections for interstellar reddening, and comparisons with stellar population models. This makes their determination more laborious than that of the luminosity function. Moreover, estimating two univariate distributions or one bivariate distribution (of  $M$  and  $\tau$ ) to a given level of statistical accuracy requires a larger sample of clusters than estimating a single univariate distribution (of  $L$ ) to the same accuracy. Consequently, we have reliable determinations of the mass and age distributions of clusters in only a few galaxies, the best example again being the Antennae. In this case, the mass and age distributions can be represented by the power laws  $\psi(M) \propto M^\beta$  with  $\beta \approx -2$  (Zhang & Fall 1999) and  $\chi(\tau) \propto \tau^\gamma$  with  $\gamma \approx -1$  (Fall et al. 2005). The exponent of  $\chi(\tau)$  quoted here is for mass-limited samples, as is required in all the formulae of this paper; for luminosity-limited samples,  $\chi(\tau)$  is significantly steeper, as a consequence of the rapid fading of

the clusters. Furthermore, over the observed domain of masses and ages,  $\psi(M)$  and  $\chi(\tau)$  are approximately independent of each other (as shown in the previous references). This fact simplifies much of the analysis in this paper.

The observations of the Antennae galaxies raise an interesting question: Why do the luminosity and mass functions of the young clusters have the same, or at least approximately the same form, i.e., why are they both power laws with the same exponent? In much of the literature on star clusters, there is a tendency to regard the luminosity function as a proxy for the mass function. As noted above, this cannot be true in general, given that clusters of different ages have different mass-to-light ratios. Yet the evidence from the Antennae galaxies suggests that  $\phi(L)$  and  $\psi(M)$  are in fact closely related. Why is this? Answering this question is the main purpose of this paper. Although our analysis is motivated by observations of the Antennae galaxies, it seems likely on theoretical grounds that our conclusions have wider applicability, possibly to most or even all galaxies with large populations of young clusters.

## 2. GENERAL RELATIONS

The general relation between the luminosity and mass functions of star clusters must also involve the age distribution. With this in mind, we introduce the following bivariate distributions:  $f(L, \tau) dL d\tau$  is the fraction of clusters with luminosities between  $L$  and  $L + dL$  and ages between  $\tau$  and  $\tau + d\tau$ , and  $g(M, \tau) dM d\tau$  is the fraction of clusters with masses between  $M$  and  $M + dM$  and ages between  $\tau$  and  $\tau + d\tau$ . In terms of these distributions, the univariate distributions of luminosity, mass, and age are

$$\phi(L) = \int_0^\infty f(L, \tau) d\tau, \quad (1)$$

$$\psi(M) = \int_0^\infty g(M, \tau) d\tau, \quad (2)$$

$$\chi(\tau) = \int_0^\infty f(L, \tau) dL = \int_0^\infty g(M, \tau) dM. \quad (3)$$

We now make our *first simplifying assumption*, that the mass-to-light ratios of clusters, denoted by  $\mu$ , depend only on their ages:

$$\mu(\tau) = M(\tau)/L(M, \tau). \quad (4)$$

In principle,  $\mu$  could vary among clusters of the same age if, for example, they had different stellar mass functions. In practice, such variations are found or assumed to be small, and equation (4) is thus the basis for all determinations of the mass functions of clusters from multiband photometry. In this case, the bivariate distributions are related by

$$f(L, \tau) = g(M, \tau) |(\partial M / \partial L)_\tau| = g(M, \tau) \mu(\tau). \quad (5)$$

Combining equations (1), (4), and (5) yields

$$\phi(L) = \int_0^\infty g[\mu(\tau)L, \tau] \mu(\tau) d\tau. \quad (6)$$

Given  $g(M, \tau)$ , equations (2), (3), and (6) determine fully the relations between  $\phi(L)$ ,  $\psi(M)$ , and  $\chi(\tau)$ . These relations in general are quite complicated.

We now make our *second simplifying assumption*, that the mass and age distributions of clusters are independent of each other:

$$g(M, \tau) = \psi(M) \chi(\tau). \quad (7)$$

This is valid if the rates of formation and disruption of the clusters are independent of their masses. For very young clusters, which dominate the bright parts of the luminosity function, the main disruptive effect is loss of interstellar matter by the energy and momentum input from young stars (photoionization, stellar winds and jets, and supernovae remnants). These inputs should be roughly proportional to the masses of the clusters and hence also to the amount of material to be removed. Thus, we expect the disruption rate to be roughly independent of the masses of the clusters, as prescribed by equation (7) above (see Fall et al. 2005 for further discussion of this and related issues). This is consistent with our observations of the young clusters in the Antennae galaxies, where we have determined  $g(M, \tau)$  for  $\tau \lesssim 10^7 (M/10^4 M_\odot)^{1.3}$  yr. Once the evaporation of stars by two-body relaxation becomes important, the disruption rate is mass dependent, and equation (7) is no longer valid. This occurs for  $\tau \gtrsim 5 \times 10^8 (M/10^4 M_\odot)$  yr, i.e., well outside the observed mass-age domain of clusters in the Antennae galaxies (see § 4 below).

When we insert equation (7) into equation (6), we obtain

$$\phi(L) = \int_0^\infty \psi[\mu(\tau)L] \chi(\tau) \mu(\tau) d\tau. \quad (8)$$

It is helpful at this point to use the mass-to-light ratio  $\mu$  as a substitute for the age  $\tau$ . With this in mind, we introduce the corresponding distribution, defined such that  $\theta(\mu) d\mu$  is the fraction of clusters with mass-to-light ratios between  $\mu$  and  $\mu + d\mu$ . This is related to the age distribution by

$$\theta(\mu) = \chi(\tau) |d\tau/d\mu|. \quad (9)$$

Combining this with equation (8) gives

$$\phi(L) = \int_0^\infty \psi(\mu L) \theta(\mu) \mu d\mu. \quad (10)$$

This is the simplest form of the general relation between the luminosity, mass, and age (or mass-to-light ratio) distributions subject to the two simplifying assumptions embodied in equations (4) and (7). To make further progress, we must specify  $\psi(M)$  and/or  $\theta(\mu)$ .

## 3. ILLUSTRATIVE EXAMPLES

We now consider four simple, illustrative examples, based on specific assumptions about the distributions of masses and mass-to-light ratios (delta functions and power laws). We could extend the list of examples indefinitely, but those presented here are sufficient to illustrate our main conclusions.

*Example 1: delta-function distribution of mass-to-light ratios*  $\theta(\mu) = \delta(\mu - \mu_0)$ , and arbitrary distribution of masses  $\psi(M)$ . In this case, all clusters have the same mass-to-light ratio  $\mu_0$  and hence the same age  $\tau_0$  given by  $\mu_0 = \mu(\tau_0)$ . The luminosity function, from equation (10), is

$$\phi(L) = \mu_0 \psi(\mu_0 L). \quad (11)$$

Here,  $\phi(L)$  and  $\psi(M)$  are identical apart from a rescaling of their arguments by  $\mu_0$ . This is appropriate for populations of clusters that are observed long after they formed, with typical ages much

larger than their age spreads (i.e.,  $\tau_0 \gg \Delta\tau$ ). This is probably a reasonable approximation for old globular clusters. However, it is a poor approximation for populations of clusters that are still forming, such as that in the Antennae galaxies, since in this case the typical age is comparable to the age spread (i.e.,  $\tau_0 \sim \Delta\tau$ ).

*Example 2: power-law distribution of mass-to-light ratios  $\theta(\mu) = A\mu^\epsilon$ , and arbitrary distribution of masses  $\psi(M)$ .* According to equation (10), the luminosity function is

$$\phi(L) = AL^{-(2+\epsilon)} \int_0^\infty \psi(M) M^{1+\epsilon} dM. \quad (12)$$

In this case,  $\phi(L)$  is a power law with exponent  $\alpha = -(2 + \epsilon)$  irrespective of the form of  $\psi(M)$ . This example illustrates in an extreme way that  $\phi(L)$ , in general, depends on  $\chi(\tau)$ , and not on  $\psi(M)$  alone. Equation (12) is valid so long as the integral on the right-hand side exists. This condition is satisfied whenever  $\psi(M)$  is finite over a finite range of  $M$  and zero elsewhere, as is true for real populations of star clusters. A power-law model of  $\psi(M)$ , if extended both to  $M = 0$  and  $M \rightarrow \infty$ , would, however, cause an artificial divergence of the integral in equation (12).

It is worth exploring this example in a little more detail. Stellar population models indicate that the mass-to-light ratio evolves approximately as a power law  $\mu(\tau) \propto \tau^\delta$ , over a wide range of ages  $\tau \gtrsim \text{few} \times 10^6$  yr (after smoothing over many small wiggles), with  $\delta \approx 0.8$  for luminosities in the  $V$  band, and larger (smaller) exponents for shorter (longer) wavelengths (Leitherer et al. 1999; Bruzual & Charlot 2003; see, e.g., Fig. 1 of Fall et al. 2005). Assuming that the age distribution is also a power law  $\chi(\tau) \propto \tau^\gamma$ , we then have, from equations (9) and (12),  $\theta(\mu) \propto \mu^\epsilon$  with  $\epsilon = (\gamma - \delta + 1)/\delta$ , and  $\phi(L) \propto L^\alpha$  with

$$\alpha = -1 - (1 + \gamma)/\delta. \quad (13)$$

In this case, the exponent of  $\phi(L)$  depends on the exponent of  $\chi(\tau)$  and, except in the special case  $\gamma = -1$ , on the photometric band (through  $\delta$ ).

For the young clusters in the Antennae galaxies, equation (13) and the observed age distribution, with  $\gamma \approx -1$ , predict  $\alpha \approx -1$ , and hence a luminosity function significantly shallower than the observed one, with  $\alpha \approx -2$ . Equation (12), therefore, does *not* explain the observed form of  $\phi(L)$  in the Antennae galaxies. We emphasize that this conclusion depends crucially on the observed *decline* of  $\chi(\tau)$ . In the absence of this information, equation (12) *would* provide an acceptable explanation for the observed form of  $\phi(L)$ , as the following example shows. For  $\gamma \approx 0$ , corresponding to  $\chi(\tau) \approx \text{const.}$  and hence little if any disruption of clusters, equation (13) implies  $\alpha \approx -2$ , close to the observed exponent of  $\phi(L)$  in many galaxies. Thus, without evidence to the contrary, we could speculate that nearly uniform age distributions, rather than power-law mass functions, are responsible for the observed power-law luminosity functions.<sup>1</sup>

*Example 3: delta-function distribution of masses  $\psi(M) = \delta(M - M_0)$ , and arbitrary distribution of mass-to-light ratios  $\theta(\mu)$ .* In this case, all clusters have the same mass  $M_0$ , and the luminosity function from equation (10) is

$$\phi(L) = M_0 L^{-2} \theta(M_0/L). \quad (14)$$

Here, as in the previous example,  $\phi(L)$  is determined entirely by  $\theta(\mu)$  and hence by  $\chi(\tau)$ . Equation (14) also demonstrates that

$\phi(L)$  can be a broad function even when  $\psi(M)$  has no width at all. In particular, if  $\theta(\mu)$  is a power law with exponent  $\epsilon$ , then  $\phi(L)$  is a power law with exponent  $\alpha = -(2 + \epsilon)$ , as expected, because equation (14) is then a special case of equation (12). Once again, this is not a viable explanation for the observed luminosity function of young clusters in the Antennae galaxies.

*Example 4: power-law distribution of masses  $\psi(M) = BM^\beta$ , and arbitrary distribution of mass-to-light ratios  $\theta(\mu)$ .* According to equation (10), the luminosity function is

$$\phi(L) = BL^\beta \int_0^\infty \theta(\mu) \mu^{1+\beta} d\mu. \quad (15)$$

In this case,  $\phi(L)$  and  $\psi(M)$  have identical forms; both are power laws with the same exponent, irrespective of  $\chi(\tau)$  and the band in which luminosities are measured. This probably is a good description of the population of young clusters in the Antennae galaxies, for which the observed luminosity, mass, and age distributions are approximate power laws with exponents  $\alpha \approx \beta \approx -2$  and  $\gamma \approx -1$ , respectively. In this case, as with any power-law model of  $\chi(\tau)$ , one might wonder whether the integral in equation (15) converges at small  $\mu$ , corresponding to small  $\tau$ . However, in all the photometric bands of interest here (near-UV through near-IR),  $\mu(\tau)$  follows a power law down to a finite, minimum value  $\mu_{\min}$ , which occurs at  $\tau \approx \text{few} \times 10^6$  yr (Leitherer et al. 1999; Bruzual & Charlot 2003). This truncates  $\theta(\mu)$  for  $\mu < \mu_{\min}$  and ensures that the integral is finite.

#### 4. DISCUSSION AND CONCLUSIONS

We can now answer the question posed in the Introduction: Why does the luminosity function of young clusters in the Antennae galaxies have the same, or approximately the same form as the mass function? As the examples in the previous section show, the answer has two parts: (1) because the mass function is a power law, and (2) because it is statistically independent of the age distribution. The exact form of  $\chi(\tau)$  is immaterial in the relation between  $\phi(L)$  and  $\psi(M)$  provided only that it declines steeply enough that example 2 above is irrelevant. The independence of  $\psi(M)$  and  $\chi(\tau)$ , however, is a key element. This may at first seem like only a mathematical convenience, but in fact it has important physical implications, as the following discussion makes clear.

Star clusters are disrupted by a variety of processes, beginning with the removal of interstellar matter by stellar activity (photoionization, winds, jets, supernovae), leading to “infant mortality.” The later disruptive effects include stellar mass loss, dynamical friction, gravitational shocks, and stellar evaporation driven by two-body relaxation (see Spitzer 1987 for a review). In massive galaxies like the Milky Way and the Antennae, dynamical friction and gravitational shocks, while potentially important for some clusters near the centers of galaxies, have relatively little effect on most of the other clusters. Moreover, the mass loss by stellar evolution and gravitational shocks, even when these processes are important, tends to preserve the shape of the mass function of the clusters; in particular, an initial power law remains a power law with the same exponent (see eqs. [12] and [13] of Fall & Zhang 2001).

The dominant long-term disruptive effect for most clusters is the gradual escape of stars resulting from gravitational scattering by other stars within the same clusters (“evaporation” driven by two-body relaxation). We denote the rate of mass loss  $-dM/dt$  by this process by  $\mu_{\text{ev}}$  (not to be confused with a mass-to-light ratio). For clusters with a constant mean internal density, set by the smooth tidal field of their host galaxy,  $\mu_{\text{ev}}$  also remains constant,

<sup>1</sup> See Hogg & Phinney (1997) for further discussion of this possibility in the context of the luminosity function of galaxies.

and  $M$  decreases linearly with time  $t$ . If the clusters of current age  $\tau$  formed at a rate  $h(\tau)$  in the past, with an initial mass function  $\psi_0(M)$ , then the current bivariate distribution of masses and ages is given by

$$g(M, \tau) = \psi_0(M + \mu_{\text{ev}}\tau)h(\tau). \quad (16)$$

(This is a straightforward generalization of eq. [11] of Fall & Zhang 2001.) Evidently,  $g(M, \tau)$  is independent of  $M$  for  $M \ll \mu_{\text{ev}}\tau$ , has a turnover at  $M \approx \mu_{\text{ev}}\tau$ , and is proportional to  $\psi_0(M)$  for  $M \gg \mu_{\text{ev}}\tau$ . This form of  $g(M, \tau)$  is not covered by the examples in the previous section because it is not the product of a function of  $M$  alone and a function of  $\tau$  alone, as in equation (7). We note incidentally that this is true of *any* disruptive process for which the mass-loss rate of a cluster depends on its mass, as in all models of the form  $dM/dt = -M/t_d(M)$  with  $t_d(M) \propto M^k$ , except the special case  $k = 0$ .

It is now interesting to consider the clusters in the Antennae galaxies in the context of equation (16). Detailed dynamical calculations for massive galaxies like the Milky Way and the Antennae give  $\mu_{\text{ev}} \approx 2 \times 10^{-5} M_\odot \text{ yr}^{-1}$  and hence a turnover or “peak” in the mass function of the clusters at  $M_p \approx 2 \times 10^5 (\tau/10^{10} \text{ yr}) M_\odot$  (Fall & Zhang 2001). This means that the clusters most affected by stellar evaporation have  $\tau \gtrsim M/\mu_{\text{ev}} \approx 5 \times 10^8 (M/10^4 M_\odot) \text{ yr}$ , and thus are not included in the empirical determination of  $g(M, \tau)$ . We are thus led to a two-stage picture for the disruption of the clusters (Fall et al. 2005). In the first stage, the energy and momentum input from massive young stars removes the interstellar matter from protoclusters, leaving many of them gravitationally unbound (“infant mortality”). The empirical evidence is that this process disrupts most of the clusters relatively rapidly but largely independently of their masses, thus reducing the amplitude of the mass function while preserving its initial power-law shape. Since nearly all the observed clusters are in this phase

of evolution, the illustrative examples of the previous section apply, as does our explanation for the similarity between the luminosity and mass functions. The few clusters that survive the first stage are then liable to disruption in the second by stellar evaporation driven by two-body relaxation. As noted above, this process eventually changes the shape of the mass function into that of old globular clusters.

How general is this picture? Does it apply to the clusters in all galaxies, or only to those in a small subset of galaxies like the Antennae? The initial mass function of star clusters is probably similar to the mass spectrum of their parent molecular clouds, and it is possible that this is determined by nearly universal, fractal-like density and/or velocity fields in the interstellar medium (Elmegreen 2002 and references therein). Moreover, the disruption rates in both the first (rapid) and second (slow) stages discussed above depend mainly on the properties of the clusters, not those of their host galaxies. Thus, we expect the picture outlined here to be fairly general. Nevertheless, it is important to test it observationally in detail in at least a few more cases, especially in noninteracting, quiescent galaxies. This requires determinations of the mass and age distributions over wide ranges of mass and age, which in turn requires deep multiband photometry of large samples of clusters. This should be a higher priority than simply determining the luminosity functions of clusters in more galaxies.

Our main conclusion, at least for the young clusters in the Antennae galaxies, is that the luminosity function has the same form as the mass function because the latter is a power law and is independent of the age distribution. But the main lesson of this paper is that disentangling the relations between the luminosity, mass, and age distributions has helped to clarify our picture of the formation and disruption of the clusters.

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