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A MICROSCOPIC ANALYSIS OF SHEAR ACCELERATION

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ABSTRACT

A microscopic analysis of the viscous energy gain of energetic particles in (gradual) nonrelativistic shear flows is presented. We extend previous work and derive the Fokker-Planck coefficients for the average rate of momentum change and dispersion in the general case of a momentum-dependent scattering time $\tau(p) \propto p^\alpha$ with $\alpha \geq 0$. We show that in contrast to diffusive shock acceleration, the characteristic shear acceleration timescale depends inversely on the particle mean free path, which makes the mechanism particularly attractive for high-energy seed particles. Based on an analysis of the associated Fokker-Planck equation we show that above the injection momentum p_0, power-law differential particle number density spectra $n(p) \propto p^{-(1+\alpha)}$ are generated for $\alpha > 0$ if radiative energy losses are negligible. We discuss the modifications introduced by synchrotron losses and determine the contribution of the accelerated particles to the viscosity of the background flow. Possible implications for the plasma composition in mildly relativistic extragalactic jet sources are addressed.

Subject headings: acceleration of particles — galaxies: jets

1. INTRODUCTION

Shear flows are a natural outcome of the density and velocity gradients in extreme astrophysical environments, and in the case of active galactic nuclei (AGNs), for example, are observationally well established (see Laing & Bridle 2002; Laing et al. 2006; Rieger & Duffy 2004). The acceleration of energetic particles occurring in such flows can represent an efficient mechanism for converting a significant part of the bulk kinetic energy of the flow into nonthermal particles and radiation, as has been successfully shown in a number of contributions (e.g., Ostrowski 1990, 1998; Stawarz & Ostrowski 2002; Rieger & Duffy 2004, 2005c). Early theoretical progress in the field has been achieved based on a detailed analysis of the Boltzmann transport equation (Berezhko & Krymskii 1981; Earl et al. 1988; Webb 1989). Somewhat similar to the microscopic picture for Fermi acceleration, shear acceleration is essentially based on the fact that particles can gain energy by scattering off (small-scale) magnetic field inhomogeneities moving with different local velocities due to being embedded in a collisionless shear flow with a locally changing velocity profile (see Rieger & Duffy 2005b for a recent review). In a scattering event, particles are assumed to be randomized in direction, with their energies being conserved in the local (comoving) flow frame. As the momentum of a particle traveling across a velocity shear changes with respect to the local flow frame, scattering leads to a net increase in momentum with time for an isotropic particle distribution. In contrast to second-order Fermi acceleration, however, not the random but the systematic velocity component of the scattering centers is assumed to play the important role. In the present paper we present a microscopic analysis of shear acceleration in which the underlying physical picture becomes most transparent. Section 2 gives the derivation of the Fokker-Planck coefficients for a simple, nonrelativistic (longitudinal) gradual shear flow. Section 3 analyzes the corresponding Fokker-Planck transport equation, providing time-dependent and steady state solutions for specific cases. The contribution of the accelerated particle to the viscosity of the background flow is determined in § 4, while implications for mildly relativistic jet sources are addressed in § 5.

2. DERIVATION OF FOKKER-PLANCK COEFFICIENTS

For a gradual nonrelativistic shear flow, Jokipii & Morfill (1990) have calculated the Fokker-Planck coefficients using a microscopic treatment restricted to a mean scattering time $\tau_c = \text{const.}$ independent of momentum (see also Rieger & Duffy 2005a). However, since under realistic astrophysical conditions the scattering off magnetic turbulence structures is momentum-dependent (e.g., gyro, Kolmogorov, or Kraichnan type), we extend this analysis to the more general case where the local scattering time τ_c is a power-law function of momentum, i.e., $\tau_c(p) = \tau_0 p^\alpha$, $\alpha \geq 0$. We consider a simple nonrelativistic two-dimensional continuous shear flow with velocity profile given by $\boldsymbol{u} = u_z(x)\boldsymbol{e}_z$. Similarly, as in Jokipii & Morfill (1990) we choose a spherical coordinate system in which θ denotes the angle between the x-axis and the velocity vector $\boldsymbol{v} = (v_x, v_y, v_z)$ of the particle and ϕ is the azimuthal angle such that $\tan\phi = v_y/v_x$. Let m be the relativistic mass, $\boldsymbol{p}_1$ the initial momentum of a particle relative to its local flow frame, and $\boldsymbol{p}_2$ the corresponding momentum after the next scattering event. As the particle travels across the shear, its momentum (and thus its mean free path) changes with respect to the local flow frame. Denoting by $\tilde{\tau} = \langle\tau\rangle$ the mean scattering time averaged over momentum magnitude, one obtains

$$\tilde{\tau} = \tau_c + \frac{1}{2}\frac{\partial\tau_c(p_1)}{\partial p_1}\Delta p = \tau_c\left(1 + \frac{1}{2}\alpha\frac{\Delta p}{p_1}\right) \tag{1}$$

in the limit $\Delta p/p_1 = (p_2 - p_1)/p_1 \ll 1$, where collisions are assumed to produce only a small change in the momentum of particles, with large changes occurring only as a consequence of many small changes. Within the time $\tilde{\tau}$ a particle travels a distance $\delta\tilde{x} = v_x\tilde{\tau} = (p_1/m)\cos\theta\tilde{\tau}$ in the x-direction across the shear, and the flow velocity changes by an amount $\delta\boldsymbol{u} = \delta\tilde{u}\boldsymbol{e}_z$, where

$$\delta\tilde{u} = \frac{\partial u_z}{\partial x}\delta\tilde{x} = \delta u\left(1 + \frac{1}{2}\alpha\frac{\Delta p}{p_1}\right) \tag{2}$$

and $\delta u \equiv (\partial u_z/\partial x)(p_1/m)\cos\theta\tau_c$. Contenting ourselves with a Galilean transformation for the nonrelativistic flow speeds involved, the particle's momentum relative to the flow will thus have changed to $\boldsymbol{p}_2 = \boldsymbol{p}_1 + m\delta\boldsymbol{u}$; i.e., one obtains

$$\begin{aligned} p_2^2 &= p_1^2 + 2m\,\delta\tilde{u}\,p_{1,z} + m^2(\delta\tilde{u})^2 \\ &= p_1^2\left(1 + 2\frac{m\,\delta\tilde{u}}{p_1}\sin\theta\cos\phi + \frac{m^2\,\delta\tilde{u}^2}{p_1^2}\right). \end{aligned} \tag{3}$$

As the next scattering event is assumed to preserve the magnitude of the particle momentum relative to the local flow frame, the particle magnitude will have this value in the local frame. To second order in δu, one thus finds

$$\begin{aligned} p_2 \simeq p_1\Bigg[1 &+ \frac{m}{p_1}\,\delta u\,\sin\theta\cos\phi + \frac{m^2}{p_1^2}\,\delta u^2 \\ &- \frac{1}{2}\frac{m^2}{p_1^2}\,\delta u^2\,(1-\alpha)\sin^2\theta\cos^2\phi\Bigg]. \end{aligned} \tag{4}$$

By using spherical coordinates and averaging over all momentum directions for an isotropic particle distribution, the Fokker-Planck coefficients (Chandrasekhar 1943) describing the average rate of momentum change and the average rate of momentum dispersion (associated with a broadening of the distribution) can be determined as follows:

$$\left\langle\frac{\Delta p}{\Delta t}\right\rangle \equiv \frac{2\langle p_2 - p_1\rangle}{\tau_c} = \frac{4+\alpha}{15}p\left(\frac{\partial u_z}{\partial x}\right)^2\tau_c, \tag{5}$$

$$\left\langle\frac{(\Delta p)^2}{\Delta t}\right\rangle \equiv \frac{2\langle(p_2 - p_1)^2\rangle}{\tau_c} = \frac{2}{15}p^2\left(\frac{\partial u_z}{\partial x}\right)^2\tau_c, \tag{6}$$

where the index 1 has been dropped on the right-hand side, revealing that a net increase in momentum proportional to the square of the flow velocity gradient occurs with time. Obviously, the stronger the shear and the larger the particle mean free path, the higher the possible impact of scattering and thus the higher the rate of momentum change. For a constant scattering time, i.e., for $\alpha = 0$, these expressions reduce to those found by Jokipii & Morfill (1990) apart from the factor of 2 that properly takes the time average into account. The Fokker-Planck coefficients are related by the equation

$$\left\langle\frac{\Delta p}{\Delta t}\right\rangle = \frac{1}{2p^2}\frac{\partial}{\partial p}\left[p^2\left\langle\frac{(\Delta p)^2}{\Delta t}\right\rangle\right] = \frac{\Gamma}{p^2}\frac{\partial}{\partial p}\left(p^4\tau_c\right), \tag{7}$$

where Γ on the right-hand side denotes the shear flow coefficient (see Earl et al. 1988), which, for the flow profile chosen, is given by

$$\Gamma = \frac{1}{30}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 - \frac{2}{45}\frac{\partial u_i}{\partial x_i}\frac{\partial u_j}{\partial x_j} = \frac{1}{15}\left(\frac{\partial u_z}{\partial x}\right)^2. \tag{8}$$

Equations (5) and (7) indicate that for the adopted scaling $\tau_c \propto p^\alpha$, acceleration occurs as long as $\alpha > -4$. Comparison with previous approaches shows that the average rate of momentum increase $\langle\Delta p/\Delta t\rangle = (4+\alpha)\Gamma p\tau_c$ as given by equation (5) agrees with the nonrelativistic limit of the full shear acceleration coefficient derived in Rieger & Duffy (2004) and the results found by Berezhko & Krymskii (1981) and Earl et al. (1988). Equation (5) implies a characteristic acceleration timescale for particle acceleration in nonrelativistic gradual shear flows of

$$t_{\rm acc} = \frac{p}{\langle\Delta p/\Delta t\rangle} = \frac{c}{(4+\alpha)\Gamma\lambda}, \tag{9}$$

which is inversely proportional to the particle mean free path $\lambda \simeq c\tau_c$, in remarkable contrast to the diffusive shock acceleration in which $t_{\rm acc} \propto \lambda$ (e.g., Kirk & Dendy 2001). As the mean free path of a particle increases with energy, it follows that higher energy particles will be accelerated more efficiently than lower energy particles. This characteristic behavior makes shear acceleration particularly attractive for the acceleration of high-energy seed particles and the production of ultra-high-energy cosmic rays (e.g., Rieger & Duffy 2005c). In the case of a particle mean free path proportional to the gyroradius r_g, for example, the acceleration timescale scales with the particle Lorentz factor in the same way as the timescales for synchrotron and inverse Compton losses.

3. PARTICLE TRANSPORT AND POWER-LAW FORMATION

The propagation of energetic charged particles in nonrelativistic shear flows can be cast into a Fokker-Planck-type momentum diffusion equation (see Melrose 1980). Taking synchrotron losses into account the isotropic phase-space distribution function $f(p,t)$ (averaged over all momentum directions) then satisfies

$$\begin{aligned} \frac{\partial f(p,t)}{\partial t} = &-\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\left(\left\langle\frac{\Delta p}{\Delta t}\right\rangle + \langle\dot{p}_s\rangle\right)f(p,t)\right] \\ &+ \frac{1}{2p^2}\frac{\partial^2}{\partial p^2}\left[p^2\left\langle\frac{(\Delta p)^2}{\Delta t}\right\rangle f(p,t)\right] + \tilde{Q}(p,t), \end{aligned} \tag{10}$$

where $\tilde{Q}(p,t)$ denotes the source term, $\langle\dot{p}_s\rangle$ is the synchrotron loss term given by

$$\langle\dot{p}_s\rangle = -\beta_s p^2 \tag{11}$$

with $\beta_s = 4B^2e^4/(9m^4c^6)$ when expressed in Gaussian units, and where the coefficients are related by equation (7). In the absence of losses and injection, equation (10) reduces to a momentum space diffusion equation,

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D\frac{\partial f}{\partial p}\right), \tag{12}$$

where $D = \Gamma p^{2+\alpha}\tau_0$ is the momentum space diffusion coefficient. Equation (7) is known as the principle of detailed balance (e.g., Blandford & Eichler 1987) and is a condition that must be satisfied for the Fokker-Planck equation to reduce to the case of pure diffusion. It is interesting to note that second-order Fermi acceleration as a result of scattering from forward and reverse Alfvén waves is also described by momentum space diffusion (Skilling 1975). It is also possible to analyze such a process on the microscopic level, deriving the Fokker-Planck coefficients and demonstrating the principle of detailed balance (see Duffy & Blundell 2005 for details). In the following, equation (10) is used to analyze the evolution of the particle distribution function for some particular cases.

3.1. *Time-dependent Solutions for an Impulsive Source*

First consider the case in which radiative synchrotron losses are negligible and particles are injected monoenergetically at

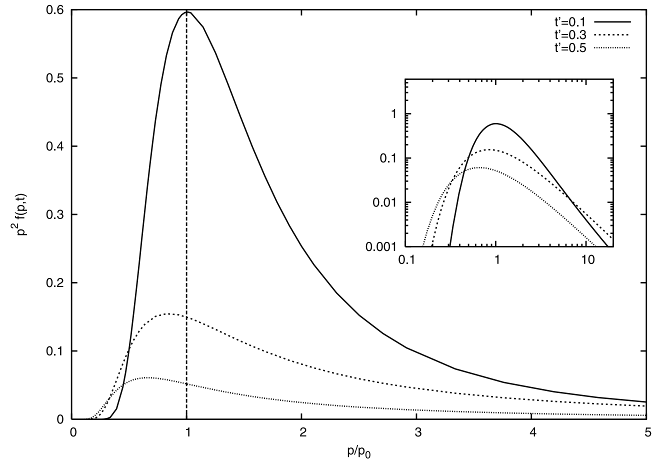

FIG. 1.—Characteristic evolution of the differential particle number density $n(p, t) \propto p^2 f(p, t)$ for $\alpha = 1$ (see eq. [14]) as a function of momentum p/p_0 in the case of impulsive injection of particles with p_0 at $t = 0$. The distribution function is plotted at three different times t where $t = t' t_c$ with $t_c = 1/(\Gamma \tau_0 p_0)$. The inlay shows the same in double logarithm representation and already indicates the formation of a power-law tail $n(p, t) \propto p^{-2}$ for $t' \geq 0.3$.

time $t_0 = 0$ with momentum p_0, i.e., $\tilde{Q}(p, t) = Q\delta(p - p_0)\delta(t)$. Equation (10) then reduces to

$$\frac{\partial f(p, t)}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[\Gamma p^{4+\alpha}\tau_0 \frac{\partial f(p, t)}{\partial p}\right] + Q\delta(t)\delta(p - p_0). \tag{13}$$

The same equation is obtained if one uses the generalized Parker transport equation (Earl et al. 1988, eq. [6]) for the shear flow profile under consideration. As shown in the Appendix, the mathematical solution of equation (13) for $\alpha \neq 0$ takes the form (see eq. [A6])

$$f(p, t) = \frac{Q p_0^{-(\alpha+1)}}{|\alpha|\Gamma\tau_0 t}\left(\frac{p_0}{p}\right)^{(3+\alpha)/2} \exp\left(-\frac{p^{-\alpha} + p_0^{-\alpha}}{\alpha^2 \Gamma \tau_0 t}\right) \times I_{|1+3/\alpha|}\left[\frac{2}{\alpha^2 \Gamma \tau_0 p_0^{\alpha} t}\left(\frac{p}{p_0}\right)^{-\alpha/2}\right], \tag{14}$$

where $I_\nu(z)$ is the modified Bessel function of the first kind (see Abramowitz & Stegun 1972). The corresponding differential particle number density $p^2 f$ is illustrated in Figure 1 using a momentum dependence of $\alpha = 1$. Obviously, the position of the peak shifts to the left and decreases with time (thus ensuring particle number conservation), while the distribution becomes broader and the relative strength of its tail increases (as a consequence of dispersion and acceleration). For small $z \to 0$ the modified Bessel function is known to approach $I_\nu(z) \sim (z/2)^\nu/\tilde{\Gamma}(\nu + 1)$ for $\nu \neq -1, -2, \ldots$ with $\tilde{\Gamma}(\nu + 1)$ denoting the gamma function. It thus follows that for $\alpha > 0$ and $p \gtrsim p_0$, the phase-space distribution approaches a power law $f \propto p^{-(3+\alpha)}$ on a characteristic timescale $t_c = 1/(\alpha^2 \Gamma \tau_0 p_0^\alpha)$ (see Berezhko 1982). For $p \geq p_0$ the time-integrated solution of equation (13) will thus show the same power-law behavior (see eq. [19]). Note that the Fokker-Planck description employed is only valid at late and not at very early stages, since the resulting transport equation is essentially noncausal; i.e., equation (10) is a parabolic partial differential equation. Its solutions thus share with those of the heat equation the mathematical properties of an infinite speed of propagation (i.e., the distribution is nonzero everywhere for $t > 0$), a smoothing of singularities, and a decrease and broadening with time. In general, the problem of infinite propagation can be overcome by using a modified diffusion equation of the (hyperbolic) telegrapher's type (e.g., Cattaneo 1948; Morse & Feshbach 1953). Formally, equation (10) relies on the diffusion approximation and therefore only provides an adequate description of the particle transport for times much larger than the mean scattering time, i.e., as long as $\tau(p) \ll t$, which constrains the maximum momentum range for a given time t over which the solution given in equation (14) can be considered appropriate. Complementary, from a physical point of view, particle energization is expected to occur only on a characteristic timescale determined by the systematic acceleration in equation (9) (e.g., Ball et al. 1992), which constrains the characteristic maximum momentum $p(t)$ achievable within a given time interval.

3.2. *Time-integrated (Steady State) Solutions*

Defining the time-integrated phase-space distribution function

$$F(p) = \int_0^\infty f(p, t)\, dt \tag{15}$$

and integrating the Fokker-Planck equation (10) over time assuming a monoenergetic source term $\tilde{Q}(p, t) = Q\delta(p - p_0)\delta(t)$ gives

$$\frac{\partial}{\partial p}\left[\Gamma p^{4+\alpha}\tau_0 \frac{\partial F(p)}{\partial p}\right] + \frac{\partial}{\partial p}\left[\beta_s p^4 F(p)\right] = -Q p_0^2 \delta(p - p_0). \tag{16}$$

Note that the time-integrated function $F(p)$ is proportional to the steady state solution of equation (10) for continuous injection, since equation (16) coincides with equation (10) if $\partial f/\partial t = 0$. The general solution of equation (16) is of the form

$$F(p) = \frac{Q p_0^2}{\Gamma \tau_0}\left[c_1 - H(p - p_0)\right] e^{-\chi(p)} \int_{p_0}^{p} dp' \frac{e^{\chi(p')}}{p'^{4+\alpha}} + c_2 e^{-\chi(p)}, \tag{17}$$

where $H(p)$ is the Heaviside step function, c_1 and c_2 are constants to be determined by the boundary conditions, and

$$\chi(p) = \begin{cases} -\dfrac{\beta_s}{(\alpha - 1)\Gamma\tau_0} p^{-(\alpha-1)}, & \alpha \neq 1, \\ \dfrac{\beta_s}{\Gamma\tau_0} \ln p, & \alpha = 1. \end{cases} \tag{18}$$

If synchrotron losses are negligible ($\beta_s = 0$) the time-integrated phase-space distribution follows a simple power law above p_0 with momentum index $-(3 + \alpha)$ (see Rieger & Duffy 2005a), i.e., one finds

$$F(p) = \frac{Q p_0^2}{(3 + \alpha)\Gamma\tau_0}\left[\frac{1}{p^{3+\alpha}} H(p - p_0) + \frac{1}{p_0^{3+\alpha}} H(p_0 - p)\right] \tag{19}$$

for $\alpha > 0$ and boundary conditions $F(p \to \infty) \to 0$ and $\partial F/\partial p \to 0$ for $p \to 0$. Thus, in the case of a mean scattering time scaling with the gyroradius (i.e., Bohm case: $\alpha = 1$), for example, this corresponds to a differential power-law particle number density $n(p) \propto p^2 F(p) \propto p^{-2}$ above p_0 and $n(p) \propto p^2$

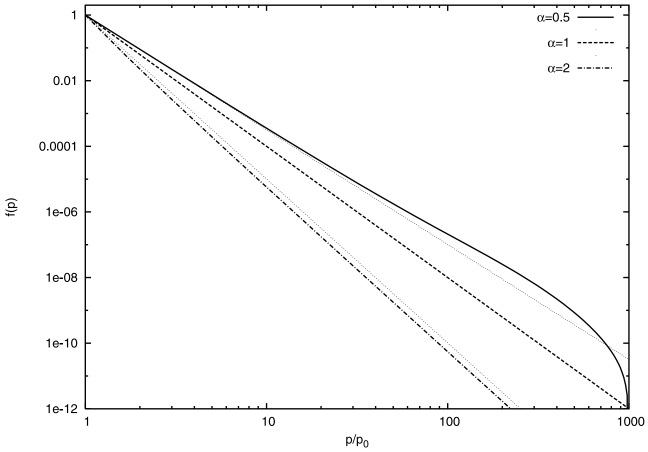

Fig. 2.—Evolution of the normalized phase-space particle distribution function $f(p)$ as a function of momentum p/p_0 for different power indices of the mean scattering time $\tau \propto p^{\alpha}$. The thin dotted lines are drawn to guide the eyes and correspond to power-law distributions $f(p) \propto p^{-3.5}$ and p^{-5}, respectively. For $\alpha = 0.5$ the maximum particle momentum, at which acceleration is balanced by losses, has been chosen to be $p_{\max} = 1000 p_0$, whereas for $\alpha = 2$ a minimum momentum $p_{\min} = p_0/2$ has been used.

below p_0. If losses are nonnegligible, the solutions become more complex. For $\alpha < 1$ we end up with expressions involving exponential integrals (see Abramowitz & Stegun 1972). The characteristic evolution of the phase-space particle distribution function is illustrated in Figure 2. For $\alpha < 1$ the acceleration efficiency is generally constrained by synchrotron losses, which results in a cutoff at $p_{\max} = [(4+\alpha)\Gamma\tau_0/\beta_s]^{1/(1-\alpha)}$, where the acceleration timescale $t_{\rm acc}$ (see eq. [9]) matches the cooling timescale $t_{\rm syn} = 1/(\beta_s p)$. If $p \ll p_{\max}$ cooling effects are negligible and the particle distribution follows a power law as described in equation (19). For $\alpha = 1$ the ratio $t_{\rm acc}/t_{\rm syn}$ becomes independent of momentum. Hence, if conditions are such that $\beta_s/(\Gamma\tau_0) < 5$ (see eq. [10]) is satisfied, particle acceleration is no longer constrained by synchrotron losses, and for $\beta_s/(\Gamma\tau_0) < 4$ the resulting particle distribution above p_0 becomes $F(p) = Qp_0^2/(4\Gamma\tau_0 - \beta_s)p^{-4}$, as expected from equation (19). For $\alpha > 1$ shear acceleration becomes possible (and is essentially unconstrained by synchrotron losses) when particles are injected with momenta above a threshold $p_{\min} = \{\beta_s/[(4+\alpha)\Gamma\tau_0]\}^{1/(\alpha-1)}$. For $p \gg p_{\min}$ cooling effects are completely negligible, and the particle distribution approaches a power law with momentum index as given in equation (19). Note that if losses are unimportant, a proper physical steady state situation may be achieved in instances where particles are continuously injected with momentum p_0 and considered to escape above a fixed momentum $p_{\max} \gg p_0$, where the associated particle mean free path becomes of order the size of the system. Formally, this can be done by adding a simple momentum-dependent escape term $Q(p_0/p_{\max})^2\delta(p - p_{\max})$ on the right-hand side of equation (16).

4. COSMIC-RAY VISCOSITY

As particles gain energy by scattering off inhomogeneities embedded in a background flow, efficient shear acceleration essentially draws on the kinetic energy of that flow. In principle, efficient cosmic-ray acceleration can thus cause a nonnegligible deceleration even for (quasi-collimated) large-scale relativistic jets, although a proper assessment of its significance in comparison with other mechanisms (e.g., entrainment) will require detailed source-specific modeling. In any case, for the purposes of our (nonrelativistic) analysis here, the resulting dynamical effects on the flow can be modeled by means of an induced viscosity coefficient $\eta_s > 0$ that describes the associated decrease in flow mechanical energy per unit time, e.g.,

$$\dot{E}_{\rm kin} = -\eta_s \int \left(\frac{\partial u_z}{\partial x}\right)^2 dV, \tag{20}$$

for our case of a (nonrelativistic) two-dimensional gradual shear flow $u_z(x)\boldsymbol{e}_z$ (Landau & Lifshitz 1982, § 16). We can determine this viscosity coefficient using $\dot{E}_{\rm kin} = -\dot{E}_{\rm cr}$, where $E_{\rm cr} = \int \epsilon_{\rm cr}\, dV$ is the energy gained by the cosmic-ray particles (see also Earl et al. 1988). If particle injection is described by a continuous source term $Q\delta(p - p_0)$, the density $\dot{\epsilon}_{\rm cr}$ of power gained becomes

$$\dot{\epsilon}_{\rm cr} = 4\pi \int_0^{\infty} p^2 E(p) \frac{\partial f(p,t)}{\partial t}\, dp - 4\pi Q E(p_0) p_0^2, \tag{21}$$

where $E(p)$ denotes the relativistic kinetic energy. Note that due to the constraints discussed at the end of § 3.1, special care has to be taken if one wishes to evaluate the integral in equation (21), as done in Earl et al. (1988) by replacing the time derivative of f with the Fokker-Planck expression of equation (13) for the chosen source term.[1] From an astrophysical point of view, we are most interested in steady state–type situations in which particles are injected quasicontinuously with momentum p_0 and, in the absence of significant radiative losses, are considered to escape above a momentum threshold $p_{\max}$ at which $\lambda(p_{\max})$ becomes larger than the width of the acceleration region. The density of power gained then becomes $\dot{\epsilon}_{\rm cr} \simeq 4\pi Q p_0^3 c(p_{\max}/p_0 - 1) \simeq 4\pi Q p_0^2 p_{\max} c$ for $p_{\max} \gg p_0 \gg m_0 c^2$. This implies a viscosity coefficient

$$\eta_s \simeq \frac{3\alpha}{15} \lambda(p_0) n_0 p_{\max}, \tag{22}$$

for $\alpha > 0$, where $\lambda(p_0) \simeq \tau_c(p_0)c$ denotes the mean free path for a particle with momentum p_0 and $n_0 = 4\pi \int_0^{p_{\max}} p^2 f(p)\, dp$ is the number density of cosmic-ray particles in the acceleration region.

5. APPLICATIONS

Efficient shear acceleration of cosmic-ray particles is likely to occur in a number of powerful jet sources, including Galactic microquasars and extragalactic FR I and FR II sources (e.g., Stawarz & Ostrowski 2002; Laing & Bridle 2002; Laing et al. 2006). Whereas a proper analysis of powerful AGN-type jets requires a fully relativistic treatment (see Rieger & Duffy 2004), application of the results derived above may allow useful insights in the case of moderately relativistic jet sources. As an example, let us consider the possible role of shear acceleration in wide-angle-tailed radio galaxies (WATs), deferring a detailed discussion of particle acceleration in microquasar jets to a subsequent paper. WATs are central cluster galaxies and appear as "hybrid" sources (Jetha et al. 2006), showing both FR I and FR II morphologies. Their inner jets extend tens of kiloparsec, seem to have central speeds in the range $v_j \simeq (0.3–0.7)c$ (provided Doppler-hiding effects can be neglected), are apparently very well collimated on the kiloparsec scale, and exhibit spectra close to $S(\nu) \propto \nu^{-0.5}$, with evidence pointing to a steeper spectrum sheath likely to be induced by the strong interactions between the jet and its environment (e.g., Katz-Stone et al. 1999; Hardcastle 1999; Hardcastle et al. 2005; Jetha et al. 2006). Phenomenological

[1] Note that as shown above we cannot independently choose a power-law index for f (e.g., as suggested in Earl et al. 1988) once we have chosen a momentum index for the particle mean free path.

studies suggest a velocity transition layer width for the large-scale jet of $\Delta r = \xi$ kpc ($\xi \lesssim 0.5$) and a fiducial jet magnetic field strength of $B = 10^{-5} b_0$ G ($b_0 \gtrsim 1$ for equipartition). As a likely scenario, let us consider the case in which energetic seed particles required for efficient shear acceleration are provided by internal shock-type (Fermi I) processes operating in the jet interior and giving rise to a power-law proton distribution $f_Q(p) = n_Q p_1/(4\pi p^4)$ in the momentum range $p_1 \simeq 2m_p c \leq p \leq p_2$, where $p_2 \gg p_1$ can be determined either from the condition of lateral confinement (e.g., $p_2 \sim 10^{10} \xi b_0 m_p c$ for $\lambda \sim r_g$) or via the balance of radiative synchrotron losses, and where n_Q is the number density of accelerated particles. The minimum (Bohm diffusion) timescale for nonrelativistic shock acceleration is of order $t_{\rm acc}(p) \sim 6 r_g c/u_s^2$, where u_s is the shock speed as measured in the upstream frame (e.g., Rieger & Duffy 2005c). For typical parameters $u_s \sim 0.1c$, $\lambda \sim r_g$, and a simple linear shear profile with $\Gamma \sim (v_j/\Delta r)^2/15$, shear acceleration (see eq. [9]) will dominate over shock-type processes for protons with Lorentz factors above $\gamma_c \sim 10^9 b_0 \xi$, so that protons with momenta $p \geq p_c = \gamma_c m_p c$ can be considered as being effectively injected into the shear acceleration mechanism, resulting in a rate of injected particles of $Q \simeq n_s/t_0$, where $n_s \simeq n_Q(p_1/p_c) \sim 2 \times 10^{-9} n_Q$, $t_0 \sim t_{\rm acc}(p_c)/P_{\rm esc}$, and $P_{\rm esc} \simeq 4u_2/c$ is the escape probability of a particle from the shock (with u_2 measured in the shock frame). This yields a viscosity coefficient $\eta_s \sim n_s p_{\max} c/(15\Gamma t_0)$. Now, the Navier-Stokes equations imply that in a viscous flow of density ρ_f and viscosity coefficient η_s, the characteristic viscous damping timescale T for the decay of velocity structures of size L is of order $T \sim \rho_f L^2/\eta_s$ (e.g., Earl et al. 1988). For $L \simeq \Delta r$ the numbers estimated above give a characteristic decay timescale T of order

$$T \sim \frac{\rho_f}{\gamma_{\max} n_s m_p} \left(\frac{v_j}{c}\right)^2 t_0 \sim 2 \times 10^4 \frac{\rho_f}{n_Q m_p} \frac{\xi}{0.3} \text{ yr.} \quad (23)$$

Requiring T to be larger than the timescale t_l set by the apparent stability of the jet flow, i.e., $t_l \sim L_j/v_j \sim 2 \times 10^5$ yr for a typical jet length $L_j \sim 30$ kpc and $v_j \sim 0.5c$, gives a minimum density ratio contrast of cold matter to energetic protons $\rho_f/(n_Q m_p) \sim 10(0.3/\xi)$. Therefore, if similar to the supernova remnant or solar wind case, at most a fraction $\sim$0.01 of the overtaken thermal particles are injected into the shock acceleration process (e.g., see Trattner & Scholer 1991; Duffy et al. 1995; Baring et al. 1999), then significant velocity decay effects are unlikely even if the total power of WAT jets should still be dynamically dominated by a cold (thermal) proton component. We note that such a case (slight thermal proton dominance) appears not to be impossible and may indeed be consistent with the notion that WATs are intermediate stages in a broader framework where FR I jets appear as pair-dominated sources and FR II jets as mainly composed of electrons and protons (e.g., Celotti 1997), although circumstantial evidence based on jet bending seems to suggest that WAT jets are rather light (Hardcastle et al. 2005; Jetha et al. 2006).

6. CONCLUSIONS

Turbulent shear flows are widely expected in astrophysical environments. Using a microscopic analysis we have shown that, in the absence of strong synchrotron losses, the acceleration of energetic particles occurring in such flows can give rise to power-law differential particle number densities $n(p) \propto p^{-(1+\alpha)}$ above the injection momentum p_0 for a scattering time $\tau_c \propto p^\alpha, \alpha > 0$. Dependent on the details of the underlying turbulence spectrum, shear acceleration can thus allow for different power-law indices. As efficient shear acceleration generally requires sufficiently energetic seed particles, this implies an interesting corollary: if energetic seed particles are provided by shock-type acceleration processes, the takeover by shear acceleration may reveal itself by a change of the power-law index above the corresponding energy threshold. Perhaps even more interesting is the fact that the characteristic timescale for particle acceleration in gradual shear flows is inversely proportional to the particle mean free path, i.e., $t_{\rm acc} \propto 1/\lambda$. Shear acceleration thus leads to a preferred acceleration of particles with higher magnetic rigidity. Indeed, detailed analyses show that in realistic astrophysical circumstances efficient shear acceleration works usually quite well for protons, but appears restricted for electrons (e.g., Rieger & Duffy 2004, 2005c). As shear acceleration essentially draws on the kinetic energy reservoir of the background flow, the associated viscous drag force can, depending on the intrinsic plasma characteristics, significantly contribute to a deceleration of the large-scale jet flow. This suggests that shear acceleration may have important implications for our understanding of the acceleration of cosmic-ray particles, the plasma composition, and the velocity evolution in astrophysical jet sources.

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APPENDIX

DERIVATION OF TIME-DEPENDENT SOLUTIONS FOR IMPULSIVE SOURCES

For an impulsive source where Q particles are assumed to be injected with momentum p_0 at time $t = 0$, the shear Fokker-Planck equation reads

$$\frac{\partial f(p,t)}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[\Gamma p^{4+\alpha} \tau_0 \frac{\partial f(p,t)}{\partial p} \right] + Q\delta(t)\delta(p - p_0). \quad \text{(A1)}$$

For $\alpha \neq 0$ we choose new variables $\hat{t} \equiv \Gamma \tau_0 t$ and $x \equiv p^{-\alpha}$, for which the homogeneous part of equation (A1) becomes

$$x \frac{\partial^2 f(x,\hat{t})}{\partial x^2} - \frac{3}{\alpha} \frac{\partial f(x,\hat{t})}{\partial x} - \frac{1}{\alpha^2} \frac{\partial f(x,\hat{t})}{\partial \hat{t}} = 0. \quad \text{(A2)}$$

Equation (A2) can be identified with the Kepinski partial differential equation (Kepinski 1906)

$$\frac{\partial^2 f}{\partial x^2} + \frac{m+1}{x}\frac{\partial f}{\partial x} - \frac{n}{x}\frac{\partial f}{\partial \hat{t}} = 0, \tag{A3}$$

for $n \equiv 1/\alpha^2$ and $m \equiv -(3+\alpha)/\alpha$. For the initial condition $f(\hat{t}=0, x) = \tilde{f}(x)$, the solution of the Kepinski partial differential equation is known and of the form (Kepinski 1906)

$$f(x, \hat{t}) = \frac{n}{\hat{t}} \int_0^\infty \left(\frac{\lambda}{x}\right)^{m/2} \exp\left(-n\frac{x+\lambda}{\hat{t}}\right) I_{|m|}\left(2n\frac{\sqrt{x\lambda}}{\hat{t}}\right) \tilde{f}(\lambda)\, d\lambda, \tag{A4}$$

where $I_\nu(z)$ denotes the modified Bessel function of the first kind (see Abramowitz & Stegun 1972). For our initial condition $Q\delta(p-p_0)$ at $t=0$ we have

$$\tilde{f}(\lambda) = Q|\alpha| p_0^{-(\alpha+1)} \delta(\lambda - \lambda_0), \tag{A5}$$

where $\lambda_0 = p_0^{-\alpha}$. Using the original set of variables (p, t), the full solution of equation (A1) for $\alpha \neq 0$ thus becomes

$$f(p,t) = \frac{Q p_0^{-(\alpha+1)}}{|\alpha|\Gamma\tau_0 t} \left(\frac{p_0}{p}\right)^{(3+\alpha)/2} \exp\left(-\frac{p^{-\alpha} + p_0^{-\alpha}}{\alpha^2 \Gamma\tau_0 t}\right) I_{|1+3/\alpha|}\left[\frac{2}{\alpha^2 \Gamma\tau_0 p_0^\alpha t}\left(\frac{p}{p_0}\right)^{-\alpha/2}\right]. \tag{A6}$$

Equation (A6) agrees with the solution presented in Berezhko (1982) and can be shown to reduce to $f(p,t) \to Q\delta(p-p_0)$ in the limit $t \to 0^+$. In the case of $\alpha = -2$ we use that

$$I_{1/2}(z) = \frac{1}{\sqrt{2\pi z}}(e^z - e^{-z}) \tag{A7}$$

to obtain

$$f(p,t) = \frac{Q p_0}{p\sqrt{4\pi\Gamma\tau_0 t}} \left\{ \exp\left[-\frac{(p-p_0)^2}{4\Gamma\tau_0 t}\right] - \exp\left[-\frac{(p+p_0)^2}{4\Gamma\tau_0 t}\right] \right\}, \tag{A8}$$

which agrees with the solution derived for this particular case by Earl et al. (1988, see their eq. [13]). For $\alpha = 0$ we use the variables $z \equiv \ln p + 3\hat{t}$, where $\hat{t} \equiv \Gamma\tau_0 t$, to obtain the characteristic diffusion equation $\partial^2 f/\partial z^2 = \partial f/\partial \hat{t}$ for the homogeneous part of equation (A1) with fundamental solution $f(z, \hat{t}) = \exp[-z^2/(4\hat{t})]/(4\pi\hat{t})^{1/2}$. Thus, for $\alpha = 0$, the solution of equation (A1) becomes (see also Kardashev 1962)

$$f(p,t) = \frac{Q}{p_0\sqrt{4\pi\Gamma\tau_0 t}} \exp\left\{\frac{-[\ln(p/p_0) + 3\Gamma\tau_0 t]^2}{4\Gamma\tau_0 t}\right\}. \tag{A9}$$

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