# A PULSATIONAL DISTANCE TO $\omega$ CENTAURI BASED ON NEAR-INFRARED PERIOD-LUMINOSITY RELATIONS OF RR LYRAE STARS ${ }^{1}$ 

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#### Abstract

We present new near-infrared ( $J$ and $K$ ) magnitudes for 114 RR Lyrae stars in the globular cluster $\omega$ Centauri (NGC 5139), which we combine with data from the literature to construct a sample of 180 RR Lyrae stars with $J$ and $K$ mean magnitudes on a common photometric system. This is currently the largest such sample in any stellar system. We also present updated predictions for $J$ - and $K$-band period-luminosity relations for both fundamental and first-overtone RR Lyrae stars, based on synthetic horizontal branch models with metal abundance ranging from $Z=0.0001$ to 0.004 . By adopting for the $\omega$ Cen variables with measured metal abundances an $\alpha$-element enhancement of a factor of $3(\approx 0.5 \mathrm{dex})$ with respect to iron, we find a true distance modulus $\mu_{0}=13.70 \pm 0.06 \pm 0.06$ (random and systematic errors, respectively), corresponding to a distance $d=5.5 \pm 0.03 \pm 0.03 \mathrm{kpc}$. Our estimate is in excellent agreement with the distance inferred for the eclipsing binary OGLEGC 17, but differ significantly from the recent distance estimates based on cluster dynamics and on high-amplitude $\delta$ Scuti stars.


Subject headings: distance scale - globular clusters: individual ( $\omega$ Centauri) — stars: evolution stars: horizontal-branch - stars: oscillations - stars: variables: other

## 1. INTRODUCTION

The period-luminosity (PL) relation for classical Cepheids and, in the near-infrared (NIR), for RR Lyrae stars is one of the most important methods of estimating distances and calibrating secondary distance indicators (Bono et al. 2001; Freedman et al. 2001; Saha et al. 2001; Cassisi et al. 2004, hereafter C04; Catelan 2004, hereafter CPS04; Gieren et al. 2005). Current trigonometric parallaxes from Hipparcos (Feast \& Catchpole 1997) and HST (Benedict et al. 2002), and from ground-based telescopes (Lane et al. 2000; Kervella et al. 2004) have provided a unique possibility of calibrating the zero point of these relations. Unfortunately, the number of Galactic calibrators with accurate trigonometric parallaxes is very limited, and for RR Lyrae stars it is restricted to a single object, namely, RR Lyr itself. Therefore, we still lack a

[^0]sound knowledge of the slope of the different PL relations based on purely geometrical methods. To overcome this problem, the astronomical community devoted a paramount observational effort to measuring the pulsation properties of these robust distance indicators. The ideal laboratories for these studies are the globular clusters (GCs) and the dwarf spheroidals (dSphs) in the Local Group, as the RR Lyrae stars in a given system are located at largely the same distance.
It has been demonstrated by Longmore et al. (1990) that cluster RR Lyrae follow a well-defined $K$-band PL relation, but we still lack a comprehensive empirical investigation of its properties. For this reason, during the last few years we undertook a detailed pulsation and evolutionary investigation of RR Lyrae stars (Bono et al. 2003 and references therein), and, at the same time, we also started a large observational project aimed at collecting new NIR data for RR Lyrae stars in Galactic (M92; Del Principe et al. 2005) and Large Magellanic Cloud clusters (Reticulum; Dall'Ora et al. 2004).

Although the theoretical and empirical scenarios have recently seen new predictions based on synthetic horizontal-branch (SHB) models (C04; CPS04) and new data (Butler 2003; Borissova et al. 2004; Storm 2004), we still lack a detailed NIR photometric analysis of a stellar system that includes a large sample of fundamental (F; RR $a b$ ) and first-overtone (FO; RR $c$ ) RR Lyrae stars (only a dozen Galactic GCs host more than 50 RR Lyrae, see, e.g., Clement et al. 2001) and that spans a broad range in metal abundances. At present, the largest NIR data set includes 74 RR Lyrae stars whose metal abundance, based on 23 objects, ranges from $[\mathrm{Fe} / \mathrm{H}] \approx-2$ to -0.7 , as observed by Borissova et al. (2004) in the inner regions of the Large Magellanic Cloud. In this context, the GC $\omega$ Cen is a perfect target, since it includes more than 186 RR Lyrae ( 86 RRab, 100 RRc; Kaluzny et al. 2004) with a metal content ranging from $[\mathrm{Fe} / \mathrm{H}] \sim-2$ to -1 (Rey et al. 2000 using 131 objects; Sollima et al. 2006 using 74 objects). Moreover, the $\omega$ Cen distance has been estimated using different distance indicators such as the first-overtone blue edge (FOBE; Caputo et al. 2002), the eclipsing binary OGLEGC 17 (Thompson et al.


Fig. 1.-Distribution of RR Lyrae stars in the field of $\omega$ Cen. Open and filled symbols show FO and F mode RR Lyrae, respectively. Solid and dashed rectangles show the fields observed with SOFI/NTT. See text for more details.

2001; Kaluzny et al. 2002), the $V$-band metallicity relation of RR Lyrae stars (Rey et al. 2000; Catelan 2006), and a dynamical analysis by van de Ven et al. (2006) based on proper motion and radial velocities of a large sample of individual stars. In this paper, we present new NIR photometry for a substantial number of the RR Lyrae stars in $\omega$ Cen, and we provide new distance estimates, which we compare with determinations already present in the literature.

## 2. OBSERVATIONS AND DATA REDUCTION

The present NIR data set includes three different samples of $J$ and $K_{s}$ data: those collected with the NIR camera SOFI, available at the New Technology Telescope (NTT; ESO, La Silla), or by Longmore et al. (1990), and those available from the Two Micron All Sky Survey (2MASS) catalog. ${ }^{13}$

NIR $J$ and $K_{s}$ images of $\omega$ Cen were collected in two different runs, namely 2001 February 6 and 8 (UT) and 2002 February 3, 4,25 , and 26 (UT). The pixel scale of SOFI is 0.292 , while the field of view is $4.94 \times 4^{\prime} .94$. We observed three different fields: field A is centered on the cluster center, field C at a distance $\sim 10$. 5 NW away from the center, and field D at a distance $\sim 10.7$ SW away from the center (see solid rectangles in Fig. 1). We collected $36 J$ and $49 K_{s}$ band images of field A, $33 J$ and $52 K_{s}$ band images offield C, and $12 J$ and $18 K_{s}$ band images of field D. For $J$ images, the exposure time was $t=3 \mathrm{~s}$, while for $K_{s}$ images, it was $t=12 \mathrm{~s}$. Owing to the crowded nature of the fields, the observing time was equally divided between observations on $\omega$ Cen and sky observations. Together with these data, we retrieved from the ESO archive a mosaic of nine $J$ and nine $K_{s}$ images of $\omega$ Cen (Sollima et al. 2004) that cover an area of $\sim 13^{\prime} \times 13^{\prime}$ across the cluster center (see dashed rectangles in Fig. 1). Note that our field A overlaps with the central pointing of this data set. These images were collected on 2000 January 13 and 14 (UT), using the same experimental equipment. Seven out of the nine fields were observed once in $J$ and $K_{s}$ bands, while the

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Fig. 2.-Difference in $K(t o p)$ and $J$ (bottom) magnitudes of Field A based on two different absolute zero-point calibrations. $J_{\text {Persson }}$ and $K_{\text {Persson }}$ secondary standards have been calibrated using four standard stars measured by Persson et al. (1998), while $J_{2 \text { MASS }}$ and $K_{2 \text { MASS }}$ secondary standards have been calibrated using $\approx 400$ local 2MASS standard stars.
other two fields were observed twice. The individual exposure times were 54 and 180 s .

The preprocessing was performed using standard IRAF procedures. To improve the photometric accuracy, we did not stack the individual images but determined the (linearly variable) pointspread function (PSF) for each frame based on more than 50 PSF stars uniformly distributed across each image. Photometry for the entire set of $J$ - and $K_{s}$-band, together with a few $V$ - and $I$-band (Dall'Ora et al. 2004; Del Principe et al. 2005) images was then simultaneously performed with DAOPHOT and ALLFRAME (Stetson 1994). A more detailed analysis concerning the reduction strategy will be discussed in a forthcoming paper (M. Del Principe et al. 2006, in preparation).

The absolute calibration in the 2MASS system (Cutri et al. 2003) was performed using a large set of $\approx 400$ local stars selected from the 2MASS Point Source Catalog. To constrain, on a quantitative basis, the intrinsic accuracy of both absolute and relative zero points, we performed an independent calibration using a set of four standard stars (Persson et al. 1998) observed during the night of 2002 February 24 at air masses that bracket the observations of fields A, C, and D. The stars selected were S791-C, S867-V, S273-E, and S870-T. Figure 2 shows the comparison between secondary standards in field A and local 2MASS stars. The observed distribution of magnitude differences has a standard deviation of $K_{2 \mathrm{M}}-K_{\mathrm{P}}=-0.012 \pm 0.076$ and $J_{2 \mathrm{M}}-J_{\mathrm{P}}=$ $0.003 \pm 0.080$, which is a measure of the typical difference for an individual star. Note that the quoted differences agree well, within the errors, with the transformations provided by Carpenter (2001). As a further independent test, we also calibrated the NIR data using standard stars collected during the other observing nights, and we found that the accuracy of the absolute zero points is $\sim 0.02 \mathrm{mag}$.

We ended up with a photometric catalog of $\sim 115,000$ stars with limiting magnitudes of $J \approx 20$ and $K_{s} \approx 19.0 \mathrm{mag}$. Figure 3 shows the NIR color-magnitude diagram (CMD) of the stars with


Fig. 3.-NIR color-magnitude diagram of $\omega$ Cen. The stars plotted in this figure are located within $\approx 13^{\prime} \times 13^{\prime}$ across the cluster center. Open and filled circles mark the sample of FO and F RR Lyrae stars for which we estimated $J$ - and $K$-band magnitudes. Error bars display intrinsic photometric errors.
a "separation index" ${ }^{14}$ larger than 3.0 and an intrinsic photometric accuracy of $\sigma_{J, K} \leq 0.08$ mag.

We identified 56 cataloged (OGLE II; Kaluzny et al. 2004) RR Lyrae stars ( $27 \mathrm{RR} a b, 29 \mathrm{RR} c$ ) in fields A, C, and D (see circles in Fig. 1) and 58 additional RR Lyrae ( $25 \mathrm{RR} a b, 33 \mathrm{RR}_{\mathrm{c}}$ ) variables (see diamonds in Fig. 1) in Sollima's fields. In order to increase the sample of RR Lyrae stars, we also included data for the 29 RR Lyrae stars (19 RR $a b, 10 \mathrm{RR} c$; see squares in Fig. 1) observed by Longmore et al. (1990). We have transformed their mean $K$-band data from the Anglo-Australian Observatory NIR system to the 2MASS $K_{s}$ system using the transformations provided by Carpenter (2001). We also cross-correlated the catalogs provided by OGLE II and by Clement et al. (2001) with the 2MASS catalog. In this way we identified 37 more cataloged RR Lyrae stars ( $10 \mathrm{RR} a b, 27 \mathrm{RR} c$; see triangles in Fig. 1), and, therefore, we ended up with a total sample of 180 RR Lyrae (81 RRab, $99 \mathrm{RR} c$ ), for which we have at least one $J$ - or $K_{s}$-band measurement.

To obtain the most accurate mean magnitudes for the RR Lyrae, we fit the individual $K_{s}$ phase points measured by ALLFRAME with a template curve (Jones et al. 1996) and compute, on the basis of this curve, the intensity-averaged mean magnitude. The method requires an accurately known ephemeris of the epoch of maximum for each star, as well as the amplitude in the optical $V$ or $B$ bands. These parameters are available from Kaluzny et al. (2004) for 167 of the variables in our sample. For each of the remaining 13 variables, all with single-epoch 2MASS measurements, the 2MASS measurement was adopted as the best estimate of the mean magnitude.

The typical accuracy of mean $K$-band magnitude of RR Lyrae stars, either with a good coverage of the light curve or with singleepoch measurements, epoch of maxima, and luminosity ampli-

[^2]tudes, is better than 0.02 mag . In order to constrain the typical uncertainty for the remaining thirteen objects (2MASS sample) with only single-epoch measurements, we estimated the difference between the mean $K$-band magnitudes of the RR Lyrae stars for which we have a very good coverage of the light curves ( $22 \mathrm{RR} a b$, $22 \mathrm{RR} c$ ) and the mean $K$-band magnitudes only based on the 2MASS measurements. We found that the individual differences are smaller than 0.2 mag for $\mathrm{RR} a b$ and 0.1 mag for $\mathrm{RR} c$.

The current RR Lyrae sample includes 16 Blazhko RR Lyrae stars (Kaluzny et al. 2004). Ten Blazhko RR Lyrae (eight in our sample and two in the Longmore sample) present a good coverage of the light curves. For four objects (three in the Sollima sample and one in the 2MASS sample), single-epoch measurements and accurate epochs are available, and therefore the mean magnitude was determined using the template curve. We adopted the single-epoch measurements for only two objects in the 2MASS sample, since recent epochs are not available. The Blazhko effect shows up as a modulation in the luminosity amplitude (Stothers 2006, and references therein) that might affect the accuracy of the mean $K$-band magnitudes estimated with the template curve. In order to account for the uncertainty in the luminosity amplitude, we estimated the mean $K$-band magnitude by increasing/ decreasing the amplitude by a factor of 2 . The difference in the mean $K$-band magnitude is of the order of 0.02 mag . The error for the two objects with single-epoch $J$ - and $K$-band magnitudes is at most of the order of $0.15-0.20 \mathrm{mag}$.

Unfortunately, no empirical template is available for the $J$-band light curves. Instead, the mean $J$-band magnitude of RR Lyrae stars with well-sampled light curves (56 RR Lyrae stars located in fields $\mathrm{A}, \mathrm{C}$, and D ) was determined from a cubic spline fit to the light curves. The fit was performed using a spline under tension (Stellingwerf 1978, and references therein). This approach presents several advantages when compared with the classical Fourier fit, particularly for noisy light curves (Del Principe et al. 2006). For the other objects, the $J$-band magnitude is based on a single epoch measurement. We found that the typical difference between the mean $J$-band magnitudes of the RR Lyrae stars for which we have a very good coverage of the light curves ( $22 \mathrm{RR} a b, 22 \mathrm{RR} c$ ) and the mean $J$-band magnitudes only based on single-epoch 2MASS measurements is systematically smaller than 0.3 mag for RRab and 0.15 mag for $\mathrm{RR} c$ stars.

Open and filled circles in Figure 3 show the location in the CMD of the entire sample of RR Lyrae stars. This is the largest sample of homogeneous NIR magnitudes for RR Lyrae stars in a stellar system ever collected.

## 3. DISTANCE DETERMINATIONS

To determine the distance to $\omega$ Cen on the basis of the observed mean dereddened $K_{0}$ and $J_{0}$ magnitudes, we need to adopt a relation between observational quantities and the absolute magnitude. From the theoretical point of view, these relations have been recently investigated (e.g., Bono et al. 2001, 2003; CPS04; C 04 ) on the basis of SHB models with promising results in good agreement with empirical calibrations. These model grids can be parameterized in different ways, the simplest relation being the $\log P-M_{K}$ relation, and the more complicated relations taking into account either the global metallicity $Z$ (e.g., Bono et al. 2001, 2003) or the Lee (1990) horizontal-branch (HB) type $\tau_{\mathrm{HB}}{ }^{15}$ (e.g., C04; CPS04).

[^3]Bono et al. (2003) showed that metallicity has a significant impact on the $K$-band magnitude of RR Lyrae stars. C04 showed that using the HB type as a parameter, at fixed metal abundance, also results in tight relations in both $J$ and $K$. The latter approach is only valid for GCs where the HB type can be determined, and it is not directly applicable to field stars. $\omega$ Cen is a peculiar object, as it is morphologically a GC, but the chemical composition and probably the ages of the stars differ significantly, making the population more similar to that of a dwarf galaxy. Consequently, the HB type is not well defined a priori, although the CMD presented in Figure 3 shows that there are very few HB stars redder than the RR Lyrae stars. This feature suggests that all the subpopulations that contribute to the HB must have a collective HB type close to $1 .{ }^{16}$

In order to determine accurate distance estimates for both F and FO pulsators, we computed a new set of SHB models with selected metal abundances $(Z=0.0001,0.0003,0.001$, and 0.004$)$ and a helium content of $Y=0.245$. We did this by adopting the detailed set of HB models recently constructed by Pietrinferni et al. (2006) and the procedure already discussed by C04. The pulsation models discussed by Di Criscienzo et al. (2004) were used to fix the boundaries of the RR Lyrae instability strip and to evaluate the periods of the pulsators. On this basis, evolutionary and pulsation predictions were used to constrain the stellar distribution along the HB and to compute for each individual simulation the HB type $\tau_{\text {HB }}$. The SHB models were constructed by accounting for the hysteresis effect (see, e.g., Bono et al. 1995) and by neglecting the pre-zero-age HB evolutionary phases (see, e.g., Piersanti et al. 2004).

Theoretical predictions were transformed into the Bessell \& Brett (1988) NIR photometric system using the bolometric corrections and color-temperature relations provided by Pietrinferni et al. (2004). They were then transformed into the 2MASS system using the transformations provided by Carpenter (2001), and eventually, for each given SHB, the predicted period-luminosity relations, as given in the form $M_{J, K}=a_{J, K}+b_{J, K} \log P+$ $c_{J, K} \tau_{\mathrm{HB}},{ }^{17}$ were derived. The coefficients and the relative errors of the least-squares solutions are listed in Table 1. The zero points and the coefficients of the above period-luminosity-metallicity (PLZ) relations further support the use of independent relations for F and FO pulsators. As a matter of fact, the slopes and the zero points between the two different groups differ on at least the $10 \sigma$ level for both the $J$ and $K$ bands. The referee suggested we check the dependence of our F and FO PL relations on the predicted effective temperature of the F blue edge and the FO red edge. We performed a few numerical experiments artificially increasing/ decreasing, by 150 K , the effective temperature of the FO red edge and of the F blue edge. We found that the change in the effective temperature of the F blue edge affects the coefficients of $J$ - and $K$-band FO PL relations by $\pm 0.02-0.03$ (a) and by $\pm 0.02-0.05(b)$, respectively. The coefficients of $J$ - and $K$-band F PL relations change by $\pm 0.02$ (a) and by $\pm 0.02-0.04$ (b), respectively. The change in the effective temperature of the FO red edge affects the $a$ and $b$ coefficients of the F and FO relations, on average, by $\pm 0.01$. The impact of these changes on the coefficient of the HB type (c) is marginal. Note that the adopted uncertainty on the location of the FO red edge and the F blue edge is supported by the comparison with RR Lyrae stars in GCs (Di Criscienzo et al. 2004). These findings further support the

[^4]TABLE 1
Predicted Near-Infrared Period-Luminosity Relations

| $Z$ | Mode | $a$ | $b$ | $c$ |
| :---: | :---: | :---: | :---: | :---: |
| $M_{K}=a+b \log P+c \tau_{\text {HB }}$ |  |  |  |  |
| $0.0001 \ldots \ldots .$. | F | $-1.32 \pm 0.02$ | $-2.19 \pm 0.01$ | $0.14 \pm 0.01$ |
|  | FO | $-1.65 \pm 0.02$ | $-2.32 \pm 0.01$ | $0.14 \pm 0.01$ |
| $0.0003 \ldots \ldots$. | F | $-1.23 \pm 0.02$ | $-2.34 \pm 0.01$ | $0.04 \pm 0.01$ |
|  | FO | $-1.55 \pm 0.02$ | $-2.38 \pm 0.01$ | $0.04 \pm 0.01$ |
| $0.001 \ldots \ldots \ldots$. | F | $-1.14 \pm 0.01$ | $-2.37 \pm 0.01$ | $0.01 \pm 0.01$ |
|  | FO | $-1.48 \pm 0.01$ | $-2.41 \pm 0.01$ | $0.01 \pm 0.01$ |
| $0.004 \ldots \ldots \ldots$. | F | $-1.05 \pm 0.01$ | $-2.46 \pm 0.01$ | 0.0 |
|  | FO | $-1.37 \pm 0.01$ | $-2.43 \pm 0.01$ | 0.0 |


| $M_{J}=a+b \log P+c \tau_{\mathrm{HB}}$ |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| $0.0001 \ldots \ldots \ldots$ | F | $-0.92 \pm 0.02$ | $-1.62 \pm 0.01$ | $0.08 \pm 0.01$ |
|  | FO | $-1.13 \pm 0.02$ | $-1.64 \pm 0.01$ | $0.07 \pm 0.01$ |
| $0.0003 \ldots \ldots .$. | F | $-0.83 \pm 0.02$ | $-1.77 \pm 0.01$ | $0.05 \pm 0.01$ |
|  | FO | $-1.10 \pm 0.02$ | $-1.83 \pm 0.01$ | $0.04 \pm 0.01$ |
| $0.001 \ldots \ldots \ldots$. | F | $-0.76 \pm 0.01$ | $-1.89 \pm 0.01$ | $0.01 \pm 0.01$ |
|  | FO | $-1.04 \pm 0.01$ | $-1.88 \pm 0.01$ | $0.01 \pm 0.01$ |
| $0.004 \ldots \ldots \ldots$. | F | $-0.70 \pm 0.01$ | $-2.13 \pm 0.01$ | 0.0 |
|  | FO | $-0.94 \pm 0.01$ | $-1.93 \pm 0.01$ | 0.0 |

Note.-The pulsation modes are indicated as F for fundamental and FO for first-overtone pulsators.
approach of deriving different PL relations for F and FO pulsators. It is also noteworthy that the use of fundamentalized FO periods causes an increase of a factor of 2 in the uncertainty of their zero points and slopes.

As a whole, these relations agree quite well with those already presented by C 04 (see their Tables 3 ). In particular, (1) the slopes of the $M_{J, K}-\log P$ relations are, at fixed metal content, independent of the HB type, while they become slightly steeper with increasing $Z$; (2) the zero points of the $M_{J, K}-\log P$ relations become, at fixed metal content, brighter when moving from blue to red HB populations - this dependence significantly decreases with increasing $Z$; and (3) the zero points of the $M_{J, K}-\log P$ relations, independently of the HB-type, become fainter with increasing $Z$. An even more striking agreement is found with the relations computed by CPS04. For the relevant mean metallicity model, $Z=0.001$ and HB type $\tau_{\mathrm{HB}}=0.934$, they find for their combined sample of F mode and FO pulsators that $M_{K}=$ $-2.388 \log P_{F}-1.133$, whereas we find for the F mode pulsators that $M_{K}=-2.37 \log P_{F}-1.13$ and for the FO pulsators that $M_{K}=-2.41 \log P_{\mathrm{FO}}-1.20$. The referee noted that, according to CPS04, the coefficients $a$ and $b$ of the PL relations present some dependence on both the HB type and the metal abundance. Unfortunately, a global comparison is hampered by the different approach adopted to derive the PL relations, i.e., multilinear leastsquares versus third-order polynomials (see their eqs. [1] and [2] and their Table 9). Moreover, the metallicity of our SHB models range from $Z=0.0001$ to 0.004 , while those from CPS04 range from $Z=0.0005$ to 0.006 .

The individual reddening values of $\mathrm{RR} a b$ stars have been empirically estimated using a period-color $(V-K)$-amplitude $\left(A_{V}\right)$ relation (Piersimoni et al. 2002; A. M. Piersimoni et al. 2006, in preparation), while for $R R c$ stars and RR Lyrae affected by amplitude modulation, we used a reddening map based on uvby Strömgren photometry (Calamida et al. 2005). Accurate mean $V$-band magnitudes for RR Lyrae in our sample have been measured either by Kaluzny et al. (2004) or by Butler et al. (1978) for the outermost ones. For the remaining objects, we adopted a


Fig. 4.-Dereddened $K$ magnitudes of RR Lyrae stars in $\omega$ Cen vs. period. Dashed and solid lines display the predicted period-luminosity relations for F ( filled circles) and FO (open circles) pulsators, respectively. The adopted metal content and HB-type are labeled. The error bars show intrinsic errors in the measured magnitudes.
mean reddening of $E(B-V)=0.11 \pm 0.02$. Selective absorptions have been estimated using the reddening law from Cardelli et al. (1989), that is, $A_{V} / E(B-V)=3.1, A_{J} / E(B-V)=0.868$, and $A_{K} / E(B-V)=0.341$.

Data plotted in Figure 4 show the fit between the dereddened $K$ magnitudes and predicted PL relations at fixed metal abundance and for $\tau_{\mathrm{HB}}=0.94$. In this figure, the asterisks mark the overluminous RR $a b$ stars that are either blended with a fainter close companion (V32, V118, V139, and V150) or have a controversial mode identification (V84), while the open triangle refers to V182, which, together with three other (V168, V181, and V183) underluminous variables with $K_{0} \geq 14 \mathrm{mag}$, presents peculiar $J-K$ colors. By excluding these variables, we estimate a true distance of $\left\langle\mu_{0}\right\rangle=13.80 \pm 0.10 \mathrm{mag}$ and $13.70 \pm 0.10 \mathrm{mag}$ with $Z=0.0001$ and 0.001 , respectively, clearly indicating that individual metallicity estimates are required to improve the distance determination.

Conveniently enough, individual metal abundances on the scale of Zinn \& West (1984), as based on the $h k$ index, for a substantial fraction (130) of the RR Lyrae plotted in Figure 4 have been measured by Rey et al. (2000). In a recent investigation based on medium resolution spectroscopy, Sollima et al. (2006) found, for 74 RR Lyrae in $\omega$ Cen, no systematic difference with the metal abundances based on the $h k$ index. To properly use the theoretical predictions, the observed $[\mathrm{Fe} / \mathrm{H}]$ values have been transformed into "global" $Z$ values by taking into account possible $\alpha$-element enhancements. ${ }^{18}$ Following to Gratton et al. (2004), who found $[\alpha / \mathrm{Fe}] \approx 0.45$ as a plausible upper limit to the $\alpha$-element enhancement, we adopt $f=10^{[\alpha / \mathrm{Fe}]}=3$.

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Fig. 5.-Individual true distance moduli versus [Fe/H] for RR Lyrae in $\omega$ Cen. The panels show distance evaluations for scaled solar $(f=1)$ and $\alpha$-enhanced ( $f=3$ ) chemical compositions, as well as with HB-type $\tau_{\mathrm{HB}}=0$ and 0.94 . The error bars in the bottom right panel display individual errors affecting $K$ magnitudes and $[\mathrm{Fe} / \mathrm{H}]$ abundances. The error bars in the other panels have been omitted to make more clear the distribution of F and FO variables.

We are now in a position to derive distance moduli of individual RR Lyrae stars with metallicity measurements from Rey et al. (2000). In Figure 5d, we show the resulting values as a function of metallicity, leading to $\left\langle\mu_{0}\right\rangle=13.70 \pm 0.06 \mathrm{mag}$. To illustrate the sensitivity of the method to possible errors in the assumed HB type or $\alpha$-enhancement, we have also computed the distance moduli for $\tau_{\text {HB }}=0$ and/or $f=1$ and plotted the results in Figure 5 . We note that the effect on the distance moduli is rather small, considering that the estimated error on the HB type, $\sigma_{\tau}=0.2$ (see star counts of HB stars in NGC 2808 listed in Table 2 of Castellani et al. 2006), and on the $\alpha$-enhancement factor, $\sigma_{f}=0.5$, are typically smaller than the values adopted here. Of particular interest is the fact that the effect of an error in the assumed HB type is to introduce a slope in these diagrams (see Figs. $5 a$ and 5b). According to the relations given in Table 1, this trend can be explained as empirical evidence that the more metal-poor RR Lyrae are connected with bluer HB populations than the more metal-rich ones.

The $K$-based distance determinations are fully supported by the $J$-band measurements. The estimated uncertainty on the mean $J$ magnitudes depends on the luminosity amplitude, $\sigma_{J}(\mathrm{RR} c) \approx$ $0.1 ; \sigma_{J}(\mathrm{RR} a b) \approx 0.2$, and it is less accurate than for the $K$-band measurements ( $\sigma_{K} \approx 0.015$ ). It is worth mentioning that we still lack empirical template curves for the $J$ band, and therefore accurate mean $J$-band magnitudes are only available for 56 out of the 180 RR Lyrae in our sample. As shown in Figure 6, the true distance moduli based on the relations given in Table 1 agree with the values based on $K$ magnitudes, but with a larger error. Although current mean $J$-band magnitudes have this drawback, this NIR band appears very promising to estimate distances, since it presents several advantages over the $K$ band and requires much less telescope time to reach similar photometric accuracy.

We can also determine the distance to the individual RR Lyrae stars using the $\mathrm{PLZ}_{K}$ relations of Bono et al. (2003). By using both field and cluster RR Lyrae stars, they found $M_{K}=-0.77-$ $2.101 \log P_{F}+0.231[\mathrm{Fe} / \mathrm{H}]$, where the fundamentalized logarithmic


Fig. 6.-As in Fig. 5, but for the mean $J$ magnitudes.
period was computed as $\log P_{F}=\log P_{\mathrm{FO}}+0.127$. Combining this relation with the subsample of RR Lyrae for which metallicity estimates from Rey et al. (2000) are available, we find $\mu_{0}=$ $13.77 \pm 0.07 \mathrm{mag}$ based on a total of $113 \mathrm{RR} a b$ and $\mathrm{RR} c$ stars.

In addition to the quoted intrinsic errors, we estimate the external, systematic error to be $\sigma_{\text {sys }}=0.06$ mag. This is the result of summing, in quadrature, the contributions from the photometric zero point ( $\sigma=0.02 \mathrm{mag}$ ) and the uncertainty on the absorption $A_{K}\left[\sigma=0.01 \mathrm{mag}\right.$, based on uncertainties of 0.1 on $R_{K}$ and 0.02 mag on $E(B-V)$ ]. The uncertainty of the reddening is based on the recent Strömgren photometric investigation by Calamida et al. (2005), while the uncertainty on the selective absorption coefficient is due to current uncertainties in the adopted extinction law (Rieke \& Lebofsky 1985; Cardelli et al. 1989, and references therein). The absolute zero point of the metallicity scale is a significant source of uncertainty, as discussed by Dall'Ora et al. (2004), due to systematic differences between metallicity scales. They estimate $\sigma=0.25$ dex on the zero point, which results in the dominating contribution to the systematic error of $\sigma=0.25 \times 0.231=0.06 \mathrm{mag}$.

As the $\omega$ Cen RR Lyrae stars exhibits a broad range of metallicities and all belong to the same cluster, we can empirically constrain the metallicity effect without having to rely on uncertain distance estimates. We have made linear three-parameter fits of the relation in equation (1) for $\mathrm{RR} a b$ and $\mathrm{RR} c$ stars separately, using only dereddened apparent magnitudes and the variables with individual $h k$ abundance evaluations. We find

$$
\begin{align*}
K_{0}(F)= & 12.70( \pm 0.06)-2.71( \pm 0.12) \log P_{F} \\
& +0.12( \pm 0.04)[\mathrm{Fe} / \mathrm{H}](N=54)  \tag{1}\\
K_{0}(\mathrm{FO})= & 12.26( \pm 0.05)-2.49( \pm 0.15) \log P_{\mathrm{FO}} \\
& +0.04( \pm 0.04)[\mathrm{Fe} / \mathrm{H}](N=48),  \tag{2}\\
J_{0}(F)= & 13.09( \pm 0.12)-2.37( \pm 0.25) \log P_{F} \\
& +0.15( \pm 0.08)[\mathrm{Fe} / \mathrm{H}](N=54)  \tag{3}\\
J_{0}(\mathrm{FO})= & 12.58( \pm 0.08)-2.09( \pm 0.21) \log P_{\mathrm{FO}} \\
& -0.01( \pm 0.05)[\mathrm{Fe} / \mathrm{H}](N=48), \tag{4}
\end{align*}
$$



Fig. 7.-Mean $K_{0}$ magnitude corrected for the empirically determined period effect as a function of metallicity for $\mathrm{FO}(\mathrm{RR} a b)$ and $\mathrm{FO}(\mathrm{RR} c)$ pulsators respectively. The best-fit lines have been overplotted.
where the symbols have their usual meaning. In Figure 7, we have plotted the observed magnitudes against metallicity after having removed the period dependency. The metallicity effect is weak for the F pulsators, and for the FO pulsators the metallicity effect is consistent with only a marginal dependence. In both cases, the relations are in reasonable agreement with the predictions from the SHB models. Our $K$-band PL relations present two substantial differences when compared with the PLZ relation derived by Bono et al. (2003): (1) we derived independent PL relations for F and FO pulsators, while Bono et al. only derived semiempirical "average" PL relations for F and fundamentalized FO pulsators; (2) our PL relations only apply to cluster variables for which we can estimate the HB type, while the PLZ relation derived by Bono et al. (2003) applies to individual RR Lyrae stars.

## 4. CONCLUSIONS AND FINAL REMARKS

In Table 2, we list recent empirical distance estimates to $\omega$ Cen, where we have corrected all the moduli for the same reddening and employed the same reddening law that we have used for the RR Lyrae stars. We note that our new near-IR results are in excellent agreement with the largely geometric distance to the eclipsing binary OGLEGC 17 from Thompson et al. (2001) and Kaluzny et al. (2002) and from the first-overtone blue edge method based on pulsation predictions from Caputo et al. (2002). Our distance determinations based on $J$ - and $K$-band measurements and on NIR PL ( $\tau_{\mathrm{HB}}$ ) relations are also in very good agreement with similar estimates based on NIR relations derived by CPS04. In particular, for $\tau_{\mathrm{HB}}=0.94$ and $f=3$, we found $13.70 \pm 0.06$ versus $13.72 \pm 0.06$ ( $K$ band) and $13.71 \pm 0.10$ versus $13.76 \pm$ 0.10 ( $J$ band). Note that distance estimates based on CPS04 relations were determined using the same solar metal abundance and we did not include a few very metal-poor RR Lyrae, since CPS04 predictions range from $Z=0.0005$ to 0.006 . This finding is also supported by most recent $M_{V^{-}}[\mathrm{Fe} / \mathrm{H}]$ relations. By adopting the mean $V$ magnitudes for RR Lyrae stars provided by Kaluzny et al. (2004) and Butler et al. (1978), together with the calibration given by Bono et al. (2003), we obtain for RR Lyrae with individual metal abundances a distance value of $(m-M)_{0}=$ $13.72 \pm 0.11$. We obtain a similar distance modulus ( $13.62 \pm$ 0.11 ) using the same magnitudes and the recent calibration of the $M_{V^{-}}[\mathrm{Fe} / \mathrm{H}]$ relation provided by Catelan (2006). The slope of such a relation was estimated by Cacciari \& Clementini (2003),

TABLE 2
Distance Determinations to $\omega$ Cen

| Method | $(m-M)_{0}{ }^{\text {a }}$ | Notes |
| :---: | :---: | :---: |
|  | $13.70 \pm 0.06$ | b |
|  | $13.71 \pm 0.10$ | b |
|  | $13.77 \pm 0.07$ | c |
|  | $13.72 \pm 0.06$ | d |
| PL- $\tau_{\mathrm{HB}}(J) . . . . . . . . . . . . . . . . . . . . . . . . . . ~$ | $13.76 \pm 0.10$ | d |
| $M_{V}-[\mathrm{Fe} / \mathrm{H}] . . . . . . . . . . . . . . . . . . . . . . . . . . ~$ | $13.72 \pm 0.11$ | e |
| $M_{V}$-[Fe/H] ......................... | $13.62 \pm 0.11$ | Catelan (2006) ${ }^{\text {f }}$ |
| Eclipsing binary .................... | $13.71 \pm 0.11$ | Thompson et al. (2001) |
| Eclipsing binary .................... | $13.75 \pm 0.04$ | Kaluzny et al. (2002) |
| FOBE .................................. | $13.74 \pm 0.11$ | Caputo et al. (2002) |
| High-amplitude $\delta$ Sct............. | $14.05 \pm 0.02$ | McNamara (2000) |
| Cluster dynamics................... | $13.41 \pm 0.13$ | van de Ven et al. (2006) |

[^6]while the zero point is based on the trigonometric parallax of RR Lyr itself (Benedict et al. 2002; Bono et al. 2002). Note that the systematic error for distance estimates based on visual magnitudes is $\sigma=0.08$, due to the significant contribution from the absorption correction.

On the other hand, we deviate by more than $3 \sigma$ from the distance evaluation based on high-amplitude $\delta$ Scuti (HADS) stars ( $\mu_{0}=14.05 \pm 0.02 ;$ McNamara 2000) and from the modulus of $\mu_{0}=13.41 \pm 0.13$ from van de Ven et al. (2006) based on clus-
ter star dynamics. It is worth noting that our uncertainties on metal abundance and on NIR photometry cannot account for such a discrepancy.

The NIR PL relations for RR Lyrae stars provide an accurate approach for determining absolute distances to globular clusters and other stellar systems. We find for $\omega$ Cen a distance modulus of $13.70 \pm 0.06 \pm 0.06$, where the error estimates are random and systematic errors, respectively. We note that for populous systems, such as globular clusters and dwarf galaxies, the dominating source of error are the systematic sources. The accuracy of NIR photometry in crowded regions significantly improved during the last few years and the sample sizes will certainly benefit from the new large field of view NIR cameras available with 4 m class telescopes (van der Bliek et al. 2004). New $J$-band light curve templates for both fundamental and first-overtone RR Lyrae are also urgently required to improve the accuracy of their mean magnitudes in this band.

It is a pleasure to thank S. Cassisi and A. Pietrinferni for several helpful discussions and for providing detailed sets of HB models. We also thank M. Marconi for many enlightening suggestions concerning the pulsation properties of RR Lyrae stars. We acknowledge an anonymous referee for his/her positive comments and insights on an early version of this paper. We are very grateful to the ESO support astronomers in La Silla and in Garching for their ongoing support and effort in dealing with a series of different observing blocks. We also thank the ESO Science Archive for their prompt support. This work was partially supported by Particle Physics and Astronomy Research Council (PPARC) and INAF/PRIN2005. This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation.

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[^1]:    ${ }^{13}$ See http://www.ipac.caltech.edu/2mass/releases/allsky/.

[^2]:    ${ }^{14}$ The "separation index" estimates the photometric contamination due to crowding (Stetson et al. 2003). Current separation index value corresponds to stars having less than $6 \%$ of their observed light contributed by known neighbors.

[^3]:    ${ }^{15}$ Here, $\tau_{\mathrm{HB}}$ is defined as the ratio $\tau_{\mathrm{HB}}=(B-R) /(B+V+R)$, where $B, V$, and $R$ refer to the number of blue, variable, and red HB stars, respectively, thus resulting in $\tau_{\mathrm{HB}}=-1$ for HBs with only red stars and $\tau_{\mathrm{HB}}=1$ for HBs with only blue stars.

[^4]:    ${ }^{16}$ V. Castellani et al. (2006, in preparation) find $\tau_{\mathrm{HB}}=0.94$, on average, for $\omega$ Cen.
    ${ }^{17}$ Note that in the following we will use $K$-band notation for $K_{s}(2 \mathrm{MASS})$ band.

[^5]:    ${ }^{18}$ Following Salaris et al. (1993), the global metallicity was estimated as $\log Z=[\mathrm{Fe} / \mathrm{H}]+\log Z_{\odot}+\log \left(0.362+0.638 f\right.$ ), where $Z_{\odot}=0.012$ (Asplund et al. 2004) and $f$ is the enhancement factor of $\alpha$-elements with respect to iron. Note that the change in the distance modulus when moving from scaled solar to $\alpha$-enhanced predictions also includes our uncertainties in the solar metallicity.

[^6]:    ${ }^{\text {a }}$ Using the same mean reddening $[E(B-V)=0.11 \pm 0.02]$ and reddening law (Cardelli et al. 1989) adopted in this paper. The error estimates do not include the contribution of systematic errors ( $\sigma_{\mathrm{sys}}^{K}=0.06 \mathrm{mag}, \sigma_{\mathrm{sys}}^{V}=0.08 \mathrm{mag}$ ).
    ${ }^{\mathrm{b}}$ Based on the PL- $\tau_{\mathrm{HB}}$ relations presented here for $\tau_{\mathrm{HB}}=0.94$, an $\alpha$-element enhancement factor $f=3$, and the new solar metal abundance.
    ${ }^{\text {c }}$ Based on the semi-empirical $K$-band PLZ relation from Bono et al. (2003).
    ${ }^{\mathrm{d}}$ Based on the PL- $\tau_{\mathrm{HB}}$ relations from CPS04 and $\tau_{\mathrm{HB}}=0.934$, an $\alpha$-element enhancement factor $f=3$, and the new solar metal abundance.
    ${ }^{\mathrm{e}}$ Based on the $M_{V}-[\mathrm{Fe} / \mathrm{H}]$ relation from Bono et al. (2003) and mean $V$ magnitudes from Kaluzny et al. (2004) and Butler et al. (1978).
    ${ }^{\mathrm{f}}$ Based on the $M_{V}-[\mathrm{Fe} / \mathrm{H}]$ relation from Catelan (2006) and mean $V$ magnitudes from Kaluzny et al. (2004) and Butler et al. (1978).

