

ROTOCHEMICAL HEATING OF NEUTRON STARS: RIGOROUS FORMALISM WITH ELECTROSTATIC POTENTIAL PERTURBATIONS

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ABSTRACT

The electrostatic potential that keeps approximate charge neutrality in neutron star matter is self-consistently introduced into the formalism for “rotochemical heating” presented in a previous paper by Fernández and Reisenegger. Although the new formalism is more rigorous, we show that its observable consequences are indistinguishable from those of the previous one, leaving the conclusions of the previous paper unchanged.

Subject headings: dense matter — pulsars: general — relativity — stars: neutron — stars: rotation

1. INTRODUCTION

Rotochemical heating is one of several mechanisms that can keep neutron stars (NSs) hot beyond their standard cooling time of $\sim 10^7$ yr (see Fernández & Reisenegger [2005, hereafter FR05] for further references and Yakovlev & Pethick [2004] for a recent review on neutron star cooling). It originates in deviations from beta equilibrium produced as the star gets compressed due to its decreasing rotation rate, implying a conversion of rotational energy into thermal energy. This effect is capable of keeping old millisecond pulsars (MSPs) at surface temperatures $\sim 10^5$ K, consistent with likely thermal ultraviolet emission from PSR J0437–4715 (Kargaltsev et al. 2004; FR05).

The heating mechanism works as follows. As a NS spins down, the centrifugal force acting on each fluid element diminishes, raising the local pressure. This compression results in a change of the equilibrium concentration of each particle species. Reactions that modify the chemical composition drive the system toward the new equilibrium configuration. But if the rate at which reactions do this task is slower than the change of the equilibrium concentrations due to spin-down compression, the matter remains out of chemical equilibrium. The excess energy stored in the chemical imbalance is dissipated, partly by enhanced neutrino emission and partly by heating the matter in the star. Thus, the star is kept warm and can radiate for long after the standard cooling time.

After its introduction (Reisenegger 1995), this heating mechanism was studied by several authors: Cheng & Dai (1996) applied it to quark stars, Reisenegger (1997) estimated the effects of superfluidity, and Iida & Sato (1997) studied the heating due to compositional transitions in the crust due to spin-down compression.

The most striking prediction associated with rotochemical heating is that if the spin-down timescale is substantially longer than any other timescale involved (with the likely exception of magnetic field decay), the star arrives at a “quasi-steady state,” in which the temperature depends only on the current, slowly changing value of $\Omega\dot{\Omega}$ (the product of the angular velocity and its time derivative) and not on the star’s previous history (Reisenegger

1995). This provides a simple way to constrain the physics involved in theoretical models, once the spin parameters and observed surface temperature of a given, sufficiently old NS are known.

In spite of the previous work, FR05 were the first to go beyond order-of-magnitude estimates and calculate the thermal evolution of NSs with rotochemical heating, taking the structure of the star fully into account in the frame of general relativity and using realistic equations of state of dense matter. They developed a general formalism to treat the evolution of the temperature and departures of chemical equilibrium, implementing it for the simplest possible core composition, namely, neutrons, protons, electrons, and muons (*npeμ* matter). Using state-of-the-art equations of state and modified Urca reactions as the dissipative process, they obtained results consistent with the observed ultraviolet emission from the millisecond pulsar PSR J0437–4715 (Kargaltsev et al. 2004).

Here we show that their formalism is not completely rigorous, in the sense of ignoring the perturbations to the electrostatic potential that must be present inside the NS in order to keep nearly exact charge neutrality among the different charged particles present inside the star.

We start in § 2 by discussing the physical origin, importance, and strength of the electrostatic potential expected inside a NS, as well as its relation to the (nearly exact) condition of local charge neutrality. Then in § 3, we review the main components of the rotochemical heating formalism developed by FR05, which are rigorously correct and not directly affected by the presence of electrostatic potential perturbations, showing that their main result, namely, the existence and properties of the quasi-steady state, are unchanged. In § 4, we introduce the electrostatic potential into the equations of FR05 that relate the different chemical imbalance variables (chemical potential imbalances and deviations from equilibrium particle abundances) to each other. The quantitative consequences of these modifications are explored in § 5, and § 6 contains our main conclusions.

2. ELECTROSTATIC POTENTIAL IN A NEUTRON STAR

It is well known that a neutron (*n*) in vacuum is an unstable particle, decaying into a proton (*p*), an electron (*e*), and an electron antineutrino ($\bar{\nu}_e$). In NSs, neutrons are stabilized by the presence of enough protons and electrons to block (through the Pauli exclusion principle) the possible final states of the decay. Neutrons and protons have high masses and are therefore held inside the star by its gravitational potential well, whose depth must

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be larger than the kinetic part of their Fermi energies. However, this potential is not deep enough to hold the much lighter electrons, whose Fermi energy is relativistic (much larger than their mass) and which would therefore tend to escape, yielding a net positive charge on the NS. However, at NS densities, already a small charge imbalance between protons and electrons causes an electrostatic field that can hold in the electrons and at the same time partially balance the gravitational force on the protons, preventing them from sinking into the center of the star, due to their relatively small kinetic energies. Thus, a nonrotating NS in hydrostatic, chemical (beta), and diffusive equilibrium contains a spherically symmetric electrostatic potential $\psi(r)$ that keeps the charged particles near local charge neutrality. The existence of such an electric potential in stars has been known to astrophysicists since the early twentieth century. A recent discussion, including early references, is found in Neslušan (2001).

More formally, and using a relativistic description of gravity, in contrast to the Newtonian description in the previous paragraph, we describe the star by the spherically symmetric metric (Schutz 1985)

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

The condition of diffusive equilibrium for each particle species i is that its total, redshifted chemical potential,

$$\mu_i^\infty = [\mu_i(r) + q_i\psi(r)]e^{\Phi(r)}, \quad (2)$$

is uniform throughout the star. Here q_i is the charge of each particle species and μ_i is its intrinsic chemical potential (Kittel & Kroemer 1980), determined by the kinetic energy of the species (or, equivalently, by their number density n_i) and their local interactions with particles of the same and other species (which are determined by all the densities). For a given metric and electrostatic potential, these conditions determine the distribution of the individual particle species by fixing the spatial dependence of the intrinsic chemical potential. For a typical density inside NSs, $n_e \sim 10^{37} \text{ cm}^{-3}$, the extremely relativistic electron chemical potential $\mu_e \approx \hbar c(3\pi^2 n_e)^{1/3} \approx 10^2 \text{ MeV}$, which sets the scale for the electrostatic potential, $|\psi| \sim \mu_e/e \sim 10^8 \text{ V}$, where $e \equiv q_p = -q_e$.

The field equation for the electrostatic potential is Maxwell's equation, $F^{\alpha\beta}_{;\beta} = 4\pi J^\alpha$, where the components of the electromagnetic field tensor can be written in terms of the 4-vector potential components A^β as $F_{\alpha\beta} = A_{\beta;\alpha} - A_{\alpha;\beta}$, and J^α are the components of the charge-current 4-vector. In the static system of interest here (ignoring currents and magnetic fields inside the neutron star), the space components of the 4-vectors can be set to zero, $A^i = 0 = J^i$, and the time component $A^0 \equiv \psi$. A substantial amount of algebra yields the relativistic Poisson equation for the electrostatic potential,

$$e^{-2\Lambda} \left[\psi'' + \left(\Phi' - \Lambda' + \frac{2}{r} \right) \psi' \right] - e^{-2\Phi} R_{00} \psi = 4\pi \sum_i n_i q_i, \quad (3)$$

where $R_{\alpha\beta}$ are the components of the Riemann curvature tensor and the primes indicate derivatives with respect to the radial coordinate r . The terms on the left-hand side (the generalization of the Laplacian operator of flat spacetime) are of the order of $|\psi|/R^2$, where $R \sim 10^6 \text{ cm}$ is the radius of the NS. Thus, a minute excess positive charge density $\sim \mu_e/(eR)^2 \sim 10^2 \text{ cm}^{-3}$ (compared to the typical density of each charged species, $\sim 10^{37} \text{ cm}^{-3}$,

mentioned above) is enough to produce the required potential. Therefore, to an excellent approximation, we can assume that the electrostatic potential is exactly that required to keep local neutrality everywhere among the charged particle species, replacing equation (3) with the condition of charge neutrality

$$\sum_i n_i q_i = 0 \quad (4)$$

imposed on the densities n_i and obtaining the *nonzero* electrostatic potential from this condition, together with equation (2).

3. CHEMICAL IMBALANCE AND ROTOCHEMICAL HEATING

The formalism of FR05 can be summarized as follows (we refer to the original paper for more details). The changing rotation rate $\Omega(t)$ of the star, assumed to be much slower than the “breakup” or “Keplerian” rate, slightly perturbs its structure, making the total, chemical-equilibrium number N_i^{eq} of particles of species i in the star change with time as

$$\dot{N}_i^{\text{eq}} = 2I_{\Omega,i}\Omega\dot{\Omega}, \quad (5)$$

where $I_{\Omega,i}$ is a constant, depending on the structure of the unperturbed, nonrotating star. The actual number of particles of the same species, N_i , is changed by nonequilibrium weak interactions in the stellar core as

$$\dot{N}_i = \int_{\text{core}} dV e^\Phi \sum_\alpha \Delta\Gamma_\alpha^i. \quad (6)$$

The integral is performed over the volume of the unperturbed, nonrotating stellar core, and the sum runs over different kinds of reactions (labeled by α), each of which creates a net number $\Delta\Gamma_\alpha^i$ of particles of species i per unit time and per unit volume. The latter quantity is a function of the local density, the temperature, and, most importantly, the local chemical imbalance, quantified by the variables $\eta_{npl} = \mu_n - \mu_p - \mu_l$, where l stands for the charged leptons: electrons ($l = e$) or muons ($l = \mu$). If the η_{npl} are nonzero, so are the net reaction rates, with their signs such that the chemical imbalances tend to be decreased. The evolution of the excess (or deficit) of particles of species i , $\delta N_i \equiv N_i - N_i^{\text{eq}}$, is therefore determined by the competition between the changing rotation rate of the star, which tends to increase its absolute value, and the reactions, tending to decrease it:

$$\delta\dot{N}_i = \int_{\text{core}} \left(e^\Phi \sum_\alpha \Delta\Gamma_\alpha^i \right) dV - 2I_{\Omega,i}\Omega\dot{\Omega}. \quad (7)$$

The evolutionary timescale of the star is assumed to be much longer than the timescales for diffusion of particles and heat across it. Thus, in particular, the redshifted temperature $T^\infty = T(r)e^{\Phi(r)}$ is considered to be uniform inside the star, except for a thin surface layer. This temperature evolves as

$$\dot{T}^\infty = \frac{1}{C} (L_H^\infty - L_\nu^\infty - L_\gamma^\infty), \quad (8)$$

where C is the total heat capacity of the star, L_H^∞ is the total power released by reactions and other heating mechanisms, L_ν^∞ is the total neutrino luminosity, and L_γ^∞ is the photon luminosity, which are functions of the chemical imbalances η_{npl} and the temperature T everywhere inside the star.

Although the particles are in diffusive equilibrium, moving easily everywhere inside the star, they are generally not in chemical equilibrium (free to convert into one another). Therefore, nonzero, but spatially uniform, redshifted chemical imbalances are set up,

$$\eta_{npl}^\infty \equiv \mu_n^\infty - \mu_p^\infty - \mu_l^\infty = \eta_{npl} e^\Phi, \quad (9)$$

not involving the electrostatic potential, which cancels out due to the equal and opposite charges of protons and leptons. Being uniform in the star, these redshifted chemical imbalances are the ideal variables to quantify the departure from chemical equilibrium and follow its time evolution. The reaction rates on the right-hand side of equations (6) and (7) can be written in terms of η_{npl}^∞ and T^∞ instead of the corresponding unredshifted, local, position-dependent variables.

In order to close the system of dynamical equations, it is necessary to write the chemical imbalances, η_{npl}^∞ , in terms of the excess numbers of particles, δN_i , which is where the inaccuracy of the previous treatment (FR05) is located, as we discuss below. However, at this point we may note that if we are only interested in the quasi-steady state reached by NSs with slow spin-down evolution, we only need to set the right-hand sides of equations (7) and (8) equal to zero, immediately yielding a nondegenerate set of algebraic equations for η_{npl}^∞ and T^∞ , which is equivalent to that of FR05 and therefore has identically the same solutions.

4. RELATIONS AMONG THE VARIABLES QUANTIFYING THE CHEMICAL IMBALANCE

When the chemical equilibrium is slightly perturbed, the total, redshifted chemical potential of equation (2) is displaced from its equilibrium value by a small amount,

$$\delta\mu_i^\infty = [\delta\mu_i + q_i\delta\psi + (\mu_i + q_i\psi)\delta\Phi]e^\Phi, \quad (10)$$

where the prefix δ refers to the respective variable being slightly perturbed with respect to the chemical equilibrium state, maintaining diffusive equilibrium (so $\delta\mu_i^\infty$ remains uniform). We note that equation (9) of FR05 considered only the first term. If some neutrons are converted into proton-electron pairs (plus escaping antineutrinos), the effect on the energy density and pressure, and therefore on the structure of the star and the space-time metric, is not large, as verified explicitly (in the Newtonian approximation) in Appendix A. We therefore invoke the relativistic Cowling approximation (e.g., Finn 1988), setting $\delta\Phi = 0$. However, the absolute value of the electrostatic potential must increase, in order to hold more protons and electrons together than before. This effect is considered explicitly in what follows.

The intrinsic chemical potential perturbations are related to local density changes, $\delta n_i = \sum_j (\partial n_i / \partial \mu_j) \delta \mu_j$, where, from equation (10), $\delta \mu_j = \delta \mu_j^\infty e^{-\Phi} - q_j \delta \psi$. Imposing conservation of local charge neutrality, $\sum_i q_i \delta n_i = 0$, yields

$$\delta\psi = \frac{\sum_{k,m} q_k (\partial n_k / \partial \mu_m) \delta \mu_m^\infty}{\sum_{r,s} q_r q_s (\partial n_r / \partial \mu_s)}. \quad (11)$$

Thus, after some algebra, the perturbation to the total number of particles of one species in the star can be written as in equation (11) of FR05,

$$\delta N_i = \int dV \delta n_i = \sum_j B_{ij} \delta \mu_j^\infty, \quad (12)$$

but with the coefficients redefined as

$$B_{ij} = \int dV e^{-\Phi} \left[\frac{\partial n_i}{\partial \mu_j} - \frac{\sum_{k,m} q_k q_m (\partial n_i / \partial \mu_m) (\partial n_k / \partial \mu_j)}{\sum_{r,s} q_r q_s (\partial n_r / \partial \mu_s)} \right]. \quad (13)$$

(The second term inside the square brackets is missing in eq. [12] of FR05.) The thermodynamic identity $\partial n_i / \partial \mu_j = \partial n_j / \partial \mu_i$ can be used to show that $B_{ij} = B_{ji}$. More important, however, is to note that the 4×4 matrix $\mathbf{B}$ is singular, since charge conservation is “built in,” in the sense that for any combination of four values for the variables $\delta \mu_j^\infty$, it is guaranteed that $\sum_i q_i \delta N_i = 0$; i.e., three of the rows of $\mathbf{B}$ are not linearly independent: $B_{pj} = B_{ej} + B_{\mu j}$. One manifestation of this singularity is the existence of “trivial” perturbations with $\delta \mu_i^\infty = q_i \delta \psi e^\Phi$, for which $\delta \mu_i = 0$, and therefore the particle distributions inside the star are not changed. These perturbations can be interpreted as redistributions of charges outside the star or just as redefinitions of the zero point of the potentials.

In order to obtain an invertible problem, we reduce our four degrees of freedom to three, by defining the new variables $\delta \tilde{\mu}_n^\infty \equiv \delta \mu_n^\infty$ and $\delta \tilde{\mu}_l^\infty \equiv \delta \mu_p^\infty + \delta \mu_l^\infty$, the latter for both $l = e, \mu$, and eliminating the row and column corresponding to the protons from the matrix $\mathbf{B}$, in this way converting it into a symmetric and invertible 3×3 matrix $\tilde{\mathbf{B}}$, with $\delta N_i = \sum_{j=n,e,\mu} \tilde{B}_{ij} \delta \tilde{\mu}_j^\infty$ for $i = n, e, \mu$. (The proton density and total number can be recovered from those of electrons and muons through the condition of charge neutrality.) The components of the inverse matrix, $\tilde{\mathbf{B}}^{-1}$, relate the chemical imbalance variables to the particle number perturbations: $\eta_{npl}^\infty = \delta \tilde{\mu}_n^\infty - \delta \tilde{\mu}_l^\infty = \sum_{i=n,e,\mu} (\tilde{B}_{ni}^{-1} - \tilde{B}_{li}^{-1}) \delta N_i$.

Finally, this can be further reduced by considering that the reactions also conserve baryon number,⁴ i.e., $\delta N_n = -\delta N_p = -\delta N_e - \delta N_\mu$, which allows us to also eliminate δN_n , writing

$$\eta_{npl}^\infty = \sum_{i=e,\mu} (\tilde{B}_{ni}^{-1} - \tilde{B}_{li}^{-1} - \tilde{B}_{nn}^{-1} + \tilde{B}_{nn}^{-1}) \delta N_i. \quad (14)$$

We can make contact with the notation of FR05 by writing

$$\eta_{npe}^\infty = -Z_{npe} \delta N_e - Z_{np} \delta N_\mu, \quad \eta_{np\mu}^\infty = -Z_{np} \delta N_e - Z_{np\mu} \delta N_\mu, \quad (15)$$

with the coefficients rewritten as

$$Z_{np} \equiv \tilde{B}_{nn}^{-1} - \tilde{B}_{ne}^{-1} - \tilde{B}_{n\mu}^{-1} + \tilde{B}_{e\mu}^{-1}, \quad (16)$$

$$Z_{npe} \equiv \tilde{B}_{nn}^{-1} - 2\tilde{B}_{ne}^{-1} + \tilde{B}_{ee}^{-1}, \quad (17)$$

$$Z_{np\mu} \equiv \tilde{B}_{nn}^{-1} - 2\tilde{B}_{n\mu}^{-1} + \tilde{B}_{\mu\mu}^{-1}, \quad (18)$$

instead of equations (54)–(56) of FR05. These redefinitions were chosen so that their equations (52) and (53) for the time evolution of the chemical imbalance variables η_{npl}^∞ remain formally correct. The coefficients W_{npl} , which determine the rate by which chemical imbalances are built up by the changing rotation rate, also remain correctly expressed by equations (57) and (58), although they are more easily written as

$$W_{npe} = Z_{npe} I_{\Omega,e} + Z_{np} I_{\Omega,\mu}, \quad W_{np\mu} = Z_{np} I_{\Omega,e} + Z_{np\mu} I_{\Omega,\mu}. \quad (19)$$

⁴ Note that, contrary to charge neutrality, which can be imposed locally everywhere in the star, baryon number is only a globally conserved quantity.

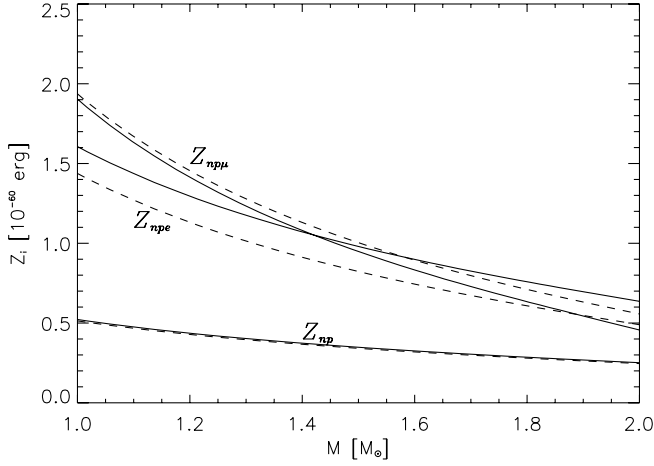


FIG. 1.—Coefficients Z_{np} , Z_{npe} , and Z_{npmu} as functions of stellar mass, for models built with the A18 + δv + UIX* equation of state (Akmal et al. 1998). Dashed lines correspond to the formalism of FR05 (without explicit account of electrostatic potentials), while solid lines are the results of the corrected formalism presented in this paper.

5. QUANTITATIVE EVALUATION AND DISCUSSION

Formally, the only changes that have to be introduced to the equations of FR05 are to replace their equations (9), (12), (13), and (54)–(56) by the corresponding equations in the previous sections. In their dynamical system for the time evolution of the variables T^∞ and η_{npl}^∞ , the only difference lies in the values of the coefficients Z_{np} , Z_{npe} , and Z_{npmu} , now defined by our equations (16)–(18), instead of their equations (54)–(56), and in the coefficients W_{npl} , which depend on the former (e.g., our eq. [19]). We argue in § 3 that the quasi-steady state eventually reached by old NSs is unaffected by these changes. However, the rate at which this state is approached (directly depending on the coefficients W_{npl} ; see, e.g., FR05, eqs. [76] and [81]) could be affected, as well as the less intuitive time evolution of younger NSs with stronger magnetic fields (e.g., Reisenegger 1995).

Figure 1 shows a comparison of the coefficients Z_{np} , Z_{npe} , and Z_{npmu} , as functions of stellar mass, for one particular equation of state (A18 + δv + UIX*; Akmal et al. 1998), for the formalism of FR05 (*dashed lines*) and the more rigorous one presented here (*solid lines*). It can be seen that Z_{np} hardly changes, whereas Z_{npe}

and Z_{npmu} show variations whose fractional amplitude increases with mass, up to a $\sim 10\%$ – 20% level at the largest masses. Calculations with other equations of state yielded very similar results.

In Figure 2, we compare the time evolution of the chemical imbalance variables, η_{npe}^∞ and η_{npmu}^∞ , as well as the redshifted internal temperature, T^∞ , as obtained in both formalisms for two different neutron star masses. For the lower mass, particles inside the star undergo only (slow) modified Urca reactions, whereas at the higher mass electrons can undergo (fast) direct Urca processes, whereas muons are still restricted to the modified Urca process. This accounts for the relatively simple evolutionary curves in the left panel, compared to the more complex one in the right panel. (See FR05 for a more detailed discussion.) It can be seen that, consistent with our previous discussion, there is no evidence for differences between the two formalisms in the asymptotic, quasi-steady regime, whereas some differences are seen at the epochs of strong time evolution, especially in the higher mass star.

6. CONCLUSIONS

We have shown that the electrostatic potential inside a NS plays an important role in determining the spatial distribution of charged particles and their nearly exact local charge neutrality everywhere in the star. We take it into account explicitly in an extension of the formalism for rotochemical heating of FR05, in which the relations between the two sets of variables quantifying chemical imbalances (chemical potential differences and departures from equilibrium abundances) are affected, without changing the rest of the formalism. The quasi-steady state reached by old NSs is strictly unaffected by these corrections, and the time-dependent evolution at earlier times shows quantitatively negligible changes.

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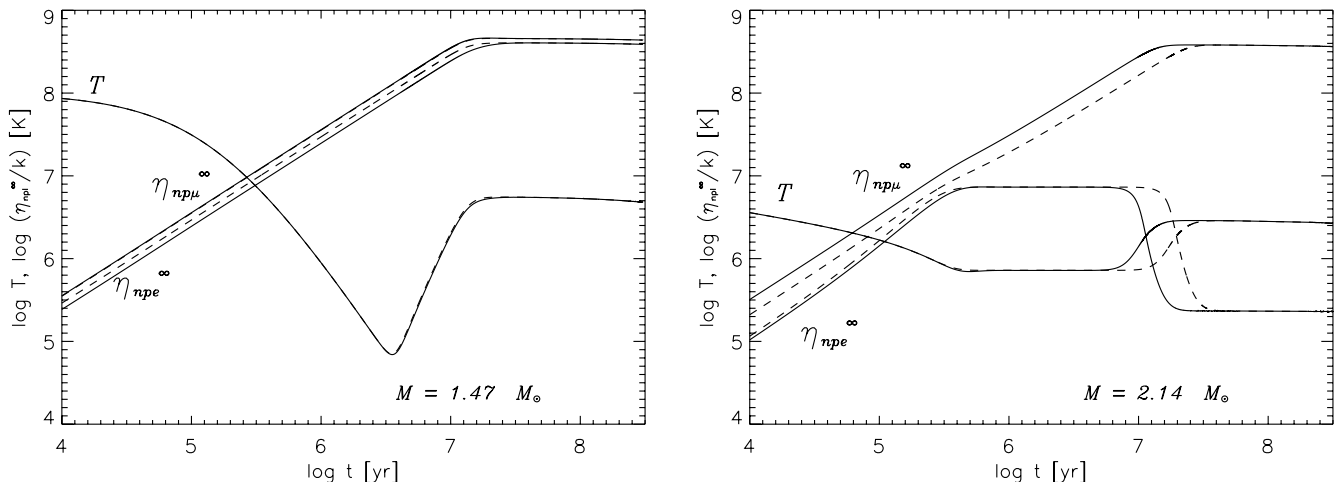


FIG. 2.—Evolution of the internal redshifted temperature, T^∞ , and chemical imbalance variables, η_{npe}^∞/k and η_{npmu}^∞/k , for NS models of $1.47 M_\odot$ (*left*) and $2.13 M_\odot$ (*right*), calculated with the A18 + δv + UIX* equation of state (Akmal et al. 1998), with initial conditions of vanishing chemical imbalances and $T_\infty = 10^8$ K at $t = 0$, and assuming magnetic dipole spin-down with field strength $B = 10^8$ G and initial period $P_0 = 1$ ms. Dashed lines represent results for the formalism of FR05, while solid lines correspond to the present, corrected formalism.

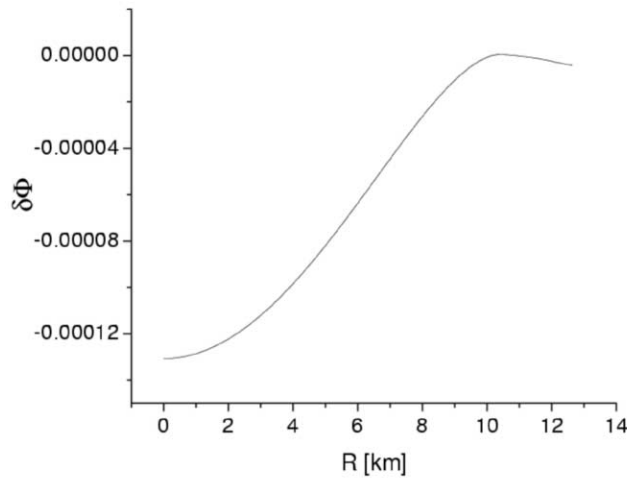


FIG. 3.—Perturbation of the gravitational potential in a neutron star model with the first PAL equation of state (Prakash et al. 1988) and total mass $M = 1.6 M_{\odot}$, subject to a chemical perturbation corresponding to a 1% increase of the electron and proton density everywhere in the star, together with a corresponding decrease in the neutron density, so as to conserve total baryon number.

APPENDIX

QUANTITATIVE EVALUATION OF GRAVITATIONAL POTENTIAL PERTURBATIONS

Here we verify whether the gravitational potential (or metric) perturbation in equation (2) can indeed be neglected. We consider a neutron star model of $1.6 M_{\odot}$, calculated in full general relativity with the first PAL equation of state (Prakash et al. 1988). For simplicity using perturbation theory in a Newtonian framework, we calculate the gravitational potential perturbation $\delta\Phi$ corresponding to an arbitrarily chosen deviation from chemical equilibrium, taken as an excess electron and proton density of 1% everywhere (muons are not present in this model), with a corresponding decrease in the neutron density, so as to conserve baryon number. Figure 3 shows $\delta\Phi$ as a function of the radial coordinate. For comparison, $\delta\mu_e/\mu_e \approx \delta n_e/(3n_e) \approx 0.003$, so the intrinsic chemical potential perturbation term in equation (2) is about 50 times larger than that corresponding to the gravitational potential.

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