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NEUTRINO-DOMINATED ACCRETION MODELS FOR GAMMA-RAY BURSTS: EFFECTS OF GENERAL RELATIVITY AND NEUTRINO OPACITY

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ABSTRACT

We first refine the fixed concept in the literature that the usage of the Newtonian potential in studies of black hole accretion is invalid and the general relativistic effect must be considered. As our main results, we then show that the energy released by neutrino annihilation in neutrino-dominated accretion flows is sufficient for gamma-ray bursts when the contribution from the optically thick region of the flow is included, and that in the optically thick region advection does not necessarily dominate over neutrino cooling because the advection factor is relevant to the geometrical depth rather than the optical depth of the flow.

Subject headings: accretion, accretion disks — black hole physics — gamma rays: bursts — neutrinos

1. INTRODUCTION

The fireball shock model (see, e.g., Mészáros 2002 and Zhang & Mészáros 2004 for reviews) has been widely accepted to interpret the gamma-ray and afterglow emitting of gamma-ray bursts (GRBs). Despite the successes of this phenomenological model, the central engine of the relativistic fireball is not yet well understood. Most popular models for the energy source of GRBs are in common invoking a hyperaccreting black hole. Accretion models in this context were first considered by Narayan et al. (1992) and have been recently discussed by Popham et al. (1999, hereafter PWF), Narayan et al. (2001), Kohri & Mineshige (2002), Di Matteo et al. (2002, hereafter DPN), and Kohri et al. (2005).

PWF introduced the concept of neutrino-dominated accretion flows (NDAFs) and showed that the energy released by neutrino annihilation was adequate for GRBs. Their calculations, however, were based on the assumption that the flow is optically thin for neutrinos. As pointed out by themselves, this assumption breaks down for the mass accretion rate $\dot{M} \gtrsim 10\ M_\odot\ \mathrm{s}^{-1}$. They mentioned that their estimate of the neutrino annihilation luminosity $\sim 2 \times 10^{53}$ ergs s^{-1} (see their Table 3) for $\dot{M} = 10\ M_\odot\ \mathrm{s}^{-1}$ should be taken as an upper limit, and the actual luminosity could be as much as a factor of 5 lower, i.e., $\sim 4 \times 10^{52}$ ergs s^{-1}. The NDAF model was reinvestigated by DPN, in which a bridging formula was adopted for calculating neutrino radiation in both the optically thin and thick cases. They showed that for $\dot{M} > 0.1\ M_\odot\ \mathrm{s}^{-1}$ there exists an optically thick inner region in the flow and argued that for $\dot{M} \gtrsim 1\ M_\odot\ \mathrm{s}^{-1}$ neutrinos are sufficiently trapped and energy advection becomes the dominant cooling mechanism, resulting in the maximum luminosity of neutrino annihilation, which is only $\sim 10^{50}$ ergs s^{-1} (see their Fig. 6). Thus, they claimed that the NDAF model cannot account for GRBs.

How do we understand the inconsistent results of PWF and DPN? We note that PWF worked in the relativistic Kerr geometry but with the a priori assumption that neutrinos are optically thin, whereas DPN calculated the optical depth for neutrinos but went back into the Newtonian potential and omitted totally the neutrino radiation from the optically thick region. The purpose of this Letter is to try to update partly the NDAF model. It is surely correct that the general relativistic effect must be considered in studies of black hole accretion; then we wish to know how important the effect of the neutrino opacity is in determining the luminosity of an NDAF.

2. ASSUMPTIONS AND EQUATIONS

For simplicity, a steady state axisymmetric black hole accretion flow is considered as in PWF and DPN. We adopt that the general relativistic effect of the central black hole is simulated by the well-known pseudo-Newtonian potential introduced by Paczyński & Wiita (1980, hereafter PW potential), i.e., $\Phi = -GM_{\mathrm{BH}}/(R - R_g)$, where M_{BH} is the black hole mass, R is the radius, and $R_g = 2GM_{\mathrm{BH}}/c^2$ is the Schwarzschild radius. Other assumptions about the flow are usual in the literature: the angular velocity is approximately Keplerian, i.e., $\Omega = \Omega_{\mathrm{K}} = (GM_{\mathrm{BH}}/R)^{1/2}/(R - R_g)$; the vertical scale height of the flow is $H = c_s/\Omega_{\mathrm{K}}$, where $c_s = (P/\rho)^{1/2}$ is the isothermal sound speed, with P and ρ being the pressure and mass density, respectively; and the kinematic viscosity coefficient is expressed as $\nu = \alpha c_s H$, where α is the constant viscosity parameter.

The basic equations describing the flow consist of the continuity, azimuthal momentum, and energy equations plus the equation of state. The continuity equation is

$$\dot{M} = -4\pi \rho H R v, \tag{1}$$

where v is the radial velocity. With the assumption $\Omega = \Omega_{\mathrm{K}}$, the azimuthal momentum equation is reduced to an algebraic form:

$$v = -\alpha c_s \frac{H}{R} f^{-1} g, \tag{2}$$

where $f = 1 - j/\Omega R^2$ and $g = -d \ln \Omega_{\mathrm{K}}/d \ln R$, with the integration constant j representing the specific angular momentum (per unit mass) accreted by the black hole. The equation of state is written as

$$P = P_{\mathrm{gas}} + P_{\mathrm{rad}} + P_{\mathrm{deg}} + P_\nu, \tag{3}$$

where P_{gas}, P_{rad}, P_{deg}, and P_ν are the gas pressure from nucleons, radiation pressure of photons, degeneracy pressure of electrons,

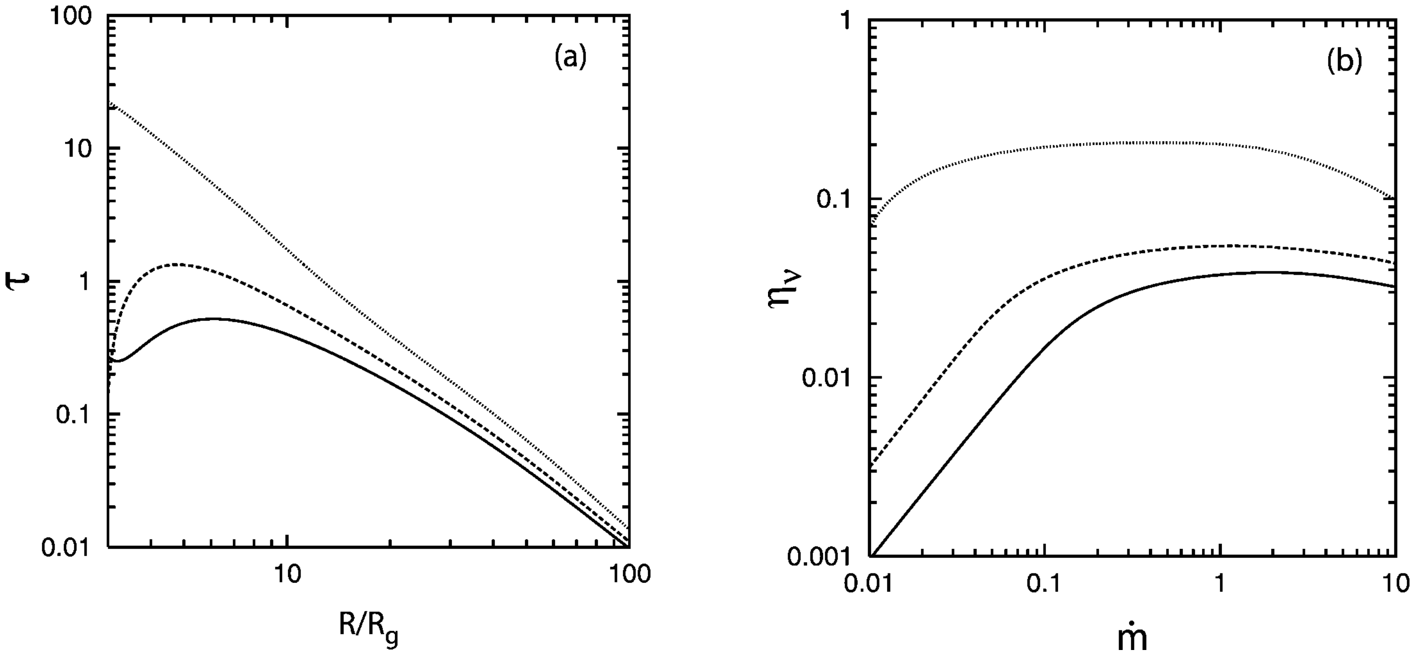

FIG. 1.—Solutions in the PW potential with the accreted specific angular momentum $j = 1.8cR_g$ (*solid line*), in the Newtonian potential with $j = 1.2cR_g$ (*dashed line*), and in the Newtonian potential with $j = 0$ (*dotted line*). (*a*) Neutrino optical depth τ as a function of radius R for the dimensionless mass accretion rate $\dot{m} = 1$. (*b*) Efficiency of neutrino radiation η_ν as a function of $\dot{m}$.

and radiation pressure of neutrinos, respectively. The energy equation is written as

$$Q_{\rm vis} = Q_{\rm adv} + Q_{\rm photo} + Q_\nu. \tag{4}$$

The viscous heating $Q_{\rm vis}$ and the advective cooling $Q_{\rm adv}$ (for a half-disk above or below the equator) are expressed as

$$Q_{\rm vis} = \frac{1}{4\pi}\dot{M}\Omega^2 fg, \tag{5}$$

$$Q_{\rm adv} = \rho H v T\frac{ds}{dR} - \xi v\frac{H}{R}T\left(\frac{11}{3}aT^3 + \frac{3}{2}\frac{\rho k}{m_p}\frac{1+3X_{\rm nuc}}{4} + \frac{4}{3}\frac{u_\nu}{T}\right), \tag{6}$$

where T is the temperature, s is the specific entropy, $X_{\rm nuc}$ is the mass fraction of free nucleons, u_ν is the neutrino energy density, and $\xi \propto -d\ln s/d\ln R$ is assumed to be equal to 1 as in DPN. The quantity $Q_{\rm photo}$ is the cooling of the photodisintegration process, and Q_ν is the cooling of the neutrino radiation. We adopt a bridging formula for calculating Q_ν, which is valid in both the optically thin and thick cases. Detailed expressions for $P_{\rm gas}$, $P_{\rm rad}$, $P_{\rm deg}$, P_ν, $Q_{\rm photo}$, $X_{\rm nuc}$, u_ν, and the bridging formula for Q_ν are given in DPN.

Equations (1)–(4) contain the four independent unknowns ρ, T, H, and v as functions of R, which can be numerically solved with given constant parameters $M_{\rm BH}$, $\dot{M}$, α, and j; then all the other quantities can be obtained. In the following calculations, we fix $M_{\rm BH} = 3\ M_\odot$ and $\alpha = 0.1$.

3. INVALIDITY OF THE USAGE OF THE NEWTONIAN POTENTIAL

Most previous calculations for NDAFs (e.g., Narayan et al. 2001; DPN; Kohri et al. 2005) adopted the Newtonian potential and did not take the integration constant j into consideration. Kohri & Mineshige (2002) also used the Newtonian potential but considered j. Only PWF worked in the relativistic Kerr geometry, as we mentioned already. In this section, we refine the fixed concept in the literature, i.e., the invalidity of the usage of the Newtonian potential. We concentrate on three solutions corresponding to the PW potential with $j = 1.8cR_g$ (just a little less than the Keplerian angular momentum at the last stable orbit, $l_{{\rm K}\,|\,3Rg} = 1.837cR_g$), the Newtonian potential with $j = 1.2cR_g$ ($l_{{\rm K}\,|\,3R_g} = 1.225cR_g$; see Kohri & Mineshige 2002), and the Newtonian potential with $j = 0$ (DPN; Kohri et al. 2005), respectively.

The variation of the optical depth τ for neutrinos with R is drawn in Figure 1*a*, for which the dimensionless mass accretion rate is $\dot{m} \equiv \dot{M}/(M_\odot\ {\rm s}^{-1}) = 1$. The figure shows that the values of τ in the Newtonian potential (*dotted and dashed lines*) are significantly larger than those in PW potential (*solid line*). The accretion flow in the PW potential is completely optically thin ($\tau < \frac{2}{3}$), whereas for the Newtonian potential with $j = 0$ there exists a wide optically thick ($\tau > \frac{2}{3}$) region of $R \lesssim 15.4R_g$. We believe that the results with the PW potential are more convincing since this potential is known to be a better description for a nonrotating black hole than the Newtonian potential. Our argument can be further confirmed by Figure 1*b*, which shows the variation of η_ν with $\dot{m}$, where $\eta_\nu \equiv L_\nu/\dot{M}c^2$ is the efficiency of energy release by neutrino radiation (before annihilation). As seen in the figure, η_ν in the Newtonian potential is much larger than that in the PW potential. For $j = 0$, the former can reach a maximum value of 0.206 at $\dot{m} = 0.45$, which is far beyond the maximum possible efficiency in the Schwarzschild geometry ($\eta = 0.057$) and is unphysical. In fact, by integrating the viscous heating $Q_{\rm vis}$ from $3R_g$ to the infinite outer boundary of the flow, we can obtain the theoretical maximum η_ν for the above three solutions: $\frac{1}{4}$ for the Newtonian potential with $j = 0$ (from eq. [14] of DPN), 1/12 for the Newtonian potential with $j = 1.225cR_g$ (from eq. [32] of Kohri & Mineshige 2002), and 1/16 for the PW potential with $j = 1.837cR_g$. Obviously, the result with the PW potential is the closest to reality (0.057), while the results with the Newtonian potential are unreasonable.

We conclude for the moment that the usage of the Newtonian potential is invalid in calculations for NDAFs at least at the following two points: (1) it would overestimate substantially the optical depth for neutrinos; (2) it would lead to an unphysical efficiency of energy release by neutrino radiation.

4. EFFECT OF THE OPTICAL DEPTH ON THE NEUTRINO ANNIHILATION LUMINOSITY

Our method for calculating neutrino annihilation is similar to many previous works (e.g., Ruffert et al. 1997; PWF; Rosswog et al. 2003). Figure 2 shows the variations of L_ν (*upper thin solid line*) and $L_{\nu\bar{\nu}}$ (*lower thick solid line*) with $\dot{m}$, where L_ν is the luminosity of neutrino radiation before annihilation and $L_{\nu\bar{\nu}}$ is the luminosity of neutrino annihilation (which is the most important from the observational point of view); both of them are calculated with the PW potential. The circles and triangles represent the results of PWF for L_ν (*open symbols*) and $L_{\nu\bar{\nu}}$ (*filled symbols*), respectively. It is seen that our results agree very well with that of PWF for $\dot{m} \lesssim 1$, because the PW potential is a good approximation for the Schwarzschild geometry. For $\dot{m} > 1$, our results are lower than that of PWF. This is because they assumed neutrinos to be optically thin, while we use the bridging formula for Q_ν, and there exists an optically thick region for $\dot{m} > 1.2$. According to our calculations, $L_{\nu\bar{\nu}}$ varies from 3.9×10^{50} to 3.6×10^{52} ergs s^{-1} for $1 < \dot{m} < 10$, which implies that, based on the energy consideration, NDAF can indeed work as the central engine for GRBs. In particular, our $L_{\nu\bar{\nu}}$ (3.6×10^{52} ergs s^{-1}) for $\dot{m} = 10$ is in good agreement with PWF's "actual luminosity" ($\sim 4 \times 10^{52}$ ergs s^{-1}, as mentioned in § 1).

For comparison, Figure 2 also shows $L_{\nu\bar{\nu}}$ in three other cases: (1) using the PW potential but omitting the contribution from the optically thick region (*dotted line*; the $\tau > \frac{2}{3}$ region appears for $\dot{m} > 1.2$); (2) using the Newtonian potential and including the contribution from the optically thick region (*dot-dashed line*); and (3) using the Newtonian potential but omitting that contribution (*dashed line*; the $\tau > \frac{2}{3}$ region appears for $\dot{m} > 0.052$). As known from § 3, the results of cases 2 and 3 are unreal because the usage of the Newtonian potential overestimates unphysically both τ and L_ν. It is seen that the omitting of the contribution from the $\tau > \frac{2}{3}$ region reduces substantially $L_{\nu\bar{\nu}}$ in case 3, i.e., even with the overestimated L_ν caused by the Newtonian potential, as well as in case 1, i.e., even when the general relativistic effect is considered. This is probably the reason why DPN obtained $L_{\nu\bar{\nu}}$ in their Newtonian calculations that is insufficient for GRBs. We think that it is unfair to ignore totally the neutrino radiation from the $\tau > \frac{2}{3}$ region. As DPN also stated, the neutrino emission is partially suppressed as the inner regions of the flow are becoming opaque. The trapping of neutrinos is a process that is strengthening gradually with increasing τ, the value of τ reaching $\frac{2}{3}$ does not mean that all neutrinos are suddenly trapped, and the use of the bridging formula for Q_ν is exactly to calculate the neutrino radiation from both the optically thin and thick regions. A similar bridging formula has been widely used for calculating the radiation of photons in both the optically thin and thick cases (e.g., Narayan & Yi 1995).

5. ENERGY ADVECTION

DPN argued that energy advection would become the dominant cooling mechanism when the flow is optically thick for neutrinos. As seen from their Figure 3, however, it is not the case. For example, for $\dot{m} = 1$ the flow is optically thick at $R = 10R_g$, but the advection factor $f_{\rm adv} \equiv Q_{\rm adv}/Q_{\rm vis}$ at this radius is only ~ 0.1. In our opinion, whether cooling is dominated by advection or by radiation is not determined by the optical depth. For accretion flows in X-ray binaries and active galactic nuclei, it is known that $f_{\rm adv}$ is relevant to the geometrical depth rather than the optical depth of the flow (Abramowicz et al. 1986):

$$f_{\rm adv} \propto \left(\frac{H}{R}\right)^2. \tag{7}$$

Such a relationship can be well checked by the four representative types of accretion models: the optically thick standard thin disk (Shakura & Sunyaev 1973) and the optically thin Shapiro-Lightman-Eardley disk (Shapiro et al. 1976) are both geometrically thin and radiation-dominated, i.e., $f_{\rm adv} \sim 0$; and the optically thick slim disk (Abramowicz et al. 1988) and the optically thin advection-dominated accretion flow (Narayan & Yi 1994) are both geometrically thick and advection-dominated, i.e., $f_{\rm adv} \sim 1$. We argue that this relationship should also work in the NDAF model with the following modification in accordance with the PW potential:

$$f_{\rm adv} \propto f_H \equiv \left(\frac{H}{R}\right)^2 f^{-1} g^{-1}, \tag{8}$$

where f_H is called by us the geometrical depth factor and $f^{-1}g^{-1}$ comes from the expression of $Q_{\rm vis}$ (eq. [5]).

As shown in Figure 3, the variation of $f_{\rm adv}$ with R (*solid line*) agrees very well with that of f_H (*dot-dashed line*) but differs

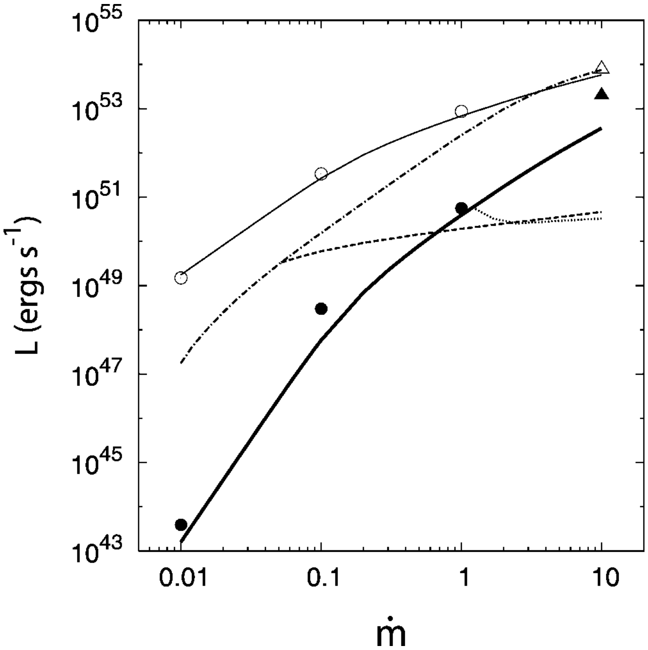

FIG. 2.—Neutrino luminosity before annihilation L_ν with the PW potential (*thin solid line*), neutrino annihilation luminosity $L_{\nu\bar{\nu}}$ with the PW potential and including the $\tau > \frac{2}{3}$ region (*thick solid line*), $L_{\nu\bar{\nu}}$ with the PW potential but omitting the $\tau > \frac{2}{3}$ region (*dotted line*), $L_{\nu\bar{\nu}}$ with the Newtonian potential and including the $\tau > \frac{2}{3}$ region (*dot-dashed line*), and $L_{\nu\bar{\nu}}$ with the Newtonian potential but omitting the $\tau > \frac{2}{3}$ region (*dashed line*) as functions of $\dot{m}$. The open circles and triangle denote L_ν of PWF, and the filled circles and triangle denote $L_{\nu\bar{\nu}}$ of PWF.

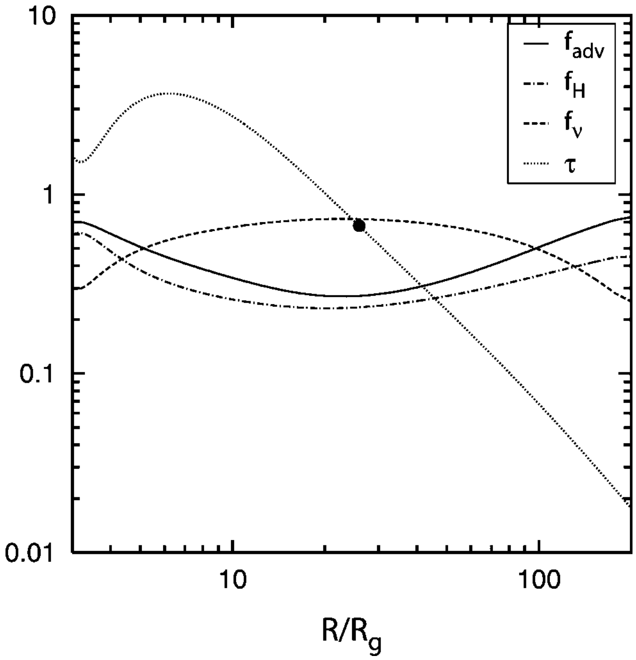

FIG. 3.—Advection factor f_{adv}, geometrical depth factor f_H, neutrino cooling factor f_ν, and τ as functions of R for $\dot{m} = 5$. The filled circle denotes the $\tau = \frac{2}{3}$ position, i.e., $R = 25.9R_g$.

significantly from that of τ (*dotted line*), clearly indicating that the strength of energy advection is relevant to the geometrical depth rather than the optical depth. It is also seen that, although the flow is optically thick for $R < 25.9R_g$, advection dominates over neutrino cooling ($f_{adv} > f_\nu$, where $f_\nu \equiv Q_\nu/Q_{vis}$ is drawn by the dashed line) only in a smaller region $R < 5.1R_g$. Once again, this result supports the view that it is important to consider the role of the optically thick but neutrino radiation-dominated region, e.g., $5.1R_g < R < 25.9R_g$ in the example of Figure 3.

6. DISCUSSION

We have shown that the usage of the Newtonian potential along with the omitting of neutrino radiation from the optically thick region would lead to unreal luminosities for NDAFs, and that when the general relativistic effect is considered and the contribution from the optically thick region is included, NDAFs can work as the central engine for GRBs.

In addition to its mass, a black hole may have its spin as the other fundamental property. We consider here only the nonrotating black hole, for which the PW potential can work. PWF has shown that a spinning (Kerr) black hole will enhance the efficiency of neutrino radiation. This strengthens our conclusion here that NDAFs into black holes can be the central engine for GRBs.

We have tried to update partly the NDAF model by considering both the effects of general relativity and neutrino opacity. There are certainly other factors that may influence the neutrino luminosity of an NDAF and we do not consider here, such as the electron degeneracy. We adopt a simple treatment for the electron degeneracy pressure in agreement with PWF and DPN. Kohri & Mineshige (2002) pointed out that it is important to include the effect of electron degeneracy that suppresses the neutrino cooling at high density and high temperature. Most recently, Kohri et al. (2005) considered the effects of both electron degeneracy and neutrino optical depth, and calculated the neutrino cooling, the electron pressure, and other physical quantities even in the delicate regime where the electron degeneracy is moderate, while in previous works as well as ours here the calculations can be made accurate only in the two opposite limits of extremely degenerate electrons and fully nondegenerate electrons. We wish to see in future studies how the electron degeneracy would affect our results here.

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